a) $\int x e^{-2x} \, dx$

$u = x$

$du = dx$

d

$$\int x e^{-2x} \, dx = x - \frac{1}{2e^{2x}} - \int -\frac{1}{2e^{2x}} \, dx$$

$$= x - \frac{1}{2e^{2x}} + \frac{1}{2} \int e^{-2x} \, dx$$

$$= -\frac{1}{2} e^{-2x} x + \frac{1}{2} \left(-\frac{1}{2e^{2x}} \right)$$

$$= -\frac{1}{2} e^{-2x} x - \frac{1}{4} e^{-2x} + C$$

Escaneado con CamScanner

b) $\int x \ln x \, dx$

$u = \ln x$ $\quad dv = x \, dx$

$du = \frac{1}{x} dx$ $\quad v = \frac{x^2}{2}$

$$\int x \ln x \, dx = \ln x \cdot \frac{x^2}{2} - \int \frac{x^2}{2} \cdot \frac{1}{x} \, dx$$

$$= \frac{x^2}{2} \ln x - \int \frac{x}{2} \, dx$$

$$= \frac{x^2}{2} \ln x - \frac{1}{2} \int x \, dx$$

$$= \frac{x^2}{2} \ln x - \frac{1}{2} \left(\frac{x^2}{2} \right)$$

$$= \frac{x^2}{2} \ln x - \frac{x^2}{4} + C$$

Escaneado con CamScanner

c) $\int \ln x^2 \, dx$

$u = \ln x \qquad dv = dx$

$du = \frac{1}{x}\, dx \qquad v = x$

$\int 2 \ln x \, dx$

$2\int \ln x \, dx = \ln x \cdot x - \int x \frac{1}{x}\, dx$

$= x \ln x - \int dx$

$= x \ln x - x + c$

$= 2(x \ln x - x) + c$

Escaneado con CamScanner

d) $\int 5x^2 e^x \, dx$

$u = x^2$ $\quad dv = e^x \, dx$

$du = 2x \, dx$ $\quad v = e^x$

$$5\int x^2 e^x \, dx = 5\left(x^2 e^x - \int e^x \, 2x \, dx \right)$$

$$= 5\left(x^2 e^x - 2\int e^x \, x \, dx \right)$$

$$= 5\left(x^2 e^x - 2\left(x e^x - e^x\right)\right)$$

$$= 5(x^2 e^x - 2x e^x + 2e^x) + C$$

$$= 5x^2 e^x - 10x e^x + 10e^x + C$$

$$\int e^x \, x \, dx$$

$u = x$ $\quad dv = e^x \, dx$

$du = dx$ $\quad v = e^x$

$$= x e^x - \int e^x \, dx$$

$$= x e^x - e^x$$

Escaneado con CamScanner

e) $\int x^3 e^{x^2} dx$

$U = x^2$ $\quad du = 2x$

$dV = x e^{x^2} dx$ $\quad V = \frac{e^{x^2}}{2}$

$$\int x^2 \, x \, e^{x^2} = x^2 \frac{e^{x^2}}{2} - \int \frac{e^{x^2}}{2} 2x$$

$$= x^2 \frac{e^{x^2}}{2} - \int e^{x^2} x \, dx$$

$$= x^2 \frac{e^{x^2}}{2} - \int \frac{1}{2} dt$$

$$= x^2 \frac{e^{x^2}}{2} - \frac{1}{2} t$$

$$= x^2 \frac{e^{x^2}}{2} - \frac{1}{2} e^{x^2}$$

$$= \boxed{\frac{x^2 e^{x^2} - e^{x^2}}{2} + C}$$

9) $\int \sec^3(5x)\,dx$

$= \int \sec^3(u)\frac{1}{5}\,du = \frac{1}{5}\int \sec^3(u)\,du = \frac{1}{5}\left(\frac{\sec^2(u)\sec(u)}{2} + \right.$

$\left.\frac{1}{2}\int \sec(u)\,dx\right)$

$= \frac{1}{5}\left(\frac{\sec^2(u)\operatorname{sen}(u)}{2} + \frac{1}{2}\ln(\tan(u)+\sec(u))\right)+c$

