

# Digital systems and basics of electronics - SYC

07 Introduction to digital systems

08 Minimization of Boolean functions

# Boolean functions

## Boolean functions (1-true, 0-false)

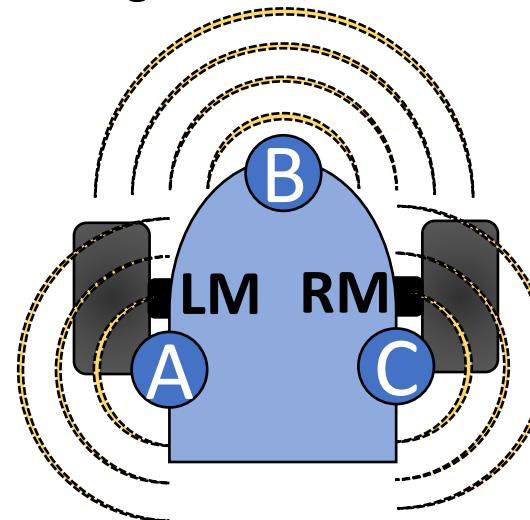
are useful in designing algorithms such as the movement of a robot.

We can create control logic that dictates a mobile robot's response to specific situations, like detecting obstacles, changing direction, or stopping at the appropriate moment.

3 distance sensor located:

- A. On left side Ⓛ
- B. In front of Ⓜ
- C. On right side Ⓝ

When obstacle is detected sensor gives logical “1”, otherwise gives “0”



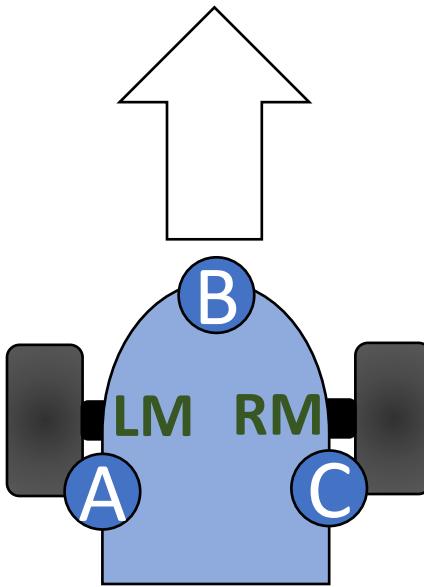
The moving platform uses differential drive:  
**left motor LM,**  
**right motor RM.**

Engine is on when logical “1” control signal is applied.

Robot should move without any collision. When robot could not continue movement should stop ( $LM=RM=0$ ).

# 1 Truth Table

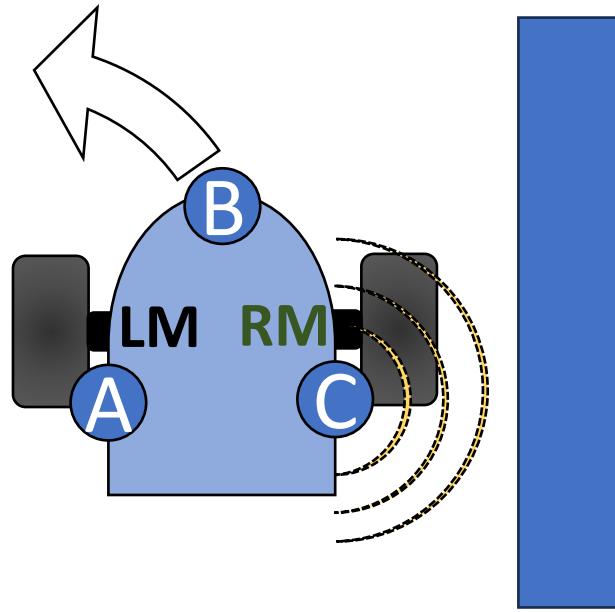
Truth Table				
A	B	C	LM	RM
0	0	0	1	1
0	0	1		
0	1	0		
0	1	1		
1	0	0		
1	0	1		
1	1	0		
1	1	1		



Obstacle is not detected (sensor gives “0”).  
The robot moves forward. The left motor  
and right motor are set to logical true.

# 1 Truth Table

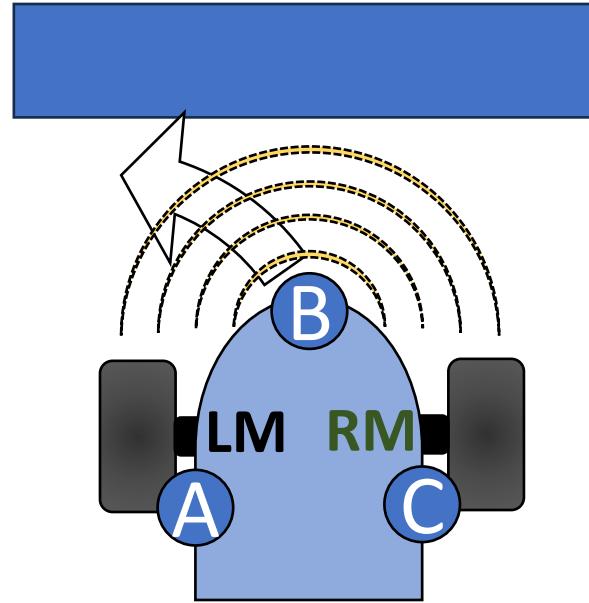
Truth Table			LM	RM
A	B	C		
0	0	0	1	1
0	0	1	0	1
0	1	0		
0	1	1		
1	0	0		
1	0	1		
1	1	0		
1	1	1		



An obstacle is detected on the right side  
(C sensor gives “1”).  
The robot turns left.  
The right motor is set to logical true.

# 1 Truth Table

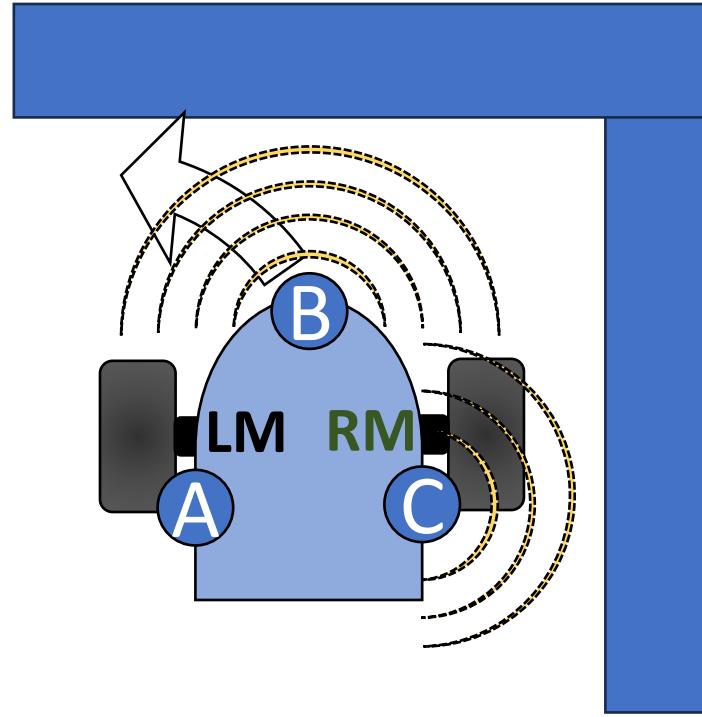
Truth Table				
A	B	C	LM	RM
0	0	0	1	1
0	0	1	0	1
0	1	0	0	1
0	1	1		
1	0	0		
1	0	1		
1	1	0		
1	1	1		



An obstacle is detected in front  
(B sensor gives “1”).  
The robot turns left.  
The right motor is set to logical true.

# 1 Truth Table

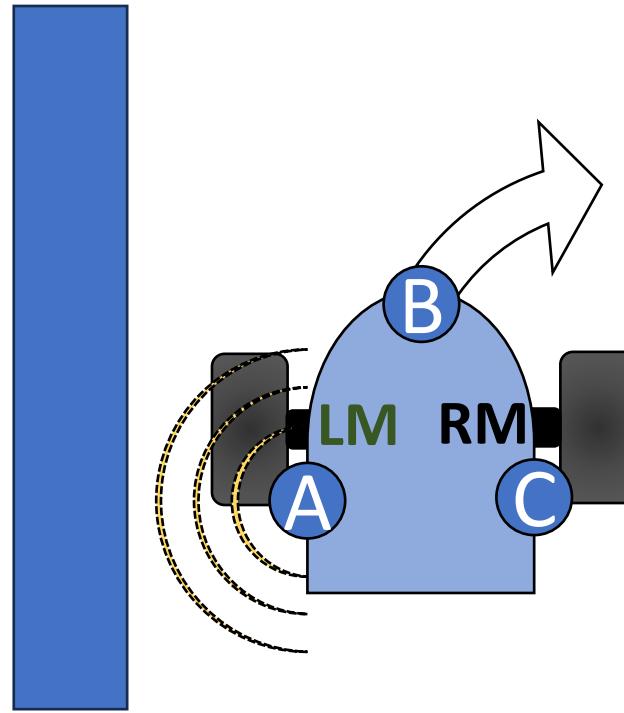
Truth Table				
A	B	C	LM	RM
0	0	0	1	1
0	0	1	0	1
0	1	0	0	1
0	1	1	0	1
1	0	0		
1	0	1		
1	1	0		
1	1	1		



An obstacle is detected in front and on the right side (B and C sensors give “1”).  
The robot turns left.  
The right motor is set to logical true.

