Probability

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Event

A Collection of outcomes of a procedure or it consists of a subset of a sample space.

Simple Event: Event with a single outcome is a simple event.

Sample Space: A collection of all simple events or all possible outcomes.

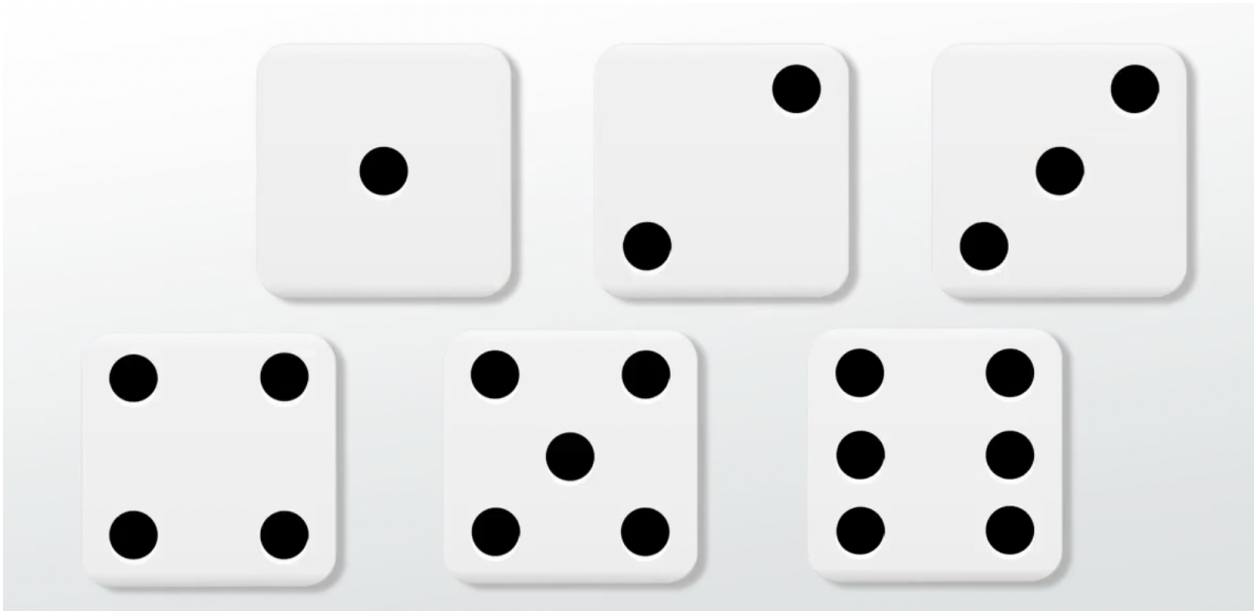
Example 1:

Toss a coin.

Procedure	Event	Sample space
Flip 1 Time	Head	{H,T}
Flip 3 Time	{H,T,T}	{H,T,T}, {H,T,H},{H,H,T}, {T,H,T},{T,T,H}, {T,H,H},{H,H,H},{T,T,T}

Example :

Roll a single die.



Procedure	Event	Sample space
Roll 1 time	3	{1,2,3,4,5,6}

Probability

The chance of an event occuring.

How likely an event is to occur.

Symbols:

1. **Probability:** P
2. **Event:** A,B,C,...
3. P(A)= The Prob. of event A is happening.

Three Types of Probability

1. **Exact Definition:**
   - **Classical Probability:** Probability based on the chance of an event occuring. (Each simple event must have an equal chance of occuring.)

$$P(A) = \frac{No.\ of\ \ times\ 'A'\ could\ occur}{No.\ of\ \ Simple\ \ Events\ (Outcomes)}$$

2. **Objective Probability**.
   - **Observed Probability / Empirical Probability:** Probability that is estimated based on observation.

$$P(A) = \frac{No.\ of\ \ times\ 'A'\ occured}{No.\ of\ \ times\ procedure\ was\ repeated}$$

3. **Subjected Probability:** Educated guess.

Examples of Classical Probability:

- Probability of rolling a 1 when a single die is rolled.

- The probability of selecting a heart ♡ from a deck of cards.

$$P(\heartsuit) = \qquad =$$

Examples of Observed Probability:

- Flip a coin 100 times, you have got 64 tails.

$$P(T) = \frac{64}{100} = 0.64$$

- Alice completed 385 out if first 528 Passes. Find the probability that Alice will complete a Pass.

$$P(\textit{Comp. a Pass}) = \qquad =$$

Examples:

- Random deck of cards. What is the probability of 'Selecting a 2'?
- What is an Event?
- What is a Procedure?
- What probability is it?

