

MCE 412- Autonomous Robotics

Probabilistic Sensor Models

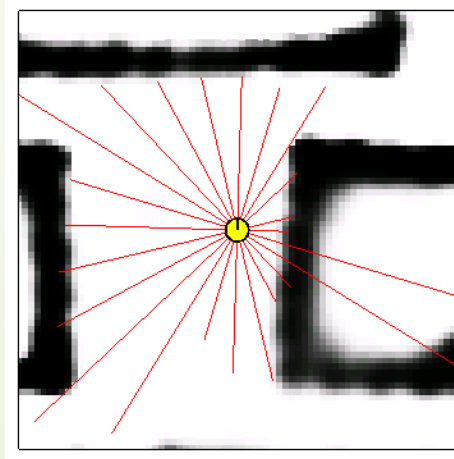


Sensors for Mobile Robots

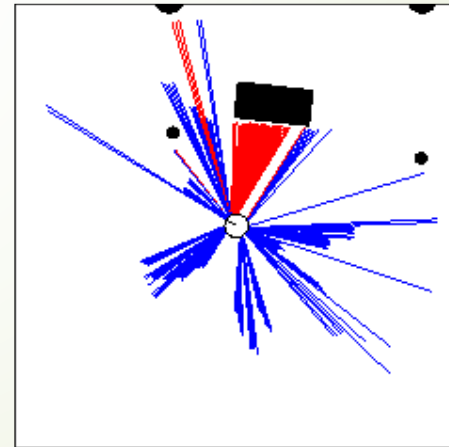
- **Contact sensors:** Bumpers
- **Proprioceptive sensors**
 - Accelerometers (spring-mounted masses)
 - Gyroscopes (spinning mass, laser light)
 - Compasses, inclinometers (earth magnetic field, gravity)
- **Proximity sensors**
 - Sonar (time of flight)
 - Radar (phase and frequency)
 - Laser range-finders (triangulation, tof, phase)
 - Infrared (intensity)
- **Visual sensors:** Cameras
- **Satellite-based sensors:** GPS

Proximity Sensors

- The central task is to determine $p(z|x)$, i.e., the probability of a measurement z given that the robot is at position x .
- Question: Where do the probabilities come from?
- Approach: Let's try to explain a measurement



Sonar Scan



Laser Scan

Beam-based Sensor Model

- Scan z consists of K measurements

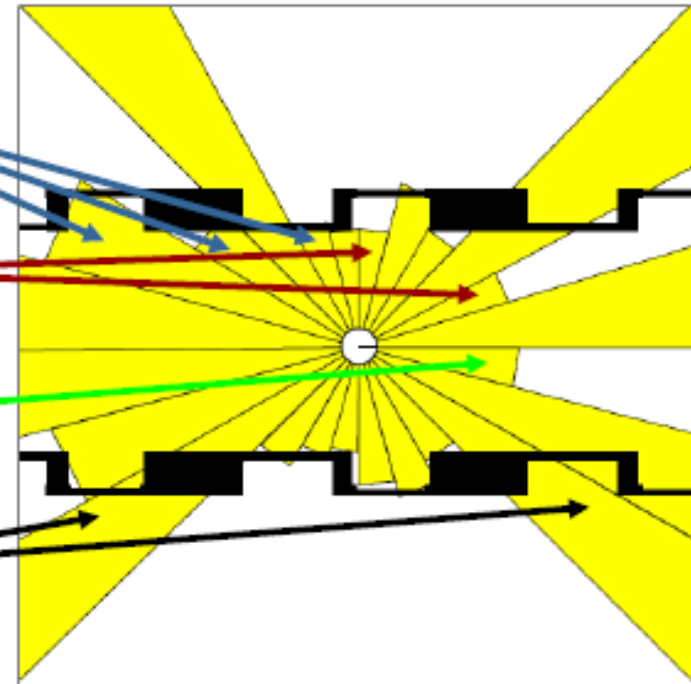
$$z = \{z_1, z_2, \dots, z_K\}$$

- Individual measurements are independent given the robot position

$$P(z|x, m) = \prod_{k=1}^K P(z_k|x, m)$$

Typical measurement Errors of Range Measurements

1. Beams reflected by obstacles
2. Beams reflected by persons / caused by crosstalk
3. Random measurements
4. Maximum range measurements