# 1 Truth Table

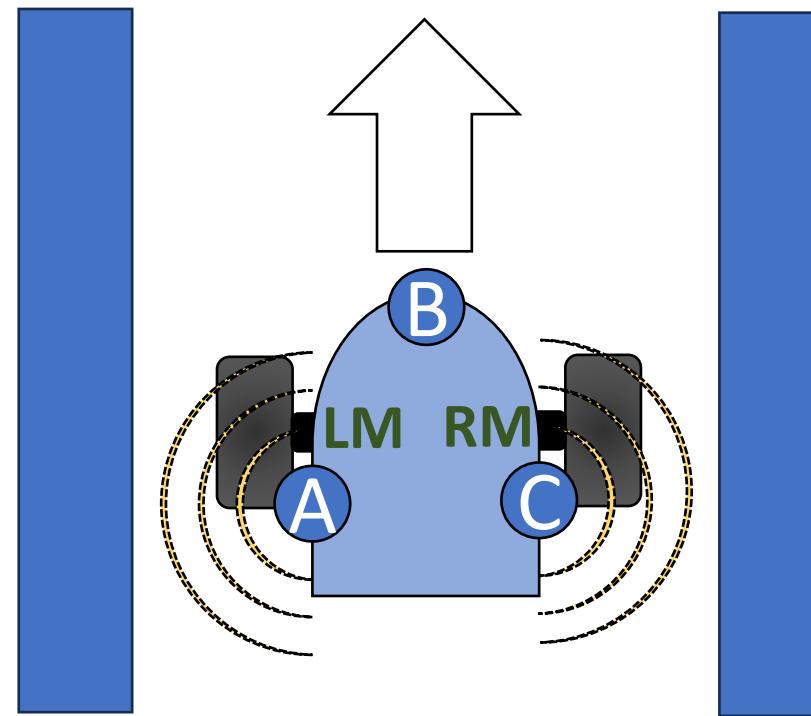
Truth Table				
A	B	C	LM	RM
0	0	0	1	1
0	0	1	0	1
0	1	0	0	1
0	1	1	0	1
1	0	0	1	0
1	0	1		
1	1	0		
1	1	1		



An obstacle is detected on the left side  
(A sensor gives “1”).  
The robot turns right.  
The left motor is set to logical true.

# 1 Truth Table

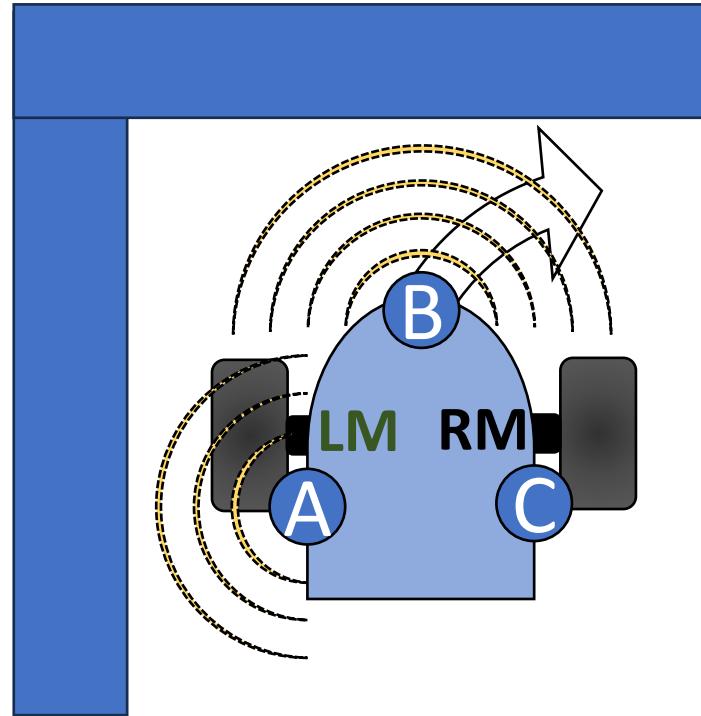
Truth Table				
A	B	C	LM	RM
0	0	0	1	1
0	0	1	0	1
0	1	0	0	1
0	1	1	0	1
1	0	0	1	0
1	0	1	1	1
1	1	0		
1	1	1		



An obstacle is detected on both sides (A and C sensors give “1”).  
The robot moves forward.  
Both motors are set to logical true.

# 1 Truth Table

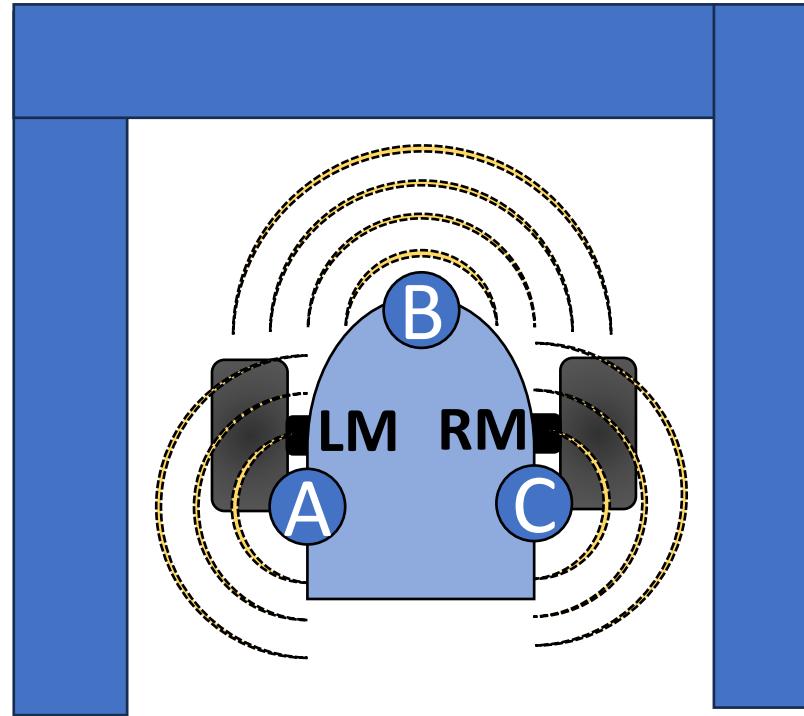
Truth Table				
A	B	C	LM	RM
0	0	0	1	1
0	0	1	0	1
0	1	0	0	1
0	1	1	0	1
1	0	0	1	0
1	0	1	1	1
1	1	0	1	0
1	1	1		



An obstacle is detected on the left side and in front (A and B sensors give “1”). The robot turns right.  
The left motor is set to logical true.

# 1 Truth Table

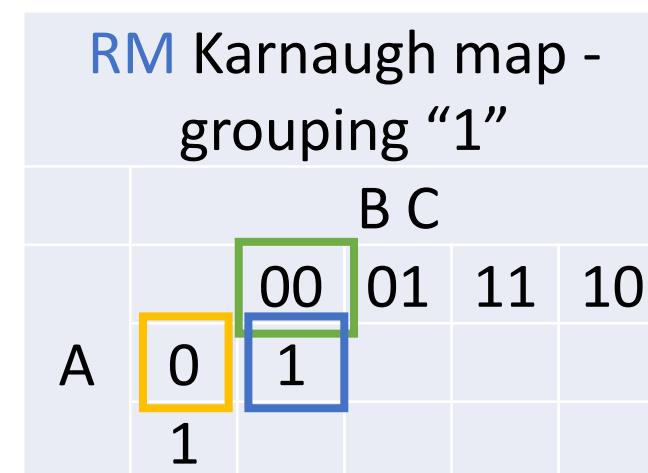
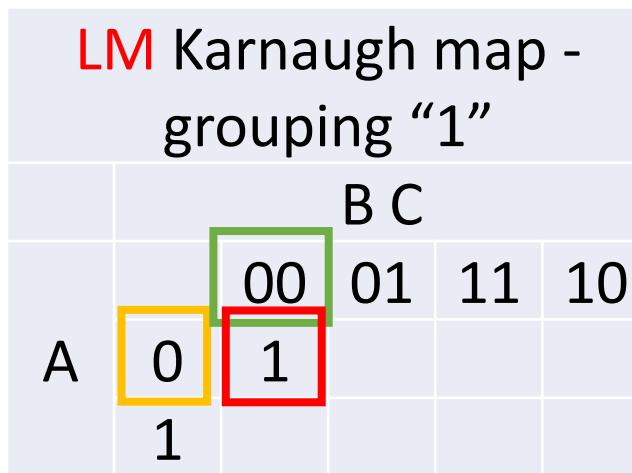
Truth Table				
A	B	C	LM	RM
0	0	0	1	1
0	0	1	0	1
0	1	0	0	1
0	1	1	0	1
1	0	0	1	0
1	0	1	1	1
1	1	0	1	0
1	1	1	0	0



An obstacle is detected on both sides and in front (A, B and C sensors give “1”).  
The robot stops.

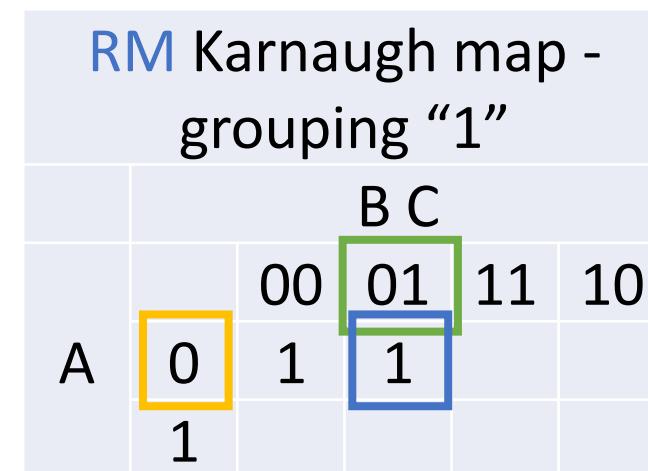
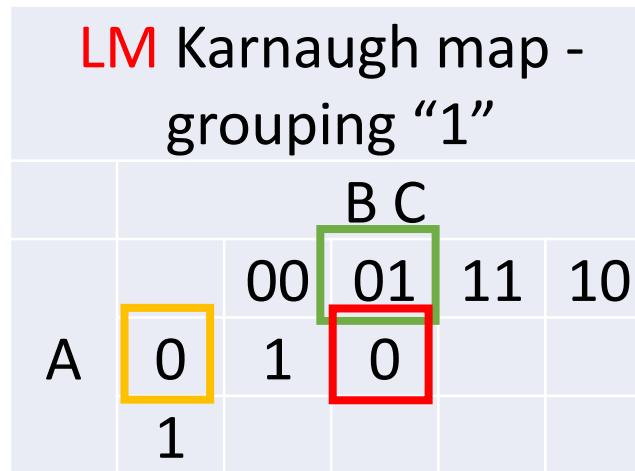
## 2. Karnaugh map – grouping

