

# Digital systems and basics of electronics - SYC




07 Introduction to digital systems

08 Minimization of Boolean functions

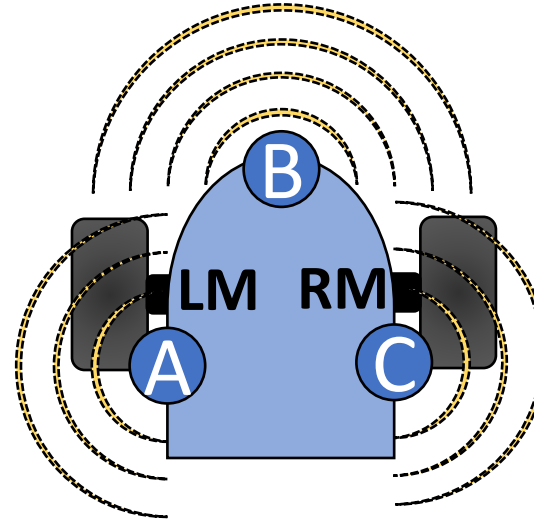
# Boolean functions

**Boolean functions (1-true, 0-false)** are useful in designing algorithms such as the movement of a robot. We can create control logic that dictates a mobile robot's response to specific situations, like detecting obstacles, changing direction, or stopping at the appropriate moment.

3 **distance sensor** located:

- A. On left side 
- B. In front of 
- C. On right side 

When obstacle is detected sensor gives logical "1", otherwise gives "0"

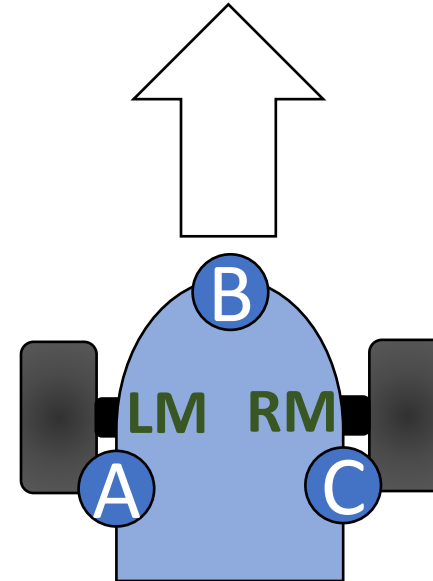


The moving platform uses differential drive: **left motor LM, right motor RM.** Engine is on when logical "1" control signal is applied.

Robot should move without any collision. When robot could not continue movement should stop ( $LM=RM=0$ ).

# 1 Truth Table

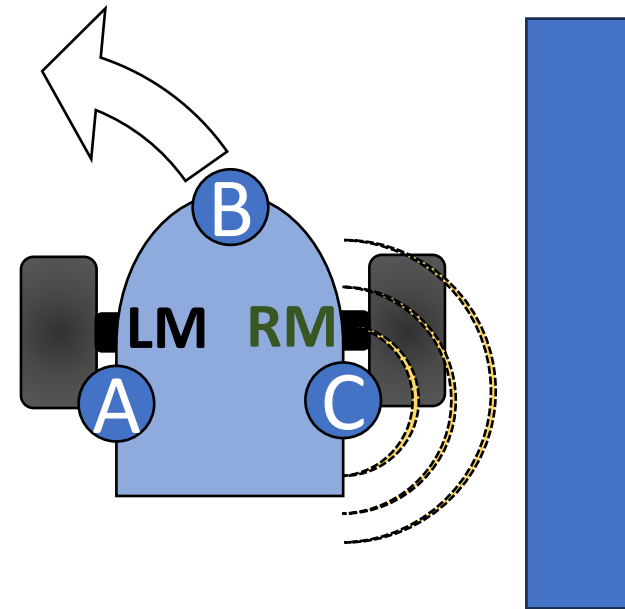
| Truth Table |   |   |    |    |
|-------------|---|---|----|----|
| A           | B | C | LM | RM |
| 0           | 0 | 0 | 1  | 1  |
| 0           | 0 | 1 |    |    |
| 0           | 1 | 0 |    |    |
| 0           | 1 | 1 |    |    |
| 1           | 0 | 0 |    |    |
| 1           | 0 | 1 |    |    |
| 1           | 1 | 0 |    |    |
| 1           | 1 | 1 |    |    |



Obstacle is not detected (sensor gives “0”).  
The robot moves forward. The left motor  
and right motor are set to logical true.

# 1 Truth Table

| Truth Table |   |   |    |    |
|-------------|---|---|----|----|
| A           | B | C | LM | RM |
| 0           | 0 | 0 | 1  | 1  |
| 0           | 0 | 1 | 0  | 1  |
| 0           | 1 | 0 |    |    |
| 0           | 1 | 1 |    |    |
| 1           | 0 | 0 |    |    |
| 1           | 0 | 1 |    |    |
| 1           | 1 | 0 |    |    |
| 1           | 1 | 1 |    |    |



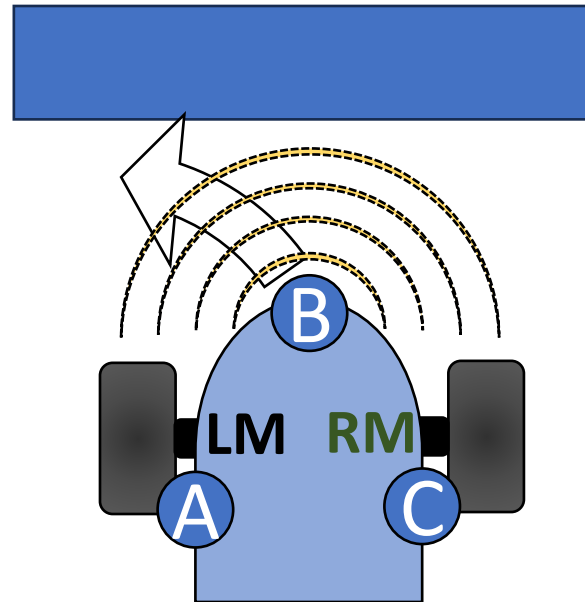
An obstacle is detected on the right side (C sensor gives "1").

The robot turns left.

The right motor is set to logical true.

# 1 Truth Table

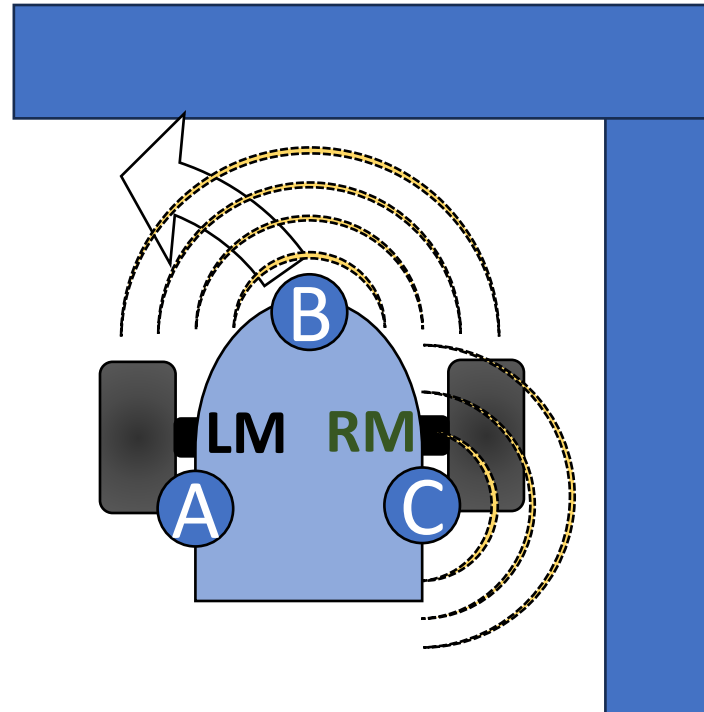
| Truth Table |   |   |    |    |
|-------------|---|---|----|----|
| A           | B | C | LM | RM |
| 0           | 0 | 0 | 1  | 1  |
| 0           | 0 | 1 | 0  | 1  |
| 0           | 1 | 0 | 0  | 1  |
| 0           | 1 | 1 |    |    |
| 1           | 0 | 0 |    |    |
| 1           | 0 | 1 |    |    |
| 1           | 1 | 0 |    |    |
| 1           | 1 | 1 |    |    |



An obstacle is detected in front  
(B sensor gives "1").  
The robot turns left.  
The right motor is set to logical true.