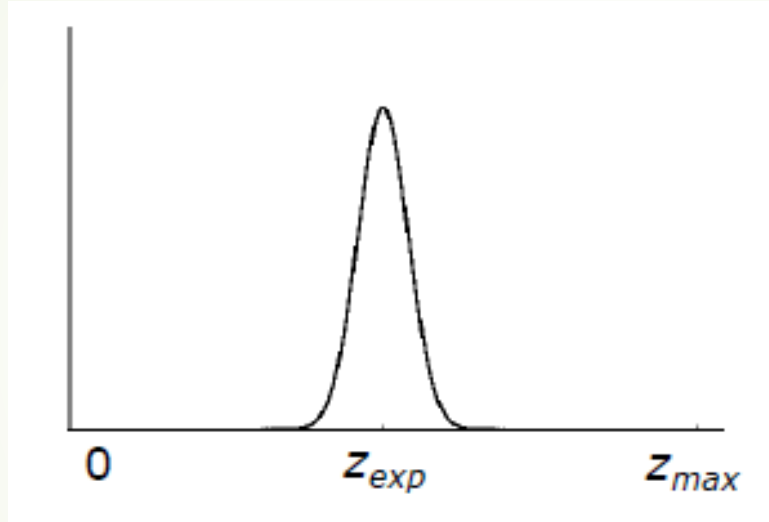




Proximity Measurement

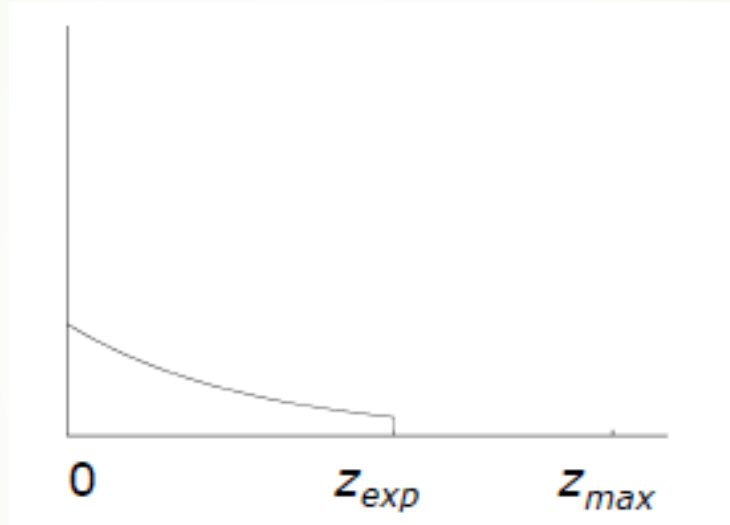
- Measurement can be caused by
 - A known obstacle
 - Cross-talk
 - An unexpected obstacle (people, furniture, ..)
 - Missing all obstacles (total reflection, glass, ..)
- Noise is due to uncertainty
 - in measuring distance to known obstacle
 - in position of known obstacles
 - in position of additional obstacles
 - Whether obstacle is missed

1- Correct range with local measurement noise



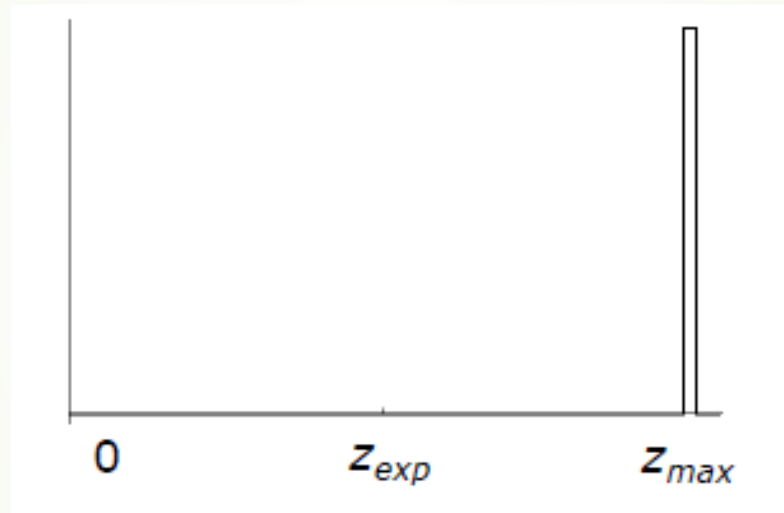
$$p_{hit}(z|x, m) = \eta \frac{1}{\sqrt{2\pi\sigma_{hit}^2}} e^{-\frac{1}{2} \frac{(z - z_{exp})^2}{\sigma_{hit}^2}}$$