$= \frac{1}{5}\left(\frac{\sec^2(5x)\operatorname{sen}(5x)}{2} + \frac{1}{2}\ln(\tan(5x)+\sec(5x))\right)+c$

i) $\int x \cdot 10^x \, dx$

$u = x$ $\qquad$ $du = 10^x$

$du = dx$ $\qquad$ $v = \dfrac{10^x}{\ln(10)}$

$$= x \cdot \frac{10^x}{\ln(10)} - \int \frac{10^x}{\ln(10)} \, dx$$

$$= x \, \frac{10^x}{\ln(10)} - \frac{1}{\ln(10)} \int 10^x \, dx$$

$$= x \, \frac{10^x}{\ln(10)} - \frac{1}{\ln(10)} \cdot \frac{10^x}{\ln(10)}$$

$$= \frac{x\,10^x}{\ln(10)} - \frac{10^x}{\ln(10)^2} + C$$

J) $\int x^2 e^{3x} dx$

$u = x^2$ $\quad dv = e^{3x} dx$

$du = 2x$ $\quad v = \frac{1}{3} e^{3x}$

$= x^2 \frac{1}{3} e^{3x} - \int \frac{1}{3} e^{3x} 2x \, dx$

$= \frac{1}{3} e^{3x} x^2 - \frac{2}{3} \int e^{3x} \cdot x \, dx$

$= \frac{1}{3} e^{3x} x^2 - \frac{2}{3} \left(\frac{1}{3} e^{3x} x - \frac{1}{9} e^{3x} \right)$

$= \frac{1}{3} e^{3x} x^2 - \frac{2}{9} e^{3x} x + \frac{2}{27} e^{3x} + C$

$u = x$ $\quad dv = e^{3x}$

$du = dx$ $\quad v = \frac{1}{3} e^{3x}$

$= x \frac{1}{3} e^{3x} - \int \frac{1}{3} e^{3x} dx$

$= \frac{1}{3} e^{3x} x - \frac{1}{3} \int e^{3x} dx$

$= \frac{1}{3} e^{3x} x - \frac{1}{3} \left(\frac{1}{3} e^{3x} \right)$

$= \frac{1}{3} e^{3x} x - \frac{1}{9} e^{3x}$

$= \frac{1}{3} e$

Escaneado con CamScanner

f) $\int (\ln x)^2\, dx = x \ln^2 x - \int x \cdot \frac{2 \ln x}{x}\, dx = x \ln^2 x - 2\int \ln x\, dx =$

$x \ln^2 x - 2\left[x \ln x - \int x \frac{1}{x}\, dx\right]$

$u = (\ln x)^2$ $\quad v' = 1$

$du = \frac{2\ln x}{x}\, dx$ $\quad v = \int dx$

$v = x$

$u = \ln x$

$du = \frac{1}{x}\, dx$

$v = x$

$x \ln^2 x - 2\left[x \ln x - \int dx\right] = x \ln^2 x - 2\left[x \ln x - x\right] + c =$

$x \ln^2 x - 2(x \ln x - x) + c = \underline{x \ln^2 x - 2x(\ln x - 1) + c}$

h) $\int e^{-5x}\cos(3x)\,dx =$ $f = \cos(3x)$ $g' = e^{-5x}$

$f' = -3\sin(3x)$ $g = -\frac{e^{-5x}}{5}$

$$= -\frac{e^{-5x}\cos(3x)}{5} - \int \frac{3e^{-5x}\sin(3x)}{5}\,dx$$

se integra por partes otra vez.

$f = -3\sin(3x)$ $g' = -\frac{e^{-5x}}{5}$

$f' = -9\cos(3x)$ $g = \frac{e^{-5x}}{25}$

$$= -\frac{e^{-5x}\cos(3x)}{5} - \left(-\frac{3e^{-5x}\sin(3x)}{25} - \int -\frac{9e^{-5x}\cos(3x)}{25}\,dx\right) =$$

$$-\frac{e^{-5x}\cos(3x)}{5} - \left(-\frac{3e^{-5x}\sin(3x)}{25} + \frac{9}{25}\int e^{-5x}\cos(3x)\,dx\right) =$$

$$\frac{3e^{-5x}\sin(3x) - 5e^{-5x}\cos(3x)}{34} = \frac{e^{-5x}(3\sin(3x) - 5\cos(3x))}{34} + C$$