# 1 Truth Table

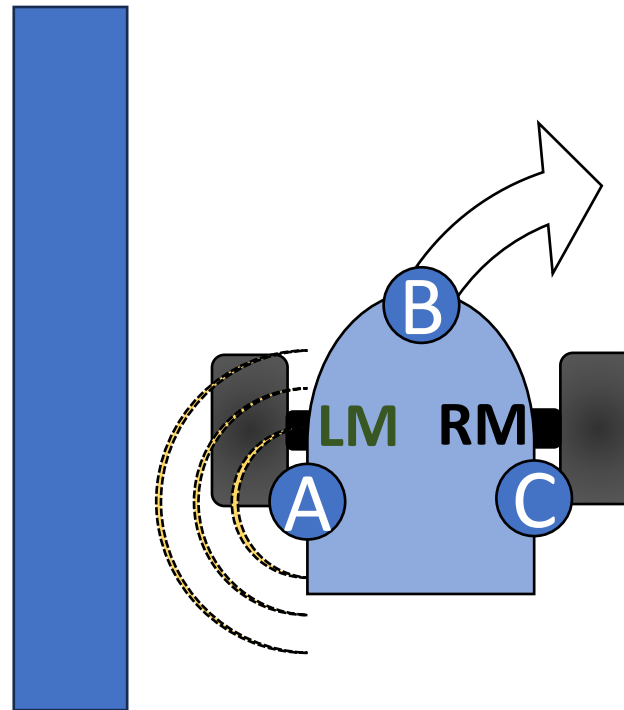
| Truth Table |   |   |    |    |
|-------------|---|---|----|----|
| A           | B | C | LM | RM |
| 0           | 0 | 0 | 1  | 1  |
| 0           | 0 | 1 | 0  | 1  |
| 0           | 1 | 0 | 0  | 1  |
| 0           | 1 | 1 | 0  | 1  |
| 1           | 0 | 0 |    |    |
| 1           | 0 | 1 |    |    |
| 1           | 1 | 0 |    |    |
| 1           | 1 | 1 |    |    |



An obstacle is detected in front and on the right side (B and C sensors give “1”).  
The robot turns left.  
The right motor is set to logical true.

# 1 Truth Table

| Truth Table |   |   |    |    |
|-------------|---|---|----|----|
| A           | B | C | LM | RM |
| 0           | 0 | 0 | 1  | 1  |
| 0           | 0 | 1 | 0  | 1  |
| 0           | 1 | 0 | 0  | 1  |
| 0           | 1 | 1 | 0  | 1  |
| 1           | 0 | 0 | 1  | 0  |
| 1           | 0 | 1 |    |    |
| 1           | 1 | 0 |    |    |
| 1           | 1 | 1 |    |    |



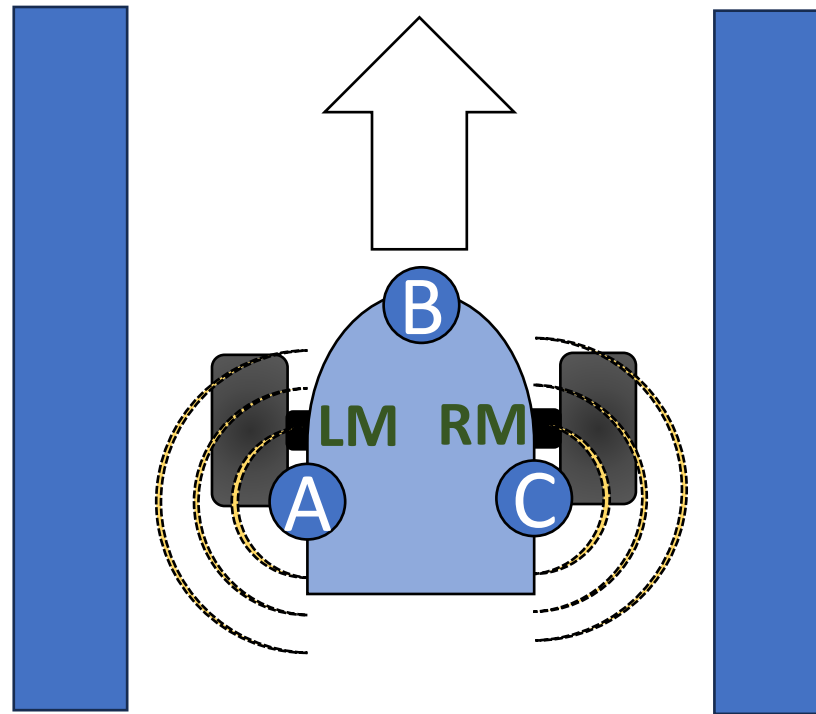
An obstacle is detected on the left side (A sensor gives “1”).

The robot turns right.

The left motor is set to logical true.

# 1 Truth Table

| Truth Table |   |   |    |    |
|-------------|---|---|----|----|
| A           | B | C | LM | RM |
| 0           | 0 | 0 | 1  | 1  |
| 0           | 0 | 1 | 0  | 1  |
| 0           | 1 | 0 | 0  | 1  |
| 0           | 1 | 1 | 0  | 1  |
| 1           | 0 | 0 | 1  | 0  |
| 1           | 0 | 1 | 1  | 1  |
| 1           | 1 | 0 |    |    |
| 1           | 1 | 1 |    |    |



An obstacle is detected on both sides (A and C sensors give "1").

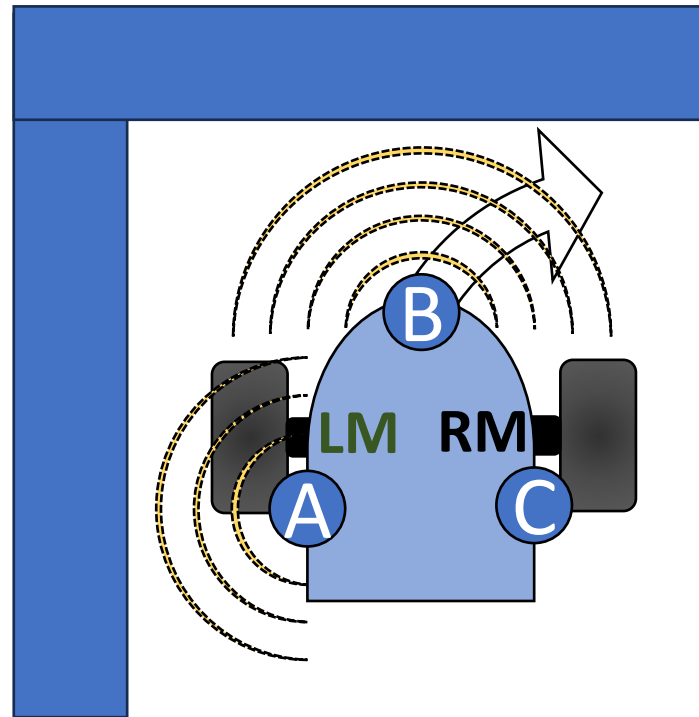
The robot moves forward.

Both motors are set to logical true.



# 1 Truth Table

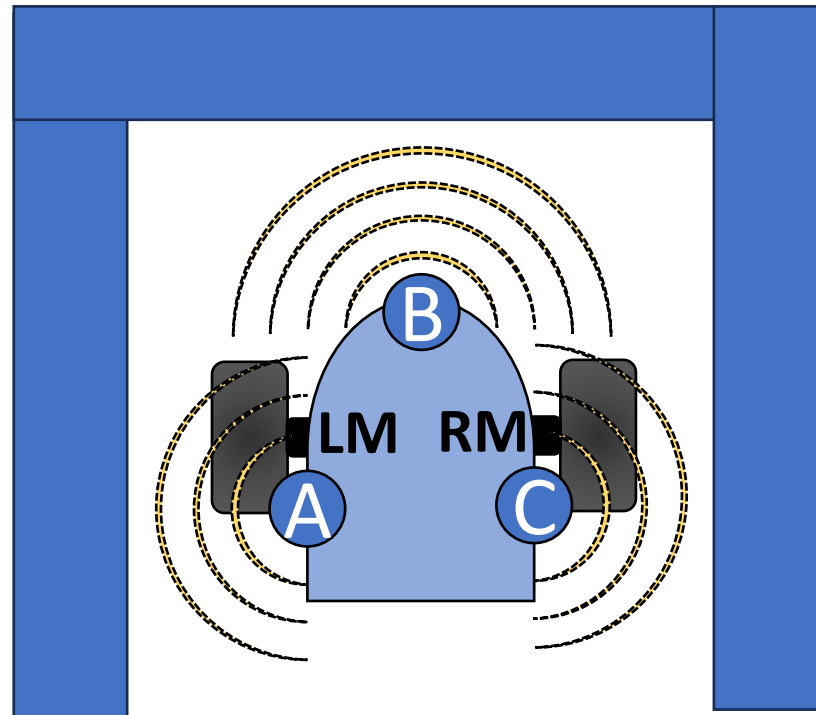
| Truth Table |   |   |    |    |
|-------------|---|---|----|----|
| A           | B | C | LM | RM |
| 0           | 0 | 0 | 1  | 1  |
| 0           | 0 | 1 | 0  | 1  |
| 0           | 1 | 0 | 0  | 1  |
| 0           | 1 | 1 | 0  | 1  |
| 1           | 0 | 0 | 1  | 0  |
| 1           | 0 | 1 | 1  | 1  |
| 1           | 1 | 0 | 1  | 0  |
| 1           | 1 | 1 |    |    |



An obstacle is detected on the left side and in front (A and B sensors give “1”). The robot turns right. The left motor is set to logical true.

# 1 Truth Table

| Truth Table |   |   |    |    |
|-------------|---|---|----|----|
| A           | B | C | LM | RM |
| 0           | 0 | 0 | 1  | 1  |
| 0           | 0 | 1 | 0  | 1  |
| 0           | 1 | 0 | 0  | 1  |
| 0           | 1 | 1 | 0  | 1  |
| 1           | 0 | 0 | 1  | 0  |
| 1           | 0 | 1 | 1  | 1  |
| 1           | 1 | 0 | 1  | 0  |
| 1           | 1 | 1 | 0  | 0  |



An obstacle is detected on both sides and in front (A, B and C sensors give "1").  
The robot stops.

## 2. Karnaugh map – grouping

| Truth Table |    |    |    |
|-------------|----|----|----|
| A           | BC | LM | RM |
| 0           | 00 | 1  | 1  |
| 0           | 01 | 0  | 1  |
| 0           | 10 | 0  | 1  |
| 0           | 11 | 0  | 1  |
| 1           | 00 | 1  | 0  |
| 1           | 01 | 1  | 1  |
| 1           | 10 | 1  | 0  |
| 1           | 11 | 0  | 0  |

| LM Karnaugh map - grouping "1" |     |    |    |    |    |
|--------------------------------|-----|----|----|----|----|
|                                | B C |    |    |    |    |
|                                |     | 00 | 01 | 11 | 10 |
| A                              | 0   | 1  |    |    |    |
|                                | 1   |    |    |    |    |

| RM Karnaugh map - grouping "1" |     |    |    |    |    |
|--------------------------------|-----|----|----|----|----|
|                                | B C |    |    |    |    |
|                                |     | 00 | 01 | 11 | 10 |
| A                              | 0   | 1  |    |    |    |
|                                | 1   |    |    |    |    |

## 2. Karnaugh map – grouping

| Truth Table |    |    |    |
|-------------|----|----|----|
| A           | BC | LM | RM |
| 0           | 00 | 1  | 1  |
| 0           | 01 | 0  | 1  |
| 0           | 10 | 0  | 1  |
| 0           | 11 | 0  | 1  |
| 1           | 00 | 1  | 0  |
| 1           | 01 | 1  | 1  |
| 1           | 10 | 1  | 0  |
| 1           | 11 | 0  | 0  |

| LM Karnaugh map - grouping "1" |   |     |    |    |    |
|--------------------------------|---|-----|----|----|----|
|                                |   | B C |    |    |    |
| A                              |   | 00  | 01 | 11 | 10 |
|                                |   | 0   | 0  |    |    |
|                                | 1 |     |    |    |    |

| RM Karnaugh map - grouping "1" |   |     |    |    |    |
|--------------------------------|---|-----|----|----|----|
|                                |   | B C |    |    |    |
| A                              |   | 00  | 01 | 11 | 10 |
|                                |   | 0   | 1  |    |    |
|                                | 1 |     |    |    |    |

## 2. Karnaugh map – grouping

| Truth Table |    |    |    |
|-------------|----|----|----|
| A           | BC | LM | RM |
| 0           | 00 | 1  | 1  |
| 0           | 01 | 0  | 1  |
| 0           | 10 | 0  | 1  |
| 0           | 11 | 0  | 1  |
| 1           | 00 | 1  | 0  |
| 1           | 01 | 1  | 1  |
| 1           | 10 | 1  | 0  |
| 1           | 11 | 0  | 0  |

| LM Karnaugh map - grouping "1" |   |     |    |    |    |
|--------------------------------|---|-----|----|----|----|
|                                |   | B C |    |    |    |
|                                |   | 00  | 01 | 11 | 10 |
| A                              | 0 | 1   | 0  |    | 0  |
|                                | 1 |     |    |    |    |

| RM Karnaugh map - grouping "1" |   |     |    |    |    |
|--------------------------------|---|-----|----|----|----|
|                                |   | B C |    |    |    |
|                                |   | 00  | 01 | 11 | 10 |
| A                              | 0 | 1   | 1  |    | 1  |
|                                | 1 |     |    |    |    |

## 2. Karnaugh map – grouping

| Truth Table |    |    |    |
|-------------|----|----|----|
| A           | BC | LM | RM |
| 0           | 00 | 1  | 1  |
| 0           | 01 | 0  | 1  |
| 0           | 10 | 0  | 1  |
| 0           | 11 | 0  | 1  |
| 1           | 00 | 1  | 0  |
| 1           | 01 | 1  | 1  |
| 1           | 10 | 1  | 0  |
| 1           | 11 | 0  | 0  |

| LM Karnaugh map -<br>grouping "1" |     |    |    |    |    |
|-----------------------------------|-----|----|----|----|----|
| A                                 | B C |    |    |    |    |
|                                   |     | 00 | 01 | 11 | 10 |
|                                   | 0   | 1  | 0  | 0  | 0  |
|                                   | 1   | 1  | 1  | 0  | 1  |

| RM Karnaugh map -<br>grouping "1" |     |    |    |    |    |
|-----------------------------------|-----|----|----|----|----|
| A                                 | B C |    |    |    |    |
|                                   |     | 00 | 01 | 11 | 10 |
|                                   | 0   | 1  | 1  | 1  | 1  |
|                                   | 1   | 0  | 1  | 0  | 0  |

## Disjunctive Normal Form (DNF)

| $a$ | $b$ | $f(a, b)$ |
|-----|-----|-----------|
| 0   | 0   | 0         |
| 0   | 1   | 1         |
| 1   | 0   | 1         |
| 1   | 1   | 0         |

- Disjunctive Normal Form (DNF) is a disjunction, where a clause is a conjunctive of literals. : function  $f$  is a sum of products  
 $f = \dots (\dots \wedge \dots \wedge \dots) \vee (\dots \wedge \dots \wedge \dots) \vee (\dots \wedge \dots \wedge \dots) \dots$ ,
- Expression in bracket (product) corresponds to one one (“1”)
- In our case:  $f(a, b) = (\bar{a} \wedge b) \vee (a \wedge \bar{b})$ ,
- Decimal description:  $f(a, b) = \sum\{1, 2\}$ .

## The most popular logic gates



| A | NOT A |
|---|-------|
| 0 | 1     |
| 1 | 0     |



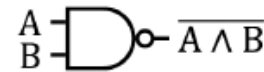
| A | B | A AND B |
|---|---|---------|
| 0 | 0 | 0       |
| 0 | 1 | 0       |
| 1 | 0 | 0       |
| 1 | 1 | 1       |



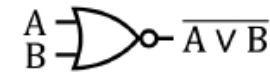
| A | B | A OR B |
|---|---|--------|
| 0 | 0 | 0      |
| 0 | 1 | 1      |
| 1 | 0 | 1      |
| 1 | 1 | 1      |



| A | B | A XOR B |
|---|---|---------|
| 0 | 0 | 0       |
| 0 | 1 | 1       |
| 1 | 0 | 1       |
| 1 | 1 | 0       |



| A | B | A NAND B |
|---|---|----------|
| 0 | 0 | 1        |
| 0 | 1 | 1        |
| 1 | 0 | 1        |
| 1 | 1 | 0        |



| A | B | A NOR B |
|---|---|---------|
| 0 | 0 | 1       |
| 0 | 1 | 0       |
| 1 | 0 | 0       |
| 1 | 1 | 0       |



### 3. Boolean function $f(a,b,c)$ from Karnaugh map:

LM Karnaugh map -  
grouping "1"

|   |   | B C |    |    |    |
|---|---|-----|----|----|----|
| A |   | 00  | 01 | 11 | 10 |
|   | 0 | 1   | 0  | 0  | 0  |
|   | 1 | 1   | 1  | 0  | 1  |

$(\underline{\text{not } a} \text{ and not } b \text{ and not } c) \text{ or } (\underline{a} \text{ and not } b \text{ and not } c) =$   
 $(\underline{\text{not } a} \text{ or } \underline{a}) \text{ and not } b \text{ and not } c = 1 \text{ and not } b \text{ and not } c = \text{not } b \text{ and not } c$

