$$\begin{vmatrix} x & y & z \\ x+1 & y+1 & z+1 \\ x+2 & y+2 & z+2 \end{vmatrix}$$

$= x(y+1)(z+2) + y(z+1)(x+2) + z(x+1)(y+2) - z(y+1)(x+2) -$

$y(x+1)(z+2) - x(z+1)(y+2)$

$= [xy + x - yx - y](z+2) + [yz + y - zy - z](x+2) + [zx + z - xz - x](y+2)$

$= (x-y)(z+2) + (y-z)(x+2) + (z-x)(y+2)$

$= zx + 2x - yz - 2y + yx + 2y - zx - 2z + zy + 2z - xy - 2x$

$= 0$

$$\begin{vmatrix} 1 & a^2 & a^3 \\ 0 & b^2 - a^2 & b^3 - a^3 \\ 0 & c^2 - a^2 & c^3 - a^3 \end{vmatrix}$$

$= (b^2 - a^2)(c^3 - a^3) - (c^2 - a^2)(b^3 - a^3)$

$= (b-c)(c-a)[(b+a)(c^2 + ac + a^2) - (c+a)(b^2 + ab + a^2)]$

$= (b-a)(a-c)[bc^2 + ac^2 - b^2c - ab^2]$

$= (b-a)(a-c)(c-b)(ab + bc + ca)$

$$\begin{vmatrix} \frac{1}{3} & \frac{2}{5} & -1 \\ 3 & \frac{21}{5} & 3 \\ -\frac{1}{7} & -\frac{1}{7} & \frac{5}{7} \end{vmatrix}$$

$$= \frac{1}{3} \times \frac{21}{5} \times \frac{5}{7} + \frac{2}{5} \times 3 \times \left(-\frac{1}{7}\right) + (-1) \times 3 \times \left(-\frac{1}{7}\right) - (-1) \times \frac{21}{5} \times \left(-\frac{1}{7}\right) - \frac{2}{5} \times 3 \times \left(\frac{5}{7}\right) - \frac{1}{3} \times 3 \times \left(-\frac{1}{7}\right)$$

$$= 1 - \frac{6}{35} + \frac{3}{7} - \frac{21}{35} - \frac{6}{7} + \frac{1}{7}$$

$$= -\frac{2}{35}$$