Chapter 7: Central Limit Theorem

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Sampling Distribution

Idea: We use sample to estimate a population.

Sampling distribution of Statistic (such as Mean, Standard Deviation, proportion etc):
Assume you take all possible samples of size 'n' and find the required statistics for each sample. If you organize all those statistic of each different sample in to a table, This is a sampling disribution.

Example: Take the set of data.
$\{1, 2, 3, 4, 5, 6, 7\}$

Average: $\mu = 4$, Standard Deviation: $\sigma = 2$

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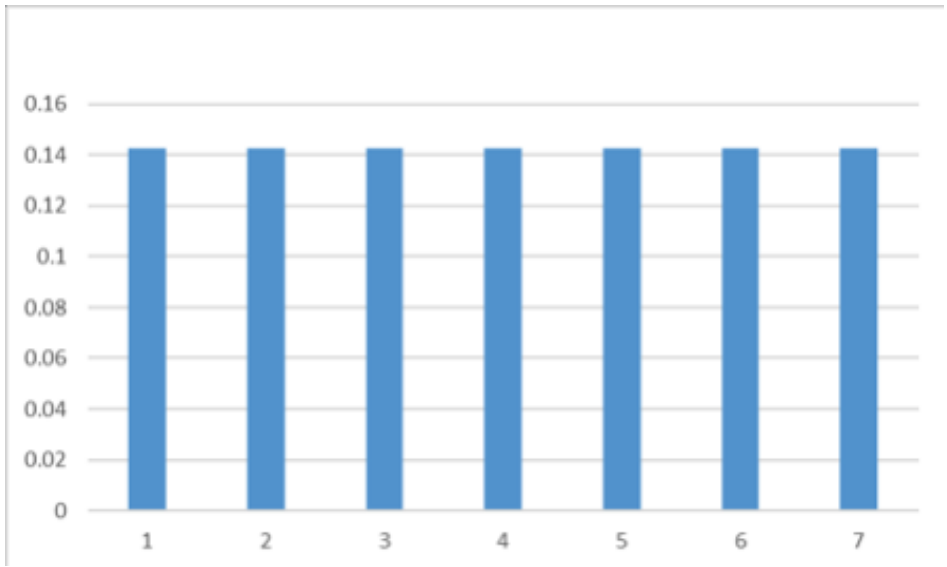
Histogram:



There are 35 subsets of size 3

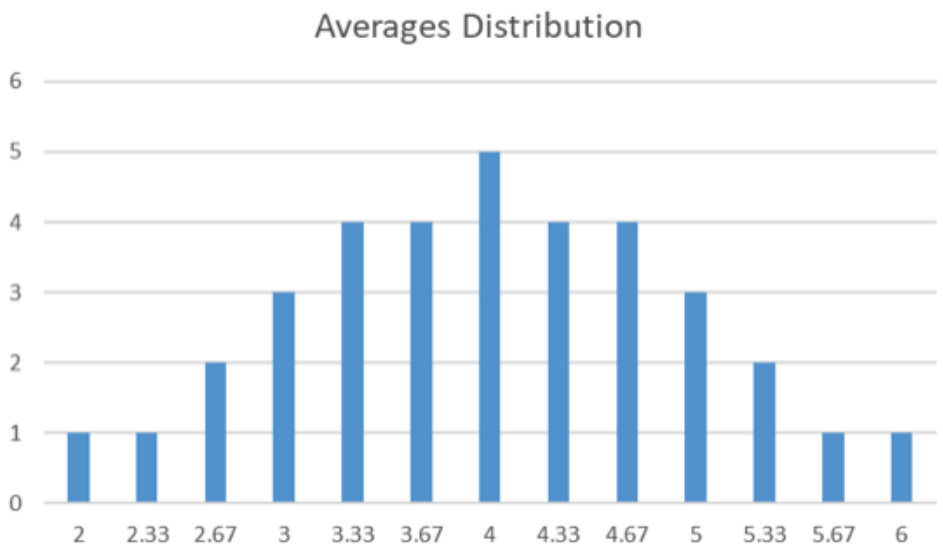
1,2,3	1,2,4	1,2,5	1,2,6	1,2,7
1,3,4	1,3,5	1,3,6	1,3,7	1,6,7
1,4,5	1,4,6	1,4,7	1,5,6	1,5,7
2,3,4	2,3,5	2,3,6	2,3,7	2,6,7
2,4,5	2,4,6	2,4,7	2,5,6	2,5,7
3,4,5	3,4,6	3,4,7	3,5,6	3,5,7
3,6,7	4,5,6	4,5,7	4,6,7	5,6,7