RM Karnaugh map -  
grouping "1"

|   |   | B C |    |    |    |
|---|---|-----|----|----|----|
| A |   | 00  | 01 | 11 | 10 |
|   | 0 | 1   | 1  | 1  | 1  |
|   | 1 | 0   | 1  | 0  | 0  |

### 3. Boolean function $f(a,b,c)$ from Karnaugh map:

LM Karnaugh map -  
grouping "1"

|   |   | B C |    |    |    |
|---|---|-----|----|----|----|
|   |   | 00  | 01 | 11 | 10 |
| A | 0 | 1   | 0  | 0  | 0  |
|   | 1 | 1   | 1  | 0  | 1  |

$(\underline{\text{not } a} \text{ and not } b \text{ and not } c) \text{ or } (\underline{a} \text{ and not } b \text{ and not } c) =$   
 $(\underline{\text{not } a} \text{ or } \underline{a}) \text{ and not } b \text{ and not } c = 1 \text{ and not } b \text{ and not } c = \text{not } b \text{ and not } c$

$(a \text{ and } \underline{\text{not } b} \text{ and not } c) \text{ or } (a \text{ and } \underline{b} \text{ and not } c) =$   
 $a \text{ and } (\underline{\text{not } b} \text{ or } \underline{b}) \text{ and not } c = a \text{ and } 1 \text{ and not } c =$   
 $= a \text{ and not } c$

RM Karnaugh map -  
grouping "1"

|   |   | B C |    |    |    |
|---|---|-----|----|----|----|
|   |   | 00  | 01 | 11 | 10 |
| A | 0 | 1   | 1  | 1  | 1  |
|   | 1 | 0   | 1  | 0  | 0  |

$a \text{ and not } b$

### 3. Boolean function $f(a,b,c)$ from Karnaugh map:

**LM** Karnaugh map - grouping "1"

|   |   | B C |    |    |    |
|---|---|-----|----|----|----|
|   |   | 00  | 01 | 11 | 10 |
| A | 0 | 1   | 0  | 0  | 0  |
|   | 1 | 1   | 1  | 0  | 1  |

**RM** Karnaugh map - grouping "1"

|   |   | B C |    |    |    |
|---|---|-----|----|----|----|
|   |   | 00  | 01 | 11 | 10 |
| A | 0 | 1   | 1  | 1  | 1  |
|   | 1 | 0   | 1  | 0  | 0  |

not b and c

not b and not c

a and not c

a and not b

$(\text{not } a \text{ and not } b \text{ and } \underline{\text{not } c}) \text{ or } (\text{not } a \text{ and not } b \text{ and } \underline{c}) \text{ or}$   
 $(\text{not } a \text{ and } b \text{ and } \underline{c}) \text{ or } (\text{not } a \text{ and } b \text{ and } \underline{\text{not } c}) =$   
 $[\text{not } a \text{ and not } b \text{ and } (\underline{\text{not } c} \text{ or } \underline{c})] \text{ or } [\text{not } a \text{ and } b \text{ and } (\underline{c} \text{ or } \underline{\text{not } c})] =$   
 $[\text{not } a \text{ and not } b \text{ and } 1] \text{ or } [\text{not } a \text{ and } b \text{ and } 1] =$   
 $[\text{not } a \text{ and } \underline{\text{not } b}] \text{ or } [\text{not } a \text{ and } \underline{b}] =$   
 $\text{not } a \text{ and } (\underline{\text{not } b} \text{ or } \underline{b}) = \text{not } a \text{ and } 1 = \mathbf{\text{not } a}$

4. Using Logisim, create LM and RM logic circuits (use OR, AND, NOT gates) and check it.

LM  $f(a,b,c) =$

$= (\text{not } b \text{ and not } c) \text{ or } (a \text{ and not } c) \text{ or } (a \text{ and not } b)$

RM  $f(a,b,c) =$

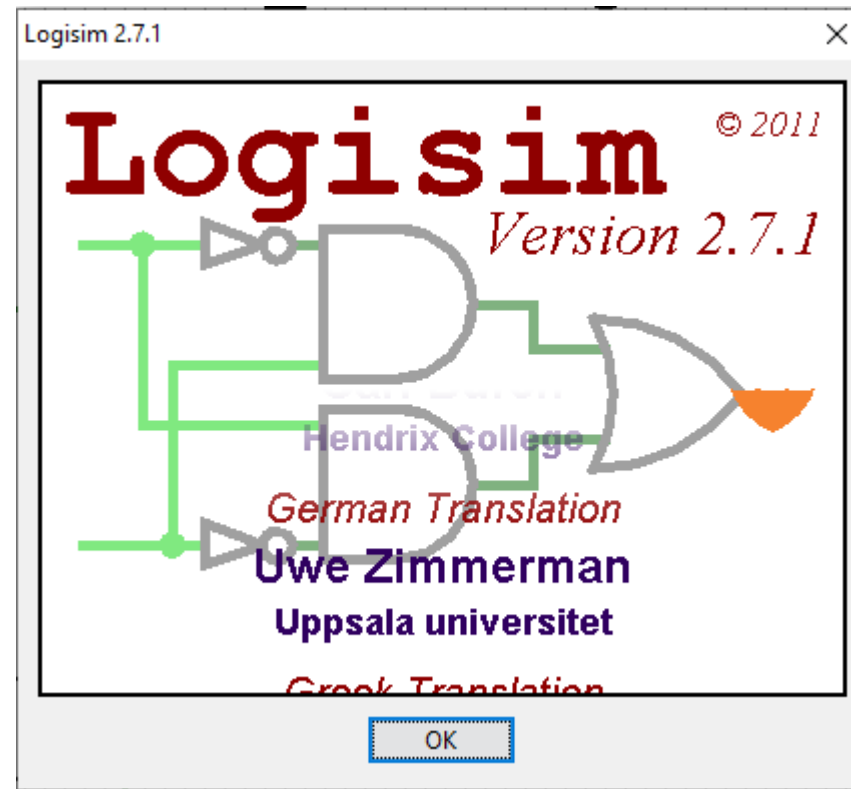
$= (\text{not } a) \text{ or } (\text{not } b \text{ and } c)$

# Start logisim

Icon



Version



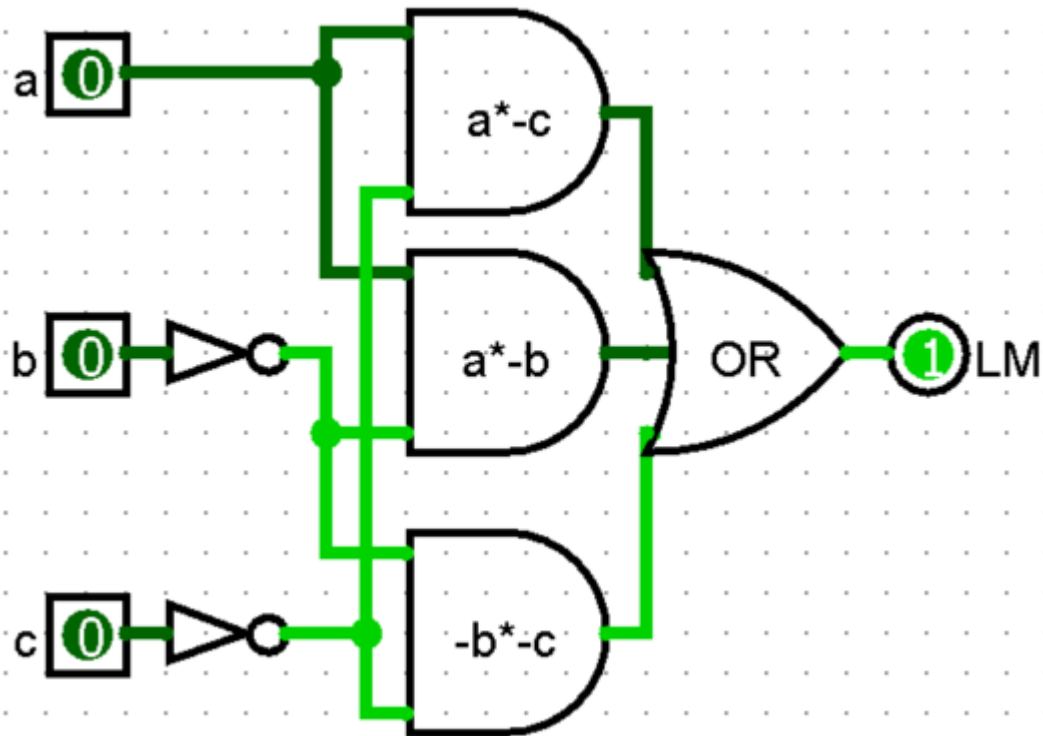
Open : [mmajew/SYC/07 and 08/07 and 08 lab mobile robot.circ](#)

