

3 Practice and Apply

Use Practice Book pp. 249–252

Assignment Guide	
Decelerated	1–8, 13–20, 33–44, 59–62
Average	2–58 Even, 59–62
Accelerated	9–12, 21–32, 45–62

Before assigning the exercises on Practice Book pages 249–252, work through the examples in the teaching display. Note that in each example shown, both solutions are roots of the original equation. Remind students that this is not always the case—that sometimes there are extraneous solutions. Then discuss how to identify extraneous solutions.

When assigning exercises 1–12, remind students to first write each equation in the form $ax^2 + bx + c = 0$. Review the factoring methods and special patterns that students have learned. Note that not all of the exercises are factorable over the rational numbers. Stress the importance of checking solutions in the original equation. Point out that the answers are given in set notation.

Elicit from students that they can use what they learned in the two previous lessons in order to predict the number of real-number solutions. Remind them, however, that not all real numbers are rational. For example, the equation in exercise 3 has 2 real-number solutions that are not rational; hence, students cannot solve it by factoring.

Errors Commonly Made

Some students may be confused by the concept of an equation's having 0, 1, or 2 solutions. Review the different possibilities of a parabola's having 0, 1, or 2 x -intercepts, and relate that fact to solving quadratic equations.

10-3 Solve Quadratic Equations by Factoring

Name _____ Date _____

Solve: $3x^2 = -20x + 63$

$3x^2 + 20x - 63 = 0$ ← Write the equation in standard form.

$(x + 9)(3x - 7) = 0$ ← Factor.

$x + 9 = 0$ or $3x - 7 = 0$ ← Apply the Zero-Product Property.

$x = -9$ or $3x = 7$ ← Apply the Addition and Subtraction Properties of Equality.

$x = \frac{7}{3}$ ← Use the Division Property of Equality.

Check:

$$3x^2 = -20x + 63$$

$$3x^2 = -20x + 63$$

$$3(-9)^2 \stackrel{?}{=} -20(-9) + 63 \quad 3\left(\frac{7}{3}\right)^2 \stackrel{?}{=} -20\left(\frac{7}{3}\right) + 63$$

$$3(81) \stackrel{?}{=} 180 + 63$$

$$3\left(\frac{49}{9}\right) \stackrel{?}{=} \frac{-140}{3} + 63$$

$$243 = 243 \text{ True}$$

$$\frac{49}{3} = \frac{49}{3} \text{ True}$$

So -9 and $\frac{7}{3}$ are the roots of the equation.

Solve: $x = \sqrt{14x - 45}$

$x^2 = (\sqrt{14x - 45})^2$ ← Square both sides of the equation.

$x^2 = 14x - 45$ ← Simplify.

$x^2 - 14x + 45 = 0$ ← Write in standard form.

$(x - 9)(x - 5) = 0$ ← Factor.

$x - 9 = 0$ or $x - 5 = 0$ ← Use the Zero-Product Property.

$x = 9$ or $x = 5$ ← Use the Addition Property of Equality.

Check:

$$x = \sqrt{14x - 45}$$

$$x = \sqrt{14x - 45}$$

$$9 \stackrel{?}{=} \sqrt{14(9) - 45}$$

$$5 \stackrel{?}{=} \sqrt{14(5) - 45}$$

$$9 \stackrel{?}{=} \sqrt{126 - 45}$$

$$5 \stackrel{?}{=} \sqrt{70 - 45}$$

$$9 \stackrel{?}{=} \sqrt{81}$$

$$5 \stackrel{?}{=} \sqrt{25}$$

$$9 = 9 \text{ True}$$

$$5 = 5 \text{ True}$$

So 5 and 9 are the roots of the equation.

Solve each equation by factoring. Check the solution on a separate sheet of paper. Check students' work.

