Contents

Chapter 7: Polynomials

In this chapter we will start looking at polynomials. Polynomials will show up in pretty much every section of every chapter in the remainder of this material and so it is important that you understand them.

0.1 Polynomials in one variable

Definition 0.1.1 *Polynomials in one variable are algebraic expressions that consist of terms in the form ax^n where is a **whole number (non negative integer)** is called exponent and a is a real number and is called the **coefficient** of the term. The **degree** of a polynomial in one variable is the **largest exponent** in the polynomial.*

The following are examples of polynomials in one variable and their degrees:

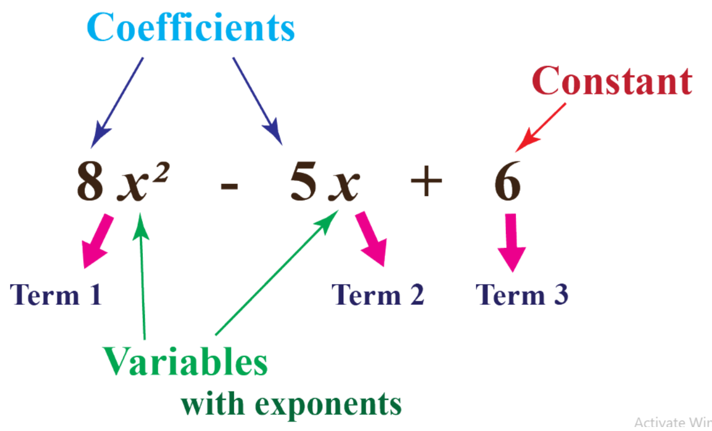
polynomial	degree	number of terms	coefficients	
$5x^{12} - 2x^6 + x^5 - 18x + 15$	12	5	$5, -2, 1, -18, 15$	
$x^4 - 17x + 1.5$	4	3	$1, -17, 1.5$	Trinomial
$\sqrt{17}x^9$	9	1	$\sqrt{17}$	Monomial
$17x - \frac{4}{3}$	1	2	$17, -\frac{4}{3}$	Binomial
$\sqrt[3]{17}x^6 + \frac{2}{5}x + 4.5$	6			

The following examples represent some expressions they are not polynomials in one variable:

$6x^{-2} + 4x$	the variable x in first term has a negative exponent.
$\sqrt{x} - 4x^2$	the variable x in first term has a rational exponent
$\frac{3}{x} - 2x$	the variable x in first term has a negative exponent

Standard form: The polynomials written in the standard form as:

$$a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$$



Typically, we arrange terms of polynomials in **descending** order based on the degree of each term. The **leading coefficient** is the coefficient of the variable with the highest power, in this case, a_n.

Example 0.1.2 *Write in standard form:*
$-2x^4 + 3x + 5x^3 - 4x^2 + 7$

$$\begin{aligned} &3x - 4x^2 + 5x^3 + 7 - 2x^4 \\ &= 3x + (-4)x^2 + 5x^3 + 7 + (-2)x^4 \\ &= -2x^4 + 5x^3 - 4x^2 + 3x + 7 \end{aligned}$$

0.1.1 Polynomial with more than one variables

Lemma 0.1.3 *If the polynomial has more than one variable. Then*

- *The degree of each term will be the sum of its exponents.*
- *The degree of polynomial will be the greatest sum of exponents of each term.*

Example 0.1.4 *Find the degree of the polynomial* $3x^2y^3 + 2xy + 5xy^3$
The degree of first term is 5
The degree of second term is 2
The degree of third term is 4
Therefore, the degree of the polynomial is 5 *(it is a greatest sum).*

0.2 Addition & subtraction of polynomials

To add or subtract polynomials we simply combine (add or subtract) any like terms together ... so what is a like term?

