MCE 412- Autonomous Robotics

Wheeled Locomotion

A mobile robot is a combination of various physical (hardware) and computational (software) components. In terms of hardware components, a mobile robot can be considered as a collection of subsytems for:

- **Locomotion**: How the robot moves through its environment
- **Sensing**: How the robot measures properties of itself and its environment
- **Reasoning**: How the robot maps these measurements into actions
- **Communication**: How the robot communicates with an outside operator.

- Given the control inputs, how does the robot move? This is known as the **forward kinematics** problem if the solution does not consider the forces involved in moving the vehicle.
- Given a desired motion, which control inputs should be chosen? This is known as the **inverse kinematics** problem if the forces to be applied are ignored.
- Understanding how a vehicle moves in response to its controls is essential for many navigation tasks and underlies the process of **dead reckoning** – estimating the path that the vehicle will travel, given the commands.

Based on the application domain, four broad categories of mobile robots have emerged:

- Terrestrial
- Aquatic
- Airborne
- Space

- Batteries : Store chemical energy and convert this to electrical enery.
  - For large vehicles: gell cell
  - For smaller vehicles: lithium-ion, nickel-cadmium (NiCad) vs
- Motors: Convert electrical energy into mechanical energy. Revolute devices such as most servo or stepper motors are designed to generate rotational /revolute motion.
  - Stepper motors: provide a mechanism for position (shaft orientation) and speed control without the use of complex sensors to monitor the motion of the motor.
  - Servo motor: combine a standard electrical motor with a shaft orientation sensor.
- Servo Motor Controllers:
  - proportional integral derivative (PID) controller
- Gears

Wheeled Mobile Robots (WMR)

- For a WMR to exhibit rolling motion there must exist a point around which each wheel on the vehicle follows a circular course. This point is known as the instantaneous center of curvature (ICC).
- A vehicle located on a plane has three degrees of freedom: an (x, y) position and a heading or orientation θ. This triplet (x, y, θ) is the pose of the robot on the plane.
- Mobile robots usually do not have complete independent control over all three pose parameters and must undergo complex maneuvers in order to reach a particular pose, e.g. Car parking.

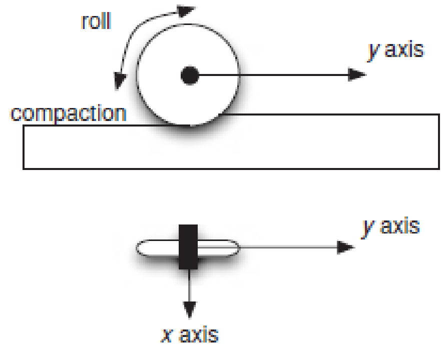


Figure 3.4. Side view and top view of an idealized rolling wheel. For a wheel rotating about the x axis, motion along the y axis is known as roll. Any component of motion in the x direction, perpendicular to the direction in which the wheel is rolling, is known as lateral slip. An ideal wheel moves along the roll direction only.

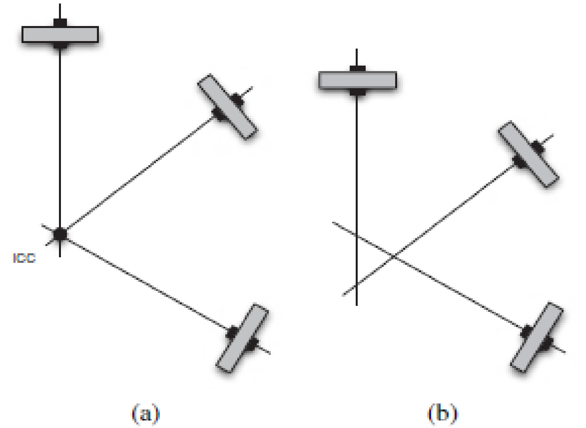
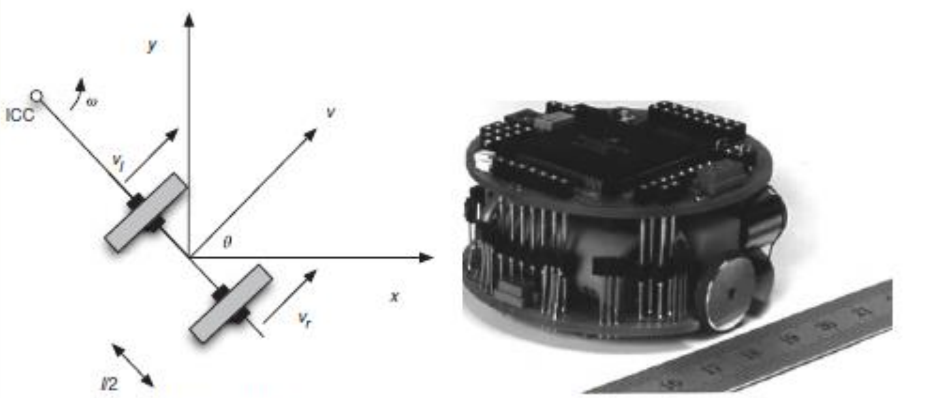