1. $x^2 + 9x = 10$

2. $x^2 + 8x = 20$

3. $7x = -11x^2 - 1$

4. $3x = -2x^2 - 7$

$$\begin{aligned} x^2 + 9x - 10 &= 0 \\ (x + 10)(x - 1) &= 0 \\ x + 10 &= 0 \text{ or } x - 1 = 0 \\ x &= -10 \text{ or } x = 1 \\ \{-10, 1\} \end{aligned}$$

$$\begin{aligned} x^2 + 8x - 20 &= 0 \\ (x + 10)(x - 2) &= 0 \\ x + 10 &= 0 \text{ or } x - 2 = 0 \\ x &= -10 \text{ or } x = 2 \\ \{-10, 2\} \end{aligned}$$

$$\begin{aligned} 11x^2 + 7x + 1 &= 0 \\ \text{not factorable over} \\ \text{the rational numbers} \end{aligned}$$

$$\begin{aligned} 2x^2 + 3x + 7 &= 0 \\ \text{not factorable over} \\ \text{the rational numbers} \end{aligned}$$

5. $-6x = x^2 - 91$

6. $8x = x^2 - 48$

7. $25x^2 - 64 = 0$

8. $64x^2 - 121 = 0$

$$\begin{aligned} x^2 + 6x - 91 &= 0 \\ (x + 13)(x - 7) &= 0 \\ x + 13 &= 0 \text{ or } x - 7 = 0 \\ x &= -13 \text{ or } x = 7 \\ \{-13, 7\} \end{aligned}$$

$$\begin{aligned} x^2 - 8x - 48 &= 0 \\ (x + 4)(x - 12) &= 0 \\ x + 4 &= 0 \text{ or } x - 12 = 0 \\ x &= -4 \text{ or } x = 12 \\ \{-4, 12\} \end{aligned}$$

$$\begin{aligned} (5x + 8)(5x - 8) &= 0 \\ 5x + 8 &= 0 \text{ or } 5x - 8 = 0 \\ x &= -\frac{8}{5} \text{ or } x = \frac{8}{5} \\ \left\{-\frac{8}{5}, \frac{8}{5}\right\} \end{aligned}$$

$$\begin{aligned} (8x + 11)(8x - 11) &= 0 \\ 8x + 11 &= 0 \text{ or } 8x - 11 = 0 \\ x &= -\frac{11}{8} \text{ or } x = \frac{11}{8} \\ \left\{-\frac{11}{8}, \frac{11}{8}\right\} \end{aligned}$$

9. $-10x^2 = 19x - 15$

10. $-21x^2 = -22x - 8$

11. $25x^2 = 70x - 49$

12. $8x^2 = 72x - 162$

$$\begin{aligned} 10x^2 + 19x - 15 &= 0 \\ (5x - 3)(2x + 5) &= 0 \\ 5x - 3 &= 0 \text{ or } 2x + 5 = 0 \\ x &= \frac{3}{5} \text{ or } x = -\frac{5}{2} \\ \left\{\frac{3}{5}, -\frac{5}{2}\right\} \end{aligned}$$

$$\begin{aligned} 21x^2 - 22x - 8 &= 0 \\ (3x - 4)(7x + 2) &= 0 \\ 3x - 4 &= 0 \text{ or } 7x + 2 = 0 \\ x &= \frac{4}{3} \text{ or } x = -\frac{2}{7} \\ \left\{\frac{4}{3}, -\frac{2}{7}\right\} \end{aligned}$$

$$\begin{aligned} 25x^2 - 70x + 49 &= 0 \\ (5x - 7)^2 &= 0 \\ 5x - 7 &= 0 \\ x &= \frac{7}{5} \\ \left\{\frac{7}{5}\right\} \end{aligned}$$

$$\begin{aligned} 8x^2 - 72x + 162 &= 0 \\ 2(4x^2 - 36x + 81) &= 0 \\ 2(2x - 9)^2 &= 0 \\ 2x - 9 &= 0; \quad x = \frac{9}{2} \\ \left\{\frac{9}{2}\right\} \end{aligned}$$

Use with

SOURCEBOOK Lesson 10-3, pages 254–257.

Chapter 10 249



Solve each equation by factoring, if possible.

