Chapter 8: Confidence Interval - Population Proportion

Dr. Siddiqua Mazhar

Mid Michigan College

Oct 28, 2021

Introduction

- Inferential Statistics
- Results on Sample $\rightarrow$ Results on Population

Introduction

- Results on Sample $\rightarrow$ Results on Population
- Our results will be Averages or Proportions

Confidence Intervals for Population Proportion

Estimating Population Proportion from a Sample Proportion

- Random Sample
- Employing Binomial Distribution Conditions
- Fixed number of Trials
- Trials are Independent
- Two outcomes: Success / Failure
- $np \geq 5$, $nq > 5$

Point Estimate

A numerival value for a sample that estimates numerical value for a population.

Altervatively, a single value used to approximate a population parameter.

Example The sample has a mean weight of 172 pounds so we estimate the population has a mean weight of 172 pounds.

Symbols

- p= Population Proportion of Success
- $\hat{p}$ = Sample Proportion of Success. $\hat{p} = \frac{X}{n}$
- $\hat{q}$ = Sample Proportion of Failure. $\hat{q} = 1 - \hat{p}$

Note: We will use $\hat{p}$ as a point estimate for p.

Deficit of Point Estimate

Suppose that a sample of 800 students are selcted at random from Mid Michigan college and they are given a shot to prevent from certain type of Flu. They are exposed to the flu, and 600 of them do not get the flu.

$$\hat{p} = \frac{600}{800} = 0.75$$

What is the probability p that the shot will be successful for any single student selected at random from entire Mid Michigan College students?
If p is 0.85 is that OK?
Should p be within 0.1 margin or could it be within 0.05 margin difference?

Confidence Interval

Confidence Interval is a range of numbers where we expect the population proportion to be.

Confidence Level

Confidence Level is a level of how confident we are that the confidence interval actually contains the population parameter.

$$1 - \alpha$$

where α is the complement of the confidence level.

Most common:

- .90 or 90% $\rightarrow \alpha = .10$
- .95 or 95% $\rightarrow \alpha = .05$
- .99 or 99% $\rightarrow \alpha = .01$

Example

The 95% confidence interval for p probability that the shot will be successful for any single student selected at random from Mid Michigan College students is

$$0.7 < p < 0.8$$

Note: I don't know what p actually is, however I am 95% sure that it falls in that range.

Critical value:

A z-score is the number such that area under the standard normal curve between $-z_{\alpha/2}$ and $z_{\alpha/2}$ equals the confidence level. Alternatively, a z-score that seperates the likely region from the unlikely region.

Z-value / Critical Values

Confidence Level			
99%			
95%			
92%			
90%			
84%			
80%			
50%			

Computing bounds of Margin of Error

The maximum difference between $\hat{p}$ and p or magnitude of $\hat{p} - p$ is called margin of error.
Distribution will be well approximated by a normal curve for large samples. We use

$$\mu = p \quad nnd \quad standard\ error, \quad \sigma = \sqrt{pq/n}$$

Explanation

3 Situations

- Z interval (For mean when σ is known)
- T interval (For mean when σ is unknown)
- Proportional Interval (For Proportion)

2 Questions

- Find the Confidence Interval (and interpret)
- Find Sample size

Confidence Interval for Proportion

Steps:

- Find $\hat{p}$, $\hat{q}$, n.
- Use Confidence Level to find $z_{\frac{\alpha}{2}}$ (Critical Value)
- Find marginal Error $E = z_{\frac{\alpha}{2}} \sqrt{\frac{\hat{p}.\hat{q}}{n}}$
- Find Confidence Interval: $\hat{p} - E < p < \hat{p} + E$

Confidence Interval for Proportion

On Calculator Press the STAT key, select Tests

Choose option A: 1-PropZInt

X= number od successes

n= Sample Size

C-Level: Confidence Level

Example 1

Touch therapist do 280 Trails, 123 found correct identification.
Construct a 95% confidence interval for population proportion.

Example 2

A Pew research poll found that 871 out of 1502 U.S. adults surveyed said that "The U.S. must be at the forefront of future space exploration." Estimate the true proportion of U.S. adults who think that "The U.S. must lead future space exploration." with 92% confidence.

Example 3

In a poll of 1000 likely voters conducted by Rasmussen Reports in June of 2018, 512 likely voters said that the Democratic Party is the party of "identity politics and victimology". Find a 95% confidence interval for the true proportion of voters who feel this way.

Example 4

A Rasmussen Report on October 18, 2017 showed that 67% of a sample of 1000 U.S. adults oppose tax breaks for NFL teams. Construct a 97% confidence interval for the proportion of U.S. adults who oppose tax breaks for the NFL.

Finding Required Sample Size

Given an 'E', you can find the sample size needed to get that 'E'.

$$E = z_{\frac{\alpha}{2}} \cdot \sqrt{\frac{\hat{p}.\hat{q}}{n}}$$

$$n = \frac{\hat{p}\hat{q}}{E^2}(z_{\frac{\alpha}{2}})^2$$

Note: In a worst case, when you know nothing about your sample proportion, we use

$$n = \frac{0.25}{E^2}(z_{\frac{\alpha}{2}})^2$$

Example 1

We want to determine the percentage of U.S. who use email 95%. What does the sample size need to ensure a margin of error of 4%.

- 16.9% of people used email in 1997

Example 1

We want to determine the percentage of U.S. who use email 95%. What does the sample size need to ensure a margin of error of 4%.

- We know nothing about previous email usage.