Figure 3.5. Instantaneous center of curvature. (a) The three wheels are arranged such that the line drawn through the roll axis of each wheel intersects at a single point (the ICC). (b) No such point exists. A robot relying on the wheels in (a) can exhibit rolling motion, while a robot relying on the wheel arrangement in (b) cannot.

Differential Drive

- Differential drive is the simplest possible drive mechanism for a ground contact mobile robot
- A differential drive robot consists of two wheels mounted on a common axis controlled by separate motors.



(a) Differential drive (b) Khepera robot

Figure 3.7. Differential drive kinematics. A differential drive robot controls its pose by providing independent velocity control to the left (v_l) and right (v_r) wheels. Most differential drive robots use castor wheels for stability. (b) Appears with the kind permission of A. Herzog.

$$w\left(R+\frac{l}{2}\right)=v_r \qquad w=\frac{v_r-v_l}{l}$$

$$w\left(R-\frac{l}{2}\right)=v_l \qquad R=\frac{l}{2}\frac{(v_r+v_l)}{(v_r-v_l)}$$

Special cases:

- $v_r = v_l$, the radius R is infinite and the robot moves in a straight line
- $v_r = -v_l$, the radius is zero, the robot rotates in place.

Forward Kinematics for Differential Drive Robots

- Determining the pose that is reachable given the control parameters is known as the forward kinematics problem for the robot.
- Suppose that the robot is at some position (x, y) and facing along a line making an angle θ with the x axis

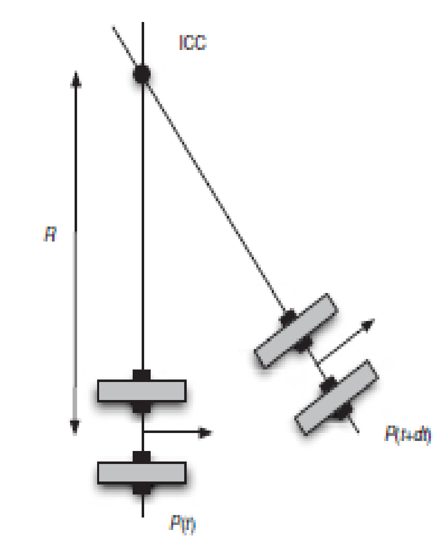


Figure 3.8. Forward kinematics geometry. The ICC is located at $(x, y) + R(\cos(\theta + \pi/2), \sin(\theta + \pi/2))$, which simplifies to $(x - R\sin(\theta),\ y + R\cos(\theta))$. To compute the position of the robot at $t + \delta t$, the robot must be rotated about the ICC by an amount $\omega\,\delta t$. Mathematically this can be accomplished by translating the ICC to the origin and rotating the robot about the origin by $\omega\,\delta t$ and then translating back to the ICC [see equation (3.2)].

$$ICC = (x - R\sin(\theta), y + R\cos(\theta))$$

The pose of the robot at $t + \delta t$

$$\begin{bmatrix} x' \\ y' \\ \theta' \end{bmatrix} = \begin{bmatrix} \cos(w\delta t) & -\sin(w\delta t) & 0 \\ \sin(w\delta t) & \cos(w\delta t) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x - ICC_x \\ y - ICC_y \\ \theta \end{bmatrix} + \begin{bmatrix} ICC_x \\ ICC_y \\ w\delta t \end{bmatrix}$$

