Exercise 1: Locomotion A robot equipped with a differential drive starts at position x = 1.0m, y = 2.0m and with heading $\theta = \pi$ 2 . It has to move to the position x = 1.5m, y = 2.0m, $\theta = \pi$ 2 (angles in radians). The movement of the vehicle is described by steering commands (vl =speed of left wheel, vr = speed of right wheel, t = driving time). a) What is the minimal number of steering commands (vl, vr, t) needed to guide the vehicle to the desired target location? b) What is the length of the shortest trajectory under this constraint? c) Which sequence of steering commands guides the robot on the shortest trajectory to the desired location if an arbitrary number of steering commands can be used? The maximum velocity of each wheel is v and the distance between both wheels is l. d) What is the length of this trajectory? Note: the length of a trajectory refers to the traveled distance along the trajectory.

Solution: A robot equipped with a differential drive starts at position x = 1.0m, y = 2.0m and with heading $\theta = \pi/2$ It has to move to the position x = 1.5m, y = 2.0m, $\theta = \pi/2$ (all angles in radians). The movement of the vehicle is described by steering commands (vl = speed of left wheel, vr = speed of right wheel, t = driving time).

a. What is the minimal number of steering commands (vl , vr, t) needed to guide the vehicle to the desired target location? Each command of a differential drive only allows us to follow a portion of a circular trajectory (or to go straight, which is in the limit of infinite radius). While it is possible to reach the second location (x, y) with a single half-circle, it is impossible to do so with the correct orientation. We thus require at least two commands to reach it. One of the possible ways to do so is to follow two half-circles, one with diameter u, the other with diameter v, such that their sum equals the distance between the poses, i.e. u + v = d = 0.5m.
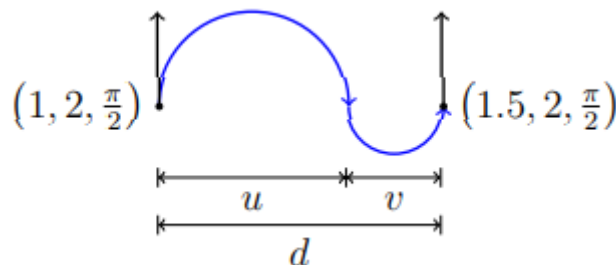
Do note that this is valid even when setting u = 0 or v = 0 (rotation on the spot).

b. What is the length of the shortest trajectory under this constraint? The robot performs two half-circles, one with diameter u and arc length s1 and one with diameter v and arc length s2. The diameters u and v can be arbitrarily chosen, as long as they add up to d.

s1 = πu/2   s2 = πv/2   u + v = d

The length of the trajectory is the sum of the arc lengths of both half-circles: s = s1 + s2 = πu 2 + πv 2 = π(u + v) 2 = πd 2 = π 4 The trajectory length is independent of the particular u and v chosen.

c. Which sequence of steering commands guides the robot on the shortest trajectory to the desired location if an arbitrary number of steering commands can be used? The maximum velocity of each wheel is v and the distance between both wheels is l.

The traveled distance cannot be smaller than the Euclidean distance between the two poses. We can achieve this lower bound by rotating the robot right by π/2, going forward for 0.5m and rotating left by π/2, for a total of three commands. For the turning commands on the spot, both wheels need to turn at opposite speeds (to the right: vl = v and vr = −v. To the left, vl = −v and vr = v). To go straight, both wheels need to turn at the same speed. From the differential drive kinematics, we know that the translational and rotational speeds vrob and ωrob of the robot depend on the wheel speeds vl and vr and the distance l between both wheels:

$$ v_{\text{rob}} = \frac{\sqrt{\Delta x^2 + \Delta y^2}}{t} = \frac{v_r + v_l}{2} \qquad \omega_{\text{rob}} = \frac{\Delta\theta}{t} = \frac{v_r - v_l}{l} $$

We can use the equations above to calculate the required time t to travel a certain rotation $\Delta\theta$ or distance $\sqrt{\Delta x^2 + \Delta y^2}$.

The shortest trajectory can be achieved by the following steering commands (vl , vr, t):

(1) $\left(v, -v, \frac{\pi l}{4v}\right)$

(2) $\left(v, v, \frac{d}{v}\right)$

(3) $\left(-v, v, \frac{\pi l}{4v}\right)$

d. What is the length of this trajectory?

      It's the Euclidean distance between the two poses, 0.5m.