Definition 0.2.1 LikeTerms*: are the terms of polynomial whose variables (and their exponents such as the* 2 *in* x^2*) are the same.*

To add or subtract to polynomials, it is enough to combine like terms as in the following examples:

Example 0.2.2 *Add*

$$\begin{aligned} & (-2x^4+3x-4x^2+5x^3+7)+(7x^4+5x+5x^3+12) \\ = \; & -2x^4+3x-4x^2+5x^3+7+7x^4+5x+5x^3+12 \\ = \; & 5x^4+10x^3-4x^2+8x+19 \end{aligned}$$

Example 0.2.3 *Subtract*

$$(-2x^4+3x-4x^2+5x^3+7)-(7x^4-5x+5x^3+12)$$
$$=-2x^4+3x-4x^2+5x^3+7-7x^4+5x-5x^3-12$$

$$=-9x^4-4x^2+8x-5$$

Example 0.2.4 *Add*

$$(3xy+2y^2x-4)+(4yx+3x^2y-10)$$
$$=3xy+2y^2x-4+4yx+3x^2y-10$$
$$=7xy+2y^2x+3x^2y-14.$$

0.3 Multiplication of two polynomials

In multiplying polynomials yo can perform the operation horizontally or vertically as in the following examples:

Example 0.3.1 *Multiply* $(3x^2-4x+2)(2x-3)$ *horizontally*

$$\begin{aligned} & (3x^2-4x+2)(2x-3) \\ & =(3x^2)(2x-3)+(-4x)(2x-3)+2(2x-3) \\ & =(3x^2)(2x)+(3x^2)(-3)+(-4x)(2x)+(-4x)(-3)+2(2x)+2(-3) \\ & =6x^3-9x^2-8x^2+12x+4x-6 \\ & =6x^3-17x^2+16x-6 \end{aligned}$$

Example 0.3.2 *Multiply* $(8x^2+6x-3)(3x^3-5x^2+3)$ *vertically*

$$\begin{array}{r}
8x^2+6x-3 \\
3x^3-5x^2+3 \\
\hline
24x^5+18x^4-9x^3 \\
+\ -40x^4-30x^3+15x^2 \\
24x^2+18x-9 \\
\hline
24x^5-22x^4-39x^3+39x^2+18x-9
\end{array}$$

0.4 Division of two polynomials:

We will discuss the division of polynomials in two cases

0.4.1 Divide polynomial by a monomial.

In this case, we will deal with the division problem as a fraction, and simply we will use the distributive property of the numerator over the denominator to perform the operation as follows:

Example 0.4.1 *Divide* $(3x^2-4x+2)\div 2x$

$$\begin{aligned}
(3x^2-4x+2)\div 2x &= \frac{3x^2-4x+2}{2x} \\
&= \frac{3x^2}{2x}+\frac{-4x}{2x}+\frac{2}{2x} \\
&= \frac{3}{2}x-2+\frac{1}{x}
\end{aligned}$$

0.4.2 Divide polynomial by a binomial.

In this case, we will use long division to perform the operation as follows:

Example 0.4.2 $(8x^3 + 4x^2 + 2x - 6) \div (2x + 4)$

$$
\begin{array}{r|l}
 & 4x^2 - 6x + 13 \\ \cline{2-2}
\underline{2x+4} & 8x^3 + 4x^2 + 2x - 6 \\
 & -\ 8x^3 + 16x^2 \\ \cline{2-2}
 & -\ -12x^2 + 2x \\
 & \quad -12x^2 - 24x \\ \cline{2-2}
 & \qquad 26x - 6 \\
 & \qquad 26x + 52 \\ \cline{2-2}
 & \qquad\ -58
\end{array}
$$

Example 0.4.3 $(4w^2 - 44w + 121) \div (2w - 11)$

$$
\begin{array}{r|l}
 & \qquad 2w - 11 \\ \cline{2-2}
\underline{2w-11} & 4w^2 - 44w + 121 \\
 & \quad 4w^2 - 22w \\ \cline{2-2}
 & \quad -22w + 121 \\
 & \quad -22w + 121 \\ \cline{2-2}
 & \qquad\quad 0
\end{array}
$$

Example 0.4.4 $(12x^2 + 5x - 72) \div (3x + 8)$

$$
\begin{array}{r|l}
 & \ 4x - 9 \\ \cline{2-2}
\underline{3x+8} & 12x^2 + 5x - 72 \\
 & 12x^2 + 32x \\ \cline{2-2}
 & -27x - 72 \\
 & -27x - 72 \\ \cline{2-2}
 & \qquad 0
\end{array}
$$