Mini-Workshop on
Geometry and Mathematical Science

Osaka City University (Building E of Faculty of Science, Lecture Room E408)

July 28–30, 2018

Organizers

Wayne Rossman (Kobe University)
Masashi Yasumoto (Osaka City University Advanced Mathematical Institute, chair)

Supports

– JSPS Grant-in-Aid for Scientific Research on Innovative Areas, No.18H04489
Principal Investigator: Masashi Yasumoto, 2018-2020.
– Osaka City University Advanced Mathematical Insttitute

Invited Speakers

Short Lectures (2 talks)

Miyuki Koiso (IMI, Kyushu University, Japan)
Yasuo Matsushita (OCAMI, Japan)
Kosuke Naokawa (Hiroshima Institute of Technology, Japan)
Yoshihiro Ohnita (OCAMI & Osaka City University, Japan)
Pascal Romon (Université Paris-Est-Marne-la-Vallée, France)

Poster Presentations

Shintaro Akamine (Nagoya University, Japan)
Joseph Cho (Kobe University, Japan)
Masahiro Kawamata (Hiroshima University, Japan)
Yuta Ogata (National Institute of Technology, Okinawa College, Japan)
Naoya Suzuki (National Institute of Technology, Akita College, Japan)
Yasushi Teruya (Kyushu University, Japan)

Program

July 28th, 2018

13:45-14:00	**Registration**
14:00-14:10	**Opening**
14:10-14:55	Miyuki Koiso (1) (IMI, Kyushu University)
Title	Geometry of anisotropic surface energy and crystalline variational problem
14:55-15:40	**Coffee Break**
15:40-16:25	Yasuo Matsushita (1) (OCAMI)
Title	Bending of Beams by Calculus of Variations — An Example of Applied Mathematics in University Curriculum —
16:55-17:40	Pascal Romon (1) (Université Paris-Est-Marne-la-Vallée)
Title	Discrete minimal and constant mean curvature surface via integrable systems
18:30-	**Banquet**

July 29th, 2018

10:00-10:45	Yoshihiro Ohnita (1) (OCAMI & Osaka City University)
Title	Geometry of Harmonic Maps and Integrable System Approach
11:15-12:00	Miyuki Koiso (2) (IMI, Kyushu University)
Title	Geometry of anisotropic surface energy and crystalline variational problem
12:00-14:00	**Lunch**
14:00-14:45	Kosuke Naokawa (1) (Hiroshima Institute of Technology)
Title	Developable Möbius strips and their discretizations
14:45-16:25	**Coffee Break and Poster Session**
Presenters	Shintaro Akamine, Joseph Cho, Masahiro Kawamata, Yuta Ogata, Naoya Suzuki, Yasushi Teruya
16:25-17:10	Yasuo Matsushita (2) (OCAMI)
Title	Bending of Beams by Calculus of Variations — An Example of Applied Mathematics in University Curriculum —

July 30th, 2018

10:00-10:45	Yoshihiro Ohnita (2) (OCAMI & Osaka City University)
Title	Geometry of Harmonic Maps and Integrable System Approach
11:15-12:00	Kosuke Naokawa (2) (Hiroshima Institute of Technology)
Title	Developable Möbius strips and their discretizations
14:00-14:45	Pascal Romon (2) (Université Paris-Est-Marne-la-Vallée)
Title	How can we compute the curvatures of a discretized surface when the normal are poorly approximated?
14:45-14:50	**Closing**

Title and Abstract

Miyuki Koiso (IMI,Kyushu University)

Title: Geometry of anisotropic surface energy and crystalline variational problem

Abstract

We study a variational problem for surfaces in the euclidean space with an anisotropic surface energy. An anisotropic surface energy is the integral of an energy density that depends on the surface normal over the considered surface, which was introduced to model the surface tension of a small crystal. The minimizer of such an energy among all closed surfaces enclosing the same volume is unique and it is (up to rescaling) so-called the Wulff shape. The Wulff shape and equilibrium surfaces of this energy for volume-preserving variations are generalizations of the round sphere and constant mean curvature surfaces, respectively. However, they are not smooth in general. In this talk, we give a suitable formulation of piecewise-smooth surfaces and discuss geometry of equilibrium surfaces. Especially, we give recent studies on this subject and open questions.

Yasuo Matsushita (OCAMI)

Title: Bending of Beams by Calculus of Variations
— An Example of Applied Mathematics in University Curriculum —

Abstract

From mathematical viewpoints, bending beams in elasticity can be analyzed by the theory of calculus of variations. The lectures on this issue in almost universities in Japan and many text-books, however, describe the subjects not by the calculus of variations, but by certain traditional way of teaching on elastic theory. In the present Lectures it will be shown that the theory of elastic bending of beams can be described simply and completely by means of calculus of variations, with paying attention on the comparison to the traditional way of teaching.

Kosuke Naokawa (Hiroshima Institute of Technology)

Title: Developable Möbius strips and their discretizations

Abstract

In Euclidean 3-space, a surface generted by a smooth motion of a line segment is called a ruled surface, and a ruled surface is said to be developable if it has zero Gaussian curvature. Roughly speaking, developable surfaces can be regarded as a mathematical model of surfaces made of "paper", such as planes, cylindrical surfaces and conical surfaces. Therefore, one can easily make a developable Möbius (non-orientable) strip by using a paper strip, twisting it a half-integer times and attaching the ends of the strip. In the first talk, we introduce several properties and known facts in the topology and differential geometry of developable Möbius strips, involving singularity

theory. In the second talk, we give discretizations of developability based on characterizations via differential geometry, and also give applications to topologies of some kinds of discrete developable Möbius strips. If we have a time, we show some works related to singularities of discrete developable surfaces.