2 – Unexpected Obstacles



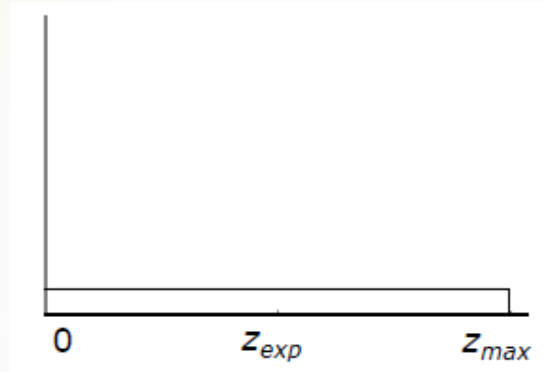
$$p_{unexp}(z|x, m) = \begin{cases} \eta \lambda e^{-\lambda z} & z < z_{exp} \\ 0 & \text{otherwise} \end{cases}$$

3 - Failures



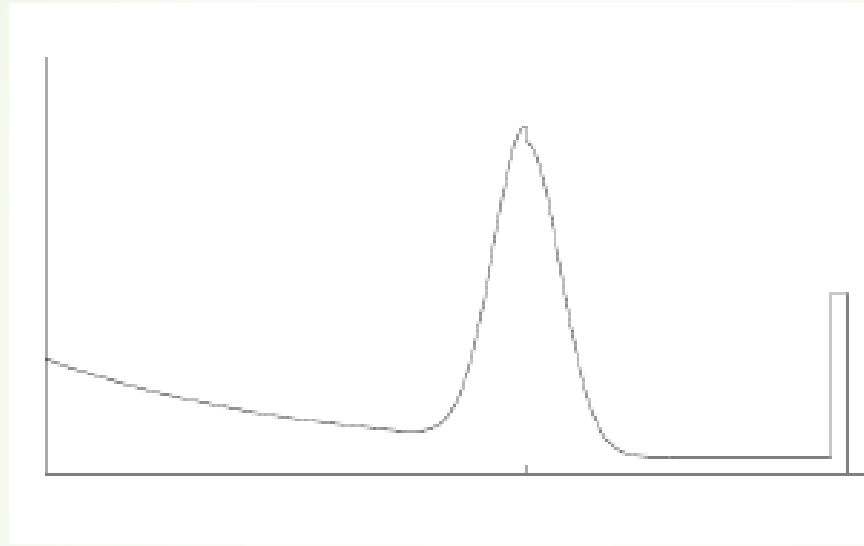
$$p_{max}(z|x, m) = \eta \frac{1}{z_{small}}$$