Truth Table			
A	BC	LM	RM
0	00	1	1
0	01	0	1
0	10	0	1
0	11	0	1
1	00	1	0
1	01	1	1
1	10	1	0
1	11	0	0



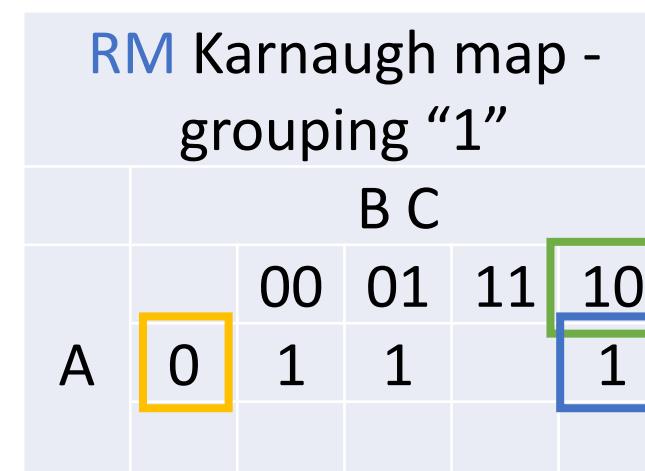
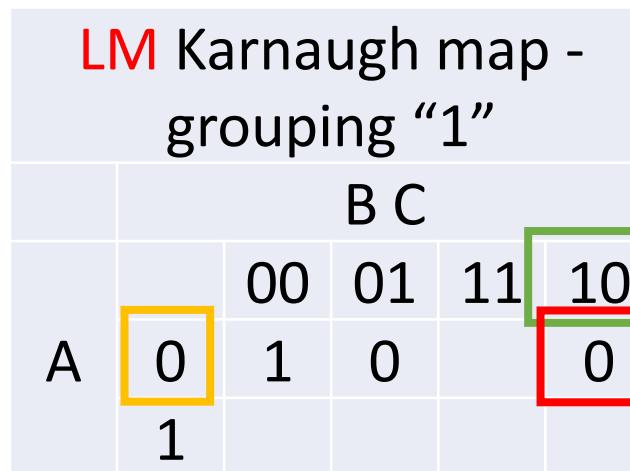
## 2. Karnaugh map – grouping

Truth Table			
A	BC	LM	RM
0	00	1	1
0	01	0	1
0	10	0	1
0	11	0	1
1	00	1	0
1	01	1	1
1	10	1	0
1	11	0	0



## 2. Karnaugh map – grouping

Truth Table			
A	BC	LM	RM
0	00	1	1
0	01	0	1
0	10	0	1
0	11	0	1
1	00	1	0
1	01	1	1
1	10	1	0
1	11	0	0



## 2. Karnaugh map – grouping

Truth Table			
A	BC	LM	RM
0	00	1	1
0	01	0	1
0	10	0	1
0	11	0	1
1	00	1	0
1	01	1	1
1	10	1	0
1	11	11	0

LM Karnaugh map - grouping “1”				
	B C			
	00	01	11	10
A	0	1	0	0
	1	1	0	1

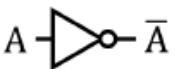
RM Karnaugh map - grouping “1”				
	B C			
	00	01	11	10
A	0	1	1	1
	1	0	1	0

## Disjunctive Normal Form (DNF)

$a$	$b$	$f(a, b)$
0	0	0
0	1	1
1	0	1
1	1	0

- Disjunctive Normal Form (DNF) is a disjunction, where a clause is a conjunctive of literals. : function  $f$  is a sum of products  $f = \dots (\dots \wedge \dots \wedge \dots) \vee (\dots \wedge \dots \wedge \dots) \vee (\dots \wedge \dots \wedge \dots) \dots,$
- Expression in bracket (product) corresponds to one one (“1”)
- In our case:  $f(a, b) = (\bar{a} \wedge b) \vee (a \wedge \bar{b}),$
- Decimal description:  $f(a, b) = \sum\{1, 2\}.$

## The most popular logic gates



A	NOT A
0	1
1	0

A	B	A AND B
0	0	0
0	1	0
1	0	0
1	1	1

A	B	A OR B
0	0	0
0	1	1
1	0	1
1	1	1



A	B	A XOR B
0	0	0
0	1	1
1	0	1
1	1	0

A	B	A NAND B
0	0	1
0	1	1
1	0	1
1	1	0

A	B	A NOR B
0	0	1
0	1	0
1	0	0
1	1	0

### 3. Boolean function $f(a,b,c)$ from Karnaugh map:

		B	C			
		00	01	11	10	
A		0	1	0	0	0
		1	1	1	0	1

(not a and not b and not c) or (a and not b and not c) =  
(not a or a) and not b and not c = 1 and not b and not  
c = **not b and not c**

		B	C			
		00	01	11	10	
A		0	1	1	1	1
		1	0	1	0	0

### 3. Boolean function $f(a,b,c)$ from Karnaugh map:

		B	C				
		00	01	11	10		
		A	0	1	0	0	0
		1	1	1	0	1	

(not a and not b and not c) or (a and not b and not c) =  
(not a or a) and not b and not c = 1 and not b and not  
c = **not b and not c**

(a and not b and not c) or (a and b and not c)=  
a and (not b or b) and not c = a and 1 and not c= = **a and not c**

		B	C				
		00	01	11	10		
		A	0	1	1	1	1
		1	0	1	0	0	0

**a and not b**

### 3. Boolean function $f(a,b,c)$ from Karnaugh map:

		B	C			
		00	01	11	10	
A		0	1	0	0	0
		1	1	1	0	1

not b and not c

a and not c

a and not b

		B	C			
		00	01	11	10	
A		0	1	1	1	1
		1	0	1	0	0

not b and c

(not a and not b and not c) or (not a and not b and c) or  
 (not a and b and c) or (not a and b and not c) =  
 [not a and not b and (not c or c)] or [not a and b and (c or not c)] =  
 [not a and not b and 1] or [not a and b and 1] =  
 [not a and not b] or [not a and b] =  
 not a and (not b or b) = not a and 1 = **not a**

4. Using Logisim, create LM and RM logic circuits  
(use OR, AND, NOT gates) and check it.

LM  $f(a,b,c) =$

$$= (\text{not } b \text{ and not } c) \text{ or } (a \text{ and not } c) \text{ or } (a \text{ and not } b)$$

RM  $f(a,b,c) =$

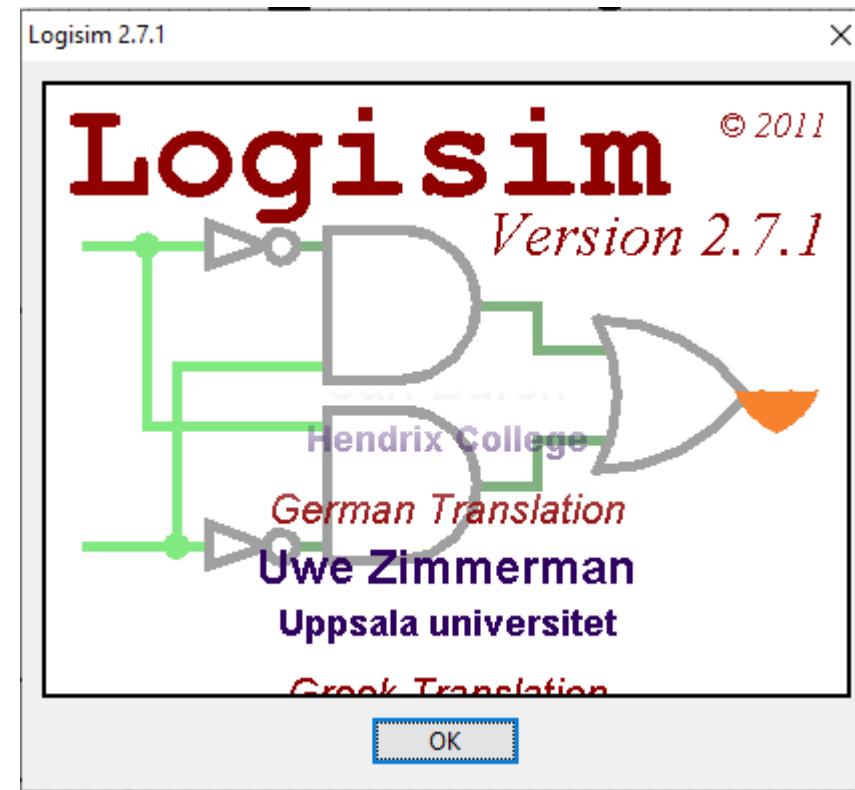
$$= (\text{not } a) \text{ or } (\text{not } b \text{ and } c)$$

# Start logisim

Icon



Version



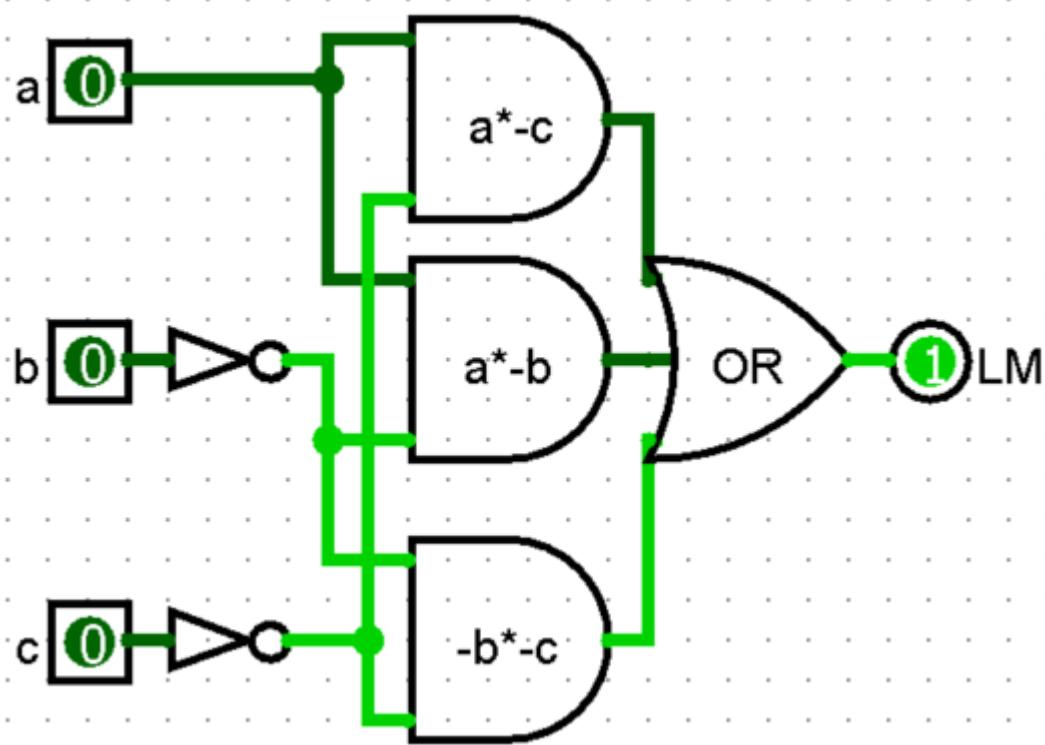
Open : mmajew/SYC/07 and 08/07 and 08 lab mobile robot.circ

