



Mo Tu We Th Fr Sa Su

Memo No. _____

Date ____/____/____

★ 1. $B \sim \text{diag}(\rho(\lambda_1), \rho(\lambda_2), \rho(\lambda_3))$
 $\Rightarrow \text{diag}(-5, 0, -3)$

2. $\det(B+4E) = \det(\text{diag}(-1, 4, 1))$
 $= -4 \Leftrightarrow \underline{D}$

★ 2. $|A - \lambda E| = 0 \Leftrightarrow \begin{vmatrix} 1-\lambda & 1 & 1 & 1 \\ 1 & 1-\lambda & 1 & 1 \\ 1 & 1 & 1-\lambda & 1 \\ 1 & 1 & 1 & 1-\lambda \end{vmatrix} = 0$

即 $-\lambda^3(4-\lambda) = 0$, $\lambda = 4$ 或 $0 \Leftrightarrow \underline{C}$

★ 3. 根据特征值定义: $A\alpha = \lambda\alpha$

$\Leftrightarrow A(1, -1, 2)^T = \lambda(1, -1, 2)^T$

$\Leftrightarrow (5, 2a-b+2, 7-a)^T = \lambda(1, -1, 2)^T$

$\begin{cases} \lambda = 5 \\ -\lambda = 2a - b + 2 \\ 2\lambda = 7 - a \end{cases} \Rightarrow \begin{cases} a = -3 \\ b = 1 \end{cases} \Leftrightarrow \underline{B}$

★ 4. 由定义, $A\alpha = \lambda_0 \alpha$

$$\Leftrightarrow PBP^T \alpha = \lambda_0 \alpha$$

$$\Leftrightarrow B(P^T \alpha) = \lambda_0 (P^T \alpha), \text{ 选 C.}$$

本题也可根据选项代入求解.

★ 5. $k_1 \alpha_1 + k_2 A(\alpha_1 + \alpha_2) = 0$

$$\Leftrightarrow (k_1 + \lambda_1 k_2) \alpha_1 + (k_2 \lambda_2) \alpha_2 = 0$$

$$\Rightarrow \begin{cases} k_1 + \lambda_1 k_2 = 0 \\ k_2 \lambda_2 = 0 \end{cases}$$

由 $k_1 = k_2 = 0$, 仅有零解, 得 $\lambda_2 \neq 0$, ~~选 D~~

否则 k_2 可取任意值, 此时 $k_1 = -\lambda_1 k_2$, 与题设矛盾.

★ 6. $|A - \lambda E| = 0$

$$\text{即 } (2-\lambda)((a-\lambda)(3-\lambda)-4) = 0$$

$$\Leftrightarrow \lambda_1 = 1, \lambda_2 = 2, \lambda_3 = b$$

$$\text{分别代入: } \lambda_1 = 1 \Rightarrow a = 3$$

$$\lambda_2 = 2 \Rightarrow 0 = 0$$

$$\lambda_3 = b \Rightarrow b = 1 \text{ 或 } 5$$

$$\because |A| = |B| = 10 = 2b$$

$$\therefore b = 5, \text{ 选 } \underline{B}$$



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★ 7. A 特征值为 $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}$

$\Rightarrow B$ 特征值为 A^{-1} 的特征值: $2, 3, 4, 5$

$\Rightarrow B^{-1} - E$ 的特征值为: $1, 2, 3, 4$

$\Rightarrow \det(B^{-1} - E) = 2 \times 3 \times 4 = 24, \underline{A}$

★ 8. 首先排除非对称阵 C ,

可对角化的充要条件是有 n 个线性无关的特征向量

\Rightarrow 对于 B 选项: $|B - \lambda E| = 0$

$$\Leftrightarrow (1 - \lambda)^2 (2 - \lambda) = 0$$

$$\Rightarrow \lambda_1 = \lambda_2 = 1, \lambda_3 = 2$$

看重根 $\lambda=1$ 对应两个差别的特征向量, 代入 $\lambda=1$, 得

$$\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \text{基础解系: } \eta_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

$$\text{或 } \begin{cases} x_2 = x_3 = 0 \\ x_1 = k \end{cases}$$

仅有一个特征向量 η_1 , 因此不可对角化, 选 B

9. A) $P^{-1}AP = B$

$$\Rightarrow P^T A^T (P^{-1})^T = B^T$$

$$\Rightarrow ((P^T)^{-1})^{-1} A^T ((P^T)^{-1}) = B^T$$

$$\text{即 } A^T \sim B^T$$

B) $P^{-1}AP = B$

$$\Rightarrow A^{-1} = PBP^{-1}$$

$$\Rightarrow A^{-1} = (PBP^{-1})^{-1} = P B^{-1} P^{-1}$$

$$\Leftrightarrow A^{-1} \sim B^{-1}$$

C) $A^* = A^{-1} \cdot |A|$

由 $A \sim B$, $\Rightarrow A^{-1} \sim B^{-1}$, 且 $|A| = |B|$

$$\Rightarrow |A| \cdot A^{-1} \sim |B| \cdot B^{-1}$$

$$\Leftrightarrow A^* \sim B^*$$

D) 无法得证, 选 D

★ 10. A, B, D 均为性质

C), 仅可推出 $|A - \lambda E| = |B - \lambda E| = 0$

$\nRightarrow A - \lambda E = B - \lambda E$. 选 C



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★ 11 $A\alpha = \alpha$

$$\Leftrightarrow \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

$$\Leftrightarrow \begin{cases} a-b=1 \\ c-d=-1 \end{cases} \Rightarrow \begin{cases} a=b+1 \\ d=c+1 \end{cases}$$