Check the solution on a separate sheet of paper. **Check students' work.**

13. $49 = x^2$

$$\begin{aligned} x^2 - 49 &= 0 \\ (x + 7)(x - 7) &= 0 \\ x + 7 = 0 \text{ or } x - 7 = 0 \\ x = -7 \text{ or } x = 7 \\ \{-7, 7\} \end{aligned}$$

14. $225 = x^2$

$$\begin{aligned} x^2 - 225 &= 0 \\ (x + 15)(x - 15) &= 0 \\ x + 15 = 0 \text{ or } x - 15 = 0 \\ x = -15 \text{ or } x = 15 \\ \{-15, 15\} \end{aligned}$$

15. $2x^2 + 5x = 3$

$$\begin{aligned} 2x^2 + 5x - 3 &= 0 \\ (2x - 1)(x + 3) &= 0 \\ 2x - 1 = 0 \text{ or } x + 3 = 0 \\ x = \frac{1}{2} \text{ or } x = -3 \\ \left\{-3, \frac{1}{2}\right\} \end{aligned}$$

16. $3x^2 - 4x = 4$

$$\begin{aligned} 3x^2 - 4x - 4 &= 0 \\ (3x + 2)(x - 2) &= 0 \\ 3x + 2 = 0 \text{ or } x - 2 = 0 \\ x = -\frac{2}{3} \text{ or } x = 2 \\ \left\{-\frac{2}{3}, 2\right\} \end{aligned}$$

17. $5x^2 = 35x - 50$

$$\begin{aligned} 5x^2 - 35x + 50 &= 0 \\ 5(x^2 - 7x + 10) &= 0 \\ 5(x - 5)(x - 2) &= 0 \\ x - 5 = 0 \text{ or } x - 2 = 0 \\ x = 5 \text{ or } x = 2 \\ \{2, 5\} \end{aligned}$$

18. $4x^2 = 44x - 112$

$$\begin{aligned} 4x^2 - 44x + 112 &= 0 \\ 4(x^2 - 11x + 28) &= 0 \\ 4(x - 4)(x - 7) &= 0 \\ x - 4 = 0 \text{ or } x - 7 = 0 \\ x = 4 \text{ or } x = 7 \\ \{4, 7\} \end{aligned}$$

19. $3x^2 + 66 = -39x$

$$\begin{aligned} 3x^2 + 39x + 66 &= 0 \\ 3(x^2 + 13x + 22) &= 0 \\ 3(x + 11)(x + 2) &= 0 \\ x + 11 = 0 \text{ or } x + 2 = 0 \\ x = -11 \text{ or } x = -2 \\ \{-11, -2\} \end{aligned}$$

20. $7x^2 + 140 = -84x$

$$\begin{aligned} 7x^2 + 84x + 140 &= 0 \\ 7(x^2 + 12x + 20) &= 0 \\ 7(x + 2)(x + 10) &= 0 \\ x + 2 = 0 \text{ or } x + 10 = 0 \\ x = -2 \text{ or } x = -10 \\ \{-10, -2\} \end{aligned}$$

21. $12x^2 = -60x - 72$

$$\begin{aligned} 12x^2 + 60x + 72 &= 0 \\ 12(x^2 + 5x + 6) &= 0 \\ 12(x + 3)(x + 2) &= 0 \\ x + 3 = 0 \text{ or } x + 2 = 0 \\ x = -3 \text{ or } x = -2 \\ \{-3, -2\} \end{aligned}$$

22. $7x^2 = -63x - 56$

$$\begin{aligned} 7x^2 + 63x + 56 &= 0 \\ 7(x^2 + 9x + 8) &= 0 \\ 7(x + 8)(x + 1) &= 0 \\ x + 8 = 0 \text{ or } x + 1 = 0 \\ x = -8 \text{ or } x = -1 \\ \{-8, -1\} \end{aligned}$$

23. $8x^2 - 64x = 56$

$$\begin{aligned} 8x^2 - 64x - 56 &= 0 \\ 8(x^2 - 8x - 7) &= 0 \\ 8(x - 7)(x + 1) &= 0 \\ x - 7 = 0 \text{ or } x + 1 = 0 \\ x = 7 \text{ or } x = -1 \\ \{7, 1\} \end{aligned}$$