4 – Random Measurements



$$p_{rand}(z|x, m) = \eta \frac{1}{z_{max}}$$

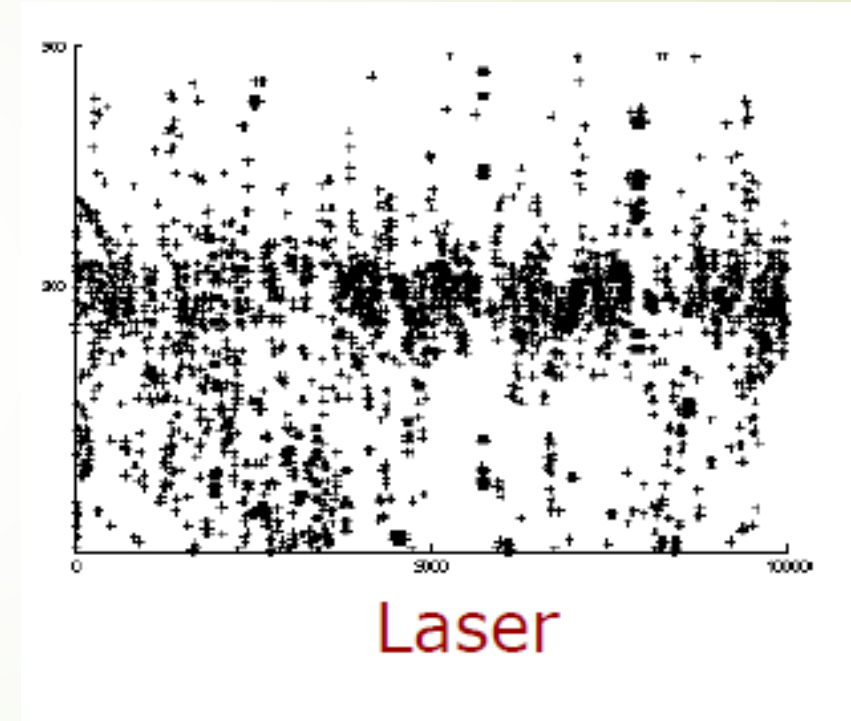
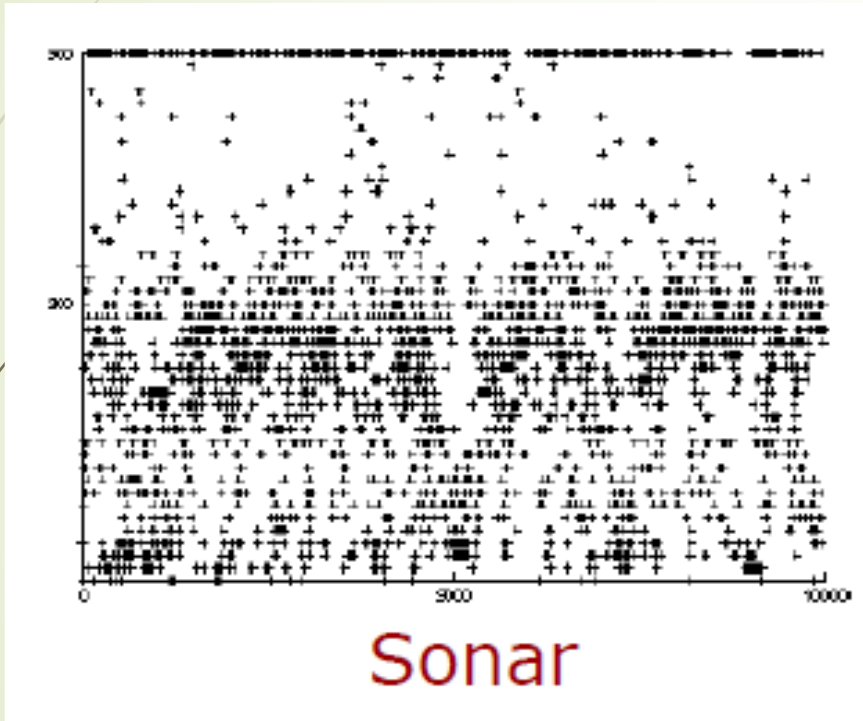
Resulting Mixture Density



$$p(z|x, m) = \begin{pmatrix} \alpha_{hit} \\ \alpha_{unexp} \\ \alpha_{max} \\ \alpha_{rand} \end{pmatrix}^T \cdot \begin{pmatrix} p_{hit}(z|x, m) \\ p_{unexp}(z|x, m) \\ p_{max}(z|x, m) \\ p_{rand}(z|x, m) \end{pmatrix}$$

How can we determine the model parameters?

