Probability

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Sep 23, 2021

Multiplication Rule

P(A AND B)= Prob. of 'A' occuring And then 'B' occuring in Successive Trials.

Example

- Dr. Siddiqua Mazhar drives BMW. (T / F)
- My Favourite color is :
  a. Red, b. Blue
  c. Green d. Black
  e. Yellow

If you Guess Randomly, find probability of getting both questions are correct.

Sample space: (T, a), (T, b), (T, c), (T, d), (T, e)
(F, a), (F, b), (F, c), (F, d), (F, e)

P(Both right) $=\frac{1}{10}$

Example

	Didn't do	Did it
Guilty	11	72
NonGuilty	85	9

(Without Replacement): Probability of slecting 'Guilty' and then 'Not Guilty'.

P(Guilty)=

AND then

P(Not Guilty)=

Conditional Probability

The Probability of an event occuring given that same other event has already occured.

$$P(B|A) = \textit{The probability of event } B \textit{ occuring. Given that event } A \textit{ has already occured.}$$

Independant vs. Dependant Events

Independant Event The occurrance of one event does not affect the occurrance of another event.

OR

A and B are independent if the event of A happening does not affect the probability of B happening.

Note:

- Events that are not independent events are dependent.
- If A B are independent.

$$P(B|A) = P(B)$$

Example:

Rolling a Die:

$$P(2|3) = P(2) = \frac{1}{6} \quad (Independent)$$

Example:

Drawing Cards:

$$P(Q|9) = \qquad (With\ replacement)$$

$$P(Q|9) = \qquad (Without\ replacement)$$

Example:

Drawing Cards:

$$P(Q|Q) = \qquad = \qquad (With\ replacement)$$

$$P(Q|Q) = \qquad\qquad (Without\ replacement)$$

Example:

Drawing Cards:

$$P(\heartsuit|J\diamondsuit) = \frac{13}{52} \quad (With\ replacement)$$

$$P(\heartsuit|J\diamondsuit) = \frac{13}{51} \approx \quad (Without\ replacement)$$

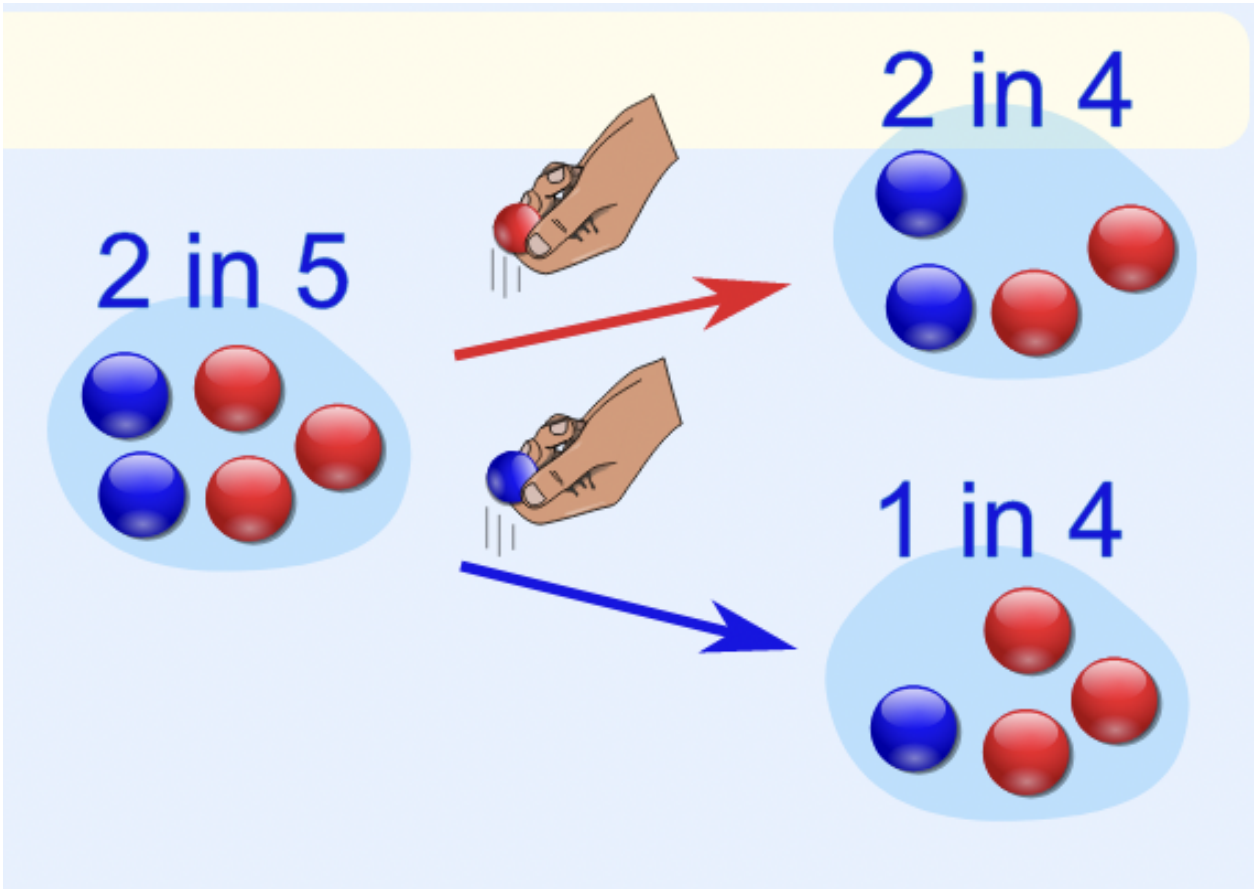
Multiplication Rule:

$$P(A \ AND \ B) = P(A).P(B|A)$$

Recall: If independent, $P(B|A) = P(B)$

$$P(A \ AND \ B) = P(A).P(B)$$

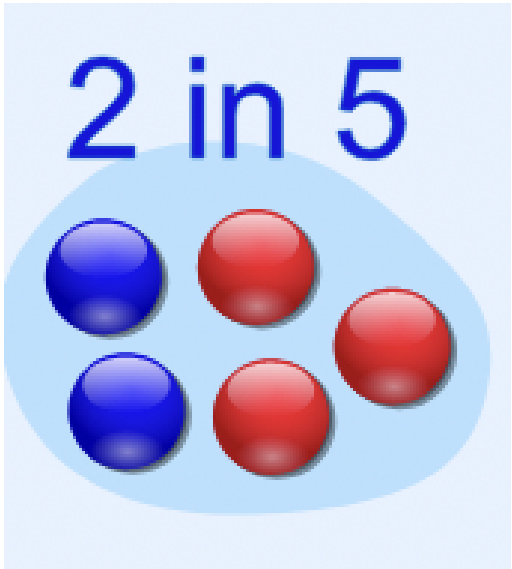
Example:



Example:

2 blue and 3 red marbles are in a bag.

With Replacement:

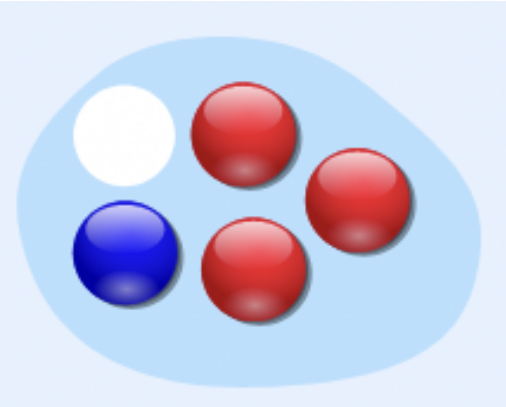


$$P(B \ AND \ R) = P(B).P(R)$$

$$P(B \ AND \ R) = \frac{2}{5}.\frac{3}{5}$$

Example

Without Replacement:



$$P(B\ AND\ R) = P(B).P(R|B))$$

$$P(B\ AND\ R) = \frac{2}{5}.\frac{3}{4}$$

Bag of Barbles

A bag contains 3 Red, 2 Blue and 4 Green marbles. **With Replacement:**

$$P(G\ AND\ B) =$$

Without Replacement:

$P(G\ AND\ B) =$

$P(R\ AND\ R) =$

$P(B\ AND\ B\ AND\ B) =$

Problems

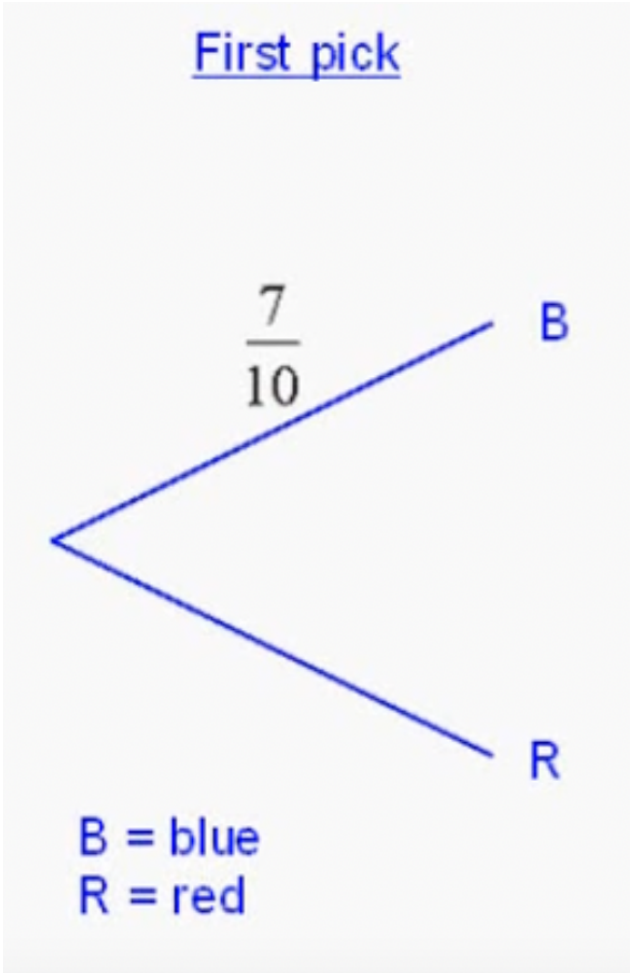
- P(1 AND 2 AND 3 AND 4)=?
- P(A AND K AND Q AND J AND 10)=? without replacement

Tree Diagram (With Replacement)