For a robot capable of moving in a particular direction θ(t) at a given velocity V(t)

$$x(t) = \int_0^t V(\sigma)\cos(\theta(\sigma))d\sigma \qquad x(t) = \frac{1}{2}\int_0^t (v_r(\sigma) + v_l(\sigma))\cos(\theta(\sigma))d\sigma$$

$$y(t) = \int_0^t V(\sigma)\sin(\theta(\sigma))d\sigma \qquad y(t) = \frac{1}{2}\int_0^t (v_r(\sigma) + v_l(\sigma))\sin(\theta(\sigma))d\sigma$$

$$\theta(t) = \int_0^t w(\sigma)d\sigma \qquad \theta(t) = \frac{1}{l}\int_0^t (v_r(\sigma) - v_l(\sigma))d\sigma$$

Inverse Kinematics for Differential Drive Robots

- How can the control parameters be selected so as to have the robot obtain a specific global pose or follow a specific trajectory? This is known as the task of determining the vehicle's inverse kinematics: inverting the kinematic relationship between control inputs and behavior
- If it is assumed that $v_r(t) = v_r$, $v_l(t) = v_l$ and $v_l \neq v_r$ (where (x,y, θ)=(0,0,0) at t=0)

$$x(t) = \frac{l}{2}\frac{v_r + v_l}{v_r - v_l}\sin(\frac{t}{l}(v_r - v_l)\Big)$$

$$y(t) = -\frac{l}{2}\frac{v_r+v_l}{v_r-v_l}\cos(\frac{t}{l}(v_r - v_l)\Big) + \frac{l}{2}\frac{v_r+v_l}{v_r-v_l}$$

$$\theta(t) = \frac{t}{l}(v_r - v_l\Big)$$

$$w = \frac{v_r - v_l}{l}$$

$$R = \frac{l}{2}\frac{(v_r + v_l)}{(v_r - v_l)}$$

If $v_l = v_r = v$, the robot moves in a straight line

$$\begin{bmatrix} x' \\ y' \\ \theta' \end{bmatrix} = \begin{bmatrix} x + v\cos(\theta)\delta t \\ y + v\sin(\theta)\delta t \\ \theta \end{bmatrix}$$

If $-v_l = v_r = v$, the robot rotates in place

$$\begin{bmatrix} x' \\ y' \\ \theta' \end{bmatrix} = \begin{bmatrix} x \\ y \\ \theta + 2v\delta t/l \end{bmatrix}$$

To drive the robot to some pose (x, y, θ), the robot can be spun in place until it is aimed at (x, y), then driven forward until it is at (x, y), and then spun in place until the required goal orientation θ is met

Synchronous Drive

- Each wheel is capable of both drive and steering.
- The vehicle uses two independent motors- one that rolls all of the wheels forwards and one that rotates them (for turning)
- All the wheels remain parallel, synchronous drive robots always rotate about the center and serves as a convenient model for the idealized point robot.
- ICC is always at infinity

Forward kinematics

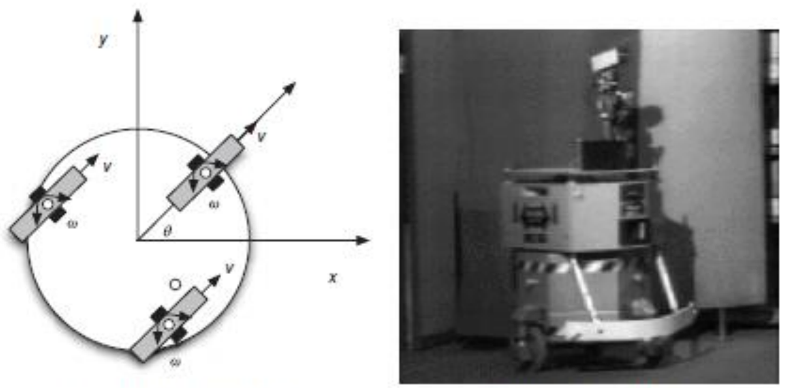
$$x(t) = \int_0^t v(\sigma)\cos(\theta(\sigma))\,d\sigma$$

$$y(t) = \int_0^t v(\sigma)\sin(\theta(\sigma))\,d\sigma$$

$$\theta(t) = \int_0^t w(\sigma)\,d\sigma$$

Translational speed

Turning rate



(a) Synchronous drive (b) ARK-1 robot

Figure 3.9. Synchronous drive kinematics. A synchronous drive robot controls its pose by providing velocity and orientation control to each of its wheels. The ARK-1 robot is built around the Cybermotion Navmaster platform, which utilizes a synchronous drive.

Because changes in orientation can be completely decoupled from translation, the inverse kinematics of a synchronous drive vehicle are very similar to the special case of the inverse kinematics of the differential drive robot.