Raw Sensor Data



Measured distances for expected distance of 300 cm



Scan-based Model (Likelihood Field Model)

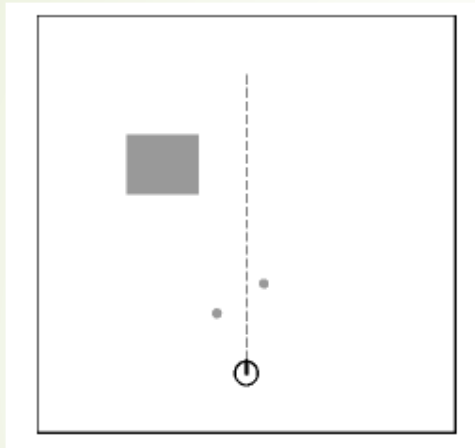
- Beam-based model is
 - Not smooth for small obstacles and at edges
 - Not very efficient
- **Idea:** Instead of following along the beam, just check the end point.



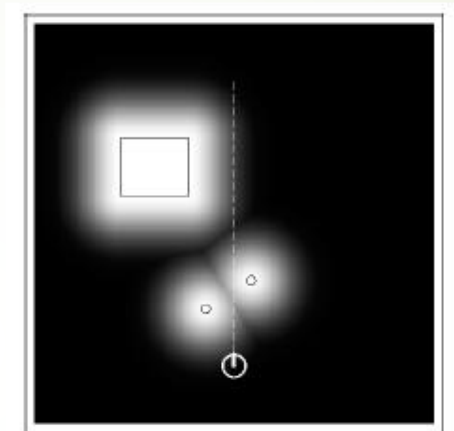
Scan-based Model

- Probability is a mixture of ...
 - A Gaussian distribution with mean at distance to closest obstacle
 - A uniform distribution for random measurements and,
 - A small uniform distribution for max range measurements
- Again, independence between different components is assumed.