4. Using Logisim, create LM and RM logic circuits (use OR, AND, NOT gates) and check it.

LM  $f(a,b,c) =$

$= (\text{not } b \text{ and not } c) \text{ or } (a \text{ and not } c) \text{ or } (a \text{ and not } b)$

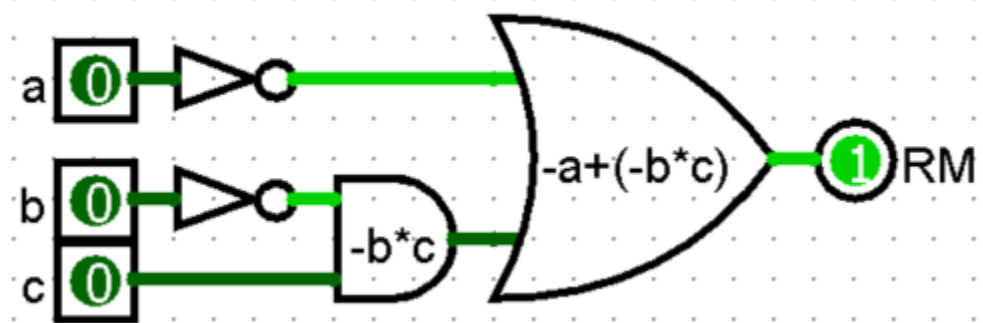
LEFT MOTOR



RM  $f(a,b,c) =$

$= (\text{not } a) \text{ or } (\text{not } b \text{ and } c)$

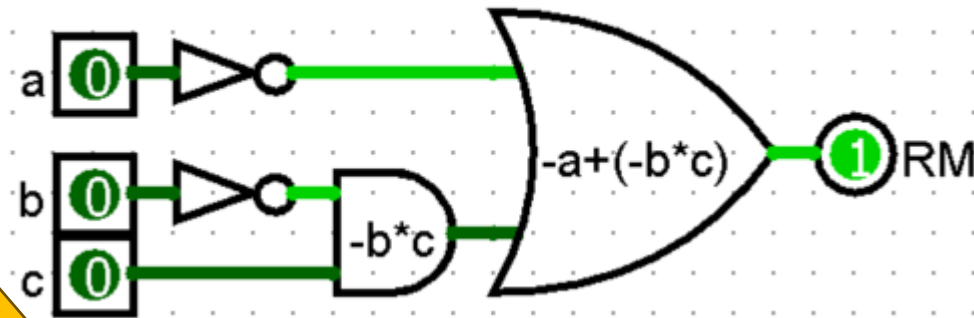
RIGHT MOTOR



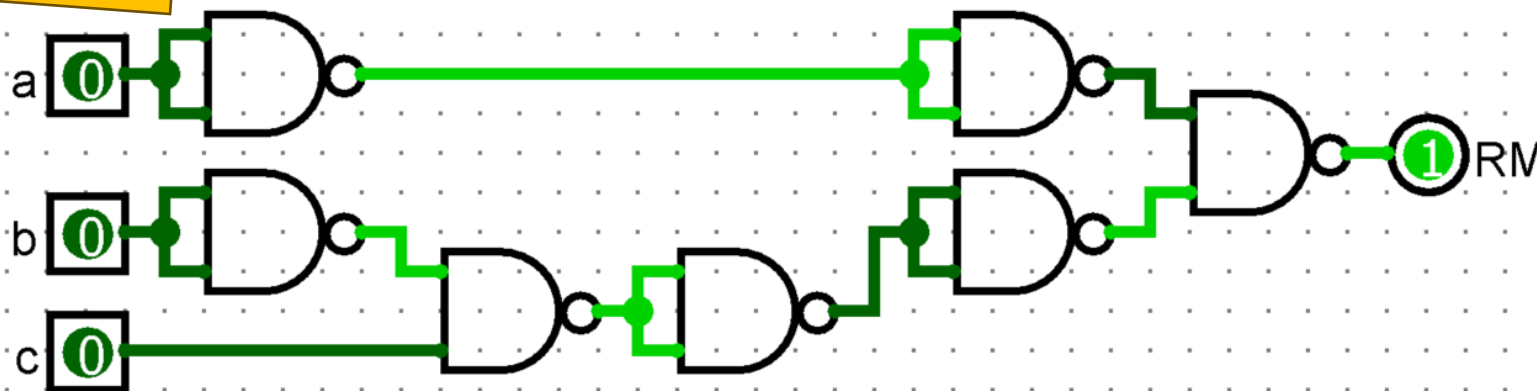
let's do it

5. In Logisim create RM logic circuits. Replace OR, AND, NOT by NAND and check it.

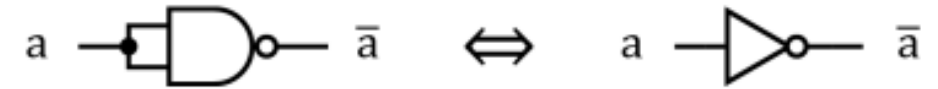
RIGHT MOTOR



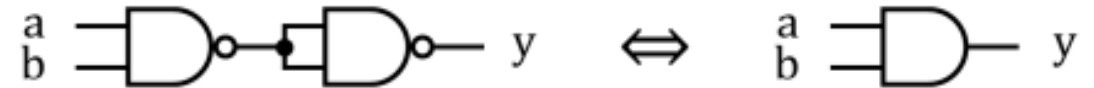
NAND substitution, without  
simplifying/minimizing the function



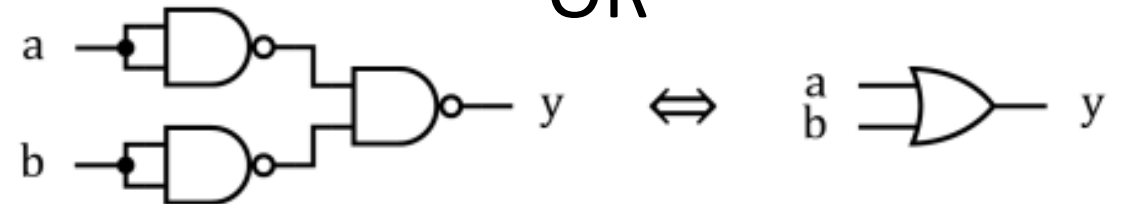
NOT



AND



OR



it's quite complicated!

## 6. Simplify boolean function $f(a,b,c)$ using NOT, NAND gates only:

...with simplifying/minimizing the function

LM  $f(a,b,c) =$

$= (\text{not } b \text{ and not } c) \text{ or } (a \text{ and not } c) \text{ or } (a \text{ and not } b) =$

RM  $f(a,b,c) =$

$= (\text{not } a) \text{ or } (\text{not } b \text{ and } c) =$



## 6. Simplify boolean function $f(a,b,c)$ using NOT, NAND gates only:

...with simplifying/minimizing the function

**Double negation**

$$Q = \text{not}[\text{not}(Q)]$$

$$\text{LM } f(a,b,c) =$$

$$= (\text{not } b \text{ and not } c) \text{ or } (a \text{ and not } c) \text{ or } (a \text{ and not } b) =$$

$$= \text{NOT}\{ \text{NOT}[(\text{not } b \text{ and not } c) \text{ or } (a \text{ and not } c) \text{ or } (a \text{ and not } b)] \}$$

$$\text{RM } f(a,b,c) =$$

$$= (\text{not } a) \text{ or } (\text{not } b \text{ and } c) =$$

$$= \text{NOT}\{ \text{NOT}[(\text{not } a) \text{ or } (\text{not } b \text{ and } c)] \}$$

## 6. Simplify boolean function $f(a,b,c)$ using NOT, NAND gates only:

...with simplifying/minimizing the function

**Double negation**    **De Morgan's law**

$$Q = \text{not}[\text{not}(Q)] \quad \text{not}(P \text{ or } Q) = (\text{not } P) \text{ and } (\text{not } Q)$$

$$\text{not}(P \text{ or } Q \text{ or } R) = (\text{not } P) \text{ and } (\text{not } Q) \text{ and } (\text{not } R)$$

$$\text{LM } f(a,b,c) =$$

$$= (\text{not } b \text{ and not } c) \text{ or } (a \text{ and not } c) \text{ or } (a \text{ and not } b) =$$

$$= \text{NOT}\{ \text{NOT}[(\text{not } b \text{ and not } c) \text{ or } (a \text{ and not } c) \text{ or } (a \text{ and not } b)] \} =$$

$$= \text{NOT}\{ \text{NOT}(\text{not } b \text{ and not } c) \text{ and } \text{NOT}(a \text{ and not } c) \text{ and } \text{NOT}(a \text{ and not } b) \}$$

$$\text{RM } f(a,b,c) =$$

$$= (\text{not } a) \text{ or } (\text{not } b \text{ and } c) =$$

$$= \text{NOT}\{ \text{NOT}[(\text{not } a) \text{ or } (\text{not } b \text{ and } c)] \} =$$

$$= \text{NOT}\{ \text{NOT}(\text{not } a) \text{ and } \text{NOT}(\text{not } b \text{ and } c) \}$$