4. Using Logisim, create LM and RM logic circuits  
(use OR, AND, NOT gates) and check it.

LM  $f(a,b,c) =$

$$= (\text{not } b \text{ and not } c) \text{ or } (a \text{ and not } c) \text{ or } (a \text{ and not } b)$$

LEFT MOTOR

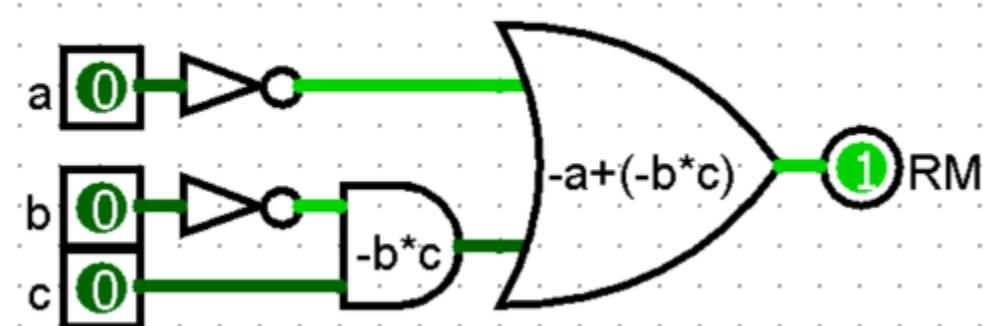


RM  $f(a,b,c) =$

$$= (\text{not } a) \text{ or } (\text{not } b \text{ and } c)$$

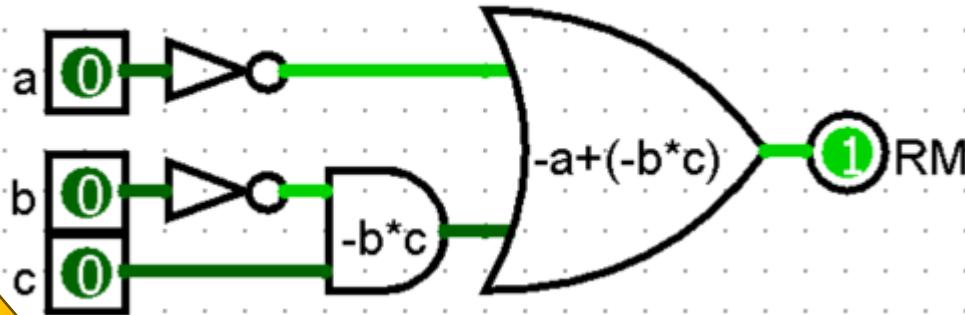
let's do it

RIGHT MOTOR



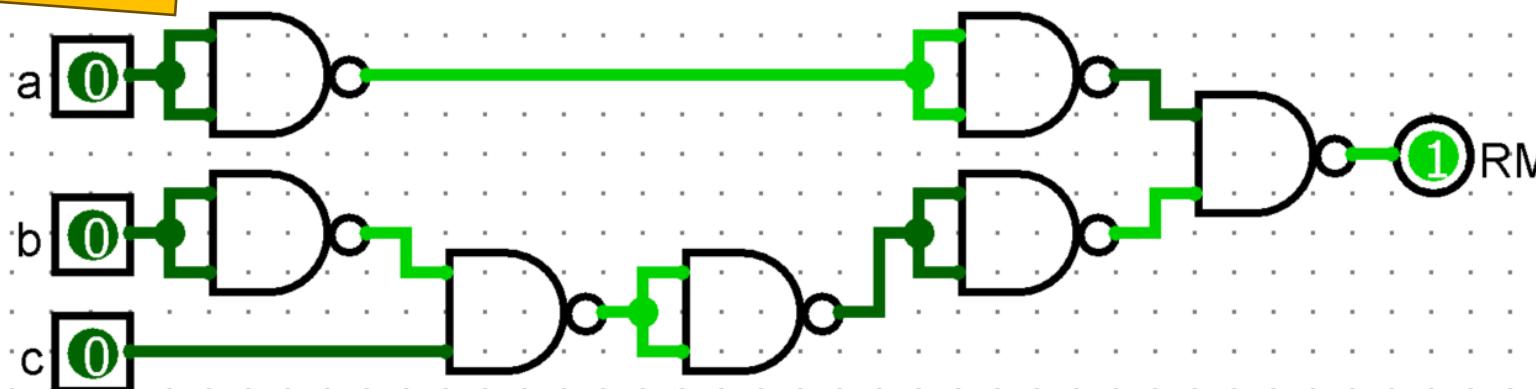
5. In Logisim create RM logic circuits. Replace OR, AND, NOT by NAND and check it.

RIGHT MOTOR

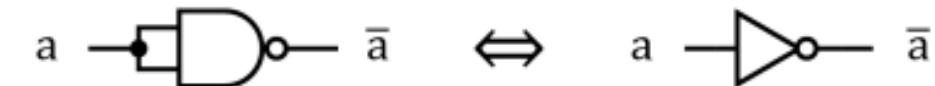


let's do it

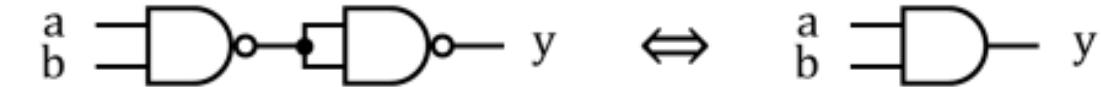
NAND substitution, without  
simplifying/minimizing the function



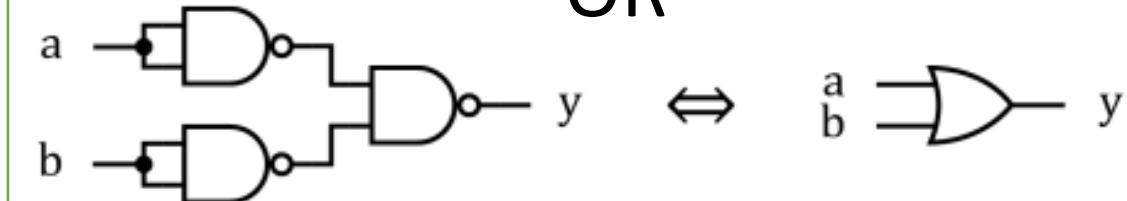
NOT



AND



OR



it's quite complicated!

# 6. Simplify boolean function $f(a,b,c)$ using NOT, NAND gates only:

...with simplifying/minimizing the function

LM  $f(a,b,c) =$

$$= (\text{not } b \text{ and not } c) \text{ or } (a \text{ and not } c) \text{ or } (a \text{ and not } b) =$$

RM  $f(a,b,c) =$

$$= (\text{not } a) \text{ or } (\text{not } b \text{ and } c) =$$

# 6. Simplify boolean function $f(a,b,c)$ using NOT, NAND gates only:

...with simplifying/minimizing the function

**Double negation**

$$Q = \text{not}[\text{not}(Q)]$$

$$\text{LM } f(a,b,c) =$$

$$= (\text{not } b \text{ and not } c) \text{ or } (a \text{ and not } c) \text{ or } (a \text{ and not } b) =$$

$$= \text{NOT}\{\text{NOT}[(\text{not } b \text{ and not } c) \text{ or } (a \text{ and not } c) \text{ or } (a \text{ and not } b)]\}$$

$$\text{RM } f(a,b,c) =$$

$$= (\text{not } a) \text{ or } (\text{not } b \text{ and } c) =$$

$$= \text{NOT}\{\text{NOT}[(\text{not } a) \text{ or } (\text{not } b \text{ and } c)]\}$$

# 6. Simplify boolean function $f(a,b,c)$ using NOT, NAND gates only:

...with simplifying/minimizing the function

**Double negation   De Morgan's law**

$$Q = \text{not}[\text{not}(Q)] \quad \text{not}(P \text{ or } Q) = (\text{not } P) \text{ and } (\text{not } Q)$$
$$\text{not}(P \text{ or } Q \text{ or } R) = (\text{not } P) \text{ and } (\text{not } Q) \text{ and } (\text{not } R)$$

$$\text{LM } f(a,b,c) =$$

$$= (\text{not } b \text{ and not } c) \text{ or } (a \text{ and not } c) \text{ or } (a \text{ and not } b) =$$

$$= \text{NOT}\{ \text{NOT}[ (\text{not } b \text{ and not } c) \text{ or } (a \text{ and not } c) \text{ or } (a \text{ and not } b) ] \} =$$

$$= \text{NOT}\{ \text{NOT}(\text{not } b \text{ and not } c) \text{ and } \text{NOT}(a \text{ and not } c) \text{ and } \text{NOT}(a \text{ and not } b) \}$$

$$\text{RM } f(a,b,c) =$$

$$= (\text{not } a) \text{ or } (\text{not } b \text{ and } c) =$$

$$= \text{NOT}\{ \text{NOT}[ (\text{not } a) \text{ or } (\text{not } b \text{ and } c) ] \} =$$

$$= \text{NOT}\{ \text{NOT}(\text{not } a) \text{ and } \text{NOT}(\text{not } b \text{ and } c) \}$$

