Practice Book

3 Practice and Apply

Use Practice Book pp. 205–206

Assignment Guide	
Decelerated	1–9, 13–18
Average	2–28 Even
Accelerated	10–12, 19–30

■ Before assigning the exercises, work through the example in the teaching display at the top of Practice Book page 205. Point out to students that they may have to try different groupings in order to find the correct factorization.

Errors Commonly Made

Some students may become frustrated when the first grouping they try does not work. Suggest they try combinations in a systematic way, such as grouping the first and second, then the first and third, and finally the first and fourth, to be sure they have attempted all possibilities.

■ In exercises 1–12, remind students to check their answers by multiplying. Point out that in exercise 12, it is possible to combine like terms, but that the polynomial can be easily factored by grouping as written.

■ In exercises 13–27, note that sometimes it may be convenient to reorder the terms of the polynomial before grouping.

■ Students should recall that the area of a rectangle is the product of the length and width. Hence, $a + b$ is one of the factors of the polynomial in exercise 28, and the width is the other. Similarly, $x + 2y^2$ is one factor of the polynomial in exercise 29, and the width is the other.

CHALLENGE

■ Students should recognize that the first two terms are a difference of two squares.

8-6 Factor by Grouping

Name _____ Date _____

Factor: $4r^5 + 4 + r^3 + 16r^2$

$(4r^5 + 4) + (r^3 + 16r^2)$ ← Arrange in groups with common factors.

$4(r^5 + 1) + r^2(r + 16)$ ← Factor the GCF from each group.

Think

There is no common binomial factor. Use the Commutative and Associative Properties of Equality to rearrange and regroup the terms differently.

$(4r^5 + 16r^2) + (r^3 + 4)$ ← Group the first and fourth terms.

$4r^2(r^3 + 4) + 1(r^3 + 4)$ ← Factor the GCF from each group.

$(4r^2 + 1)(r^3 + 4)$ ← Apply the Distributive Property.

Remember: When a polynomial contains four or more terms and two groups have the same binomial factor, factor by grouping.

Check:

$4r^5 + 4 + r^3 + 16r^3 \stackrel{?}{=} (4r^2 + 1)(r^3 + 4)$

F O I L

$\stackrel{?}{=} 4r^5 + 16r^2 + r^3 + 4$

$= 4r^5 + 4 + r^3 + 16r^2$ **True**

Factor each polynomial by grouping. Check your answer. Check students' work.

1. $21b^3 + 133b^2 + 3b + 19$

$7b^2(3b + 19) + 1(3b + 19)$
$(7b^2 + 1)(3b + 19)$
Check:
$= 21b^3 + 133b^2 + 3b + 19$

2. $55a^3 + 5a^2 + 22a + 2$

$5a^2(11a + 1) + 2(11a + 1)$
$(5a^2 + 2)(11a + 1)$

3. $7a^3 + 5a + 28a^2 + 20$

$a(7a^2 + 5) + 4(7a^2 + 5)$
$(a + 4)(7a^2 + 5)$

4. $9c^3 + 4c + 18c^2 + 8$

$c(9c^2 + 4) + 2(9c^2 + 4)$
$(c + 2)(9c^2 + 4)$

5. $12m^3 - 8m - 10 + 15m^2$

$4m(3m^2 - 2) - 5(2 - 3m^2)$
$4m(3m^2 - 2) + 5(3m^2 - 2)$
$(4m + 5)(3m^2 - 2)$

6. $12p^3 - 14p - 63 + 54p^2$

$2p(6p^2 - 7) - 9(7 - 6p^2)$
$2p(6p^2 - 7) + 9(6p^2 - 7)$
$(2p + 9)(6p^2 - 7)$

7. $-65w + 78w^4 - 6w^3 + 5$

$13w(-5 + 6w^3) - 1(6w^3 - 5)$
$13w(6w^3 - 5) - 1(6w^3 - 5)$
$(13w - 1)(6w^3 - 5)$