24. $11x^2 - 154x = 165$

$$\begin{aligned} 11x^2 - 154x - 165 &= 0 \\ 11(x^2 - 14x - 15) &= 0 \\ 11(x - 15)(x + 1) &= 0 \\ x - 15 = 0 \text{ or } x + 1 = 0 \\ x = 15 \text{ or } x = -1 \\ \{-1, 15\} \end{aligned}$$

25. $1 - 144x^2 = 0$

$$\begin{aligned} (1)^2 - (12x)^2 &= 0 \\ (1 - 12x)(1 + 12x) &= 0 \\ 1 \pm 12x &= 0 \\ x = \frac{1}{12} \text{ or } x = -\frac{1}{12} \\ \left\{-\frac{1}{12}, \frac{1}{12}\right\} \end{aligned}$$

26. $9 - 400x^2 = 0$

$$\begin{aligned} (3)^2 - (20x)^2 &= 0 \\ (3 - 20x)(3 + 20x) &= 0 \\ 3 \pm 20x &= 0 \\ x = \frac{3}{20} \text{ or } x = -\frac{3}{20} \\ \left\{-\frac{3}{20}, \frac{3}{20}\right\} \end{aligned}$$

27. $0 = 15 + 5x^2$

$$\begin{aligned} 5(3 + x^2) &= 0 \\ \text{not factorable over} & \\ \text{the rational numbers} & \end{aligned}$$

28. $0 = 7 + 28x^2$

$$\begin{aligned} 7(1 + 4x^2) &= 0 \\ \text{not factorable over} & \\ \text{the rational numbers} & \end{aligned}$$

29. $36x^2 = 132x - 121$

$$\begin{aligned} 36x^2 - 132x + 121 &= 0 \\ (6x - 11)^2 &= 0 \\ 6x - 11 = 0; \quad x = \frac{11}{6} \\ \left\{\frac{11}{6}\right\} \end{aligned}$$

30. $-18x^2 = 60x + 50$

$$\begin{aligned} 18x^2 + 60x + 50 &= 0 \\ 2(9x^2 + 30x + 25) &= 0 \\ 2(3x + 5)^2 &= 0 \\ 3x + 5 = 0; \quad x = -\frac{5}{3} \\ \left\{-\frac{5}{3}\right\} \end{aligned}$$

31. $72 = 8x^2 + 36x$

$$\begin{aligned} 8x^2 + 36x - 72 &= 0 \\ 4(2x^2 + 9x - 18) &= 0 \\ 4(2x - 3)(x + 6) &= 0 \\ 2x - 3 = 0 \text{ or } x + 6 = 0 \\ x = \frac{3}{2} \text{ or } x = -6 \\ \left\{-6, \frac{3}{2}\right\} \end{aligned}$$

32. $105 = 12x^2 - 69x$

$$\begin{aligned} 12x^2 - 69x - 105 &= 0 \\ 3(4x^2 - 23x - 35) &= 0 \\ 3(4x + 5)(x - 7) &= 0 \\ 4x + 5 = 0 \text{ or } x - 7 = 0 \\ x = -\frac{5}{4} \text{ or } x = 7 \\ \left\{-\frac{5}{4}, 7\right\} \end{aligned}$$

■ As students solve exercises 13–32, remind them that it is sometimes possible to simplify their work by dividing both sides of an equation by a common numerical factor. For example, both sides of the equation in exercise 17 can be divided by 5.

Have students examine the equation in exercise 27 and note that it has the form $0 = (\text{positive number}) + (\text{nonnegative number})$. Ask them what that suggests about the number of possible solutions. Then have them discuss how that equation is different from the equation in exercise 25, which has the form $(\text{positive number}) - (\text{nonnegative number}) = 0$.

Errors Commonly Made

Students may assume that if a quadratic equation cannot be solved by factoring, it has no solutions. When students come upon an equation where $ax^2 + bx + c$ is not factorable, have them apply the graphic methods they learned in the two previous lessons in order to determine if the equation has any real roots.