# 6. Simplify boolean function $f(a,b,c)$ using NOT, NAND gates only:

...with simplifying/minimizing the function

**Double negation   De Morgan's law**

$$\begin{aligned} Q = \text{not[ not( } Q \text{ ) ]} \quad & \text{not}(P \text{ or } Q) = (\text{not } P) \text{ and } (\text{not } Q) \\ & \text{not}(P \text{ or } Q \text{ or } R) = (\text{not } P) \text{ and } (\text{not } Q) \text{ and } (\text{not } R) \end{aligned}$$

$$\begin{aligned} \text{LM } f(a,b,c) &= \\ &= (\text{not } b \text{ and not } c) \text{ or } (a \text{ and not } c) \text{ or } (a \text{ and not } b) = \\ &= \text{NOT}\{ \text{NOT[ (not } b \text{ and not } c) \text{ or } (a \text{ and not } c) \text{ or } (a} \\ &\text{and not } b \text{ )] } = \\ &= \text{NOT}\{ \text{NOT(not } b \text{ and not } c) \text{ and NOT(a and not } c)} \\ &\text{and NOT(a and not } b)\} = \\ &= \text{NOT}\{ (\text{not } b \text{ Nand not } c) \text{ and (a Nand not } c) \text{ and (a} \\ &\text{Nand not } b)\} \end{aligned}$$

$$\begin{aligned} \text{RM } f(a,b,c) &= \\ &= (\text{not } a) \text{ or } (\text{not } b \text{ and } c) = \\ &= \text{NOT}\{ \text{NOT[ (not } a) \text{ or (not } b \text{ and } c) ] } = \\ &= \text{NOT}\{ \text{NOT(not } a) \text{ and NOT(not } b \text{ and } c) \} = \\ &= \text{NOT}\{ a \text{ and (not } b \text{ Nand } c) \} \end{aligned}$$

# 6. Simplify boolean function $f(a,b,c)$ using NOT, NAND gates only:

...with simplifying/minimizing the function

**Double negation   De Morgan's law**

$$\begin{aligned} Q = \text{not[ not( } Q \text{ ) ]} \quad & \text{not}(P \text{ or } Q) = (\text{not } P) \text{ and } (\text{not } Q) \\ & \text{not}(P \text{ or } Q \text{ or } R) = (\text{not } P) \text{ and } (\text{not } Q) \text{ and } (\text{not } R) \end{aligned}$$

$$\text{LM } f(a,b,c) =$$

$$= (\text{not } b \text{ and not } c) \text{ or } (a \text{ and not } c) \text{ or } (a \text{ and not } b) =$$

$$= \text{NOT}\{ \text{NOT[ (not } b \text{ and not } c) \text{ or } (a \text{ and not } c) \text{ or } (a \text{ and not } b) \text{ ]} \} =$$

$$= \text{NOT}\{ \text{NOT(not } b \text{ and not } c) \text{ and NOT(a and not } c) \text{ and NOT(a and not } b) \} =$$

$$= \text{NOT}\{ (\text{not } b \text{ Nand not } c) \text{ and } (\text{a Nand not } c) \text{ and } (\text{a Nand not } b) \} =$$

$$= \text{NOT}\{ (\text{not } b \text{ Nand not } c) \text{ and } (\text{a Nand not } c) \text{ and } (\text{a Nand not } b) \} =$$

$$= (\text{not } b \text{ Nand not } c) \text{ Nand } (\text{a Nand not } c) \text{ Nand } (\text{a Nand not } b)$$

$$\text{RM } f(a,b,c) =$$

$$= (\text{not } a) \text{ or } (\text{not } b \text{ and } c) =$$

$$= \text{NOT}\{ \text{NOT[ (not } a) \text{ or } (\text{not } b \text{ and } c) \text{ ]} \} =$$

$$= \text{NOT}\{ \text{NOT(not } a) \text{ and NOT(not } b \text{ and } c) \} =$$

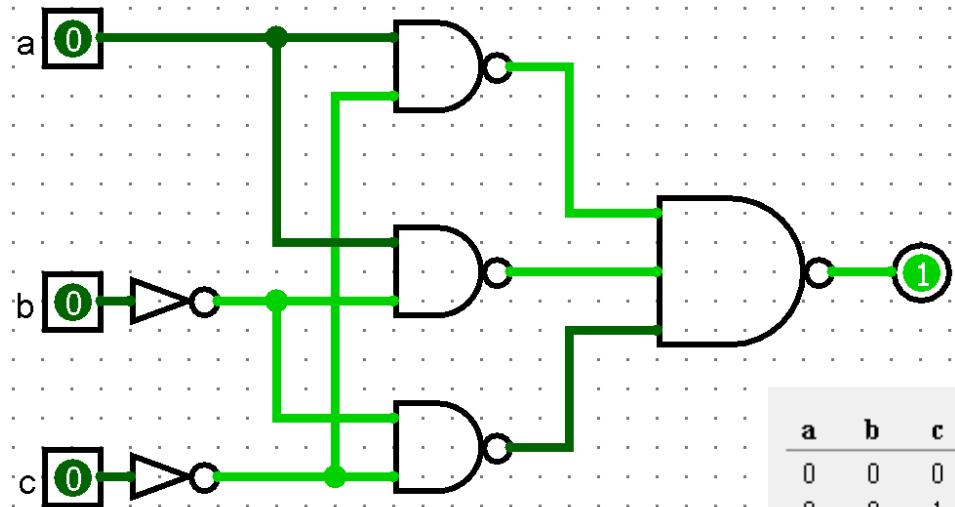
$$= \text{NOT}\{ a \text{ and } (\text{not } b \text{ Nand } c) \} =$$

$$= a \text{ Nand } (\text{not } b \text{ Nand } c)$$

7. Using Logisim, create above LM and RM logic circuits (NAND, NOT gates) and check it.

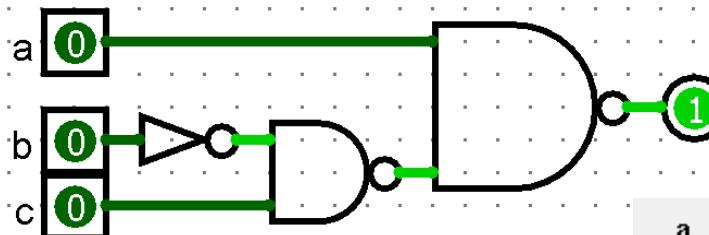
LM  $f(a,b,c) = (\text{not } b \text{ Nand not } c) \text{ Nand } (\text{a Nand not } c)$

Nand (a Nand not b)



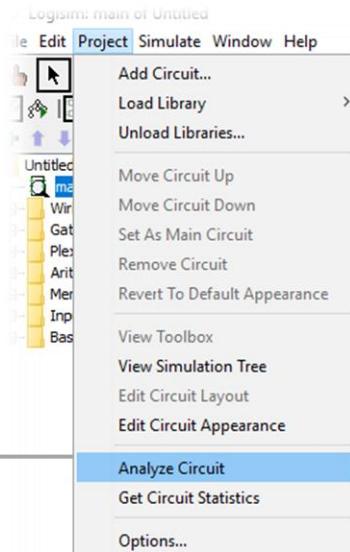
a	b	c	x
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	0

RM  $f(a,b,c) = a \text{ Nand } (\text{not } b \text{ Nand } c)$



Logisim: main of Untitled

a	b	c	x
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	0

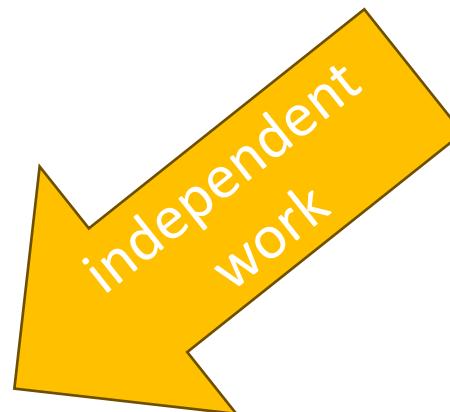
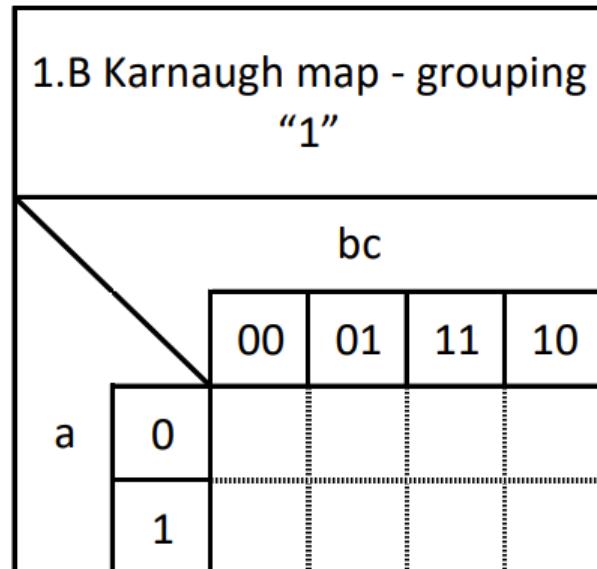


let's do it

Student number		Number of points	
----------------	--	------------------	--

1. Solve 1.B to 1.F tasks

1.A Truth table			
a	b	c	$f(a,b,c)$
0	0	0	
0	0	1	
0	1	0	
0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	




1.C Boolean function  $f(a,b,c)$  from Karnaugh map:

$$f(a,b,c) =$$


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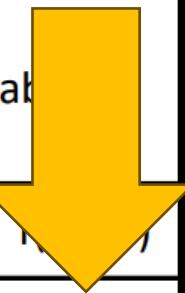