- If $v(t) = 0$ and $w(t) = w$ for some period δt, then the robot rotates in place by an amount $w\ \delta t$
- If $w(t) = 0$ and $v(t) = v$ for some period δt, then the robot moves in the direction it is currently pointing a distance $v\delta t$

Ackerman Steering (Car, Tricycle, Bogey and Bicycle Drive)

- Robots that do not use either differential or synchronous drive technologies typically have one or more wheels that can be 'steered' and one or more wheels whose rotational axis cannot be changed.
- The robot motion is controlled by the steering direction α and velocity v provided through the front wheel.
- Calculating the potential region of the ICC can be more complex.

Forward kinematics

$$R = d\tan\alpha \qquad w = v/(d^2+R^2)^{\frac{1}{2}}$$

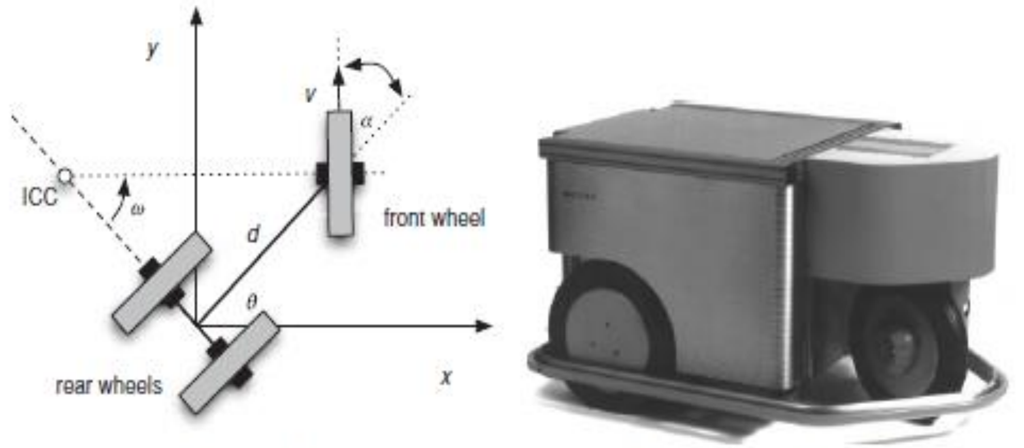
$$ICC = (x - R\sin(\theta), y + R\cos(\theta))$$

$$\begin{bmatrix} x' \\ y' \\ \theta' \end{bmatrix} = \begin{bmatrix} \cos(w\delta t) & -\sin(w\delta t) & 0 \\ \sin(w\delta t) & \cos(w\delta t) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x - ICC_x \\ y - ICC_y \\ \theta \end{bmatrix} + \begin{bmatrix} ICC_x \\ ICC_y \\ w\delta t \end{bmatrix}$$

Inverse kinematics

If α=0

$$\begin{bmatrix} x' \\ y' \\ \theta' \end{bmatrix} = \begin{bmatrix} x + v\cos(\theta)\delta t \\ y + v\sin(\theta)\delta t \\ \theta \end{bmatrix} \qquad \begin{bmatrix} x' \\ y' \\ \theta' \end{bmatrix} = \begin{bmatrix} x \\ y \\ \theta \pm v\delta t/d \end{bmatrix}$$



(a) Tricycle schematic (b) Mecos mobile robot

Figure 3.11. (a) Tricycle kinematics. (b) Tricycle mobile robot. Copyright Mecos Robotics AG. Used with permission.

