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PSYC214: Statistics
Lecture 3 – Assumptions of ANOVA and follow-up procedures – Part I

Michaelmas Term
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Assumptions of ANOVA and follow-up procedures

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Agenda/Content for Lecture 3

- Assumptions of ANOVA
  - Assumption of independence
  - Assumption of normality
  - Assumption of homogeneity of variance
- Data transformations
- Pairwise between-level comparisons
  - Planned comparisons
  - Post-hoc tests

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The assumptions of ANOVA

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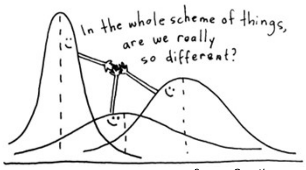
- The analysis of variance (ANOVA) is a parametric test
- ANOVAs have a set of assumptions, which should be met
- These are often ignored by researchers, because ANOVAs are typically very robust!
- Even small/moderate deviations



Source: Questionpro

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The assumptions of ANOVA

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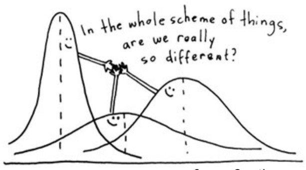
- It is unlikely that highly significant results, e.g., $p < .01$, will drastically change because of small violations
- Marginally significant results, i.e., those around $p = .05$ value, however, may be affected by even small violations!



Source: Questionpro

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In a perfect world…

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- You would have equal number of participants per level (e.g., per condition)
- Your data would be on an interval/ratio scale

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Assumptions underlying the ANOVA

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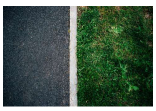
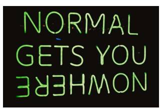
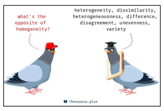
1. Assumption of independence
2. Assumption of normality
3. Assumption of homogeneity of variance



Independence



Normality



Homogeneity of variance

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Assumptions underlying the ANOVA

1. Assumption of independence
2. Assumption of normality
3. Assumption of homogeneity of variance


Independence

Normality

Homogeneity of variance

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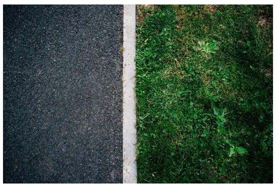
1. Assumption of independence

What is it?

- Participants should be randomly assigned to a group
- Participants should not cluster, sharing a classification variable
  - Gender
  - Skill level
- There should be no influence across one data point to another



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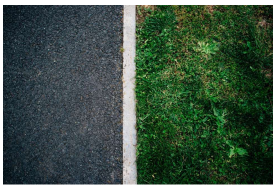
1. Assumption of independence

Consequences of violation

- Becomes difficult to interpret results
- Did the manipulation have an effect, or was this driven by classification clustering or influence?



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1. Assumption of independence

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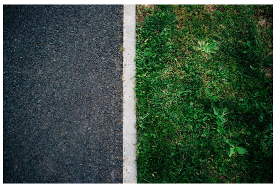
How to avoid it?

- Always randomly allocate participants to a condition
- Try to allocate equal numbers to each condition
- You can test to see whether you have significant differences on important classification variables



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Assumptions underlying the ANOVA

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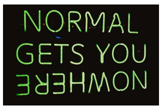
1. Assumption of independence
2. Assumption of normality
3. Assumption of homogeneity of variance

Independence



Normality

Homogeneity of variance

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2. Assumption of normality

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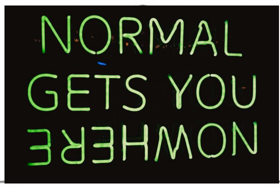
What is it?

- You want the overall data and the data for each subgroup to normally distributed
- This is because ANOVAs rely on the mean – and for skewed and bimodal data the mean is unlikely the best measure of central tendency



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2. Assumption of normality

Consequences of violation

- If data are slightly skewed this is unlikely to cause problems
- If data are skewed by roughly the same degree in the same direction – unlikely a problem
- If skewed in different directions, this is a problem. Lead to type I and II errors!