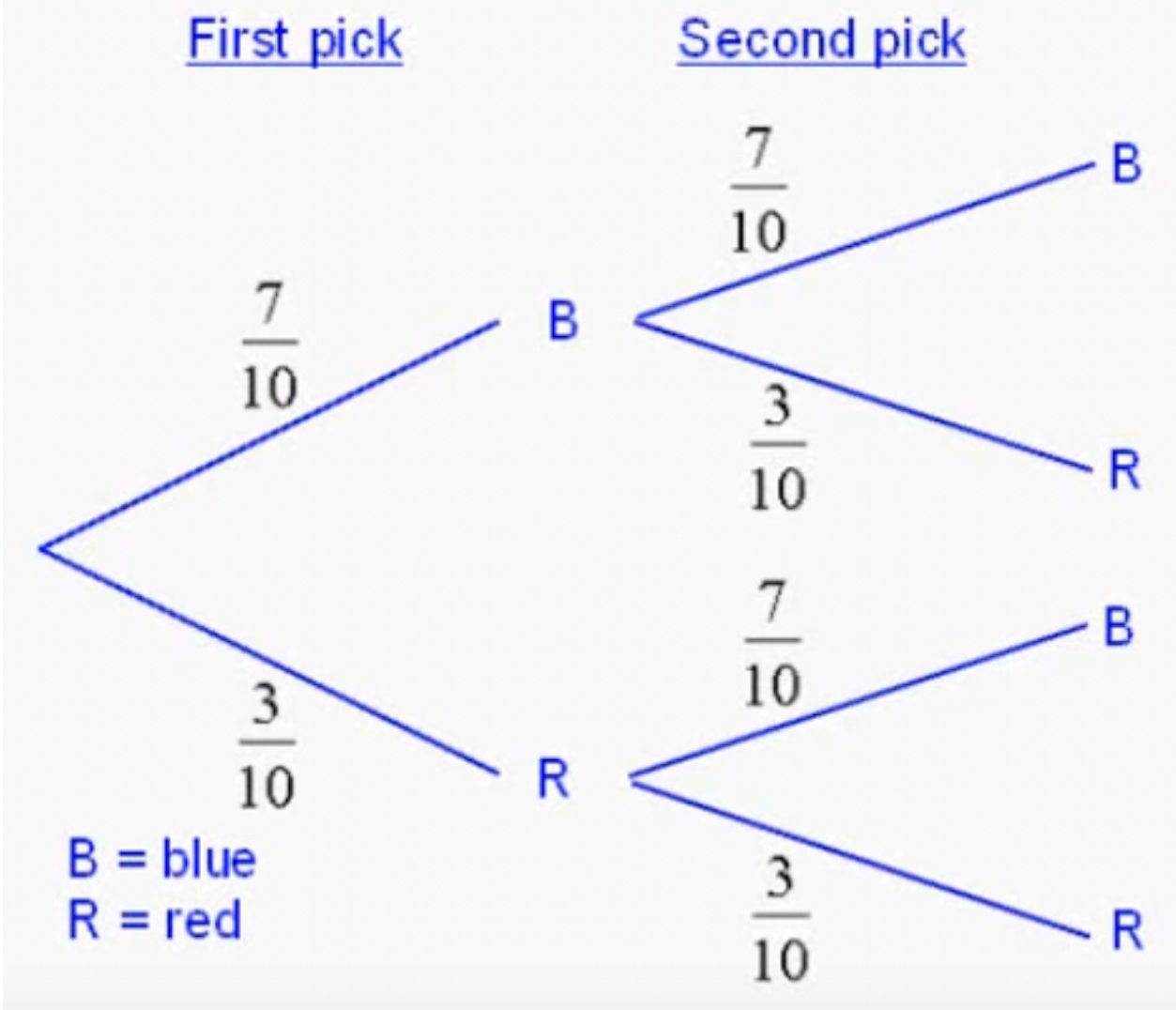
Jenny has a bag with 7 blue sweet and 3 red sweets in it. She picks a sweet at random from the bag, repalces it and then picks at random. Draw a Tree diagram to represent this situation and use it to replace the probabilities that she picks.:

- Two red sweets
- No red sweets
- at least one blue sweet
- one sweet of each color