Complex Wheels

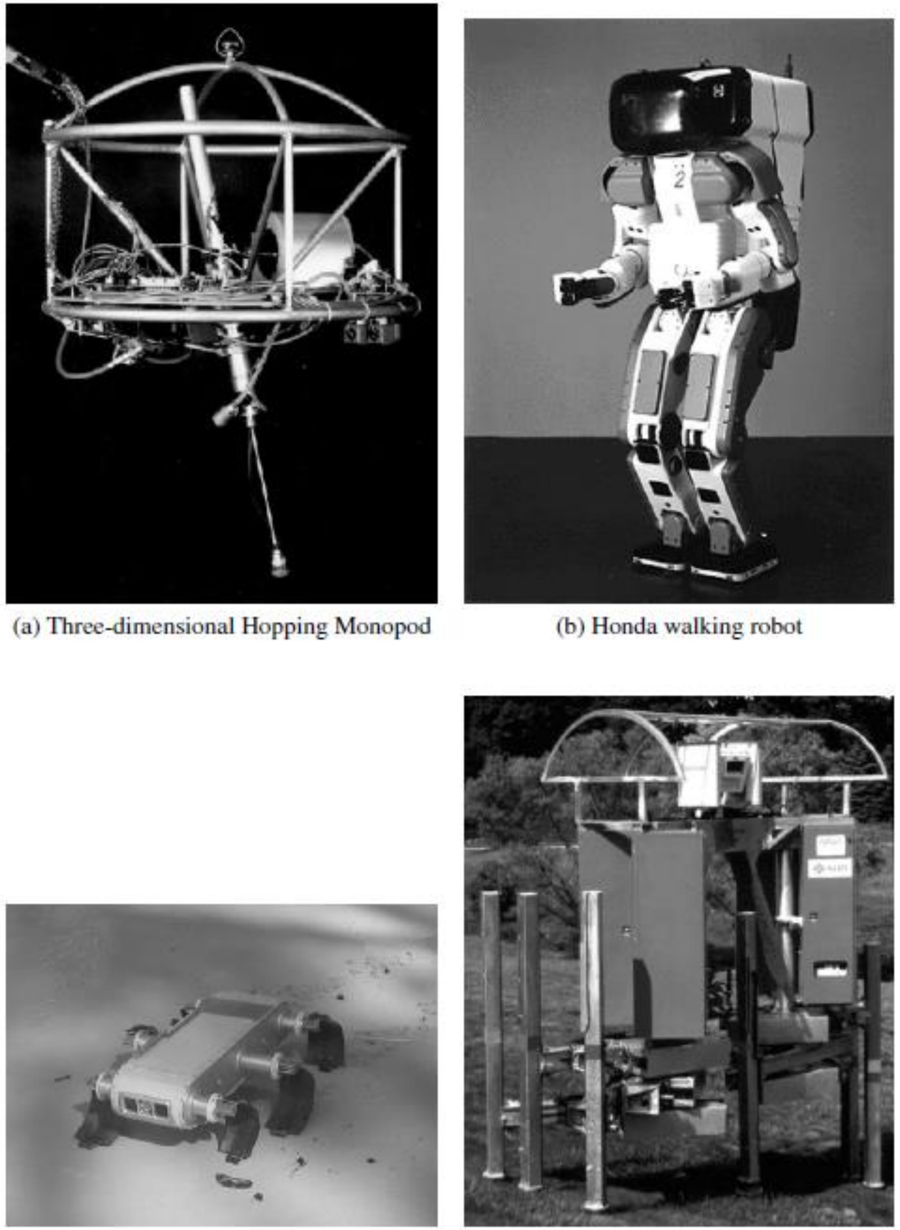
- Complex wheels are wheels that exhibit more than one preferred rolling direction.
- Such wheels can result in omnidirectional robots.

Tracked vehicles

- Similar to differential drive vehicles in terms of their kinematics but are more robust to terrain variations
- ‘extended wheels’ cause a larger amount of slip between the treads and the ground and make it impossible to predict accurately a tracked vehicle’s motion from the motion of its treads.

Legged Robots

- The primary limitation associated with wheeled or tracked vehicles is the need for ground support along the entire path of motion.
- In rough terrains such as those found in forests, near natural disasters or in planetary exploration, it is not always possible to guarantee physical support for the robot along a continuous path.



(a) Three-dimensional Hopping Monopod

(b) Honda walking robot

(c) AQUA

(d) Ambler

Figure 3.15. Walking robots. (a) Courtesy of MIT Leg Laboratory. Copyright Marc Raibert, 1984. (b) Copyright Honda. Used with permission. (c) Copyright the AQUA Project. Used with permission. (d) Copyright Carnegie Mellon University, 1992.

Non – Holonomic Constraints

- Non holonomic constraints limit the possible incremental movements within the configuration space of the robot.
- Robots with differential drive or synchronous drive on a circular trajectory and cannot move sideways.
- Mecanum-wheeled robots can move sideways (they have no non-holonomic constraints)

Summary

- Introduced different types of drives for wheeled robots
- Math to describe the motion of the basic drives given the speed of the wheels
- Non-holonomic constraints

References

- Introduction to Autonomous Mobile Robots, Roland Siegwart and Illah R. Nourbakhsh, 2004, Chapter 2 and 3
- Computational Principles of Mobile Robotics, Gregory Dudek and Michael Jenkin, 2010, Chapter 3