8. $-11z^3 + 264z^4 - 48z + 2$

$11z^3(-1 + 24z) - 2(24z - 1)$
$11z^3(24z - 1) - 2(24z - 1)$
$(11z^3 - 2)(24z - 1)$

9. $21x^2 + 14x^2y - 2xy - 3x$

$7x^2(3 + 2y) - x(2y + 3)$
$7x^2(2y + 3) - x(2y + 3)$
$(7x^2 - x)(2y + 3)$

10. $32b^2 - 3ab - 4b + 24ab^2$

$24ab^2 + 32b^2 - 3ab - 4b$
$8b^2(3a + 4) - b(3a + 4)$
$(8b^2 - b)(3a + 4)$

11. $14c^3 - 35c^2d + 6cd^3 - 15d^4$

$7c^2(2c - 5d) + 3d^3(2c - 5d)$
$(7c^2 + 3d^3)(2c - 5d)$

12. $95a^2 - 57ab + 10ab - 6b^2$

$19a(5a - 3b) + 2b(5a - 3b)$
$(19a + 2b)(5a - 3b)$

Use with SOURCEBOOK Lesson 8-6, pages 214–215.

Factor each polynomial by grouping. Check your answer. Check students' work.

13. $m^2 + np - mp - mn$
$m^2 - mp + np - mn$
$(m^2 - mp) + (np - mn)$
$m(m - p) + n(p - m)$
$m(m - p) - n(m - p)$
$(m - n)(m - p)$

14. $2ab + bc - b^2 - 2ac$
$2ab - 2ac + bc - b^2$
$2a(b - c) + b(c - b)$
$2a(b - c) - b(b - c)$
$(2a - b)(b - c)$

15. $3a^2 + 2bc + 6ac + ab$
$3a^2 + 6ac + 2bc + ab$
$3a(a + 2c) + b(2c + a)$
$(3a + b)(a + 2c)$

16. $5m^2 + 7np + 35mn + mp$
$5m^2 + mp + 7np + 35mn$
$m(5m + p) + 7n(p + 5m)$
$(m + 7n)(5m + p)$

17. $gj - hk + gk - hj$
$gj + gk - hk - hj$
$g(j + k) - h(k + j)$
$(g - h)(j + k)$

18. $fm - tz + ft - mz$
$fm + ft - tz - mz$
$f(m + t) - z(t + m)$
$(f - z)(m + t)$

19. $3ac - 8bd - 4ad + 6bc$
$3ac + 6bc - 8bd - 4ad$
$3c(a + 2b) - 4d(2b + a)$
$(3c - 4d)(a + 2b)$

20. $10ce - 18df - 12cf + 15de$
$10ce + 15de - 18df - 12cf$
$5e(2c + 3d) - 6f(3d + 2c)$
$(5e - 6f)(2c + 3d)$

21. $x^2z + w^2y + x^2w^2 + yz$
$x^2z + x^2w^2 + w^2y + yz$
$x^2(z + w^2) + y(w^2 + z)$
$(x^2 + y)(w^2 + z)$

22. $ip + j^3q^2 + iq^2 + j^3p$
$ip + iq^2 + j^3q^2 + j^3p$
$i(p + q^2) + j^3(q^2 + p)$
$(i + j^3)(p + q^2)$

23. $-28r^2u + 12su + 14r^2t - 6st$
$-28r^2u + 14r^2t + 12su - 6st$
$-7r^2(4u - 2t) + 3s(4u - 2t)$
$(3s - 7r^2)(4u - 2t)$

24. $10x^3z - 14w^2z - 15x^3y + 21w^2y$
$10x^3z - 15x^3y - 14w^2z + 21w^2y$
$5x^3(2z - 3y) - 7w^2(2z - 3y)$
$(5x^3 - 7w^2)(2z - 3y)$