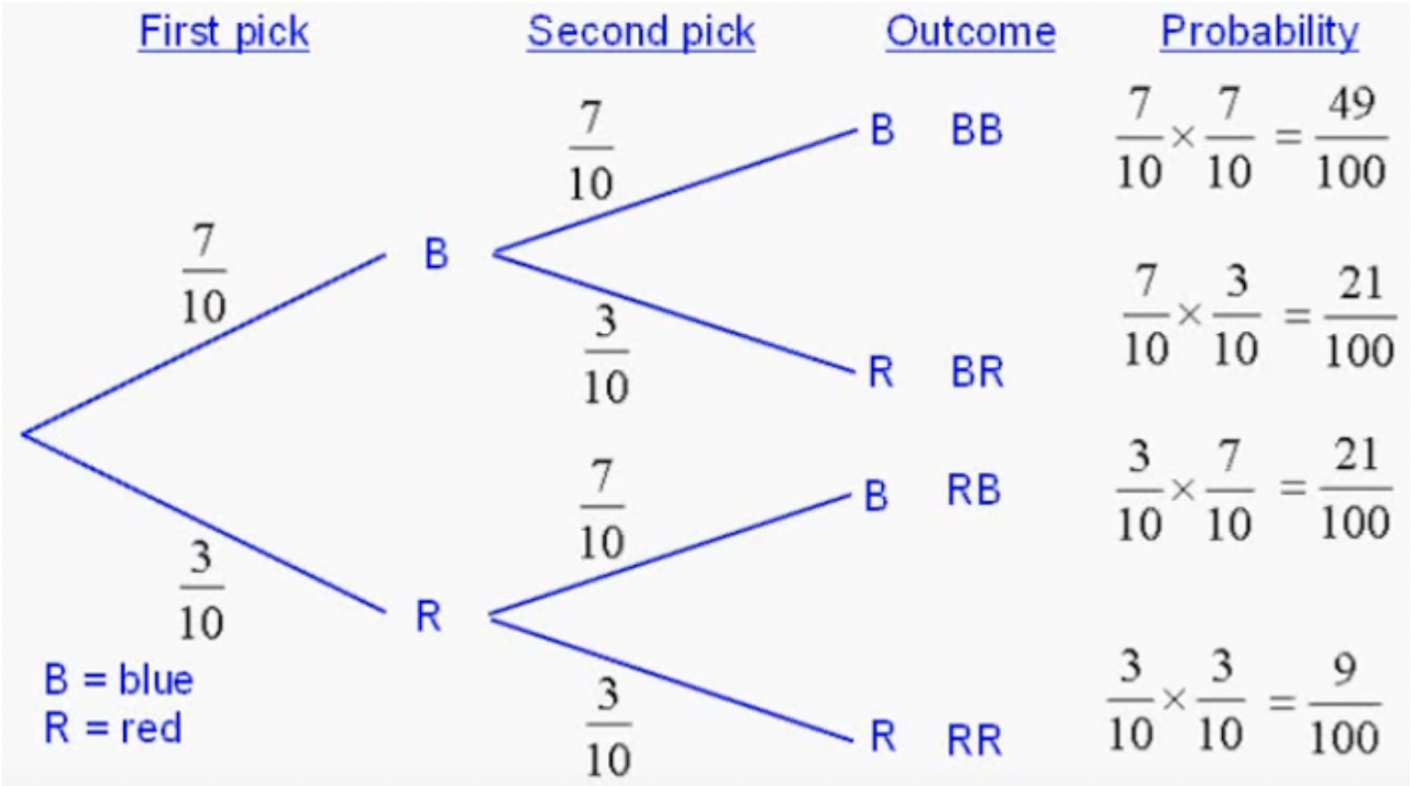
Tree Diagram (With Replacement)



Tree Diagram (With Replacement)



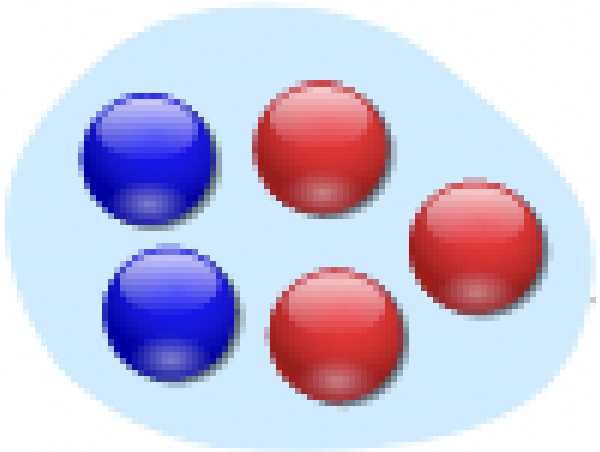
Tree Diagram (With Replacement)