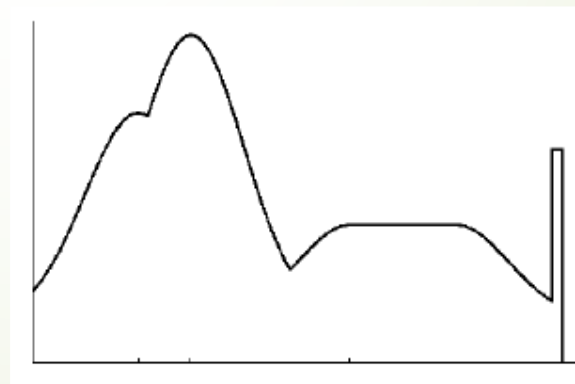
Example



Map m



Likelihood field

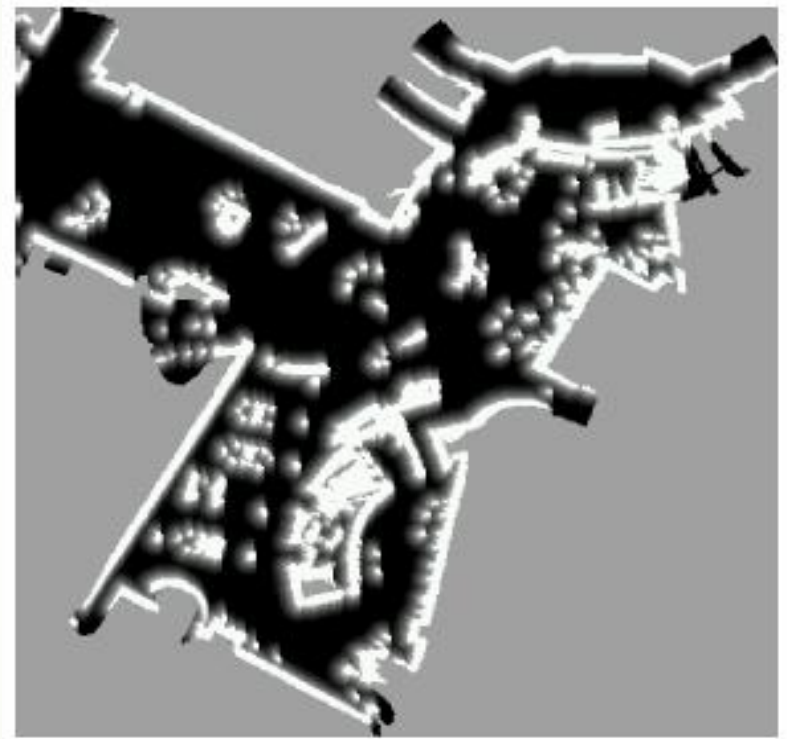


$P(z | x, m)$

San Jose Tech Museum



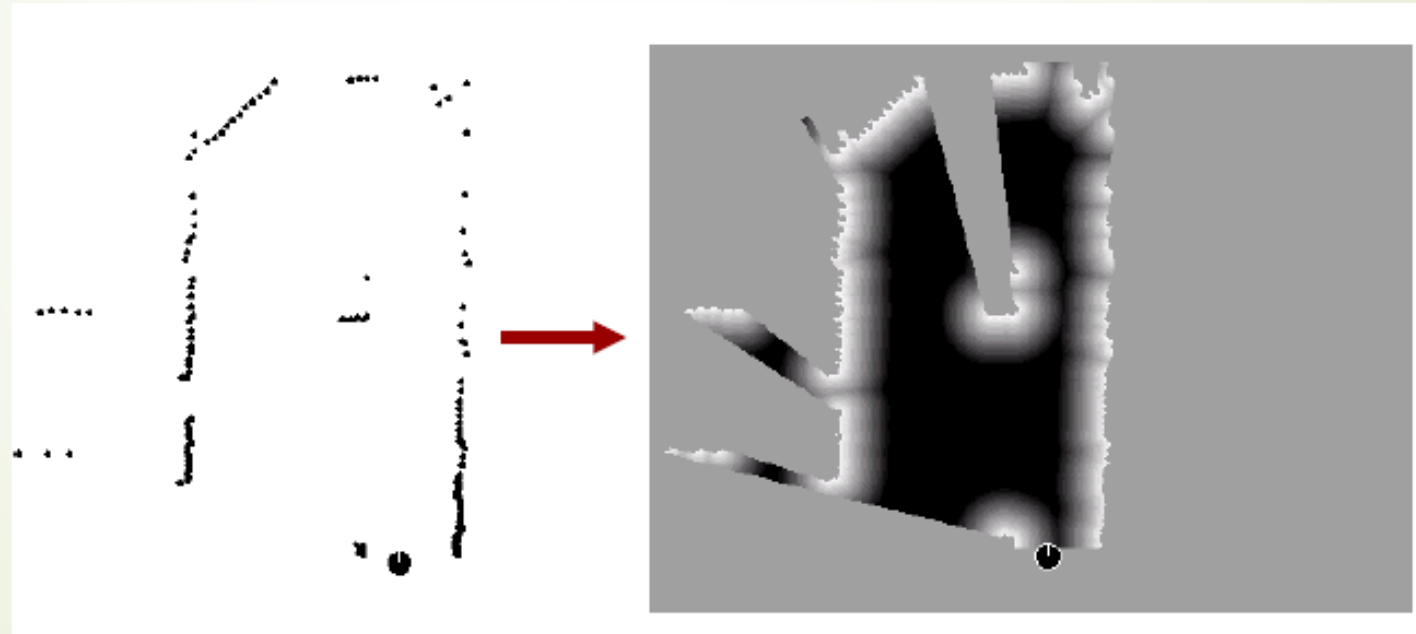
Occupancy grid map



Likelihood field

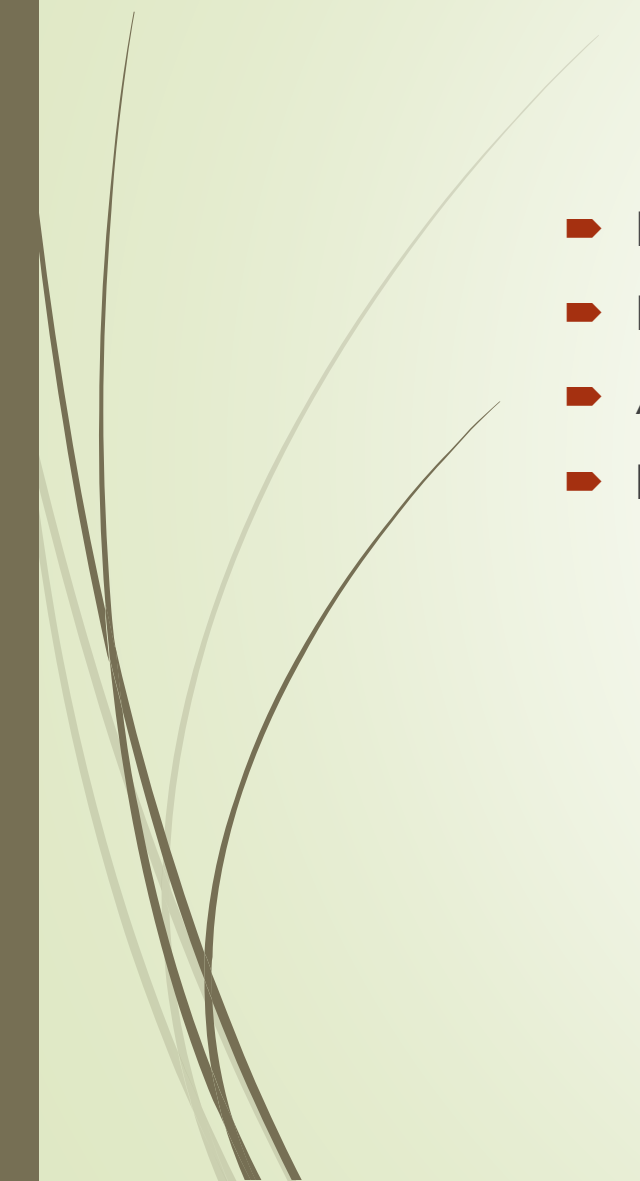
Scan Matching

- Extract likelihood field from scan and use it to match different scan





Properties of Scan-based Model

- Highly efficient, uses 2D tables only
 - Distance grid is smooth w.r.t. to small changes in robot position
 - Allows gradient descent, scan matching.
 - Ignores physical properties of beams
- 



Additional Models of Proximity Sensors

- **Map matching (sonar, laser):** generate small, local maps from sensor data and match local maps against global model
- **Scan matching (laser):** map is represented by scan endpoints, match scan into this map
- **Features (sonar, laser, vision):** Extract features such as doors, hallways from sensor data
 - For range sensors: identify lines, corners
 - For cameras: identify edges, distinct patterns



Landmarks



- Active beacons (e.g. Radio, GPS)
- Passive (e.g., visual, Retro-reflective)
- Sensor provides
 - Distance, or
 - Bearing, or
 - Distance and bearing
 - A signature may be generated

Calculation the probability of a feature with known corresponsce

```
1:  Algorithm landmark_modelknown_correspondence( $f_t^i, c_t^i, x_t, m$ ):  