# 6. Simplify boolean function $f(a,b,c)$ using NOT, NAND gates only:

...with simplifying/minimizing the function

**Double negation**    **De Morgan's law**

$$Q = \text{not}[\text{not}(Q)] \quad \text{not}(P \text{ or } Q) = (\text{not } P) \text{ and } (\text{not } Q)$$

$$\text{not}(P \text{ or } Q \text{ or } R) = (\text{not } P) \text{ and } (\text{not } Q) \text{ and } (\text{not } R)$$

$$\text{LM } f(a,b,c) =$$

$$= (\text{not } b \text{ and not } c) \text{ or } (a \text{ and not } c) \text{ or } (a \text{ and not } b) =$$

$$= \text{NOT}\{ \text{NOT}[(\text{not } b \text{ and not } c) \text{ or } (a \text{ and not } c) \text{ or } (a \text{ and not } b)] \} =$$

$$= \text{NOT}\{ \text{NOT}(\text{not } b \text{ and not } c) \text{ and } \text{NOT}(a \text{ and not } c) \text{ and } \text{NOT}(a \text{ and not } b) \} =$$

$$= \text{NOT}\{ (\text{not } b \text{ Nand not } c) \text{ and } (a \text{ Nand not } c) \text{ and } (a \text{ Nand not } b) \}$$

$$\text{RM } f(a,b,c) =$$

$$= (\text{not } a) \text{ or } (\text{not } b \text{ and } c) =$$

$$= \text{NOT}\{ \text{NOT}[(\text{not } a) \text{ or } (\text{not } b \text{ and } c)] \} =$$

$$= \text{NOT}\{ \text{NOT}(\text{not } a) \text{ and } \text{NOT}(\text{not } b \text{ and } c) \} =$$

$$= \text{NOT}\{ a \text{ and } (\text{not } b \text{ Nand } c) \}$$

## 6. Simplify boolean function $f(a,b,c)$ using NOT, NAND gates only:

...with simplifying/minimizing the function

**Double negation**    **De Morgan's law**

$$Q = \text{not}[\text{not}(Q)] \quad \text{not}(P \text{ or } Q) = (\text{not } P) \text{ and } (\text{not } Q)$$

$$\text{not}(P \text{ or } Q \text{ or } R) = (\text{not } P) \text{ and } (\text{not } Q) \text{ and } (\text{not } R)$$

$$\text{LM } f(a,b,c) =$$

$$= (\text{not } b \text{ and not } c) \text{ or } (a \text{ and not } c) \text{ or } (a \text{ and not } b) =$$

$$= \text{NOT}\{ \text{NOT}[(\text{not } b \text{ and not } c) \text{ or } (a \text{ and not } c) \text{ or } (a \text{ and not } b)] \} =$$

$$= \text{NOT}\{ \text{NOT}(\text{not } b \text{ and not } c) \text{ and } \text{NOT}(a \text{ and not } c) \text{ and } \text{NOT}(a \text{ and not } b) \} =$$

$$= \text{NOT}\{ (\text{not } b \text{ Nand not } c) \text{ and } (a \text{ Nand not } c) \text{ and } (a \text{ Nand not } b) \} =$$

$$= (\text{not } b \text{ Nand not } c) \text{ Nand } (a \text{ Nand not } c) \text{ Nand}$$

$$(a \text{ Nand not } b)$$

$$\text{RM } f(a,b,c) =$$

$$= (\text{not } a) \text{ or } (\text{not } b \text{ and } c) =$$

$$= \text{NOT}\{ \text{NOT}[(\text{not } a) \text{ or } (\text{not } b \text{ and } c)] \} =$$

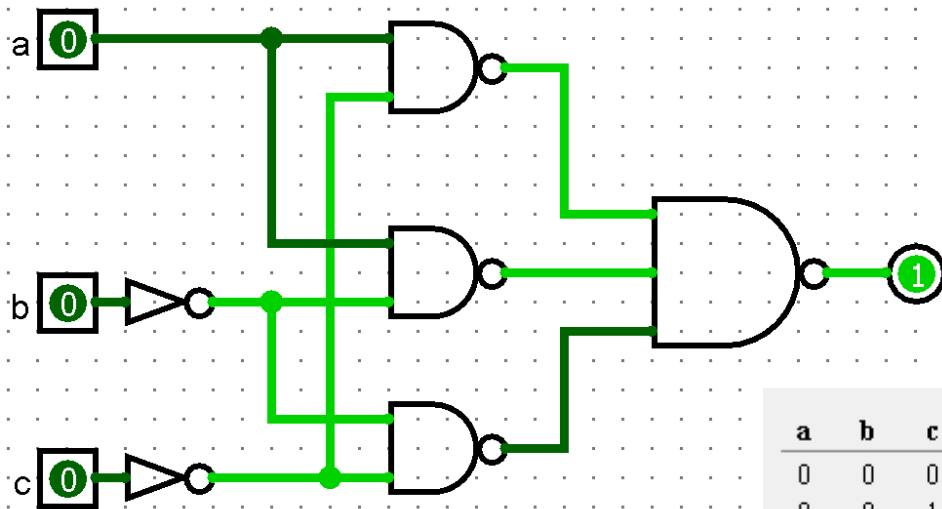
$$= \text{NOT}\{ \text{NOT}(\text{not } a) \text{ and } \text{NOT}(\text{not } b \text{ and } c) \} =$$

$$= \text{NOT}\{ a \text{ and } (\text{not } b \text{ Nand } c) \} =$$

$$= a \text{ Nand } (\text{not } b \text{ Nand } c)$$

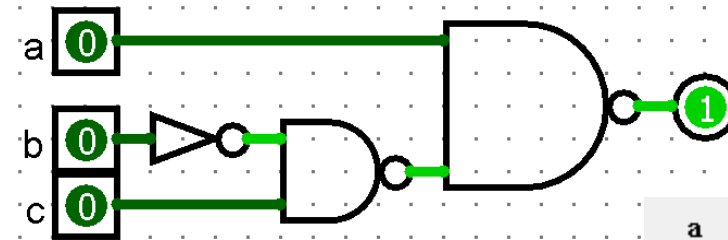
# 7. Using Logisim, create above LM and RM logic circuits (NAND, NOT gates) and check it.

LM  $f(a,b,c) = (\text{not } b \text{ Nand not } c) \text{ Nand } (a \text{ Nand not } c)$   
 $\text{Nand } (a \text{ Nand not } b)$



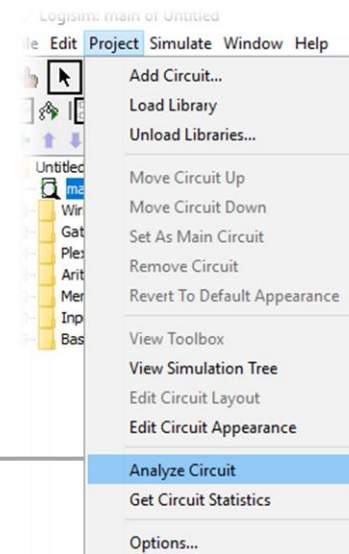
| a | b | c | x |
|---|---|---|---|
| 0 | 0 | 0 | 1 |
| 0 | 0 | 1 | 0 |
| 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 0 |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 1 | 1 |
| 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 0 |

RM  $f(a,b,c) = a \text{ Nand } (\text{not } b \text{ Nand } c)$



| a | b | c | x |
|---|---|---|---|
| 0 | 0 | 0 | 1 |
| 0 | 0 | 1 | 1 |
| 0 | 1 | 0 | 1 |
| 0 | 1 | 1 | 1 |
| 1 | 0 | 0 | 0 |
| 1 | 0 | 1 | 1 |
| 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 0 |

let's do it



Student number

Number of points

1. Solve 1.B to 1.F tasks

1.A Truth table

| a | b | c | f(a,b,c) |
|---|---|---|----------|
| 0 | 0 | 0 |          |
| 0 | 0 | 1 |          |
| 0 | 1 | 0 |          |
| 0 | 1 | 1 |          |
| 1 | 0 | 0 |          |
| 1 | 0 | 1 |          |
| 1 | 1 | 0 |          |
| 1 | 1 | 1 |          |