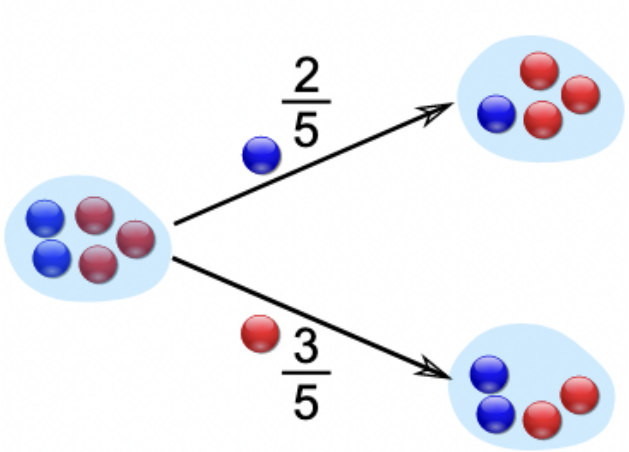
Explanation

Tree Diagram (Without Replacement)

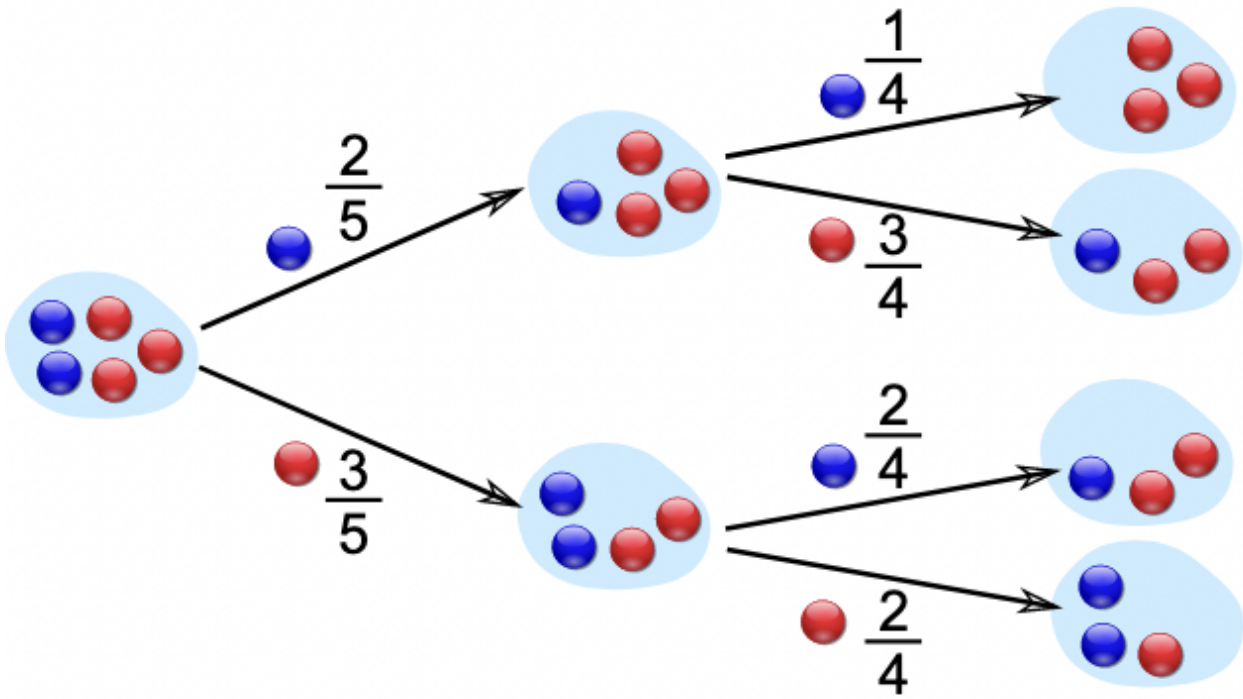
Suppose there are five balls in a bag. They are identical except the color. Three of the balls are red and two are blue. You are instructed to draw out one ball, note its color, and set it aside. Then you are to draw out another ball and note its color. What are the outcomes of the experiment? What is the probability of each outcome?



