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Notation:

r - value of a pixel before applying the transformation

s - value of a pixel after applying the transformation
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Log Transformation

Formula:

$$s = c \log(1 + r)$$

Here, c is a constant parameter.

Properties:

- This transformation maps a narrow range of low intensity values in the input into a wider range of output levels. (e.g. $[0, L/4]$ \rightarrow $[0, 3L/4]$)
- Higher values of input levels are mapped to a narrower range in the output.

Why use log transformation?

To expand the values of dark pixels in an image, while compressing the higher level values.

** In practical, after applying log transformation given a value of c, we need to normalize the output values to $[0, 255]$

Normalizing process

Reference: [How to scale down a range of numbers with a known min and max value - Stack Overflow](#) (check the most voted comment)

Here,

$$\text{min} = c \log(1 + 0) = 0 \quad [\text{Minimum value of output image to scale}]$$

$$\text{max} = c \log(1 + 255) = c \log(256) \quad [\text{Maximum value of output image to scale}]$$

** We want to convert [min, max] to [0, 255]

Formula for normalization:

$$f(x) = \frac{(r - l)(x - \min)}{\max - \min} + l$$

Where, $l = 0$, $r = 255$

$\min = 0$, $\max = c \log(256)$

$$\Rightarrow f(x) = \frac{255 * x}{c \log(256)}$$