2:       $j = c_t^i$   
3:       $\hat{r} = \sqrt{(m_{j,x} - x)^2 + (m_{j,y} - y)^2}$   
4:       $\hat{\phi} = \text{atan2}(m_{j,y} - y, m_{j,x} - x)$   
5:       $q = \text{prob}(r_t^i - \hat{r}, \sigma_r^2) \cdot \text{prob}(\phi_t^i - \hat{\phi}, \sigma_\phi^2) \cdot \text{prob}(s_t^i - s_j, \sigma_s^2)$   
6:      return  $q$ 
```

Table 6.4 Algorithm for computing the likelihood of a landmark measurement. The algorithm requires as input an observed feature $f_t^i = (r_t^i \ \phi_t^i \ s_t^i)^T$, and the true identity of the feature c_t^i , the robot pose $x_t = (x \ y \ \theta)^T$, and the map m . It's output is the numerical probability $p(f_t^i \mid c_t^i, m, x_t)$.



Summary of Sensor Models

- Explicitly modeling uncertainty in sensing is key to robustness.
- In many cases, good models can be found by the following approach:
 - Determine parametric model of noise free measurement
 - Analyze sources of noise
 - Add adequate noise to parameters (eventually mix in densities for noise)
 - Learn (and verify) parameters by fitting model to data
 - Likelihood of measurement is given by probabilistically comparing the actual with expected measurement.
- This holds for motion models as well.
- It is extremely important to be aware of the underlying assumptions!

References

- ▀ Probabilistic Robotics, Sebastian Thrun, Wolfram Burgard and Dieter Fox, Chapter 6