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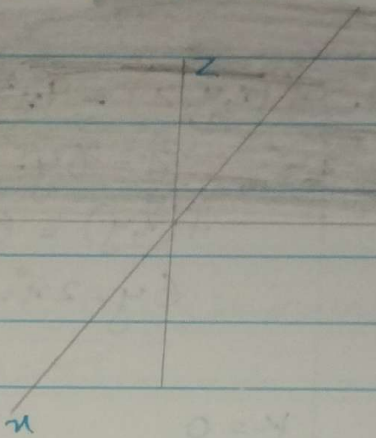
Assignment #01

MVC

Date: 27/2/25

Q1 a) $f(x,y,z) = xy\ln z$

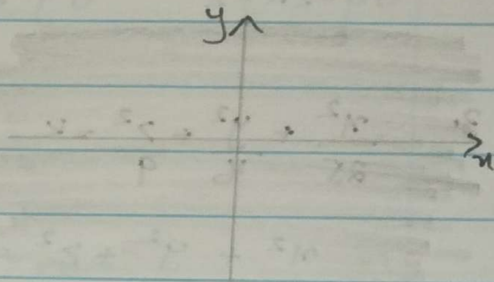


$z > 0$

Domain $= \{x,y,z \in R^3 \mid z>0\}$

b) $f(x,y,z) = \ln(x^2+y^2)$

$x^2+y^2 > 0$



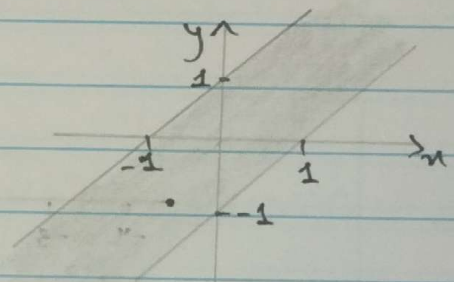
Domain $= \{(x,y,z) \in R^3 \mid x^2+y^2>0\}$