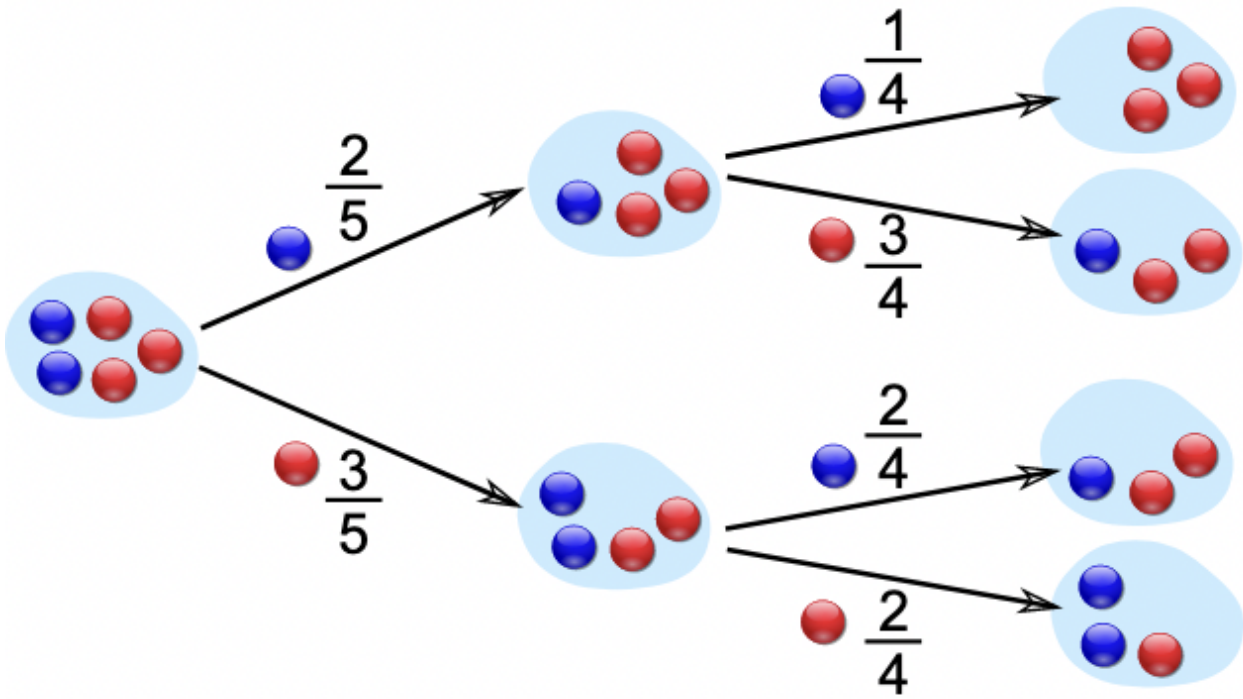
Tree Diagram (Without Replacement)



Tree Diagram (Without Replacement)



Tree Diagram (Without Replacement)



Tree Diagram (Without Replacement)

Complement Rule

'At Least One' means one or more.

The **Complement** of at least one is **'None'**

$$P('At\ Least\ One') = 1 - P('None')$$

$$P(A) = 1 - P(\overline{A})$$

Example

Flip coin 3 times. What is the probability of at least one Head?

$$P(At\ Least\ One\ Head) = 1 - P(No\ Head)$$

Example

Flip coin 20times. What is the probability of getting at least one Head?

$$P(At\ Least\ One\ Head) = 1 - P(No\ Head)$$

General Rule

General Rule can be translated into probabilities:

$$P(A|B) = \frac{P(A \ AND \ B)}{P(B)}$$

Example

	Didn't do	Did it
Guilty	11	72
NonGuilty	85	9

Find the following probabilities:

- $P(Guilty \mid Did\ it)$
- $P(Did\ it \mid Guilty)$
- $P(Didn't\ do\ it \mid Not\ Guilty)$