1.B Karnaugh map - grouping "1"

|   |   | bc |    |    |    |
|---|---|----|----|----|----|
|   |   | 00 | 01 | 11 | 10 |
| a | 0 |    |    |    |    |
|   | 1 |    |    |    |    |

independent work



1.C Boolean function f(a,b,c) from Karnaugh map:

f(a,b,c) =

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Student number

Number of points

1. Solve 1.B to 1.F tasks

1.A Truth table

| a | b | c | f(a,b,c) |
|---|---|---|----------|
| 0 | 0 | 0 | 1        |
| 0 | 0 | 1 | 1        |
| 0 | 1 | 0 | 0        |
| 0 | 1 | 1 | 1        |
| 1 | 0 | 0 | 0        |
| 1 | 0 | 1 | 1        |
| 1 | 1 | 0 | 0        |
| 1 | 1 | 1 | 0        |

1.B Karnaugh map - grouping "1"

|   |   | bc |    |    |    |
|---|---|----|----|----|----|
|   |   | 00 | 01 | 11 | 10 |
| a | 0 |    |    |    |    |
|   | 1 |    |    |    |    |

1.C Boolean function  $f(a,b,c)$  from Karnaugh map:

$f(a,b,c) =$

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1

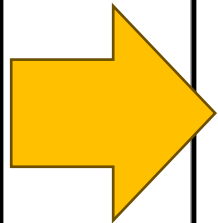
Student number

Number of points

1. Solve 1.B to 1.F tasks

1.A Truth table

| a | b | c | f(a,b,c) |
|---|---|---|----------|
| 0 | 0 | 0 | 1        |
| 0 | 0 | 1 | 1        |
| 0 | 1 | 0 | 0        |
| 0 | 1 | 1 | 1        |
| 1 | 0 | 0 | 0        |
| 1 | 0 | 1 | 1        |
| 1 | 1 | 0 | 0        |
| 1 | 1 | 1 | 0        |



1.B Karnaugh map - grouping "1"

|   |   | bc |    |    |    |
|---|---|----|----|----|----|
|   |   | 00 | 01 | 11 | 10 |
| a | 0 |    |    |    |    |
|   | 1 |    |    |    |    |

1.C Boolean function f(a,b,c) from Karnaugh map:

f(a,b,c) =

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1



Student number

Number of points

1. Solve 1.B to 1.F tasks

1.A Truth table

| a | b | c | f(a,b,c) |
|---|---|---|----------|
| 0 | 0 | 0 | 1        |
| 0 | 0 | 1 | 1        |
| 0 | 1 | 0 | 0        |
| 0 | 1 | 1 | 1        |
| 1 | 0 | 0 | 0        |
| 1 | 0 | 1 | 1        |
| 1 | 1 | 0 | 0        |
| 1 | 1 | 1 | 0        |

1.B Karnaugh map - grouping "1"

|   |   | bc |    |    |    |
|---|---|----|----|----|----|
|   |   | 00 | 01 | 11 | 10 |
| a | 0 | 1  |    |    |    |
|   | 1 |    |    |    |    |

1.C Boolean function f(a,b,c) from Karnaugh map:

f(a,b,c) =

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**1**

Student number

Number of points

1. Solve 1.B to 1.F tasks

1.A Truth table

| a | b | c | f(a,b,c) |
|---|---|---|----------|
| 0 | 0 | 0 | 1        |
| 0 | 0 | 1 | 1        |
| 0 | 1 | 0 | 0        |
| 0 | 1 | 1 | 1        |
| 1 | 0 | 0 | 0        |
| 1 | 0 | 1 | 1        |
| 1 | 1 | 0 | 0        |
| 1 | 1 | 1 | 0        |

1.B Karnaugh map - grouping "1"

|   |   | bc |    |    |    |
|---|---|----|----|----|----|
|   |   | 00 | 01 | 11 | 10 |
| a | 0 | 1  | 1  |    |    |
|   | 1 |    |    |    |    |

1.C Boolean function f(a,b,c) from Karnaugh map:

f(a,b,c) =

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**1**

Student number

Number of points

1. Solve 1.B to 1.F tasks

1.A Truth table

| a | b | c | f(a,b,c) |
|---|---|---|----------|
| 0 | 0 | 0 | 1        |
| 0 | 0 | 1 | 1        |
| 0 | 1 | 0 | 0        |
| 0 | 1 | 1 | 1        |
| 1 | 0 | 0 | 0        |
| 1 | 0 | 1 | 1        |
| 1 | 1 | 0 | 0        |
| 1 | 1 | 1 | 0        |

1.B Karnaugh map - grouping "1"

|   |   | bc |    |    |    |
|---|---|----|----|----|----|
|   |   | 00 | 01 | 11 | 10 |
| a | 0 | 1  | 1  |    | 0  |
|   | 1 |    |    |    |    |

1

1.C Boolean function f(a,b,c) from Karnaugh map:

f(a,b,c) =

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Student number

Number of points

1. Solve 1.B to 1.F tasks

1.A Truth table

| a | b | c | f(a,b,c) |
|---|---|---|----------|
| 0 | 0 | 0 | 1        |
| 0 | 0 | 1 | 1        |
| 0 | 1 | 0 | 0        |
| 0 | 1 | 1 | 1        |
| 1 | 0 | 0 | 0        |
| 1 | 0 | 1 | 1        |
| 1 | 1 | 0 | 0        |
| 1 | 1 | 1 | 0        |

1.B Karnaugh map - grouping "1"

|   |   | bc |    |    |    |
|---|---|----|----|----|----|
|   |   | 00 | 01 | 11 | 10 |
| a | 0 | 1  | 1  | 1  | 0  |
|   | 1 |    |    |    |    |

1.C Boolean function f(a,b,c) from Karnaugh map:

f(a,b,c) =

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1

Student number

Number of points

1. Solve 1.B to 1.F tasks

1.A Truth table

| a | b | c | f(a,b,c) |
|---|---|---|----------|
| 0 | 0 | 0 | 1        |
| 0 | 0 | 1 | 1        |
| 0 | 1 | 0 | 0        |
| 0 | 1 | 1 | 1        |
| 1 | 0 | 0 | 0        |
| 1 | 0 | 1 | 1        |
| 1 | 1 | 0 | 0        |
| 1 | 1 | 1 | 0        |

1.B Karnaugh map - grouping "1"

|   |   | bc |    |    |    |
|---|---|----|----|----|----|
|   |   | 00 | 01 | 11 | 10 |
| a | 0 | 1  | 1  | 1  | 0  |
|   | 1 | 0  |    |    |    |

1.C Boolean function f(a,b,c) from Karnaugh map:

f(a,b,c) =

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1

Student number

Number of points

1. Solve 1.B to 1.F tasks

1.A Truth table

| a | b | c | f(a,b,c) |
|---|---|---|----------|
| 0 | 0 | 0 | 1        |
| 0 | 0 | 1 | 1        |
| 0 | 1 | 0 | 0        |
| 0 | 1 | 1 | 1        |
| 1 | 0 | 0 | 0        |
| 1 | 0 | 1 | 1        |
| 1 | 1 | 0 | 0        |
| 1 | 1 | 1 | 0        |

1.B Karnaugh map - grouping "1"

|   |   | bc |    |    |    |
|---|---|----|----|----|----|
|   |   | 00 | 01 | 11 | 10 |
| a | 0 | 1  | 1  | 1  | 0  |
|   | 1 | 0  | 1  |    |    |

1.C Boolean function f(a,b,c) from Karnaugh map:

f(a,b,c) =

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1

Student number

Number of points

1. Solve 1.B to 1.F tasks

1.A Truth table

| a | b | c | f(a,b,c) |
|---|---|---|----------|
| 0 | 0 | 0 | 1        |
| 0 | 0 | 1 | 1        |
| 0 | 1 | 0 | 0        |
| 0 | 1 | 1 | 1        |
| 1 | 0 | 0 | 0        |
| 1 | 0 | 1 | 1        |
| 1 | 1 | 0 | 0        |
| 1 | 1 | 1 | 0        |

1.B Karnaugh map - grouping "1"

|   |   | bc |    |    |    |
|---|---|----|----|----|----|
|   |   | 00 | 01 | 11 | 10 |
| a | 0 | 1  | 1  | 1  | 0  |
|   | 1 | 0  | 1  |    | 0  |

1.C Boolean function f(a,b,c) from Karnaugh map:

f(a,b,c) =

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1

Student number

Number of points

1. Solve 1.B to 1.F tasks

1.A Truth table

| a | b | c | f(a,b,c) |
|---|---|---|----------|
| 0 | 0 | 0 | 1        |
| 0 | 0 | 1 | 1        |
| 0 | 1 | 0 | 0        |
| 0 | 1 | 1 | 1        |
| 1 | 0 | 0 | 0        |
| 1 | 0 | 1 | 1        |
| 1 | 1 | 0 | 0        |
| 1 | 1 | 1 | 0        |