c) $f(x,y) = \sin^{-1}(y-x)$

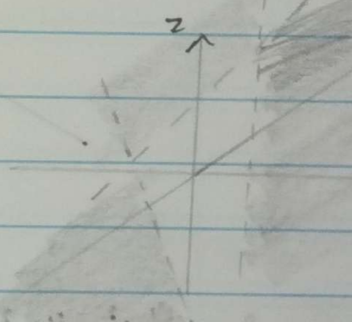
$-1 \le y-x \le 1$

$y \geq x-1, \quad y \le 1+x$

Domain $= \{(x,y,z) \in R^3 \mid -1 \le y-x \le 1\}$



d) $f(x,y,z) = \dfrac{1}{x+1} + \dfrac{1}{y-1} + \dfrac{1}{x+y-2}$



$x \neq -1, \; y \neq 1, \; x+y-2 \neq 0$

Domain $= \{(x,y,z) \in R^3 \mid x\neq -1, y\neq 1, z \neq x+y\}$

Q2 1. $f(x,y) = 4x^2+y^2+1$

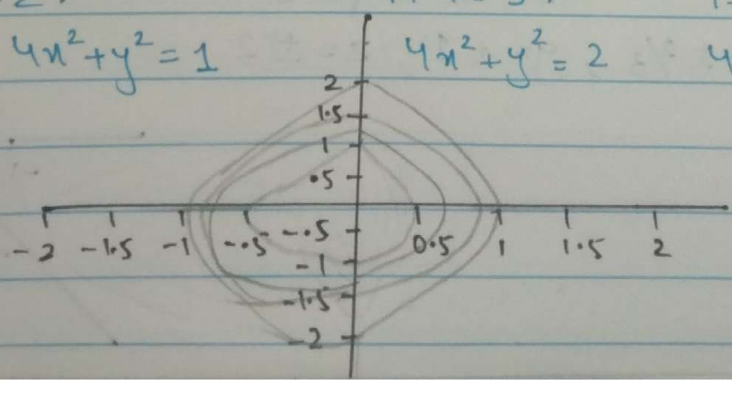
$4x^2+y^2+1 = k$

if $k=2$: $4x^2+y^2=1$

if $k=3$: $4x^2+y^2=2$

if $k=4$: $4x^2+y^2=3$

if $k=5$: $4x^2+y^2=4$



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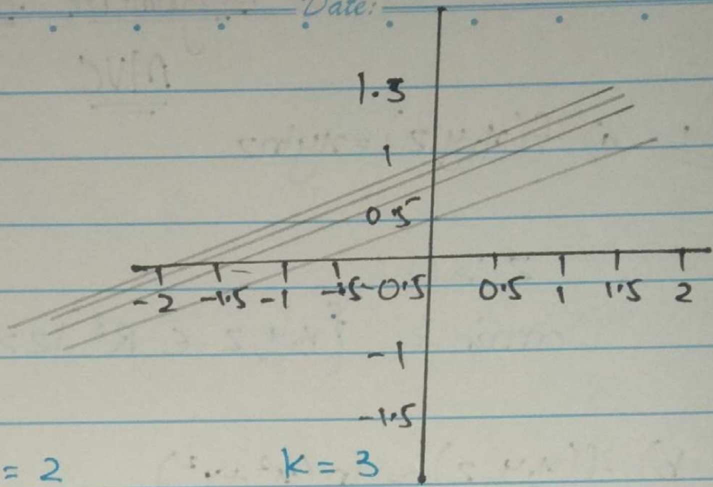
2. $f(x,y,z) = 9x^2+4y^2+z^2, \; k=4$

$z = 6y - 2x - 2$

$f(x,y) = 6y - 2x - 2$

$6y - 2x = k+2$

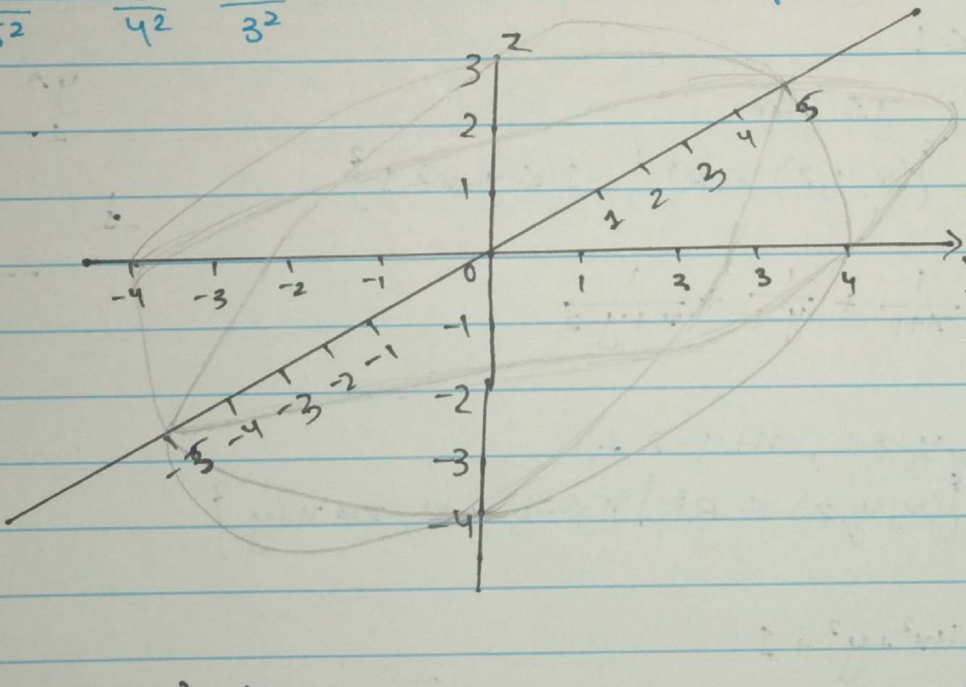


$k=0$ $\quad$ $k=1$ $\quad$ $k=2$ $\quad$ $k=3$

$6y-2x=2$ $\quad$ $6y-2x=3$ $\quad$ $6y-2x=4$ $\quad$ $6y-2x=5$

3. $\dfrac{x^2}{25} + \dfrac{y^2}{16} + \dfrac{z^2}{9} = k$

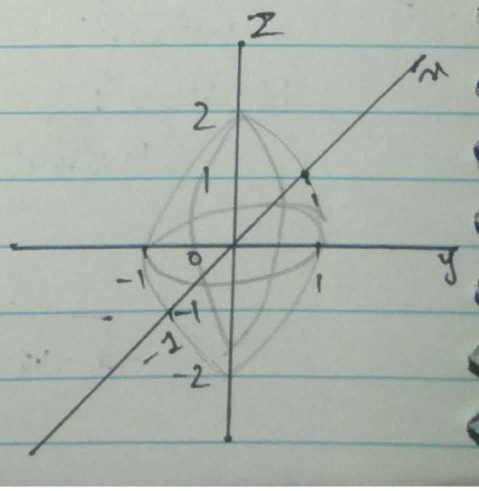
$\dfrac{x^2}{5^2} + \dfrac{y^2}{4^2} + \dfrac{z^2}{3^2} = 1$ so it's an Ellipsoid