■ Stress the need to check solutions for exercises 33–52 in the original equations. Use an example, such as why -2 is not part of the solution set in exercise 35. Emphasize the difference between an equation with no real solution (as in exercise 41) and an equation that cannot be factored over the set of rational numbers (as in exercise 45).

Errors Commonly Made

When solving equations involving radical expressions, students may check the solutions in the equations that result after the squaring process, but not in the original equations. Tell students that the process of squaring both sides of an equation can often introduce extraneous solutions because the squares of two opposite numbers are equal.

■ In exercises 53–56, discuss why there are two pairs of numbers that are solutions in exercises 53 and 54, but only one pair in exercises 55 and 56.

Problem Solving

■ Some students may take a straightforward approach when solving problems 57 and 58: multiply, combine like terms, and factor. An alternative approach is to examine the relationships between the terms and use other variables to simplify the problem.

SPIRAL REVIEW

■ Suggest that students review Lessons 9-2, 9-6, 7-6, and 6-1 before completing exercises 59–62, respectively.

Solve each equation by factoring. Then check the solution on a separate sheet of paper. **Check students' work.**

33. $x = \sqrt{7x - 6}$

$$\begin{aligned} x^2 &= (\sqrt{7x - 6})^2 \\ x^2 &= 7x - 6 \\ 0 &= x^2 - 7x + 6 \\ 0 &= (x - 6)(x - 1) \\ x - 6 &= 0 \text{ or } x - 1 = 0 \\ x &= 6 \text{ or } x = 1 \\ \{6, 1\} \end{aligned}$$

34. $x = \sqrt{7x - 12}$

$$\begin{aligned} x^2 &= (\sqrt{7x - 12})^2 \\ x^2 &= 7x - 12 \\ 0 &= x^2 - 7x + 12 \\ 0 &= (x - 3)(x - 4) \\ x - 3 &= 0 \text{ or } x - 4 = 0 \\ x &= 3 \text{ or } x = 4 \\ \{3, 4\} \end{aligned}$$

35. $\sqrt{3x + 10} = x$

$$\begin{aligned} (\sqrt{3x + 10})^2 &= x^2 \\ 3x + 10 &= x^2 \\ x^2 - 3x - 10 &= 0 \\ (x - 5)(x + 2) &= 0 \\ x - 5 &= 0 \text{ or } x + 2 = 0 \\ x &= 5 \text{ or } x = -2 \\ \{5\} \end{aligned}$$

36. $\sqrt{2x + 48} = x$

$$\begin{aligned} (\sqrt{2x + 48})^2 &= x^2 \\ 2x + 48 &= x^2 \\ x^2 - 2x - 48 &= 0 \\ (x + 6)(x - 8) &= 0 \\ x + 6 &= 0 \text{ or } x - 8 = 0 \\ x &= -6 \text{ or } x = 8 \\ \{8\} \end{aligned}$$

37. $x = \sqrt{11x - 24}$

$$\begin{aligned} x^2 &= 11x - 24 \\ 0 &= x^2 - 11x + 24 \\ 0 &= (x - 3)(x - 8) \\ x - 3 &= 0 \text{ or } x - 8 = 0 \\ x &= 3 \text{ or } x = 8 \\ \{3, 8\} \end{aligned}$$

38. $x = \sqrt{17x - 30}$

$$\begin{aligned} x^2 &= 17x - 30 \\ 0 &= x^2 - 17x + 30 \\ 0 &= (x - 2)(x - 15) \\ x - 2 &= 0 \text{ or } x - 15 = 0 \\ x &= 2 \text{ or } x = 15 \\ \{2, 15\} \end{aligned}$$

39. $x = \sqrt{8 - 2x}$

$$\begin{aligned} x^2 &= 8 - 2x \\ 0 &= x^2 + 2x - 8 \\ 0 &= (x + 4)(x - 2) \\ x + 4 &= 0 \text{ or } x - 2 = 0 \\ x &= -4 \text{ or } x = 2 \\ \{2\} \end{aligned}$$