1.B Karnaugh map - grouping "1"

|   |   | bc |    |    |    |
|---|---|----|----|----|----|
|   |   | 00 | 01 | 11 | 10 |
| a | 0 | 1  | 1  | 1  | 0  |
|   | 1 | 0  | 1  | 0  | 0  |

1.C Boolean function f(a,b,c) from Karnaugh map:

f(a,b,c) =

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1



Student number

Number of points

1. Solve 1.B to 1.F tasks

1.A Truth table

| a | b | c | f(a,b,c) |
|---|---|---|----------|
| 0 | 0 | 0 | 1        |
| 0 | 0 | 1 | 1        |
| 0 | 1 | 0 | 0        |
| 0 | 1 | 1 | 1        |
| 1 | 0 | 0 | 0        |
| 1 | 0 | 1 | 1        |
| 1 | 1 | 0 | 0        |
| 1 | 1 | 1 | 0        |

1.B Karnaugh map - grouping "1"

|   |   | bc |    |    |    |
|---|---|----|----|----|----|
|   |   | 00 | 01 | 11 | 10 |
| a | 0 | 1  | 1  | 1  | 0  |
|   | 1 | 0  | 1  | 0  | 0  |

(not a and not b and not c) or (not a and not b and c)

not a and not b and (not c or c) =  
not a and not b and 1 =  
not a and not b

1.C Boolean function f(a,b,c) from Karnaugh map:

f(a,b,c) = (not a and not b) or

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Student number

Number of points

1. Solve 1.B to 1.F tasks

1.A Truth table

| a | b | c | f(a,b,c) |
|---|---|---|----------|
| 0 | 0 | 0 | 1        |
| 0 | 0 | 1 | 1        |
| 0 | 1 | 0 | 0        |
| 0 | 1 | 1 | 1        |
| 1 | 0 | 0 | 0        |
| 1 | 0 | 1 | 1        |
| 1 | 1 | 0 | 0        |
| 1 | 1 | 1 | 0        |

1.B Karnaugh map - grouping "1"

|   |   | bc |    |    |    |
|---|---|----|----|----|----|
|   |   | 00 | 01 | 11 | 10 |
| a | 0 | 1  | 1  | 1  | 0  |
|   | 1 | 0  | 1  | 0  | 0  |

1

1.C Boolean function f(a,b,c) from Karnaugh map:

$$f(a,b,c) = \text{(not a and not b) or (not a and c) or (not b and c)}$$

| 1.B Karnaugh map - grouping "1" |   |    |    |    |    |
|---------------------------------|---|----|----|----|----|
|                                 |   | bc |    |    |    |
|                                 |   | 00 | 01 | 11 | 10 |
| a                               | 0 | 1  | 0  | 0  | 1  |
|                                 | 1 | 1  | 0  | 1  | 0  |

1.C Boolean function  $f(a,b,c)$  from Karnaugh map:

$$f(a,b,c) = \text{(not a and not c) or (not b and not c) or (a and b and c)}$$

| 1.B Karnaugh map - grouping "1" |   |    |    |    |    |
|---------------------------------|---|----|----|----|----|
|                                 |   | bc |    |    |    |
|                                 |   | 00 | 01 | 11 | 10 |
| a                               | 0 | 0  | 1  | 1  | 0  |
|                                 | 1 | 1  | 1  | 1  | 0  |

1.C Boolean function  $f(a,b,c)$  from Karnaugh map:

$$f(a,b,c) = \text{(c) or (a and not b);}$$

$$\begin{aligned}
 &[\text{not a and } \underline{\text{not b}} \text{ and c}] \text{ or } [\text{not a and } \underline{\text{b}} \text{ and c}] \text{ or } [\text{a and } \mathbf{\text{not b}} \text{ and c}] \text{ or } [\text{a and } \mathbf{\text{b}} \text{ and c}] = \\
 &[\text{not a and c and } (\underline{\text{not b or b}})] \text{ or } [\text{a and c and } (\mathbf{\text{not b or b}})] = \\
 &[\text{not a and c}] \text{ or } [\text{a and c}] = \\
 &\text{c and (not a and a)} = \\
 &\text{c}
 \end{aligned}$$

Student number

Number of points

1. Solve 1.B to 1.F tasks

1.A Truth table

| a | b | c | f(a,b,c) |
|---|---|---|----------|
| 0 | 0 | 0 | 1        |
| 0 | 0 | 1 | 1        |
| 0 | 1 | 0 | 0        |
| 0 | 1 | 1 | 1        |
| 1 | 0 | 0 | 0        |
| 1 | 0 | 1 | 1        |
| 1 | 1 | 0 | 0        |
| 1 | 1 | 1 | 0        |

1.B Karnaugh map - grouping "1"

|   |   | bc |    |    |    |
|---|---|----|----|----|----|
|   |   | 00 | 01 | 11 | 10 |
| a | 0 | 1  | 1  | 1  | 0  |
|   | 1 | 0  | 1  | 0  | 0  |

1

1.C Boolean function f(a,b,c) from Karnaugh map:

$$f(a,b,c) = \text{ (not a and not b) or (not a and c) or (not b and c) }$$

1.D Boolean function  $f(a,b,c)$  using NAND gates only:

$(\text{not } a \text{ and not } b) \text{ or } (\text{not } a \text{ and } c) \text{ or } (\text{not } b \text{ and } c) =$

CAT or DOG or HOUSE

1.D Boolean function f(a,b,c) using NAND gates only:

(not a and not b) or (not a and c) or (not b and c)=

CAT or DOG or HOUSE =

*double negation*       $\longrightarrow$       Not{ Not[CAT or DOG or HOUSE] }

*$Q = \text{not}[\text{not}(Q)]$*

1.D Boolean function f(a,b,c) using NAND gates only:

(not a and not b) or (not a and c) or (not b and c)=

CAT or DOG or HOUSE =

*double negation*

$Q = \text{not}[\text{not}(Q)]$

→ Not{ Not[CAT or DOG or HOUSE] }=

*De Morgan's law*

$\text{not}(P \text{ or } Q) = (\text{not } P) \text{ and } (\text{not } Q)$

$\text{not}(P \text{ or } Q \text{ or } R) =$

$(\text{not } P) \text{ and } (\text{not } Q) \text{ and } (\text{not } R)$

→ Not{ (not CAT) and (not DOG) and (not HOUSE) }

1.D Boolean function  $f(a,b,c)$  using NAND gates only:

(not a and not b) or (not a and c) or (not b and c) =

CAT or DOG or HOUSE =

*double negation*

$Q = \text{not}[\text{not}(Q)]$

→ Not{ Not[CAT or DOG or HOUSE] } =

*De Morgan's law*

$\text{not}(P \text{ or } Q) = (\text{not } P) \text{ and } (\text{not } Q)$

→ Not{ (not CAT) and (not DOG) and (not HOUSE) } =

Not{ (not CAT) and (not DOG) and (not HOUSE) } =

(not CAT) **Nand** (not DOG) **Nand** (not HOUSE)



1.D Boolean function f(a,b,c) using NAND gates only:

(not a and not b) or (not a and c) or (not b and c) =

CAT or DOG or HOUSE =

*double negation*

$Q = \text{not}[\text{not}(Q)]$

→ Not{ Not[CAT or DOG or HOUSE] } =

*De Morgan's law*

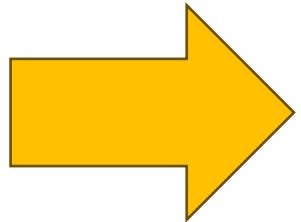
$\text{not}(P \text{ or } Q) = (\text{not } P) \text{ and } (\text{not } Q)$

→ Not{ (not CAT) and (not DOG) and (not HOUSE) } =

Not{ (not CAT) and (not DOG) and (not HOUSE) } =

(not CAT) Nand (not DOG) Nand (not HOUSE) =

(not (not a and not b) ) Nand (not (not a and c) ) Nand (not (not b and c)) =



(not a Nand not b) Nand (not a Nand c) Nand (not b Nand c)

Boolean function using Not and Nand gates

1.E Using Logisim, create a logic circuit and check it.

| 1.A Truth table |   |   |          |
|-----------------|---|---|----------|
| a               | b | c | f(a,b,c) |
| 0               | 0 | 0 | 1        |
| 0               | 0 | 1 | 1        |
| 0               | 1 | 0 | 0        |
| 0               | 1 | 1 | 1        |
| 1               | 0 | 0 | 0        |
| 1               | 0 | 1 | 1        |
| 1               | 1 | 0 | 0        |
| 1               | 1 | 1 | 0        |