mce 412 - Project

Exercise 1:

a) * $p(x_t \mid u_t, x_{t-1})$ ⇒ it allows us to predict the next state from previous state and control input/vector.

state — control input — previous state

$x_t = g(u_t, x_{t-1}) \cong \underbrace{g(u_t, \mu_{t-1})}_{\text{the mean}}$ where covariance matrix is Q_t.

* $p(z_t \mid x_t)$ ⇒ this disturibition allows us to predict the measurment using current state. The mean is $h(\mu_t)$ where R_t is the covariance matrix.

* $bel(x_t) = \underbrace{\eta\, p(z_t \mid x_t)}_{\text{measurment model}} \int \underbrace{p(x_t \mid u_t, x_{t-1})}_{\text{motion / planned system model}} \underbrace{bel(x_{t-1})}_{\text{prior belief}}\, dx_{t-1}$

From Kalman Filter

* we can deci[de] mean and covarian[ce] from the formulas

prediction ⇒ $\overline{bel}(x_t) = \int p(x_t \mid u_t, x_{t-1})\, bel(x_{t-1})\, dx_{t-1}$

correction ⇒ $bel(x_t) = \eta\, p(z_t \mid x_t)\, \overline{bel}\, x_t$

$bel(x_t)$ is the distrubition of the state giving the current estimate.

We use Gaussian as long as we start with Gaussians and perform only linear transformations. Since we're dealing with EKF which is for non-linear systems, we need to consider non Gaussian distrubitions. To do this, we have local linearization.

→ $p(x_t \mid u_t, x_{t-1})$ $p(z_t \mid x_t)$ are Gaussian if $g(u_t, \mu_{t-1})$ $h(\mu_t)$ are linear.
→ If $g(u_t, \mu_{t-1})$ $h(\mu_t)$ are non-linear, $g(u_t, \mu_{t-1})$ $h(\mu_t)$ are usually different from the mean of actual distrubitions. (EKF)

b) $g(u, x)$ ⇒ the motion model of system which estimates the current state (x_t) from previous state and control vector.

G_t ⇒ derivative / jacobian matrix G_t is defined as $G_t = \left.\dfrac{\partial g}{\partial x}\right|_{x=\mu_{t-1}}$

In KF, we have A_t (linear) which corresponds to G_t in EKF.

Q_t ⇒ Covariance matrix which is assumed to be known of zero-mean Gaussian Error corrupting the prediction of state.

$h(x)$ ⇒ the measurement model of system estimating z_t from current state var.

H_t ⇒ jacobian matrix which is defined as $H_t = \left.\dfrac{\partial h}{\partial x}\right|_{x=\mu_t}$

In KF, we have C_t corresponding to H_t in EKF.

R_t ⇒ Covariance matrix assumed to be known of zero-mean Gaussian Error corrupting the measurment. (measurment is given by h with some noise)

Σ_t ⇒ Covariance matrix of distribution state which is for estimating @ t.

(Optimal Under Linearity Assumptions)

Kalman Filter Assumptions

- Linear System measurment /observation model
- process & measurement noises ⇒ idd (independent identically distrubition)

Most of the time, we do not have linearity. Linear KF are optimal filters in terms of min - variance. Whatever the initial condition is, we can reach optimal solutions. Therefore, when we do not have linearity, we use EKF. For EKF, we do not have optimaly guarenteed. If the initial position is not close enough through starting point, we may not reach optimal solution.

Kalman Filter		Extended Kalman Filter
$\bar{\mu}_t = A_t \mu_{t-1} + B_t u_t$	$\longrightarrow$	$\bar{\mu}_t = g(u_t, \mu_{t-1})$
$\bar{\Sigma}_t = A_t \Sigma_{t-1} A^T_t + Q_t$	$\longrightarrow$	$\bar{\Sigma}_t = G_t \Sigma_{t-1} G_t^T + Q_t$
$K_t = \bar{\Sigma}_t C_t^T (C_t \bar{\Sigma}_t C_t^T + R_t)^{-1}$	$\longrightarrow$	$K_t = \bar{\Sigma}_t H_t^T (H_t \bar{\Sigma}_t H_t^T + R_t)^{-1}$
$\mu_t = \bar{\mu}_t + K_t (z_t - C_t \bar{\mu}_t)$	$\longrightarrow$	$\mu_t = \bar{\mu}_t + K_t (z_t - h(\bar{\mu}_t))$
$\Sigma_t = (I - K_t C_t) \bar{\Sigma}_t$	$\longrightarrow$	$\Sigma_t = (I - K_t H_t) \bar{\Sigma}_t$

* We just replace A_t and C_t by G_t and H_t.

$$G_t = \frac{\partial g(u_t, \mu_{t-1})}{\partial x_{t-1}} \qquad H_t = \frac{\partial h(\bar{\mu}_t)}{\partial x_t}$$

* Linearity ⇒ $L(x_1 + x_2) = L(x_1) + L(x_2) \qquad L(\lambda x) = \lambda L(x)$

Exercise 2:

a) We know $g(u_t, x_{t-1}) = g(\mu_{t-1}, u_t) + G_t (x_{t-1} - \mu_{t-1})$

$G_t = \dfrac{\partial g(u_t, \mu_{t-1})}{\partial x_{t-1}}$ ⇒ the rate of change in 'x' / slope of the function

Let's take state variables as $S_t = \begin{bmatrix} x_t \\ y_t \\ \theta_t \end{bmatrix}$ and $u_t = \begin{bmatrix} \delta_1 \\ \delta_t \\ \delta_3 \end{bmatrix}$

$$\begin{bmatrix} x_t \\ y_t \\ \theta_t \end{bmatrix} = g(S_{t-1}, u_t) = \begin{bmatrix} x_{t-1} + \delta_t \cos(\theta_{t-1} + \delta_1) \\ y_{t-1} + \delta_t \sin(\theta_{t-1} + \delta_1) \\ \theta_{t-1} + \delta_1 + \delta_2 \end{bmatrix}$$

if we do differantiation for each row according to $x_{t-1}, y_{t-1}, \theta_{t-1}$

$$G_t = \begin{bmatrix} 1 & 0 & -\delta_t \sin(\mu_{\theta,t-1} + \delta_1) \\ 0 & 1 & \delta_t \cos(\mu_{\theta,t-1} + \delta_1) \\ 0 & 0 & 1 \end{bmatrix} \begin{matrix} g_1 \\ g_2 \\ g_3 \end{matrix}$$

While we're calculating $\mu_{x,t-1}$, $\mu_{y,t-1}$, $\mu_{\theta,t-1}$ we realize that $\mu_{x,t-1}$ and $\mu_{y,t-1}$ are not seen since they're linear.

Exercise 3 :

a) We know that $h(x_t) \cong h(\mu_t) + H_t(x_t - \bar{\mu}_t)$ and $H_t = \dfrac{\partial h(\bar{\mu}_t)}{\partial x_t}$

Predicted measurement mean $z_t = \sqrt{(m_x - \bar{\mu}_{t,x})^2 + (m_y - \bar{\mu}_{t,y})^2}$

We can express landmark (ℓ) as $\ell = \begin{bmatrix} \ell_x \\ \ell_y \end{bmatrix}$ according to work frame.

$$z_t = h(s_t, \ell) = \sqrt{(x_{t-1} - \ell_x)^2 + (y_{t-1} - \ell_y)^2}$$

we add ℓ to make it paramedic

if we make differentiation

$$H_t = \begin{bmatrix} \dfrac{\bar{\mu}_{x,t} - \ell_x}{h(\bar{\mu}_t \, \ell)} & \dfrac{\bar{\mu}_{y,t} - \ell_y}{h(\bar{\mu}_t \, \ell)} & 0 \end{bmatrix}$$

Since motion model makes the position and orientation correlated, we'll be able to know the orientation eventhough h_t does not care about the orientation.

$\Rightarrow$ If we have some info about location $\rightarrow$ we get some info about orientation as we did in K.F. (K_t)