Yoshihiro Ohnita (OCAMI & Osaka City University)

Title: Geometry of Harmonic Maps and Integrable System Approach

Abstract

In these two talks I will provide an introductory lecture on fundamental results and examples related to harmonic maps of Riemann surfaces, more generally pluriharmonic maps of complex manifolds, into symmetric spaces. Concretely the loop group formulation in infinite dimensional Grassmannians and the gauge-theoretic formulation for such harmonic maps will be emphasized. I also would like to mention about some recent results and related problems.

Pascal Romon (Université Paris-Est-Marne-la-Vallée)

Title (I): Discrete minimal and constant mean curvature surface via integrable systems

Abstract

Minimal and constant mean surfaces have played a key role in classical (smooth) Riemannian geometry, but finding analogous discrete objects turns out to be difficult. Various definitions of these compete, e.g. critical points of the area functional, though that one remains unsatisfactory and breaks the maximum principle (as is obvious from the cotan Laplace operator). We will present here one based on circular quad-based nets and explain how it relates to the minimal/CMC PDE. We will also show that it does have an interpretation in terms of Lax pair, much like the smooth PDE has. As a consequence it offers a (partial) constructive approach. This is a joint work with Alexander Bobenko (TU Berlin).

Title (II): How can we compute the curvatures of a discretized surface when the normal are poorly approximated?

Abstract

There are various ways to define the Gaussian and mean curvature on a discrete surface. However, when approximating a smooth surface by a discrete one, the discrete curvatures -and even the area- do not always converge to the corresponding values on the smooth surface. This is particularly true for digital surfaces (surfaces made of small unit squares with vertices in Z^3). By working in the Grassmannian and using a corrected normal vector field, we can avoid these defects. To achieve that, we need construct an integral current in the Grassmannian that captures the relevant data. We obtain then very good and sometimes extremely fast converging (measure) estimates for the curvatures.

Joint work with Jacques-Olivier Lauchaud (University Savoie Mont-Blanc) et Boris Thibert (University Grenoble Alpes).

Title and Abstract of Poster Presentation

Shintaro Akamine (Nagoya University)

Title: Singularities on generalized timelike minimal surfaces

Abstract

A timelike minimal surface is a surface with a Lorentzian metric whose mean curvature vanishes identically, and behavior of the Gaussian curvature is closely related to the types of singularities on such surfaces. In this poster presentation, we discuss singularities on timelike minimal surfaces in the Lorentz-Minkowski 3-space. We explain some relations between the Gaussian curvature and singularities, and also prove that some types of singularities which do not appear on maximal surfaces appear on timelike minimal surfaces naturally.

Joseph Cho (Kobe University)

Title: Constant mean curvature surfaces of positon-like solutions

Abstract

The classical Bianchi-Bäcklund transformation for constant mean curvature surfaces using complex tangential line congruence, has been revisited recently in the context of Darboux transformations for isothermic surfaces and simple factor dressings of the extended frame of a constant mean curvature surface. On the other hand, the Bianchi permutability formula, also known as the superposition principle, gives an elegant way to consider successive Bianchi-Bäcklund transformations with different spectral parameters. In this presentation, we introduce a method to consider successive Bianchi-Bäcklund transformations of a constant mean curvature surface using a single spectral parameter, via the simple factor dressings of the extended frame. Then we apply such transformations to the cylinder, and prove that these transforms enjoy a surprising geometric property in terms of the finite type classification of constant mean curvature surfaces. We call these surfaces constant mean curvature surfaces of positon-like solutions, an analogue of the positons of Korteweg-de Vries equation.

Masahiro Kawamata (Hiroshima University)

Title: On a generalization of Monge-Ampère system

Abstract

It is known that Monge-Ampère system is a geometric formalization of Monge-Ampère equations by using the theory of exterior differential systems. We introduce a generalization of Monge-Ampère systems and give a relationship such systems and some partial differential equations. We also discuss a behavior of the partial differential equations under the contact transformations.

Yuta Ogata (National Institute of Technology, Okinawa College)

Title: Analysis of timelike Thomsen surface

Abstract

Minimal surfaces which are also affine minimal in the Euclidean space, called Thomsen surfaces, have been studied since the early 20th century. They include famous examples: plane, Enneper surface, helicoid, and so on. In this poster, we analyze timelike minimal surfaces in the Minkowski space which are also affine minimal. We give results about classification, deformations and singularities of them.

Naoya Suzuki (National Institute of Technology, Akita College)

Title: On the continuous cohomology and the classifying space of a semi-direct product group

Abstract

Let G be a Lie group and H be a subgroup of it. We can construct a bisimplicial manifold $NG(*) \rtimes NH(*)$ and the de Rham complex on it. This complex is a triple complex and the cohomology of its total complex is isomorphic to $H^*(B(G \rtimes H))$. We can see also that the cohomology of its horizontal complex is isomorphic to the continuous cohomology $H_c^*(G \rtimes H; S^*\mathcal{G} \times S^*\mathcal{H})$.

Yasushi Teruya (Kyushu University)

Title: Convex property of Wulff shapes and regularity of their convex integrands

Abstract

It is known that there exists a unique (up to translations) closed hypersuface which minimizes an anisotropic surface energy among all piecewise smooth closed hypersurfaces enclosing the same volume, and the minimizer coincides with (up to homothety and translation) the boundary of a closed convex body which is called the Wulff shape. The convexity of the Wulff shape is related to the regularity of the energy density of The Wulff shape. In this talk, we investigate the relationship, locally and globally.