40. $x = \sqrt{17 - 16x}$

$$\begin{aligned} x^2 &= 17 - 16x \\ 0 &= x^2 + 16x - 17 \\ 0 &= (x + 17)(x - 1) \\ x + 17 &= 0 \text{ or } x - 1 = 0 \\ x &= -17 \text{ or } x = 1 \\ \{1\} \end{aligned}$$

41. $x = \sqrt{-8x - 7}$

$$\begin{aligned} x^2 &= (\sqrt{-8x - 7})^2 \\ x^2 &= -8x - 7 \\ 0 &= x^2 + 8x + 7 \\ 0 &= (x + 1)(x + 7) \\ x + 1 &= 0 \text{ or } x + 7 = 0 \\ x &= -1 \text{ or } x = -7 \\ \text{no real solution} \end{aligned}$$

42. $x = \sqrt{-13x - 36}$

$$\begin{aligned} x^2 &= (\sqrt{-13x - 36})^2 \\ x^2 &= -13x - 36 \\ 0 &= x^2 + 13x + 36 \\ 0 &= (x + 4)(x + 9) \\ x + 4 &= 0 \text{ or } x + 9 = 0 \\ x &= -4 \text{ or } x = -9 \\ \text{no real solution} \end{aligned}$$

43. $\sqrt{15 - 4x} = 2x$

$$\begin{aligned} (\sqrt{15 - 4x})^2 &= (2x)^2 \\ 15 - 4x &= 4x^2 \\ 4x^2 + 4x - 15 &= 0 \\ (2x + 5)(2x - 3) &= 0 \\ 2x + 5 &= 0 \text{ or } 2x - 3 = 0 \\ x &= -\frac{5}{2} \text{ or } x = \frac{3}{2} \\ \left\{\frac{3}{2}\right\} \end{aligned}$$

44. $\sqrt{4 - 9x} = 3x$

$$\begin{aligned} (\sqrt{4 - 9x})^2 &= (3x)^2 \\ 4 - 9x &= 9x^2 \\ 9x^2 + 9x - 4 &= 0 \\ (3x + 4)(3x - 1) &= 0 \\ 3x + 4 &= 0 \text{ or } 3x - 1 = 0 \\ x &= -\frac{4}{3} \text{ or } x = \frac{1}{3} \\ \left\{\frac{1}{3}\right\} \end{aligned}$$

45. $5x = \sqrt{1 - 2x}$

$$\begin{aligned} 25x^2 &= 1 - 2x \\ 0 &= 25x^2 + 2x - 1 \\ \text{cannot be factored over} \\ \text{the rational numbers} \end{aligned}$$

46. $4x = \sqrt{7 - 34x}$

$$\begin{aligned} 16x^2 &= 7 - 34x \\ 0 &= 16x^2 + 34x - 7 \\ \text{cannot be factored over} \\ \text{the rational numbers} \end{aligned}$$

47. $2x = \sqrt{14 + 10x}$

$$\begin{aligned} 4x^2 &= 14 + 10x \\ 0 &= 4x^2 - 10x - 14 \\ 0 &= 2(2x - 7)(x + 1) \\ x &= \frac{7}{2} \text{ or } x = -1 \\ \left\{\frac{7}{2}\right\} \end{aligned}$$

48. $3x = \sqrt{8 + 21x}$

$$\begin{aligned} 9x^2 &= 8 + 21x \\ 0 &= 9x^2 - 21x - 8 \\ 0 &= (3x - 8)(3x + 1) \\ x &= \frac{8}{3} \text{ or } x = -\frac{1}{3} \\ \left\{\frac{8}{3}\right\} \end{aligned}$$

49. $7x = \sqrt{49x - 10}$

$$\begin{aligned} (7x)^2 &= (\sqrt{49x - 10})^2 \\ 49x^2 &= 49x - 10 \\ 0 &= 49x^2 - 49x + 10 \\ 0 &= (7x - 5)(7x - 2) \\ 7x - 5 &= 0 \text{ or } 7x - 2 = 0 \\ x &= \frac{5}{7} \text{ or } x = \frac{2}{7} \\ \left\{\frac{2}{7}, \frac{5}{7}\right\} \end{aligned}$$