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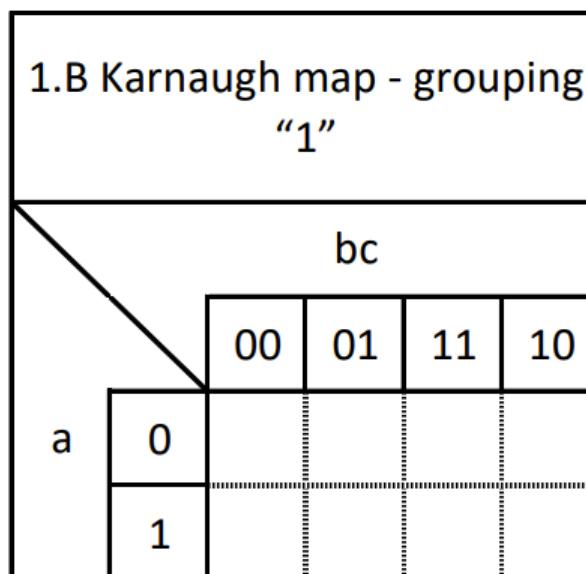
Student number		Number of points	
----------------	--	------------------	--

1. Solve 1.B to 1.F tasks

1.A Truth table



a	b	c	f
0	0	0	1
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	0



1

1.C Boolean function  $f(a,b,c)$  from Karnaugh map:

$$f(a,b,c) =$$

---

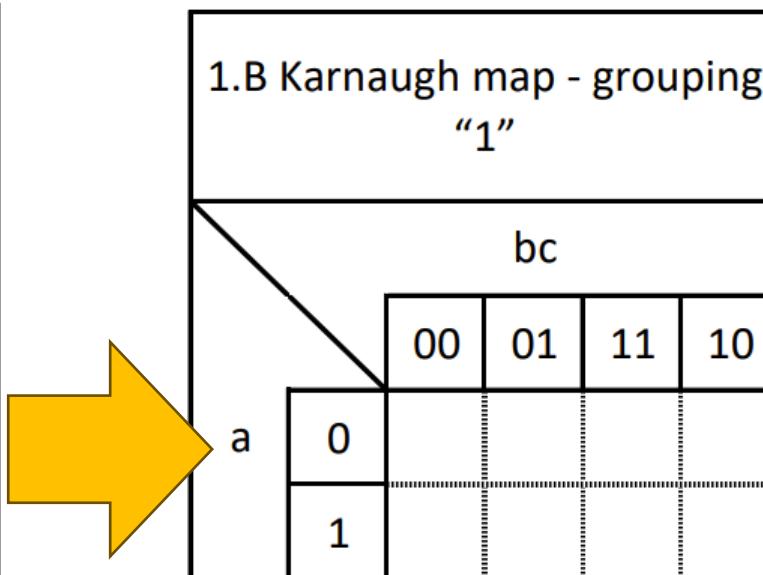
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Student number

Number of points

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0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	0



1

1.C Boolean function  $f(a,b,c)$  from Karnaugh map:

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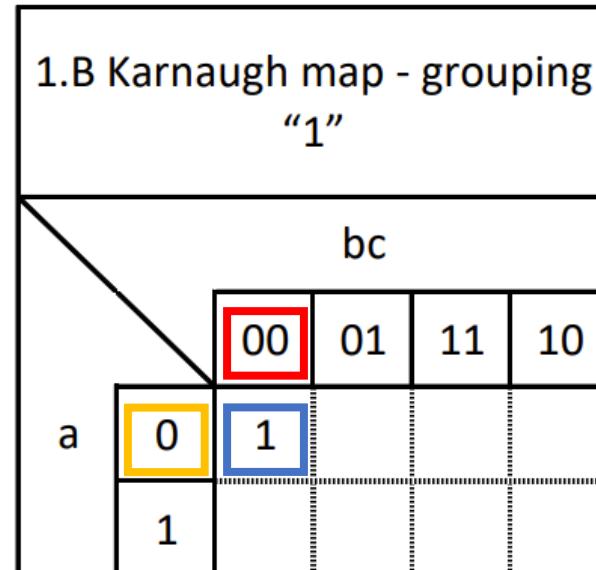
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<i>Student number</i>		<i>Number of points</i>	
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0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	0



1

1.C Boolean function  $f(a,b,c)$  from Karnaugh map:

$$f(a,b,c) =$$


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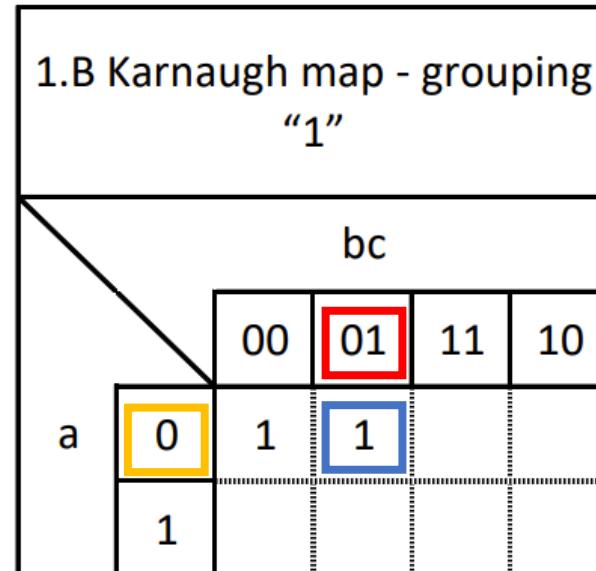


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<i>Student number</i>		<i>Number of points</i>	
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1. Solve 1.B to 1.F tasks

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0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	0



1

1.C Boolean function  $f(a,b,c)$  from Karnaugh map:

$$f(a,b,c) =$$


---



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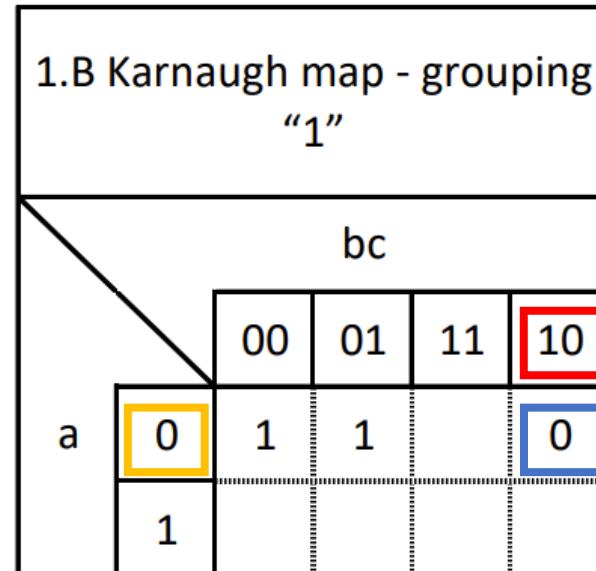


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<i>Student number</i>		<i>Number of points</i>	
-----------------------	--	-------------------------	--

1. Solve 1.B to 1.F tasks

1.A Truth table			
a	b	c	$f(a,b,c)$
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0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	0



1

1.C Boolean function  $f(a,b,c)$  from Karnaugh map:

$$f(a,b,c) =$$


---



---

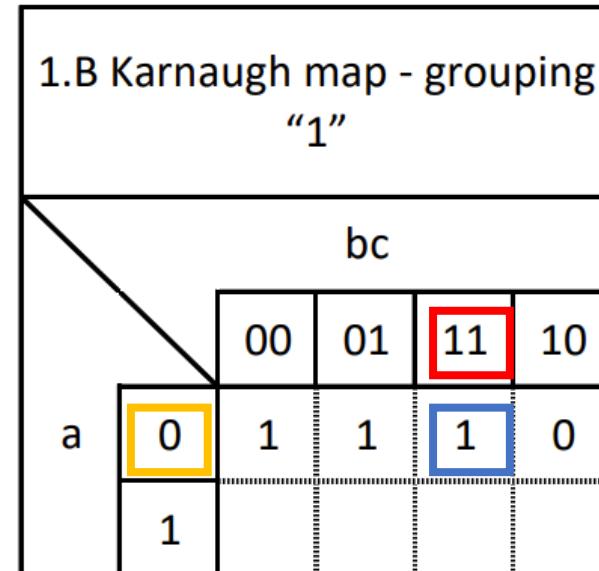


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<i>Student number</i>		<i>Number of points</i>	
-----------------------	--	-------------------------	--

1. Solve 1.B to 1.F tasks

1.A Truth table			
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0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	0



1

1.C Boolean function  $f(a,b,c)$  from Karnaugh map:

$$f(a,b,c) =$$


---



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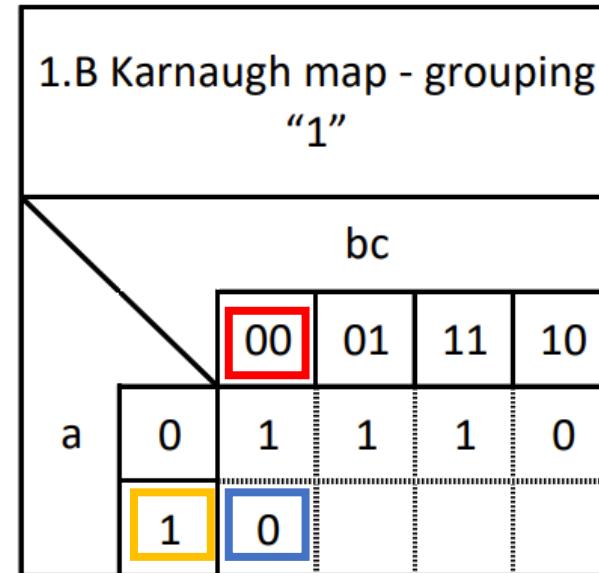


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<i>Student number</i>		<i>Number of points</i>	
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0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
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1	1	1	0



1

1.C Boolean function  $f(a,b,c)$  from Karnaugh map:

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---



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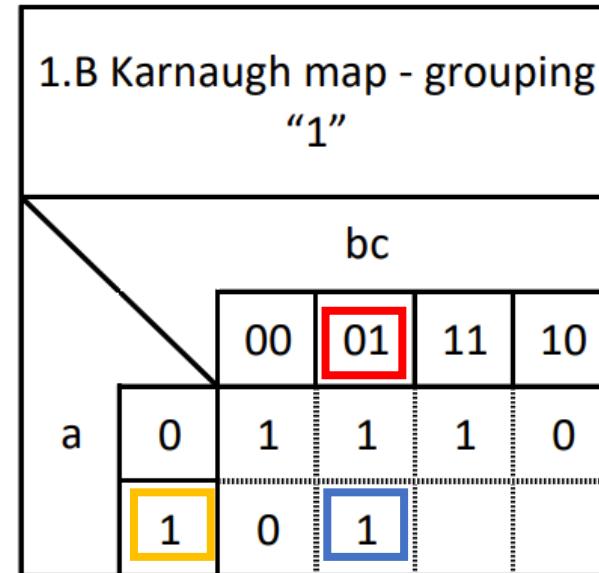


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<i>Student number</i>		<i>Number of points</i>	
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0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	0



1

1.C Boolean function  $f(a,b,c)$  from Karnaugh map:

$$f(a,b,c) =$$


---



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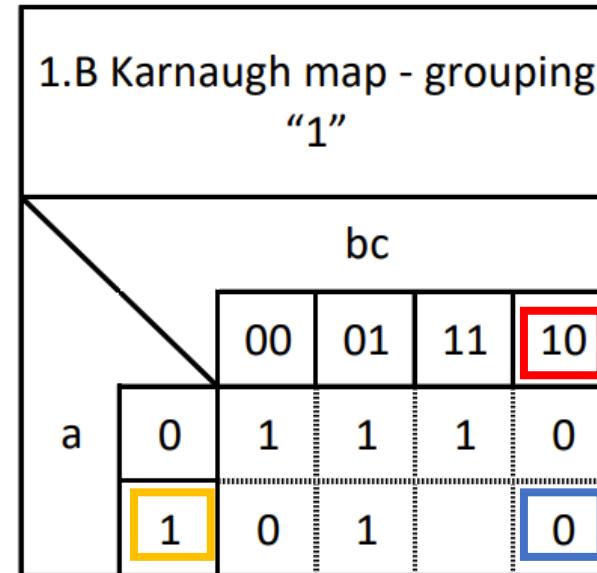


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<i>Student number</i>		<i>Number of points</i>	
-----------------------	--	-------------------------	--

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0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	0



1

1.C Boolean function  $f(a,b,c)$  from Karnaugh map:

$$f(a,b,c) =$$


---



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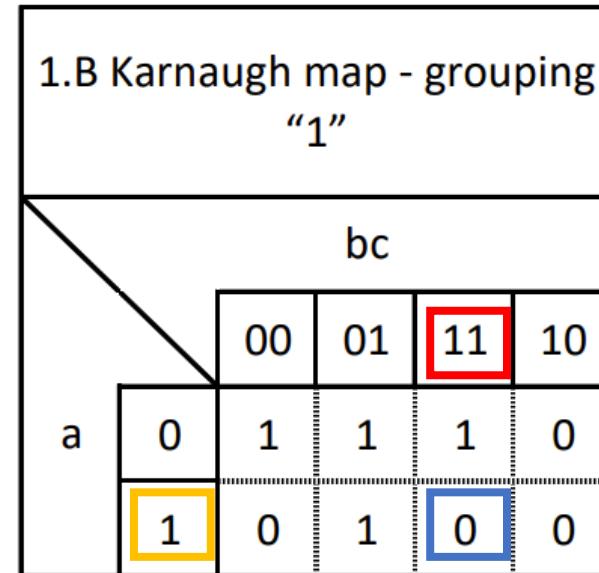


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<i>Student number</i>		<i>Number of points</i>	
-----------------------	--	-------------------------	--

1. Solve 1.B to 1.F tasks

1.A Truth table			
a	b	c	$f(a,b,c)$
0	0	0	1
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	0



1

1.C Boolean function  $f(a,b,c)$  from Karnaugh map:

$$f(a,b,c) =$$


---



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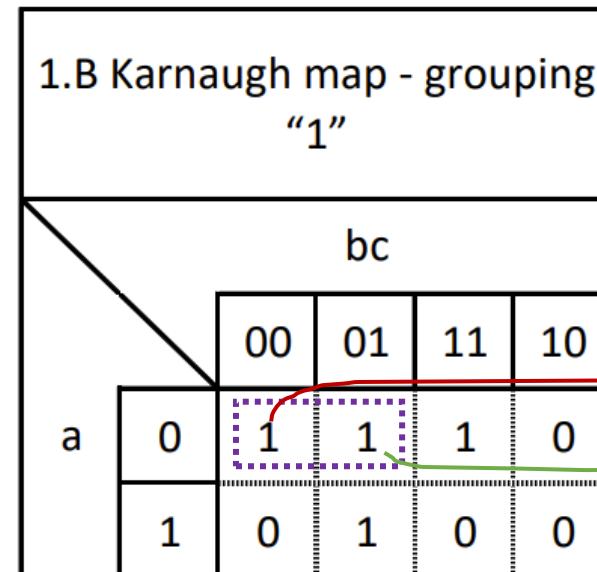


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Student number		Number of points	
----------------	--	------------------	--

1. Solve 1.B to 1.F tasks

1.A Truth table			
a	b	c	$f(a,b,c)$
0	0	0	1
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	0



(not a and not b and not c) or (not a and not b and c)

1  
not a and not b and (not c or c) =  
not a and not b and 1 =  
not a and not b

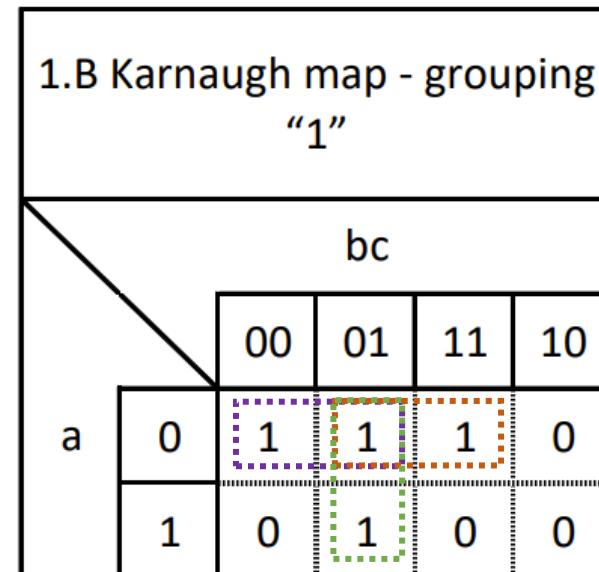
1.C Boolean function  $f(a,b,c)$  from Karnaugh map:

$$f(a,b,c) = \text{(not a and not b) or }$$

Student number		Number of points	
----------------	--	------------------	--

1. Solve 1.B to 1.F tasks

1.A Truth table			
a	b	c	$f(a,b,c)$
0	0	0	1
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	0



1

1.C Boolean function  $f(a,b,c)$  from Karnaugh map:

$$f(a,b,c) = \underline{\text{(not } a \text{ and not } b\text{)}} \text{ or } \underline{\text{(not } a \text{ and } c\text{)}} \text{ or } \underline{\text{(not } b \text{ and } c\text{)}}$$

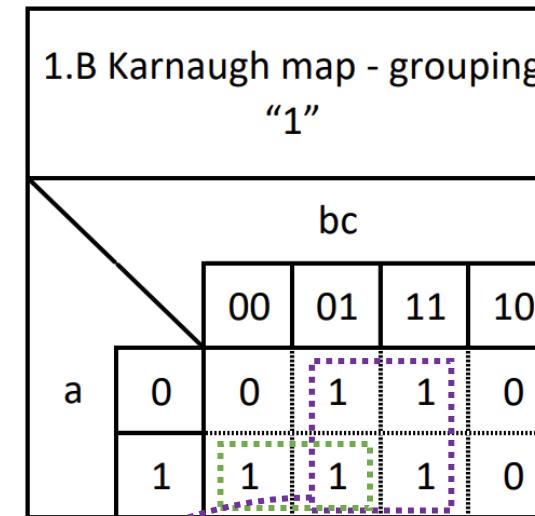
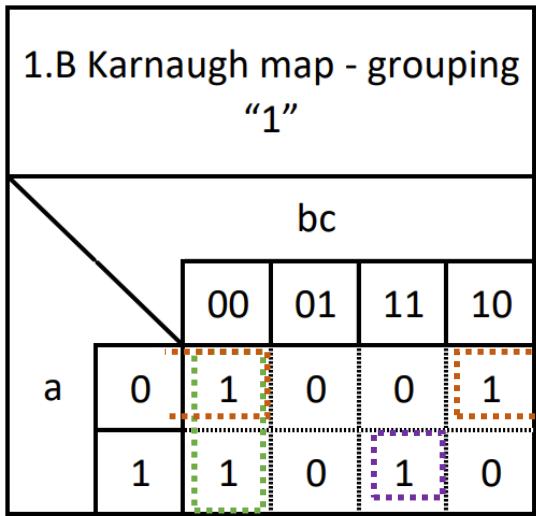

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1.C Boolean function  $f(a,b,c)$  from Karnaugh map:

$$f(a,b,c) = \underline{\text{(not } a \text{ and not } c\text{)}} \text{ or } \underline{\text{(not } b \text{ and not } c\text{)}} \text{ or } \underline{\text{(a and b and c)}}$$

1.C Boolean function  $f(a,b,c)$  from Karnaugh map:

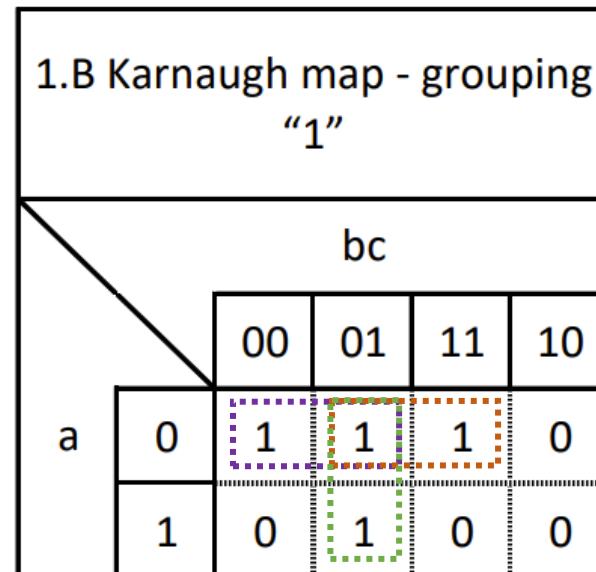
$$f(a,b,c) = \underline{\text{(c)}} \text{ or } \underline{\text{(a and not } b\text{)}}$$