Examples :

- Find probability that a couple has three kids, two will be girls. (Assuming equal chance of boy and girl)

Examples :

- Poll:

  91 people .... Cloning Good.
  901 people ... cloning Bad
  20 people ... No opinion

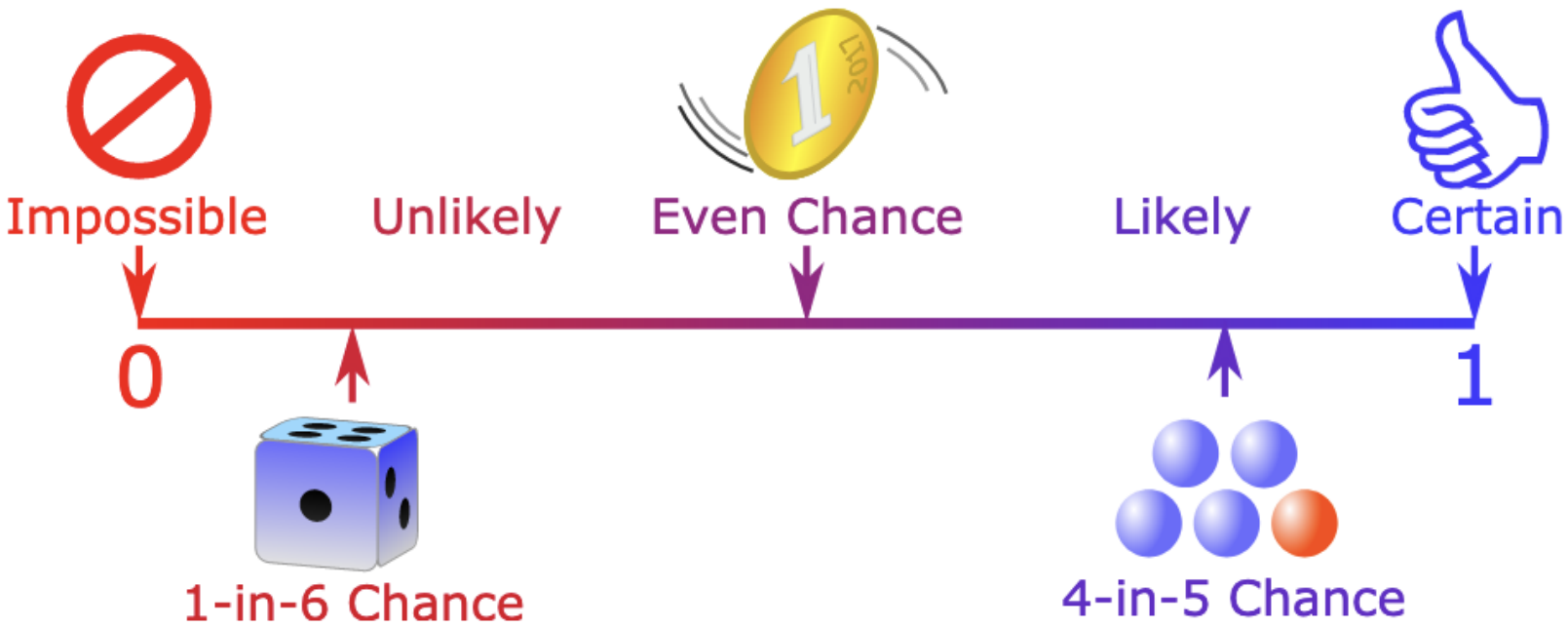
  Find the probability of random selecting a person who think cloning is a Good idea.

$$P(CloningGood) = \frac{91}{1012} = 0.090$$

  or 9%.

Allowable Probability

- Probaility is always a number (fraction or decimal) between 0 and 1.
- 0 means no chance of happening
- 1 means it will happen (always)



Probability is always between 0 and 1

Law of Large Numbers

The law of large numbers states that as more observations are collected, the proportion of occurrences with a particular outcome converges to the probability of that outcome.

OR

The more the procedure is repeated. The closer observed probability will get to classical probability.

Experiment :

Rolling a Die 100 times.

Example :

Given the large number of trials (1000) we would expect the observed number heads to be close to the expected number of heads.
The more problems you do the closer you get the expected probability.
Which of the following event would be most surprised by?

- exactly 3 heads in 10 coin flips
- exactly 3 head in 100 coin flips
- exactly 3 head in 1000 coin flips

Complement

- Complement of an event A is the set of outcomes not contained in A or is the set of all outcomes when event A does not occur.
- Complement of event A is denoted by $\overline{A}$.
- Probability of an event plus the probability of the complement must be equal to 1.

$$P(A) + P(\overline{A}) = 1$$

Example

- What is the complement of rolling a 5 on a die? Or what else could happen of rolling a die when that doesn't make a 5?
- What is the probability of Even numbers in rolling a die. Complement (RollEven) = Roll Odd
- If $P(A)$=0.64, then $P(\overline{A}) =$
- If $P(\overline{A}) = 0.44$, then $P(A) =$
- If $P(A) = 0$, then $P(\overline{A}) =$

Compound Event:

An event which joins two or more simple events.

Example: Prob. of rolling a '1' or '5'. OR, one, or the other, or both.

$$P(A\ OR\ B) = Prob.\ of\ 'A'\ occuring,\ OR\ 'B'\ occuring,\ OR\ Both$$

$$'A'\ and 'B'\ occuring\ in\ a\ SINGLE\ TRIAL$$

Example: P(Blond or Female) is not mutually exclusive.