look at the averages of each individual set

2	2.33	2.67	3	3.33
2.67	3	3.33	3.67	4.67
3.33	3.67	4	4	4.33
3	3.33	3.67	4	5
3.67	4	4.33	4.33	4.67
4	4.33	4.67	4.67	5
5.33	5	5.33	5.67	6

look at the averages of these sets

$\bar{X}$	2	2.33	2.67	3	3.33	3.67	4	4.33	4.67	5	5.33	5.67	6
$P(\bar{X})$	$\frac{1}{35}$	$\frac{1}{35}$	$\frac{2}{35}$	$\frac{3}{35}$	$\frac{4}{35}$	$\frac{4}{35}$	$\frac{5}{35}$	$\frac{4}{35}$	$\frac{4}{35}$	$\frac{3}{35}$	$\frac{2}{35}$	$\frac{1}{35}$	$\frac{1}{35}$



Key Concets

$\overline{X}$	2	2.33	2.67	3	3.33	3.67	4	4.33	4.67	5	5.33	5.67	6
$P(\overline{X})$	$\frac{1}{35}$	$\frac{1}{35}$	$\frac{2}{35}$	$\frac{3}{35}$	$\frac{4}{35}$	$\frac{4}{35}$	$\frac{5}{35}$	$\frac{4}{35}$	$\frac{4}{35}$	$\frac{3}{35}$	$\frac{2}{35}$	$\frac{1}{35}$	$\frac{1}{35}$

Average of sample mean: $\mu_{\overline{X}=4}$
Standard deviation of sample mean: $\sigma_{\overline{X}=.94}$

Central Limit Theorem: Big Idea

- Take samples of a huge population.
- While the original data may not be normally distributed, the sample averages will be.

Central Limit Theorem

- $n \geq 30$, The sampling distribution of sample means is normally distributed.

$$\mu_{\overline{X}} = \mu$$

$$\sigma_{\overline{X}} = \frac{\sigma}{\sqrt{n}}$$

- $n < 30$, and the population is normally distributed, the sampling distribution of sample means will also be normally distrubuted.

$$\mu_{\overline{X}} = \mu$$

$$\sigma_{\overline{X}} = \frac{\sigma}{\sqrt{n}}$$

How to hunt the problems

Before:

After:

Example 1

A population of men have a mean of 172 lbs and standard deviation of 10 lbs. Find the probability that a randomly selected man will weigh more than 175 lbs.

Example 2

A population of men have a mean of 172 lbs and standard deviation of 10 lbs. Find the probability that a group of 20 men will have an average weight of more than 175 lbs. Assume that weigh is normally distrubuted.

Example: Weight in the US

The average weight of adult females in the U.S. is 168.4 pounds with a standard deviation of about 45 pounds. If we sample 30 females at random, what is the probability that their average weight is between 150 and 175?