$\downarrow$   
 $[\text{not } a \text{ and } \underline{\text{not } b} \text{ and } c] \text{ or } [\text{not } a \text{ and } \underline{b} \text{ and } c] \text{ or } [a \text{ and } \underline{\text{not } b} \text{ and } c] \text{ or } [a \text{ and } \underline{b} \text{ and } c] =$   
 $[\text{not } a \text{ and } \underline{\text{and }} c \text{ and } \underline{(\text{not } b \text{ or } b)}] \text{ or } [a \text{ and } \underline{\text{and }} c \text{ and } \underline{(\text{not } b \text{ or } b)}] =$   
 $[\text{not } a \text{ and } c] \text{ or } [a \text{ and } c] =$   
 $c \text{ and } (\text{not } a \text{ and } a) =$   
 $c$

Student number		Number of points	
----------------	--	------------------	--

1. Solve 1.B to 1.F tasks

1.A Truth table			
a	b	c	$f(a,b,c)$
0	0	0	1
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	0



1

1.C Boolean function  $f(a,b,c)$  from Karnaugh map:

$$f(a,b,c) = \underline{\text{(not } a \text{ and not } b\text{)}} \text{ or } \underline{\text{(not } a \text{ and } c\text{)}} \text{ or } \underline{\text{(not } b \text{ and } c\text{)}}$$


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1.D Boolean function  $f(a,b,c)$  using NAND gates only:

(not a and not b) or (not a and c) or (not b and c)=

CAT or DOG or HOUSE

1.D Boolean function  $f(a,b,c)$  using NAND gates only:

(not a and not b) or (not a and c) or (not b and c)=

CAT or DOG or HOUSE =

*double negation*

$Q = \text{not}[\text{not}(Q)]$

→ Not{ Not[CAT or DOG or HOUSE] }

1.D Boolean function  $f(a,b,c)$  using NAND gates only:

(not a and not b) or (not a and c) or (not b and c)=

CAT or DOG or HOUSE =

*double negation*

$$Q = \text{not}[\text{not}(Q)]$$

→ Not{ Not[CAT or DOG or HOUSE] }=

*De Morgan's law*

$$\text{not}(P \text{ or } Q) = (\text{not } P) \text{ and } (\text{not } Q)$$

$$\text{not}(P \text{ or } Q \text{ or } R) =$$

$$(\text{not } P) \text{ and } (\text{not } Q) \text{ and } (\text{not } R)$$

→ Not{ (not CAT) and (not DOG) and (not HOUSE) }

1.D Boolean function  $f(a,b,c)$  using NAND gates only:

$$(\text{not } a \text{ and not } b) \text{ or } (\text{not } a \text{ and } c) \text{ or } (\text{not } b \text{ and } c) =$$

$$\text{CAT} \text{ or } \text{DOG} \text{ or } \text{HOUSE} =$$

*double negation*

$$Q = \text{not}[\text{not}(Q)]$$

$$\rightarrow \text{Not}\{\text{Not}[\text{CAT} \text{ or } \text{DOG} \text{ or } \text{HOUSE}]\} =$$

*De Morgan's law*

$$\text{not}(P \text{ or } Q) = (\text{not } P) \text{ and } (\text{not } Q)$$

$$\rightarrow \text{Not}\{(\text{not } \text{CAT}) \text{ and } (\text{not } \text{DOG}) \text{ and } (\text{not } \text{HOUSE})\} =$$

$$\text{Not}\{(\text{not } \text{CAT}) \text{ and } (\text{not } \text{DOG}) \text{ and } (\text{not } \text{HOUSE})\} =$$

$$(\text{not } \text{CAT}) \text{ Nand } (\text{not } \text{DOG}) \text{ Nand } (\text{not } \text{HOUSE})$$

1.D Boolean function  $f(a,b,c)$  using NAND gates only:

$$(\text{not } a \text{ and not } b) \text{ or } (\text{not } a \text{ and } c) \text{ or } (\text{not } b \text{ and } c) =$$

$$\text{CAT or DOG or HOUSE} =$$

*double negation*

$$Q = \text{not}[\text{not}(Q)]$$

$$\rightarrow \text{Not}\{\text{Not}[\text{CAT or DOG or HOUSE}]\} =$$

*De Morgan's law*

$$\text{not}(P \text{ or } Q) = (\text{not } P) \text{ and } (\text{not } Q)$$

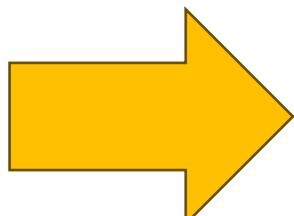
$$\rightarrow \text{Not}\{(\text{not } \text{CAT}) \text{ and } (\text{not } \text{DOG}) \text{ and } (\text{not } \text{HOUSE})\} =$$

$$\text{Not}\{(\text{not } \text{CAT}) \text{ and } (\text{not } \text{DOG}) \text{ and } (\text{not } \text{HOUSE})\} =$$

$$(\text{not } \text{CAT}) \text{ Nand } (\text{not } \text{DOG}) \text{ Nand } (\text{not } \text{HOUSE}) =$$

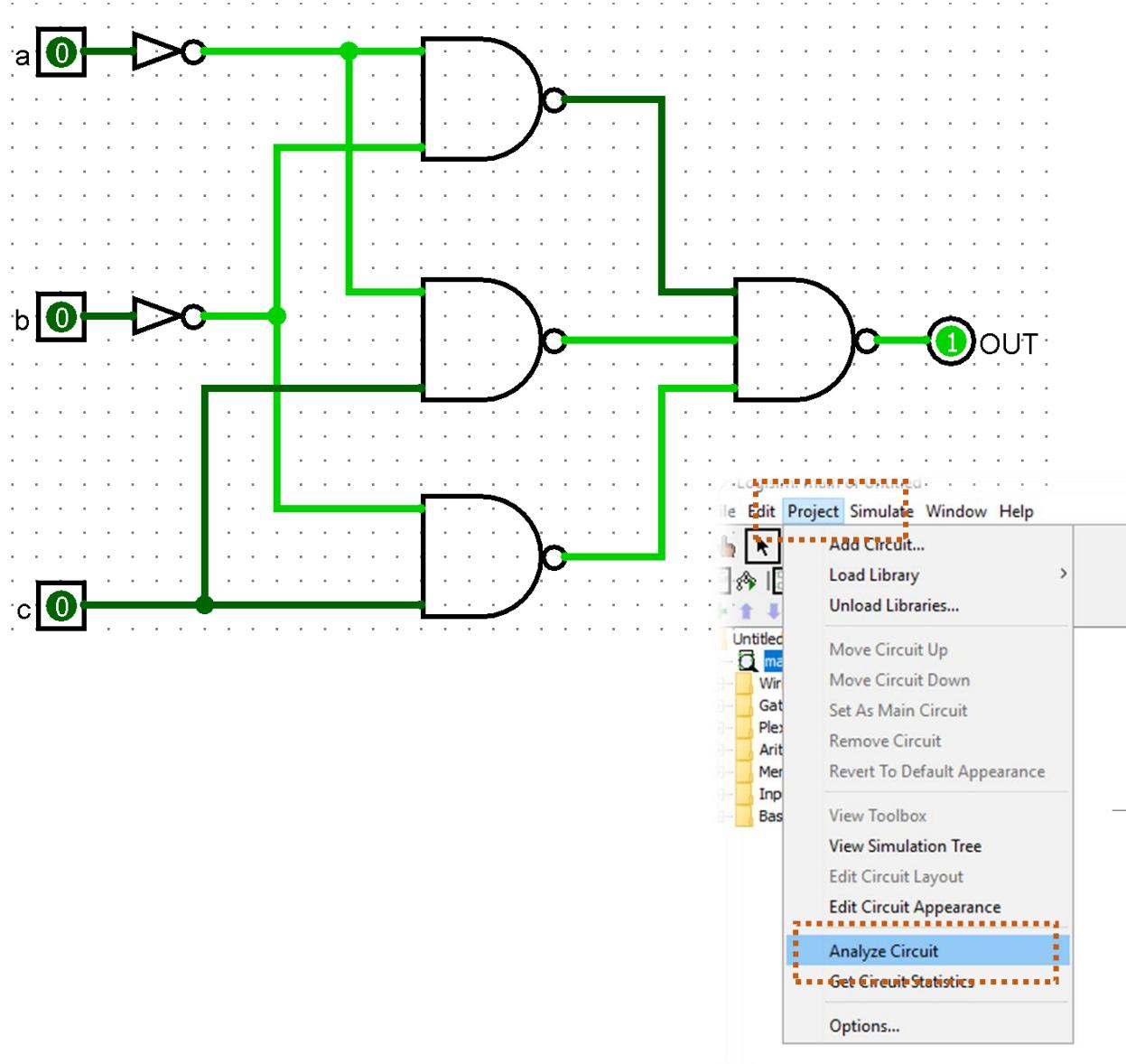
$$(\text{not } (\text{not } a \text{ and not } b)) \text{ Nand } (\text{not } (\text{not } a \text{ and } c)) \text{ Nand } (\text{not } (\text{not } b \text{ and } c)) =$$

$$(\text{not } a \text{ Nand not } b) \text{ Nand } (\text{not } a \text{ Nand } c) \text{ Nand } (\text{not } b \text{ Nand } c)$$



Boolean function using Not and Nand gates

# 1.E Using Logisim, create a logic circuit and check it.



The screenshot shows the Logisim simulation window titled 'Combinational...'. It displays a truth table with columns for inputs a, b, c and output OUT. The table shows the following values:

a	b	c	OUT
0	0	0	1
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	0

At the bottom right of the window is a 'Build Circuit' button.

The screenshot shows a truth table titled '1.A Truth table' with the following values:

a	b	c	f(a,b,c)
0	0	0	1
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	0

To the right of the table is a yellow smiley face icon.