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2. Assumption of normality

How to avoid it?

- Avoid measures which often have ceiling or floor effects
- Transform data, changing every score in a systematic way
- Use a robust ANOVA (specialized test – more complex) or non-parametric alternatives

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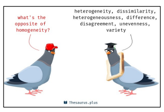
Assumptions underlying the ANOVA

1. Assumption of independence
2. Assumption of normality
3. Assumption of homogeneity of variance

Independence

Normality



Homogeneity of variance

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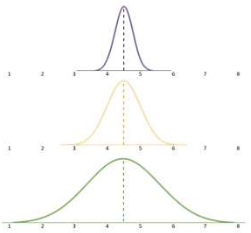
3. Homogeneity of variance

What is it?

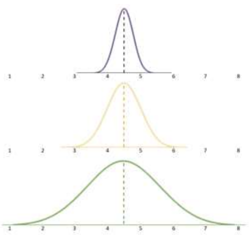
- Assumes that the variances of the distributions in the samples are equal
- Therefore the variances for each sample should not significantly vary from one another



3. Homogeneity of variance

Consequences of violation

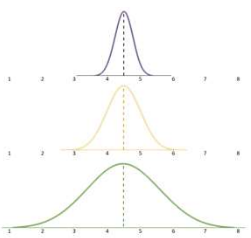
- The ANOVA tests the plausibility of the null hypothesis – i.e., all observations come from the same underlying population with the same degree of variability
- This is pointless to test when variance is already clearly different



3. Homogeneity of variance

How to avoid it?

- Difficult to avoid, but can be mitigated when testing
- As a rule of thumb, it is ok, as long as largest variance is no more than 4x the size of smallest
- Can also transform data or use non-parametric alternative



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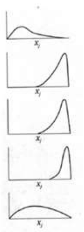
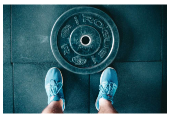
Dealing with 'rogue' data

- There are a number of strategies which may improve 'rouge' data
- None are panaceas and are unlikely to work in each situation
- If these aren't helpful, you can apply a non-parametric alternative
  - e.g., Kruskall-Wallace one-way Analysis of Variance by Ranks





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Dealing with 'rogue' data

Transforming data

- This involves taking every score from each participant and applying a uniform mathematical function to each
- Report both the original data and the transformed data

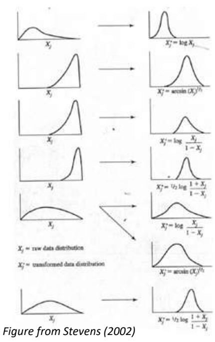

Figure from Stevens (2002)

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Dealing with 'rogue' data

How to transform data

Untransformed	Square-root transformed	Log transformed
38	6.164	1.580
1	1.000	0.000
13	3.606	1.114
2	1.414	0.301
13	3.606	1.114
20	4.472	1.301
50	7.071	1.699
9	3.000	0.954
28	5.292	1.447
6	2.449	0.778
4	2.000	0.602
43	6.557	1.633

http://www.biostathandbook.com/transformation.html

Type of Data Transformation	Nature of Data
Log Transformation $(\log(X_i))$	Whole numbers and cover wide range of values, small values with decimal fractions.
Square-root Transformation $(\sqrt{X_i})$	Small whole number & Percentage data where the range is between 0 and 30 % or between 70 and 100 %

Maidapwad & Sananse (2014)

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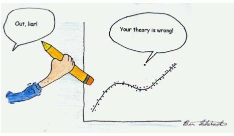
Outliers and their impact

- Outliers are data points which significantly differ from other observations
- Outliers can drastically bias/change predictive models
- Predictions can be exaggerated and present high error
- Outliers not only distort statistical analyses, they can violate assumptions