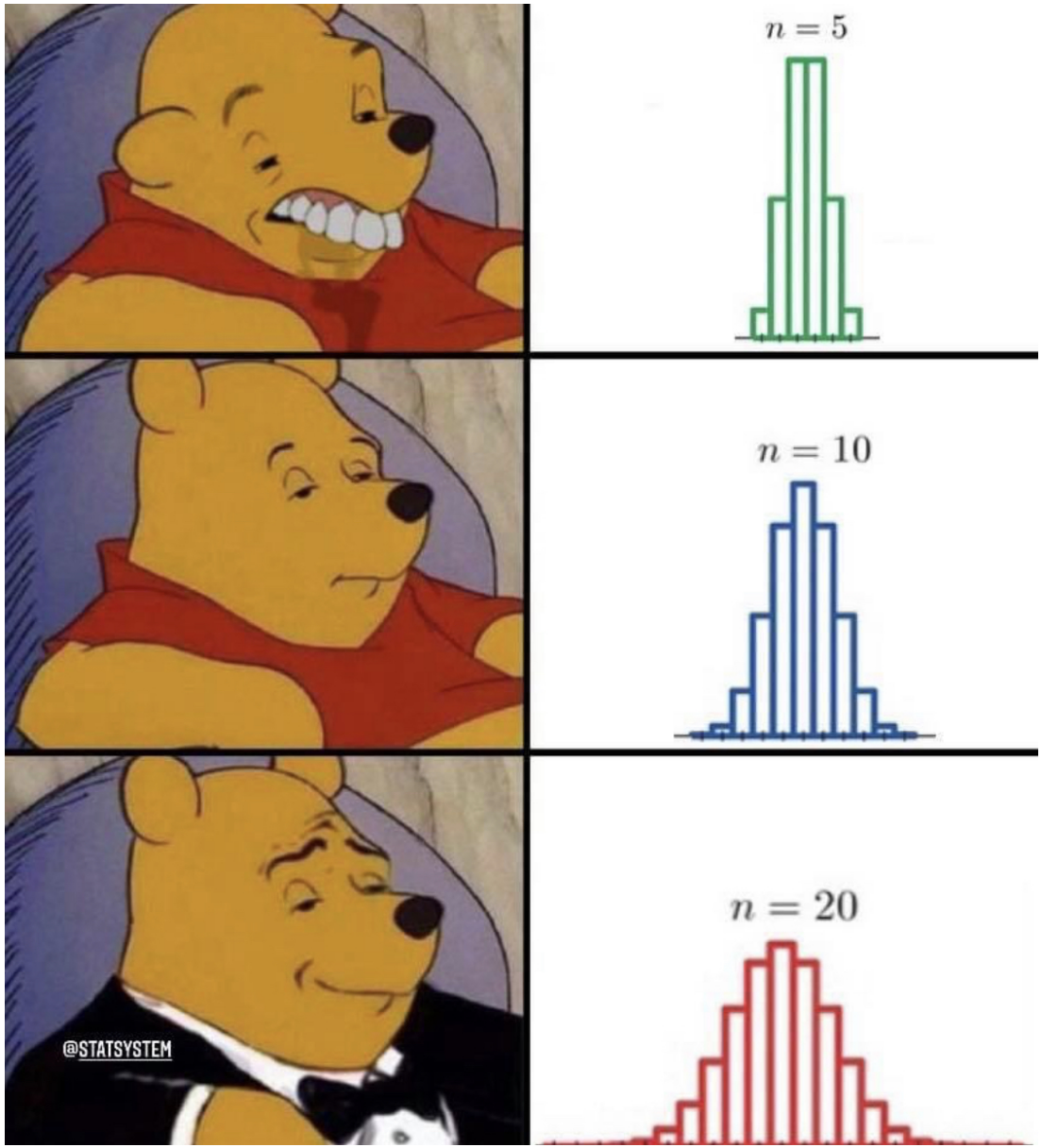
Example: Weight in the US

The average weight of adult females in the U.S. is 168.4 pounds with a standard deviation of about 45 pounds. If we sample 100 females at random, what is the probability that their average weight is between 150 and 175?

Example: Weight in the US

The average weight of adult females in the U.S. is 168.4 pounds with a standard deviation of about 45 pounds. If we sample 1000 females at random, what is the probability that their average weight is between 150 and 175?

How you can imagine :)



Example: Testosterone

According to a website, the average testosterone level for men aged 45-49 is 546 ng/dL with a standard deviation of 133. Assume the variable is normally distributed. If we sample 10 males aged 45-49, what is the probability that their average testosterone level is between 400 and 500?

Example: Testosterone

According to a website, the average testosterone level for men aged 45-49 is 546 ng/dL with a standard deviation of 133. Assume the variable is normally distributed. If we sample 40 males aged 45-49, what is the probability that their average testosterone level is between 400 and 500?

Example: Life Expectancy in the U.S.

The average life expectancy in the U.S. is about 78.74 with a standard deviation of about 15 years. If we sample 50 people at random, what is the probability that their average lifespan will be over 80 years?

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The average life expectancy in the U.S. is about 78.74 with a standard deviation of about 15 years. If we sample 500 people at random, what is the probability that their average lifespan will be over 80 years?