25. $\frac{5}{12}jk - \frac{5}{32}k\ell - \frac{7}{9}j\ell + \frac{7}{24}\ell^2$
$\frac{5}{4}k\left(\frac{1}{3}j - \frac{1}{8}\ell\right) - \frac{7}{3}\ell\left(\frac{1}{3}j - \frac{1}{8}\ell\right)$
$\left(\frac{5}{4}k - \frac{7}{3}\ell\right)\left(\frac{1}{3}j - \frac{1}{8}\ell\right)$

26. $\frac{1}{10}p^2 - \frac{2}{45}pq - \frac{5}{24}pr + \frac{5}{54}qr$
$\frac{1}{5}p\left(\frac{1}{2}p - \frac{2}{9}q\right) - \frac{5}{6}r\left(\frac{1}{4}p - \frac{1}{9}q\right)$
$\frac{2}{5}p\left(\frac{1}{4}p - \frac{1}{9}q\right) - \frac{5}{6}r\left(\frac{1}{4}p - \frac{1}{9}q\right)$
$\left(\frac{2}{5}p - \frac{5}{6}r\right)\left(\frac{1}{4}p - \frac{1}{9}q\right)$

27. $1.14ax - 0.54bx + 0.76ay - 0.36by$
$3x(0.38a - 0.18b) + 4y(0.19a - 0.09b)$
$6x(0.19a - 0.09b) + 4y(0.19a - 0.09b)$
$(6x + 4y)(0.19a - 0.09b)$

Solve. Show your work.

28. The area of a rectangle is represented by $a^2b + 2a + ab^2 + 2b$. If $a + b$ represents the length, what binomial represents the width?
$ab + 2$; Factor $a^2b + 2a + ab^2 + 2b$
$(a^2b + 2a) + (ab^2 + 2b)$; $a(ab + 2) + b(ab + 2)$
$(a + b)(ab + 2)$
So the width of the rectangle can be represented by the binomial $ab + 2$.

29. The area of a rectangle is represented by $x^3 - xy + 2x^2y^2 - 2y^3$. If $x + 2y^2$ represents the length, what binomial represents the width?
$x^2 - y$; Factor $x^3 - xy + 2x^2y^2 - 2y^3$
$(x^3 - xy) + (2x^2y^2 - 2y^3)$
$x(x^2 - y) + 2y^2(x^2 - y)$; $(x + 2y^2)(x^2 - y)$
So the length of the rectangle can be represented by the binomial $x^2 - y$.

CHALLENGE

30. Factor by grouping: $a^2 - b^2 + ax + bx$. Explain your steps.
$(a^2 - b^2) + (ax + bx)$: group the difference of two squares and the sum of ax and bx
$(a + b)(a - b) + x(a + b)$: express the difference of two squares as the product of the sum of both terms and the difference of both terms; $(a + b)(a - b + x)$
Factor out the common binomial factor $(a + b)$.

Additional Resources

ONLINE www.progressinmathematics.com
- Meeting Individual Needs Activities
- Alternative Teaching Models
- Vocabulary Activities
- Audio Glossary
- Virtual Manipulatives

Meeting Individual Needs
See page 199F.

4 Summarize/Assess

Conceptual Thinking

■ To assess whether students have conceptualized the lesson concepts, lead a discussion in which they explain how the Distributive Property is used to factor a polynomial by grouping. Have them factor polynomials such as $2x^2 + 3x + 4xy + 6y$. $(x + 2y)(2x + 3)$

MATH JOURNAL In their *Math Journals*, have students describe how to factor the polynomial $3m^2 - 11mn + 20n^2$, justifying each step. $(3m + 4n)(m - 5n)$

5 Follow-Up

Reteaching

■ To review factoring by grouping, walk through an example with the students. Create sets of cards. Write polynomials on half of the cards in each set and the factors of the polynomials on the other half.

Have students work in pairs and give each pair a set of cards. Have partners work together to match the polynomials with their factors. For each matching pair of cards, they should multiply the factors to verify their answers. Then have them write the steps to show how to factor each polynomial by grouping.

ONLINE **See Chapter 8 Alternative Teaching Models.**

End of Lesson