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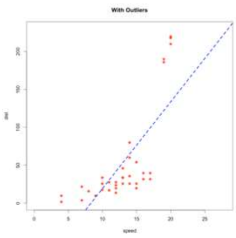
Outliers and their impact



- Given the problems outliers create, it may seem levelheaded to remove them
- However, it can be dishonest and misleading to do so if they are true scores
- It must be justifiable as to why it is necessary to remove data

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Outliers and their impact

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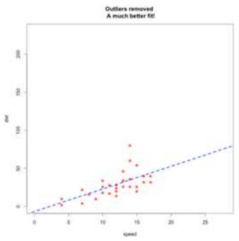

- Given the problems outliers create, it may seem levelheaded to remove them
- However, it can be dishonest and misleading to do so if they are true scores
- It must be justifiable as to why it is necessary to remove data

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The meaning of an ANOVA output

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```
##              Df Sum Sq Mean Sq F value   Pr(>F)
## Group         2   1223   611.3   12.52 6.77e-06 ***
## Residuals   237  11571    48.8
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

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The meaning of an ANOVA output

Lancaster University

```
##                Df Sum Sq Mean Sq F value   Pr(>F)
## Group           2   1223   611.3   12.52 6.77e-06 ***
## Residuals     237  11571    48.8
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

$F = \frac{\text{between-group variance}}{\text{within-group variance}}$ $F = \frac{611.3}{48.8}$ $\boxed{F = 12.52}$ $\boxed{p = 0.00000677}$

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The meaning of an ANOVA output

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P-value	Definition
> .05	▪ We accept the null hypothesis (H_0) ▪ Under H_0, the samples come from the <u>same</u> population ▪ There is no statistical difference in the population means ($\mu_1 = \mu_2 = \mu_3$) ▪ Experimental effect = 0
≤ .05	▪ We reject the null hypothesis (H_1) ▪ Under H_1, the samples come from <u>different</u> populations ▪ Population means are statistically different ($\mu_1 \neq \mu_2 \neq \mu_3$) ▪ Experimental effect ≠ 0

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Significant?

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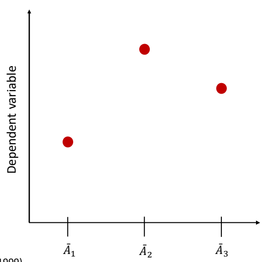

Adapted from Roberts and Russo (1999)

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Non-significant

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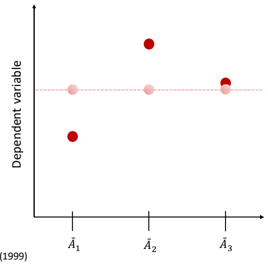

$p > .05$
There is insufficient evidence to conclude that any means significantly differs from any others

Adapted from Roberts and Russo (1999)

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Significant

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$p \le .05$
At least one of the pairs of means is significantly different. The question is, which pairs?

Adapted from Roberts and Russo (1999)

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Pairwise comparisons

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There are two strategies for following-up significant ANOVAs

- Planned comparisons
- Post-hoc comparisons

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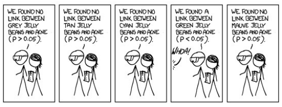
The problem of multiple comparisons

- Why not just run a bunch of t-tests?
- Multiple comparisons increase the probability of making a (familywise) type I error
- I.e., rejecting the null hypothesis when actually there was no effect



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The problem of multiple comparisons

- Type 1 error - 1 test at $p \leq 0.05$ = 0.95 (i.e., 5% chance we get noise)
- Type 1 error - 2 tests = 0.95 * 0.95, = 0.903. (10% chance)