代入选项仅有 C 满足

此时根据 $\lambda_1=1, \lambda_2=2$, 代入, 得:

$$\begin{cases} b(d-1)-b(c+1)=0 \Leftrightarrow 0=0 \\ (b-1)(d-2)-b(c+1)=0 \Leftrightarrow b+d=2 \end{cases}$$

仅有 C 满足条件, 选 C

★ 12. $A = \begin{pmatrix} 1 & 0 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

$$\therefore \alpha^T \alpha = 1+1=2$$

$$\Rightarrow A^n = \alpha \alpha^T \cdot \alpha \alpha^T \cdots \alpha \alpha^T = 2^{n-1} A$$

$$\Rightarrow aE - A^n = \begin{pmatrix} a-2^{n-1} & 0 & 2^{n-1} \\ 0 & a & 0 \\ 2^{n-1} & 0 & a-2^{n-1} \end{pmatrix}$$

$$\Rightarrow \det(aE - A^n) = a[(a-2^{n-1})^2 - (2^{n-1})^2] \\ = a^2(a-2^n)$$

选 B

★ 13. $B = \varphi(A) = A^3 - 3A^2 + 2E$

$$\varphi(\lambda) = \lambda^3 - 3\lambda^2 + 2$$

$$\Rightarrow \begin{cases} \varphi(-2) = -18 \\ \varphi(-1) = -2 \\ \varphi(2) = -2 \end{cases}$$

$$\Rightarrow |B| = -18 \times (-2) \times (-2) = -72, \text{选 } D$$

★ 14 $\begin{cases} \det(3A + 2E) = 0 \\ \det(A - E) = 0 \\ \det(3E - 2A) = 0 \end{cases} \Rightarrow \begin{cases} \lambda_1 = -\frac{2}{3} \\ \lambda_2 = 1 \\ \lambda_3 = \frac{3}{2} \end{cases}$

则 A^{-1} 的特征值为: $-\frac{3}{2}, 1, \frac{2}{3}, |A| = -1$

$$\Rightarrow \det(A^* - E) = \det(A^{-1} - E) = \left(\frac{3}{2} - 1\right)(-1 - 1)\left(-\frac{2}{3} - 1\right) = \frac{1}{2} \times (-2) \times \left(-\frac{5}{3}\right) = \frac{5}{3}, \text{选 } A$$

★ 15 $A\alpha = \lambda\alpha \Leftrightarrow -\frac{1}{2}A\alpha = -\frac{1}{2}\lambda\alpha, A \text{ 成立}$

$$\Leftrightarrow A^{-1}\alpha = \frac{1}{\lambda}\alpha$$

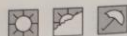
$$\Rightarrow A^{-1}|A|\alpha = \frac{|A|}{\lambda}\alpha$$

$$\Leftrightarrow A^*\alpha = \frac{|A|}{\lambda}\alpha, C \text{ 成立}$$

$$\Rightarrow \frac{1}{2}A^2\alpha = \frac{1}{2}\lambda^2\alpha$$

取逆: $\left(\frac{1}{2}A^2\right)^{-1}\alpha = \frac{2}{\lambda^2}\alpha, B \text{ 成立}$

故选 D



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$$\star 16 \quad \begin{vmatrix} 1-\lambda & 2 & 2 \\ 2 & 1-\lambda & -2 \\ 2 & 2 & 1-\lambda \end{vmatrix} = (3-\lambda)^2(\lambda+3)$$

$$\Leftrightarrow \lambda_1 = \lambda_2 = 3, \lambda_3 = -3, \text{选 } A$$

$$\star 17 \quad \beta = k_1 \alpha_1 + k_2 \alpha_2$$

$$\Rightarrow \begin{cases} k_1 + k_2 = -1 \\ k_1 = 1 \\ k_2 = -2 \end{cases} \Rightarrow \beta = \alpha_1 - 2\alpha_2$$

$$A\beta = A\alpha_1 - 2A\alpha_2 = \lambda\alpha_1 - 2\lambda\alpha_2 \\ = (-2, 4, -4)^T$$

$\star 18$. 根据定理4 特征值不等则对应的特征向量.

$\star 19$. 充分条件是几个线性无关的特征向量, 不同特征值是充分不必要条件. 选 B

$$\star 20 \quad \varphi(\lambda) = (2-\lambda)^2(3-\lambda) = 0$$

仅考虑 $\lambda=2$ 时是否有一样的特征向量

$$r(A - \lambda E) = 1$$

$$r(A_3 - \lambda E) = 1, r(A_1 - \lambda E) = 2$$

$$r(A_2 - \lambda E) = 2, r(A_4 - \lambda E) = 2$$

仅 A_3 相等, 故选 C

$\lambda E - B$

21) A): $\det(\lambda E - A) = \det(\lambda E - B) \Leftrightarrow \lambda E - A = \lambda E - B$

B): 特征向量不一定相同

C): 不一定可对角化

D): $\varphi(A) \sim \varphi(B)$, 故成立

22. 三个互不相同特征向量:

$$\varphi(\lambda) = (\lambda + 1)(1 - \lambda)^2$$

$$\lambda_1 = \lambda_2 = 1, \lambda_3 = -1$$

$$\text{代入 } \lambda = 1, A - \lambda E = \begin{pmatrix} -1 & 0 & 1 \\ x & 0 & 0 \\ 1 & 0 & -1 \end{pmatrix} \sim \begin{pmatrix} -1 & 0 & 1 \\ x & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$r(A - \lambda E) = 1$ 时有两个基础解系, 故 $x = 0$ 选 B

23. $r(E - A) = r(E - B), r(A - 3E) = r(B - 3E)$

由 $r(E - B) = 4, r(A - 3E) = 2$

\therefore 原式 = 6, 故选 B