2. $9x^2+4y^2+z^2=4$

$\dfrac{9x^2}{4} + \dfrac{y^2}{1} + \dfrac{z^2}{4} = 1$ so it's an Ellipsoid

$x = \pm 2/3, \; y = \pm 1, \; z = \pm 2$



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Q4

1. $$\lim_{(x,y)\to(0,0)} \frac{x^2 - xy}{\sqrt{x} - \sqrt{y}}$$

$$\lim_{(x,y)\to(0,0)} \frac{x(x-y)}{\sqrt{x}-\sqrt{y}} \times \frac{\sqrt{x}+\sqrt{y}}{\sqrt{x}+\sqrt{y}} \Rightarrow \frac{x\cancel{(x-y)}(\sqrt{x}+\sqrt{y})}{\cancel{(x-y)}} \Rightarrow x(\sqrt{x}+\sqrt{y})$$

Applying Limit : $0(\sqrt{0}+\sqrt{0})$.

$$\lim_{(x,y)\to(0,0)} \frac{x^2-xy}{\sqrt{x}-\sqrt{y}} = 0 \text{ Ans}$$

b) $$\lim_{(x,y)\to(0,0)} \cos \frac{x^2-y^3}{x+y+1}$$

$$\lim_{(x,y)\to(0,0)} \cos \frac{x^2-y^3}{x+y+1} \Rightarrow \text{Apply limit : } \cos\left(\frac{0-0}{0+0+1}\right) = \cos 0$$

$$\lim_{(x,y)\to(0,0)} \cos \frac{x^2-y^3}{x+y+1} = 1 \text{ Ans}$$

Q5

$h = 3.2\text{cm}$, $r = 1.5\text{cm}$, $\frac{dh}{dt} = 3 \text{ mm/s}$, $\frac{dr}{dt} = -2\text{mm/s}$

$$V = \frac{1}{3}\pi r^2 h \Rightarrow \frac{dV}{dt} = \frac{1}{3}\pi\left[2r\frac{dr}{dt}\cdot h + r^2 \cdot \frac{dh}{dt}\right]$$

$$\Rightarrow \frac{1}{3}\pi\left[2(15)(-2) + (15)^2(3)\right] = -415\pi$$

$$\boxed{\frac{dV}{dt} = -1303.76 \text{ mm}^3/\text{s}}$$

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$a = 3,\ c = 4,\ \beta = \pi/6\ ,\ \frac{da}{dt} = 0.4,\ \frac{dc}{dt} = -0.8,\ \frac{d\beta}{dt} = 0.2$

$A = \frac{1}{2} ac \sin\beta$

$$\frac{dA}{dt} = \frac{1}{2}\ \frac{da}{dt} \cdot c \sin\beta + a\frac{dc}{dt} \cdot \sin\beta + ac\cos\beta \cdot \frac{d\beta}{dt}$$

$$= \frac{1}{2}(0.2)(4)\sin\left(\frac{\pi}{6}\right) + 3(-0.8)\sin\frac{\pi}{6} + 3(4)\cos\frac{\pi}{6} \cdot (0.2)$$

$$\frac{dA}{dt} = 0.839 \text{ unit/s}$$

$pV^{1.4} = k$

$p' = p + 0.04p = 1.04p$

$V' = V - 0.015V = 0.985V$

$k' = p'(V')^{1.4}$

$= 1.04p \times (0.0985V)^{1.4}$

$= 1.04P \times 0.985^{1.4} \times V^{1.4}$

$k' = 1.017k$

$\% \text{ change} = \left(\frac{k'-k}{k}\right) \times 100\%$

$= \frac{1.017k - k}{k} \times 100\%$

$\boxed{= 1.7\%}$

8 $f_T(-15,30)$ and $f_V(-15,30)$

$$\frac{\partial W}{\partial T}(-15,30) = \lim_{\Delta T \to 0} \frac{W(-15+\Delta T, 30) - W(-15, 30)}{\Delta T}$$

with $\Delta T = 5$:

$$= \frac{W(-10,30)+26}{5} = \frac{-20+26}{5} = 1.2$$

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with $\Delta T = -5$:

$$\frac{W(-20,30)+26}{-5} = \frac{-33+26}{-5} = 1.4$$

$$\text{Average} = \frac{1.4+1.2}{2} = 1.3$$

$$\boxed{f_T(-15,30) = 1.3\ \text{km/h}}$$

$$\frac{\partial W}{\partial v}(-15,30) = \lim_{\Delta v \to 0} \frac{W(-15,30+\Delta v) - W(-15,30)}{\Delta v}$$

With $\Delta v = 10$:

$$\frac{W(-15,40)+26}{10} = \frac{-27+26}{10} = -0.1$$

With $\Delta v = -10$:

$$\frac{W(-15,20)+26}{10} = \frac{-24+26}{10} = -0.2$$

$$\text{Average} = \frac{-0.2-0.1}{2} = -0.15$$

$$\boxed{f_v(-15,30)\ \frac{\partial W}{\partial v} = -0.15\ {}^\circ C/\text{km/h}}$$

Q9 a) Meaning of $\partial h/\partial v$ and $\partial h/\partial t$?

