Discrete Probability

Dr. Siddiqua Mazhar

Mid Michigan College

Sep 29, 2021

Introduction

- Decisions are made in real life because of probabilities.
- Learn How to use probabilities to make decisions.

Lay out

- General Calculations
- Insurance Calculations
- Games of chance /casino games

Random Variable

A variable, X, that has a value for each outcome of a procedure that is determined by a chance.

Examples:

- How many heads in three flips of a coin.
- Number on a roll of the die

Probability Distribution

A table that gives the probability for each value of a random variable.

Example: Roll a die.

X	P(X)
1	1/6
2	1/6
3	1/6
4	1/6
5	1/6
6	1/6

Discrete Random Variable:

A variable with a countable or finite number of values.

Contonous Random Variable:

A Variable with an infinite number of possible values.

Histogram From Probability Distribution

Horizontal: Values of random variable.
Vertical: Probabilities.

Example: Weighted Die.

X	P(X)
1	.05
2	.15
3	.35
4	.30
5	.10
6	.05

Note: $0 \leq P(X) \leq 1$

Mean:

$$\mu = \frac{\sum(X.f)}{N}$$

$$\mu = \sum[\frac{X.f}{N}]$$

$$\mu = \sum[X.\frac{f}{N}]$$

$$\mu = \sum[X.P(X)]$$

Example

For a weighted die:

X	P(X)	X.P(X)
1	.05	.05
2	.15	.30
3	.35	1.05
4	.30	1.20
5	.10	.50
6	.05	.30

$$\mu = \sum X.P(X) = 3.4$$

Variance

$$\sigma^2 = \sum [X^2.P(X)] - \mu^2$$

OR

$$\sigma^2 = \sum (x - \mu)^2.P(X)$$

For a weighted die:

X	P(X)	X.P(X)	X^2	$X^2.P(X)$
1	.05	.05		
2	.15	.30		
3	.35	1.05		
4	.30	1.20		
5	.10	.50		
6	.05	.30		

Standard Deviation

$$\sigma = \sqrt{\sum [X^2.P(X)] - \mu^2}$$

Values are unusual if they lie outside of $\mu + 2\sigma$ and $\mu - 2\sigma$.
Which values are unusual for a weighted die.

Usual vs. Unusual

If $P(A) \leq 0.05$ or 5%, 'A' is considered unusual.

Example: Flip a coin 1000 times.

$P(Exactly\ 501\ times\ Head) = 0.0252 \leq 0.05$ (Unusual)

$P(501\ or\ more\ Head) = 0.487$ (Usual)

Example

We know 60% of people are side sleepers. Choose 5 people. X is the number who sleep on their side.

X	P(X)
0	.0102
1	.0768
2	.2304
3	.3456
4	.2592
5	.0778

Find the mean and standard deviation.

Expected Value: The theoretical average of a probability distribution.

Expected Value: Exaxmple 1:

An insurance company insures farm ground for \$300 per acre if the ground is destroyed and the crop does not come in. The probability that the farm ground is destroyed is $1/120$. The company charges \$8 per acre to insure crops. Find the expected payout and the expected profit for the insurance company.

Solution:

Expected Value: Exaxmple 2:

An insurance company insures a person for \$1.5 million if that person is abducted by aliens To get this insurance the person has to pay a premium of \$150. If the chance of the person getting abducted is one in a million, what is the insurance company's expected payout? What is the insurance company's expected profit?

Cite (www.propertycasualty360.com/2012/09/14/8-unusual-insurancecoverages).

Solution:

Assume you work at an insurance company. An average 22 year old male walks in and wants to get life insurance this year for $300,000. Based on the social security's actuarial life table, how much would the expected payout be? How much would you charge the person if the mark up is 10%?

Use Acturial Life Table.

Solution:

Expected Value: Exaxmple 4:

A lottery is held. Ten thousand tickets are sold at \$1 each. There are five prizes each for \$1000, thirty prizes each worth \$100, and fifty prizes each worth \$10. The remaining tickets are worth nothing. Find the expected payout if the person buys a ticket.

Solution:

Expected Value: Exaxmple 5:

Roll 2 dice. X is the sum of numbers on the two dice.

X	2	3	4	5	6	7	8	9	10	11	12
P(X)	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

Find the mean and standard deviation.

Expected Value: Exaxmple 6:

In a dice game, the players roll two dice. If the dice sum to 2 or 12, then the player wins \$6. If a person rolls a 7, then the player wins \$2. The game costs \$1 to play. What is the expected profit of the game? Should you play the game?

Solution:

Expected Value: Exaxmple 7:

In another dice game, the players roll two dice. If the dice sum to 6 or 8, then the player wins \$3. If a person rolls a 7, then the player wins \$1. The game costs \$1 to play. What is the expected profit of the game? Should you play the game?

Solution: