

3 Practice and Apply

Use Practice Book pp. 207–208

Assignment Guide	
Decelerated	1–9, 13–18, 24–28
Average	2–22 Even, 24–28
Accelerated	10–12, 16–28

Before assigning the exercises, review the methods students have learned for factoring polynomials. Then discuss the examples in the teaching display on Practice Book page 207.

Errors Commonly Made

Some students may be overwhelmed when presented with a polynomial that requires a variety of methods to factor completely. Suggest making a list of simple examples of polynomials that can be factored using each method and referring to it as needed.

In exercises 1–12, remind students that after they have written a polynomial as a product of two factors, they must examine each factor to determine if it can be factored further.

Emphasize the importance of checking factors by multiplication in exercises 13–21. Note that it may be helpful to reorder the terms in some of the polynomials before trying to factor.

Problem Solving

In problems 22 and 23, students should recall that the volume of a prism is the product of the area of the base and the height. Since the base is a square, one of the factors will be squared when the polynomial is factored.

Mental Math

Discuss how the Distributive Property can be used to help solve exercises 24–28.

8-7 Factor Completely

Name _____ Date _____

Factor completely: $3x^3 - 24x^2 + 48x$

$3x(x^2 - 8x + 16)$ ← The GCF is $3x$.
 $3x(x - 4)^2$ ← $x^2 - 8x + 16$ is a perfect-square trinomial.

Check: $3x^3 - 24x^2 + 48x \stackrel{?}{=} 3x(x - 4)^2$
 $\stackrel{?}{=} 3x(x^2 - 8x + 16)$
 $= 3x^3 - 24x^2 + 48x$ **True**

Factor completely: $9v^3 - 2$

$9v^3 - 2$ ← No GCF
 Not the difference of two perfect squares

$9v^3 - 2$ cannot be factored.
 It is prime.

Factor completely. Check using multiplication. If the polynomial cannot be factored using integers, label it prime.

1. $2a^2 - 162$

$$\begin{aligned} &2(a^2 - 81) \\ &2(a + 9)(a - 9) \\ \text{Check:} \\ &2a^2 - 162 \stackrel{?}{=} 2(a + 9)(a - 9) \\ &\stackrel{?}{=} (2a + 18)(a - 9) \\ &= 2a^2 - 162 \text{ True} \end{aligned}$$

2. $3b^2 - 48$

$$\begin{aligned} &3(b^2 - 16) \\ &3(b + 4)(b - 4) \end{aligned}$$

3. $8c^3 - 80c^2 + 200c$

$$\begin{aligned} &8c(c^2 - 10c + 25) \\ &8c(c - 5)^2 \end{aligned}$$

4. $2f^3 - 28f^2 + 98f$

5. $3ab^2 + 21ab - 54a$

6. $5vw^2 + 20vw - 160v$

$$\begin{aligned} &2f(f^2 - 14f + 49) \\ &2f(f - 7)^2 \end{aligned}$$

$$\begin{aligned} &3a(b^2 + 7b - 18) \\ &3a(b - 2)(b + 9) \end{aligned}$$

$$\begin{aligned} &5v(w^2 + 4w - 32) \\ &5v(w - 4)(w + 8) \end{aligned}$$

7. $-5x^2y - 25xy - 15y$

8. $-2hj^2 - 18hj - 2h$

9. $8x^2y - 4x^2 + 19y$

$$\begin{aligned} &-5y(x^2 + 5x + 3) \\ &x^2 + 5x + 3 \text{ is prime.} \\ &-5y(x^2 + 5x + 3) \end{aligned}$$

$$\begin{aligned} &-2h(j^2 + 9j + 1) \\ &j^2 + 9j + 1 \text{ is prime.} \\ &-2h(j^2 + 9j + 1) \end{aligned}$$

prime

10. $4a^3b - 9b^2 + 4a$

11. $48t^3 + 88t^2 + 24t$

12. $140m^3 + 133m^2 + 21m$

prime

$$\begin{aligned} &8t(6t^2 + 11t + 3) \\ &ac = 18; 2 + 9 = 11 \\ &8t(6t^2 + 2t + 9t + 3) \\ &8t[2t(3t + 1) + 3(3t + 1)] \\ &8t(2t + 3)(3t + 1) \end{aligned}$$

$$\begin{aligned} &7m(20m^2 + 19m + 3) \\ &ac = 60; 4 + 15 = 19 \\ &7m(20m^2 + 4m + 15m + 3) \\ &7m[4m(5m + 1) + 3(5m + 1)] \\ &7m(5m + 1)(4m + 3) \end{aligned}$$

Use with

SOURCEBOOK Lesson 8-7, pages 216–217.

Chapter 8 207



Factor completely. Check using multiplication. If the polynomial *cannot* be factored using integers, label it *prime*.

13. $7d^4 - 7$

14. $8c^4 - 128$

15. $c^4d - 13c^2d + 36d$

$$\frac{7(d^4 - 1)}{7(d^2 + 1)(d^2 - 1)} = \frac{7(d^2 + 1)(d^2 - 1)}{7(d^2 + 1)(d + 1)(d - 1)}$$

$$\frac{8(c^4 - 16)}{8(c^2 + 4)(c^2 - 4)} = \frac{8(c^2 + 4)(c^2 - 4)}{8(c^2 + 4)(c + 2)(c - 2)}$$

$$\frac{d(c^4 - 13c^2 + 36)}{d(c^2 - 9)(c^2 - 4)} = \frac{d(c + 3)(c - 3)(c + 2)(c - 2)}{d(c + 3)(c - 3)(c + 2)(c - 2)}$$

16. $a^4b - 5a^2b + 4b$

17. $27a^4 + 27a^3 - 12a^2 - 12a$

18. $64x^4 + 64x^3 - 324x^2 - 324x$

$$\frac{b(a^4 - 5a^2 + 4)}{b(a^2 - 4)(a^2 - 1)} = \frac{b(a^2 + 2)(a - 2)(a + 1)(a - 1)}{b(a^2 + 2)(a - 2)(a + 1)(a - 1)}$$

$$\frac{3a(9a^3 + 9a^2 - 4a - 4)}{3a[9a^2(a + 1) - 4(a + 1)]} = \frac{3a(9a^2 - 4)(a + 1)}{3a(9a^2 - 4)(a + 1)}$$

$$\frac{4x(16x^3 + 16x^2 - 81x - 81)}{4x[16x^2(x + 1) - 81(x + 1)]} = \frac{4x(16x^2 - 81)(x + 1)}{4x(16x^2 - 81)(x + 1)}$$

19. $a^4 - 2a^2b^2 + b^4$

20. $16x^4 - 72x^2y^2 + 81y^4$

21. $x^8 - 1$

$$\frac{(a^2 - b^2)^2}{(a^2 - b^2)(a^2 - b^2)} = \frac{(a + b)^2(a - b)^2}{(a + b)^2(a - b)^2}$$

$$\frac{(4x^2 - 9y^2)^2}{(4x^2 - 9y^2)(4x^2 - 9y^2)} = \frac{(2x + 3y)^2(2x - 3y)^2}{(2x + 3y)^2(2x - 3y)^2}$$

$$\frac{(x^4 - 1)(x^4 + 1)}{(x^2 - 1)(x^2 + 1)(x^4 + 1)} = \frac{(x - 1)(x + 1)(x^2 + 1)(x^4 + 1)}{(x - 1)(x + 1)(x^2 + 1)(x^4 + 1)}$$

Problem Solving

22. A rectangular prism has a volume of $x^3y + xy^3 - 2x^2y^2$ with a square base. If it has a lateral area of 30 in.^2 , what is the volume in cubic inches?

First factor: $xy(x^2 - 2xy + y^2)$
 $xy(x - y)^2$; the area of a lateral side is $xy(x - y) = 30$
 $5(2)(5 - 2) = 10(3) = 30$; so $x = 5$, $y = 2$
 $V = 5(2)(3)^2 = 90$; The rectangular prism has a volume of 90 in.^3 .

23. A rectangular prism has a volume of $x^3 - y^3 + x^2y - xy^2$, and a square base. If a lateral side has an area of 16 cm^2 , what is the volume in cubic centimeters?

$(x^3 + x^2y) - (xy^2 + y^3)$; $x^2(x + y) - y^2(x + y)$
 $(x^2 - y^2)(x + y)$; $(x + y)(x - y)(x + y)$; $(x + y)^2(x - y)$; the area of a lateral side is $(x + y)(x - y) = 16$ or $x^2 - y^2 = 16$
 $5^2 - 3^2 = 25 - 9 = 16$; so $x = 5$, $y = 3$; $V = (8)^2(2) = 128$
 The rectangular prism has a volume of 128 cm^3 .

MENTAL MATH

Multiply.

24. $(24)(36) = (30 - 6)(30 + 6) = 900 - 36 = 864$

25. $(91)(89) = (90 + 1)(90 - 1) = 8100 - 1 = 8099$

26. $(74)(66) = (70 + 4)(70 - 4) = 4900 - 16 = 4884$

27. $(29)^2 = (30 - 1)^2 = (30)^2 - 2(30)(1) + (1)^2 = 900 - 60 + 1 = 840 + 1 = 841$

28. $(63)^2 = (60 + 3)^2 = (60)^2 + 2(60)(3) + (3)^2 = 3600 + 360 + 9 = 3960 + 9 = 3969$

Additional Resources

- Meeting Individual Needs Activities
- Alternative Teaching Models
- Vocabulary Activities
- Audio Glossary
- Virtual Manipulatives
- Check your Progress III
- Practice Activities (Lessons 6–7)

4 Summarize/Assess

Conceptual Thinking

■ To assess whether students have conceptualized the lesson concepts, have them discuss the various methods of factoring polynomials they have learned in the chapter. Provide several polynomials for them to factor, such as $6xy^2 - 24x$ and $3a^3b + 6a^2b - 24ab$.

$$6x(y + 2)(y - 2); 3ab(a + 4)(a - 2)$$

ONLINE Check Your Progress III

Administer Check Your Progress III to assess understanding of Lessons 6–7. For additional practice, assign the online Practice Activities.

5 Follow-Up

Reteaching

■ Review the methods of factoring polynomials that the class has learned. Have each student write a factored form of a polynomial and then multiply. Students should start with simple polynomials where only one step is required to factor the polynomials. Have students write the expanded form of the polynomial on a separate sheet of paper, then exchange papers with a classmate.

Each student should then factor the polynomial completely. Have pairs of students check each other's work and help each other find and fix any errors. Have them repeat the activity using more complex problems that require two steps to factor. Be sure they include polynomials whose factorization involves a difference of two squares, a binomial squared, a product of two binomials, and grouping.

ONLINE See Chapter 8 Alternative Teaching Models.