$\frac{\partial h}{\partial v}$ illustrates how the wave height changes with variations in wind speed v, while maintaining a constant duration t.

$\frac{\partial h}{\partial t}$ examines the rate at which height h changes over time t, assuming the wind speed v remains constant.

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b) $f_v(40,15)$ and $f_t(40,15)$

$$f_v(40,15) = \frac{\partial h}{\partial v}(40,15) = \lim_{\Delta v \to 0} \frac{h(40+\Delta v, 15) - h(40,15)}{\Delta v}$$

with $\Delta v = 10$:

$$= \frac{h(50,15)-25}{10} \Rightarrow \frac{36-25}{10} \Rightarrow 1.1$$

with $\Delta v = -10$:

$$= \frac{h(30,15)-25}{10} \Rightarrow \frac{16-25}{-10} = 0.9$$

$$\text{Average} = \frac{1.1+0.9}{2}$$

$f_v(40,15) = 1$ feet per knot.

$$f_t(40,15) = \frac{\partial h}{\partial t} = \lim_{\Delta t \to 0} \frac{h(40,15+\Delta t) - h(40,15)}{\Delta t}$$

with $\Delta t = 5$:

$$= \frac{h(40,20)-25}{5} = \frac{28-25}{5} = 0.6$$

with $\Delta t = -5$:

$$= \frac{h(40,10)-25}{-5} = \frac{21-25}{-5} = 0.8$$

$$\text{Average} = \frac{0.8+0.6}{2}$$

$f_t(40,15) = 0.7$ feet per ~~second~~ hour.

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Q10 1. $f(x,y,z) = xe^y + ye^z + ze^x$, $(0,0,0)$, $v = (5,1,-2)$

$D_v f(x,y,z) = f_x(x_0,y_0,z_0)v_1 + f_y(x_0,y_0,z_0)v_2 + f_z(x_0,y_0,z_0)v_3$

$f_x = e^y + ze^x \Rightarrow f_x(0,0,0) = 1$

$f_y = xe^y + e^z \Rightarrow f_y(0,0,0) = 1$

$f_z = ye^z + e^x \Rightarrow f_z(0,0,0) = 1$

unit vector $= \sqrt{5^2 + 1^2 + 2^2} = \sqrt{30}$

" " $= \frac{5}{\sqrt{30}}\hat{i}, \frac{1}{\sqrt{30}}\hat{j}, \frac{-2}{\sqrt{30}}\hat{k}$

$D_v f(x,y,z) = \frac{5}{\sqrt{30}} + \frac{1}{\sqrt{30}} - \frac{2}{\sqrt{30}} = \boxed{\frac{2\sqrt{30}}{15}}$

2. $f(x,y,z) = \sqrt{xyz}$, $(3,2,6)$, $v = \langle -1,-2,2 \rangle$

$f_x = \frac{yz}{2\sqrt{xyz}} \Rightarrow f_x(3,2,6) = 1$

$f_y = \frac{xz}{2\sqrt{xyz}} \Rightarrow f_y(3,2,6) = 1.5$

$f_z = \frac{xy}{2\sqrt{xyz}} \Rightarrow f_z(3,2,6) = 0.5$

Magnitude $= \sqrt{1^2 + 2^2 + 2^2} = 3$

unit vector $\hat{v} = \frac{-1}{3}\hat{i}, \frac{-2}{3}\hat{j}, \frac{2}{3}\hat{k}$

$D_v f(x,y,z) = \frac{-1}{3} + \frac{3}{2}\left(\frac{-2}{3}\right) + \frac{1}{2}\left(\frac{2}{3}\right) \Rightarrow \boxed{-1}$

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3. $f(x,y) = x - \frac{y^2}{x} + \sqrt{3}\sec^{-1}(2xy)$; $(1,1)$, $v = 12i + 5j$

$f_x = 1 + \frac{y^2}{x^2} + \frac{2\sqrt{3}\,y}{\sqrt{4x^2y^2 - 1}} \Rightarrow f_x(1,1) = 4$

$f_y = \frac{-2y}{x} + \frac{2\sqrt{3}\,x}{\sqrt{4x^2y^2 - 1}} \Rightarrow f_y(1,1) = 0$

Magnitude $= \sqrt{12^2 + 5^2} = 13$

Unit vector $\hat{v} = \frac{12}{13}\hat{i}, \frac{5}{13}\hat{j}$

$D_v f(x,y,z) = 4\left(\frac{12}{13}\right) + (0)\left(\frac{5}{13}\right) = \boxed{\frac{48}{13}}$