Linear Programming
26:711:651
Final Project

Instructor: Mert Gurbuzbalaban
Due Date: Sunday December 15th, at 11:50PM

last updated on December 8, 2019

Instructions: Please submit your answers through blackboard and write your name to the submissions. Late submissions will not be accepted. All the answers should be justified properly for getting any credit. Please write your name and your project partner's name to the top of your submission. The final project should include your code with detailed comments as well.
Deadline: *December 15th, Sunday night, 11:50pm.*
Attention: *No late submissions will be accepted. Make-up final exams are not available. Please provide your code or Excel Sheet with detailed comments explaining your work.*

1. Furnco manufactures desks and chairs. Each desk uses four units of wood, and each chair uses three units of wood. A desk contributes \$30 to profit and a chair contributes \$20. Marketing restrictions require that the number of chairs produced be at least four times more the number of desks produced. There are 50 units of wood available.

   1a) Formulate the problem algebraically as a linear program, clearly identifying decision variables, the objective function and constraints.

   1b) Use a spreadsheet solver to maximize Funrnco's profits. Clearly mark the cells containing decision variables, the formula for the objective function, and the cell(s) containing formulas for the constraints. Upload your spreadsheet file separately.

   1c) Confirm graphically that the solution in part a) maximizes Furnco's profit. The simplest way is to use a graph paper and a ruler and draw your graphs, and then scan the image and drag it into your spreadsheet. Make sure your scanned images are not too large (certainly well below 1 Megabyte.) Be neat and clear. Mark each line corresponding to each constraint and shade the feasible region properly. Clearly show the direction of improvement and iso-profit (or iso-cost) lines as appropriate. Mark the optimal solution in your graph.

1d) Formulate your program from part a) in the standard form, i.e. rewrite it as

$$\min c^T x \quad \text{subject to} \quad Ax = b, x \geq 0 \tag{1}$$

for appropriately chosen vector c, matrix A and a vector b. Write the dual linear program. (Hint: you can use the slides about duality). Maximize the dual program by hand using a graph paper a ruler, drawing your graphs. What are the dual variables y_* and s_* at the optimum?

1e) Confirm that complementary slackness conditions, i.e. $x_* s_* = 0$ where x_* is your solution you computed in part c).

1f) (**Extra credit**, 5 points, medium difficulty) Find a basic feasible solution to (1) and show the iterations of the simplex method. How many iterations do you need to compute the optimal solution?

1g) (**Extra credit**, 5 points, medium difficulty) Show the iterations of the dual simplex method.

1h) (**Extra credit**, 10 points, difficulty hard) Consider the following optimization problem

$$\min \max_{i=1,2,\ldots,k} |a_i^T x - b_i|$$

where $a_i \in \mathbb{R}^n$ are vectors, b_i is a real scalar and the decision variable $x \in \mathbb{R}^n$. Formulate this problem as a linear problem.

2. Solve the **portfolio optimization** problem described in the attached pdf file using Excel, Matlab or R. Explain the steps/commenting your code.

3. **The (famous) diet problem**: After developing the simplex method for solving linear programming problems, George Dantzig needed a good problem to test it on. The problem he selected was the diet problem formulated in 1945 by Nobel economist George Stigler. The problem was to determine an adequate nutritional diet at minimum cost (which was an important military and civilian issue during World War II). Formulated as a linear programming model, the diet problem consisted of 77 unknowns and nine equations. It took nine clerks using hand-operated (mechanical) desk calculators 120 person-days to obtain the optimal simplex solution: a diet consisting primarily of wheat flour, cabbage, and dried navy beans that cost 39.69 per year (in 1939 prices). The solution developed by Stigler using his own numerical method was only 24 cents more than the optimal solution. Here, we consider a variation of this problem below. Consider the following food items with prices and nutrient information:

Breakfast Food	Calories	Fat (g)	Cholesterol (mg)	Iron (mg)	Calcium (mg)	Protein (g)	Fiber (g)	Cost
1. Bran cereal (cup)	90	0	0	6	20	3	5	$0.18
2. Dry cereal (cup)	110	2	0	4	48	4	2	0.22
3. Oatmeal (cup)	100	2	0	2	12	5	3	0.10
4. Oat bran (cup)	90	2	0	3	8	6	4	0.12
5. Egg	75	5	270	1	30	7	0	0.10
6. Bacon (slice)	35	3	8	0	0	2	0	0.09
7. Orange	65	0	0	1	52	1	1	0.40
8. Milk—2% (cup)	100	4	12	0	250	9	0	0.16
9. Orange juice (cup)	120	0	0	0	3	1	0	0.50
10. Wheat toast (slice)	65	1	0	1	26	3	3	0.07

The problem is to determine to minimize the cost of a breakfast for a person with lactose intolerance, prediabetes and gluten intolerance. The total cost is the sum of the individual costs of each food item. Let's define the decision variables.

This problem includes 10 decision variables, representing the number of standard units of each food item that can be included in each breakfast:

x1 = cups of bran cereal
x2 = cups of dry cereal
x3 = cups of oatmeal
x4 = cups of oat bran
x5 = eggs
x6 = slices of bacon
x7 = oranges
x8 = cups of milk
x9 = cups of orange juice
x10 = slices of wheat toast

We have the following constraints: The diet should containt

- at least 420 calories
- at most 20 g of fat
- at most 30 mg of cholesterol
- at least 5 mg of iron
- at least 400 mg of calcium
- at least 20 g of protein
- at least 12 g of fiber
- at most 1 slice of wheat toast (because the person has a gluten intolerance).
- at most 1 cup of milk (because the person has a lactose intolerance).
- at most 1 orange (because the person has prediabetes, needs to avoid high-sugar content items.).

- No “bran cereal” (because the person does not like the taste).

  Formulate a linear program and solve it in your favorite software from Excel, Matlab or R. Does the solution make sense to you?

- (Extra Credit)