Disjoint Events or Mutually Exclusive Events:

An events that can not occur at the same time.

Example: Event A - Rolling a 1
Event B - Rolling an even

Example: Event A - Rolling a 2 or 4
Event B - Rolling a Prime

Example:

	Didn't do	Did it
Guilty	11	72
NonGuilty	85	9

How many people are 'Guilty' or 'Did it' ?

General Addition Rule

P(A OR B) requires elimination of double count.

$$P(A\ OR\ B) = P(A) + P(B) - P(A\ AND\ B)$$

OR

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Addition Rule for Mutually Exclusive Events

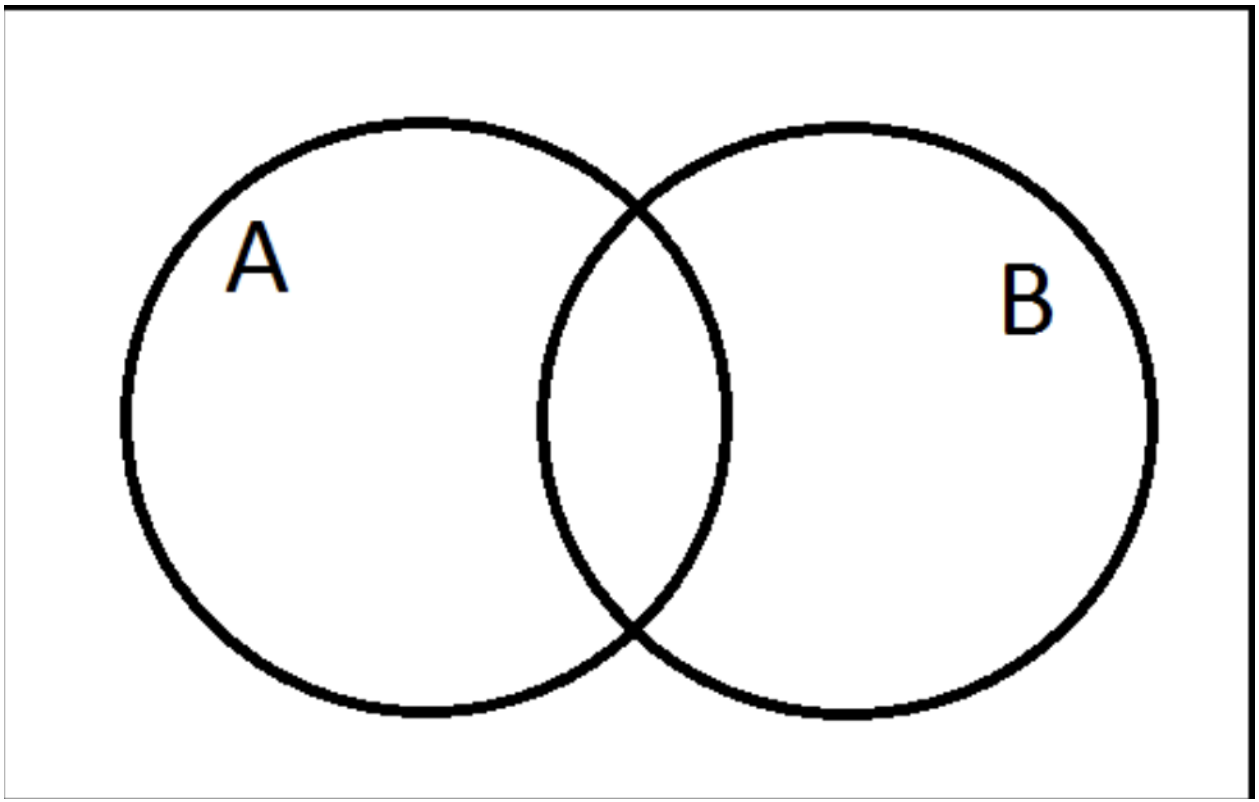
Note: For Mutually Exclusive Events or Disjoint Events
$P(A \text{ and } B) = 0$

$$P(A \cup B) = P(A) + P(B)$$

Venn Diagram

Venn Diagram is a diagram representing sets pictorially as circles or closed curves within an enclosing rectangle (the universal set).

- General Venn Diagram with two sets.



Rules

- $P(A \cup B) \geq P(A)$
- $P(A \cup B) \geq P(B)$
- $P(A) \geq P(A \cap B)$
- $P(B) \geq P(A \cap B)$

Use Venn Diagram

What is wrong here?

- $P(A \cup B) = 0.8$, $P(A) = 0.3$ and $P(A \cap B) = 0.5$
- $P(A) = 0.1$, $P(B) = 0.4$ and $P(A \cup B) = 0.2$

Examples:

- Probability of selecting a 'Heart' and a 'Spade' from a standard deck of card.

Examples:

- Probability of 'Blond' and 'Female'.

Examples:

- Probability of $\diamondsuit$ and K.

Examples:

- Probability of Complementary Events.
  $P(A \ OR \ \overline{A}) = P(A) + P(\overline{A}) - P(A \cap \overline{A})$