50. $5x = \sqrt{60x - 27}$

$$\begin{aligned} (5x)^2 &= (\sqrt{60x - 27})^2 \\ 25x^2 &= 60x - 27 \\ 0 &= 25x^2 - 60x + 27 \\ 0 &= (5x - 3)(5x - 9) \\ 5x - 3 &= 0 \text{ or } 5x - 9 = 0 \\ x &= \frac{3}{5} \text{ or } x = \frac{9}{5} \\ \left\{\frac{3}{5}, \frac{9}{5}\right\} \end{aligned}$$

51. $x - 2 = \sqrt{x - 2}$

$$\begin{aligned} (x - 2)^2 &= (\sqrt{x - 2})^2 \\ x^2 - 4x + 4 &= x - 2 \\ x^2 - 5x + 6 &= 0 \\ (x - 2)(x - 3) &= 0 \\ x - 2 &= 0 \text{ or } x - 3 = 0 \\ x &= 2 \text{ or } x = 3 \\ \{2, 3\} \end{aligned}$$

52. $x - 5 = \sqrt{x - 5}$

$$\begin{aligned} (x - 5)^2 &= (\sqrt{x - 5})^2 \\ x^2 - 10x + 25 &= x - 5 \\ x^2 - 11x + 30 &= 0 \\ (x - 5)(x - 6) &= 0 \\ x - 5 &= 0 \text{ or } x - 6 = 0 \\ x &= 5 \text{ or } x = 6 \\ \{5, 6\} \end{aligned}$$



Solve.

53. The product of two consecutive integers is 42. What are the integers?

Let x = the least integer; $x(x + 1) = 42$;
 $x^2 + x - 42 = 0$; $(x + 7)(x - 6) = 0$; $x = -7$,
 so $x + 1 = -6$; or $x = 6$, so $x + 1 = 7$;
 Possible integers are -7 and -6 or 6 and 7 .

55. The product of two consecutive positive integers is 110. What are the integers?

Let x = the least integer; $x(x + 1) = 110$
 $x^2 + x - 110 = 0$; $(x + 11)(x - 10) = 0$;
 $x = -11$ or $x = 10$
 so $x + 1 = 11$; So the two integers are 10 and 11.

Problem Solving

57. Solve for x : $(x - 4)^2 + (x + 5)^2 = (2x + 1)^2$

Adopt a different point of view:

Let $a = x - 4$, $b = x + 5$, $a + b = 2x + 1$
 Then: $a^2 + b^2 = (a + b)^2$; $a^2 + b^2 = a^2 + 2ab + b^2$
 $0 = 2ab$; $a = 0$ or $b = 0$; If $a = 0$: $x - 4 = 0$; $x = 4$
 If $b = 0$: $x + 5 = 0$; $x = -5$
 Check: If $x = 4$: $0^2 + 9^2 \stackrel{?}{=} [2(4) + 1]^2$; $81 = 81$ True
 If $x = -5$: $(-9)^2 + 0^2 \stackrel{?}{=} [2(-5) + 1]^2$; $81 = 81$ True
 The solution set for this equation is $\{-5, 4\}$.

54. The product of two consecutive integers is 156. What are the integers?

Let x = the least integer; $x(x + 1) = 156$;
 $x^2 + x - 156 = 0$; $(x + 13)(x - 12) = 0$;
 $x = -13$ so $x + 1 = -12$; or $x = 12$, so $x + 1 = 13$;
 Possible integers are -13 and -12 or 12 and 13 .

56. The product of two consecutive negative integers is 420. What are the integers?

Let x = a negative integer;
 $x(x + 1) = 420$; $x^2 + x - 420 = 0$;
 $(x + 21)(x - 20) = 0$; $x = -21$ or $x = 20$;
 so $x + 1 = -20$ or $x + 1 = 21$;
 So the two integers are -21 and -20 .

58. Solve for x : $(2x + 5)^2 - (x - 11)^2 = (3x - 6)^2$

Adopt a different point of view:

Let $a = 2x + 5$, $b = x - 11$, $a + b = 3x - 6$
 Then: $a^2 - b^2 = (a + b)^2$; $a^2 - b^2 = a^2 + 2ab + b^2$
 $0 = 2ab + 2b^2$; $0 = 2b(a + b)$; $b = 0$ or $a + b = 0$
 If $b = 0$: $x - 11 = 0$; $x = 11$; If $a + b = 0$: $3x - 6 = 0$;
 $3x = 6$; $x = 2$;
 Check: If $x = 11$: $[2(11) + 5]^2 - 0^2 \stackrel{?}{=} [3(11) - 6]^2$;
 $27^2 = 27^2$ True
 Check: If $x = 2$: $[2(2) + 5]^2 - (-9)^2 \stackrel{?}{=} 0$;
 $9^2 - 81 \stackrel{?}{=} 0$; $0 = 0$ True
 The solution set for this equation is $\{2, 11\}$.

SPIRAL REVIEW

Solve each problem.

59. Simplify: $9\sqrt{2} - 7\sqrt{3} + 11\sqrt{2} + 15\sqrt{3}$

$$20\sqrt{2} + 8\sqrt{3}$$

61. Multiply: $(3n + 1)(4n^2 + 8n - 2)$

$$3n(4n^2 + 8n - 2) + (4n^2 + 8n - 2)$$

$$12n^3 + 24n^2 - 6n + 4n^2 + 8n - 2$$

$$12n^3 + 28n^2 + 2n - 2$$

60. Find the distance between points $(-7, -1)$ and $(4, -3)$.

$$D = \sqrt{(-7 - 4)^2 + [-1 - (-3)]^2}$$

$$D = \sqrt{(-11)^2 + [2]^2}$$

$$D = \sqrt{125}; D = \sqrt{25 \cdot 5}$$

$$D = 5\sqrt{5}$$

62. Is $(9, 1)$ a solution of the inequality $y > -3x + 4$?

$$y > -3x + 4; 1 \stackrel{?}{>} -3(9) + 4; 1 > -27 + 4$$

$$1 > -23$$
 True
 Yes

Additional Resources



www.progressinmathematics.com

- Meeting Individual Needs Activities
- Alternative Teaching Models
- Vocabulary Activities
- Audio Glossary
- Virtual Manipulatives
- Check Your Progress I
- Practice Activities (Lessons 1–3)

4 Summarize/Assess

Conceptual Thinking

- To assess whether students have conceptualized the lesson concepts, have them explain the Zero-Product Property and how it can be used to solve quadratic equations such as $2x^2 - 9x - 18 = 0$. $x = 6$ and $x = -\frac{3}{2}$
- Discuss how to solve equations containing radicals, including the meaning of extraneous roots. Provide examples such as $x = \sqrt{x + 12}$ and $x = \sqrt{4 - 3x}$.
 $x = 4$ ($x = -3$ is extraneous); $x = 1$ ($x = -4$ is extraneous)

ONLINE Check Your Progress I

Administer Check Your Progress I to assess understanding of Lessons 1–3. For additional practice, assign the online Practice Activities.

5 Follow-Up

Reteaching

ONLINE Virtual Manipulatives: Grapher

- Discuss how the solutions of a quadratic equation relate to the x -intercepts of the related quadratic function. Give students a quadratic equation such as $x^2 + x - 6 = 0$. Ask, “What is the related quadratic function?” $f(x) = x^2 + x - 6$. Have them use the Grapher Virtual Manipulative, or a handheld, to graph the function and identify the zeros to find the solutions. $x = -3$ and $x = 2$. Then challenge them to write the factored form of the polynomial. $(x + 3)(x - 2)$

Provide other quadratic equations for students to solve by graphing. Include equations that have only one solution, such as $x^2 - 8x + 16 = 0$ $x = 4$. Also include equations that have no solution, such as $x^2 - 2x + 3 = 0$.

ONLINE See Chapter 10 Alternative Teaching Models.