- Type 1 error - 3 tests = 0.95 * 0.95 * 0.95 = 0.857 (14% chance)
- Type 1 error – 4 tests = 0.95 * 0.95 * 0.95 * 0.95 = 0.815 (18.5% chance)
- Type 1 error – 5 tests = 0.95 * 0.95 * 0.95 * 0.95 * 0.95 = 0.774 (22.6% chance)

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Pairwise comparisons

There are two strategies for following-up significant ANOVAs

- Planned comparisons
- Post-hoc comparisons

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The problem of multiple comparisons

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Group	$\bar{A}_1$	$\bar{A}_2$	$\bar{A}_3$	$\bar{A}_4$	$\bar{A}_5$
$\bar{A}_1$	-	-	-	-	-
$\bar{A}_2$	●	-	-	-	-
$\bar{A}_3$	●	●	-	-	-
$\bar{A}_4$	●	●	●	-	-
$\bar{A}_5$	●	●	●	●	-

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Planned comparisons

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- Focussed approach to examine specific group differences
- Perfect when certain hypotheses can be tested without comparing all combinations of means
- Should be pre-specified
- Need to keep the number of planned comparisons as low as possible to negate Type I errors – (number of levels – 1)

Group	$\bar{A}_1$	$\bar{A}_2$	$\bar{A}_3$	$\bar{A}_4$	$\bar{A}_5$
$\bar{A}_1$	-	-	-	-	-
$\bar{A}_2$	●	-	-	-	-
$\bar{A}_3$	●	●	-	-	-
$\bar{A}_4$	●	●	●	-	-
$\bar{A}_5$	●	●	●	●	-

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Planned comparisons

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Our options:

1. Run t-tests with a low number of pairs
2. Run t-tests with Bonferroni adjustment
3. ~~Specialized linear contrast~~

Group	$\bar{A}_1$	$\bar{A}_2$	$\bar{A}_3$	$\bar{A}_4$	$\bar{A}_5$
$\bar{A}_1$	-	-	-	-	-
$\bar{A}_2$	●	-	-	-	-
$\bar{A}_3$	●	●	-	-	-
$\bar{A}_4$	●	●	●	-	-
$\bar{A}_5$	●	●	●	●	-

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Planned comparisons – 1. Run t-tests

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- Accept that we have inflated our risks
- Keep the number of planned comparisons as low as possible to negate Type I errors – (number of levels – 1)
- Even with two tests, however, our chance of a Type I error is 10%!

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Planned comparisons – 1. Run t-tests

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A_1 - Robot A(lpha)

A_2- Robot B(eta)

A_3 - Robot O(mega)

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Planned comparisons

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Group	A_1	A_2	A_3
A_1	-	-	-
A_2		-	-
A_3			-

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Planned comparisons – 1. Run t-tests

A_1 - Robot A(lpha)

$$t = \frac{\bar{A}_1 - \bar{A}_2}{\sqrt{(Mean\ Square_{ERROR})\left(\frac{2}{NA}\right)}}$$

A_2- Robot B(eta)

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Planned comparisons – 1. Run t-tests

Mean differences between two levels

$$t = \frac{\boxed{\bar{A}_1} - \boxed{\bar{A}_2}}{\sqrt{(\boxed{Mean\ Square_{ERROR}})\left(\frac{2}{\boxed{NA}}\right)}}$$

Within group variance from ANOVA output

Number of scores in each levels being compared

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Planned comparisons – 1. Run t-tests

```
##   Group variable         n  mean    sd   min   max
##   <chr> <chr>        <dbl> <dbl> <dbl> <dbl> <dbl>
## 1 A     Likeability     80   2.5 0.928     1     4
## 2 A     Score           80  58.1  6.45    44    72
## 3 B     Likeability     80   4.5  1.01     2     7
## 4 B     Score           80  60.4  7.27    40    74
## 5 O     Likeability     79  2.11 0.847     1     4
## 6 O     Score           80  63.6  7.22    47    79
```

$$t = \frac{\boxed{\bar{A}_1} - \boxed{\bar{A}_2}}{\sqrt{(\boxed{Mean\ Square_{ERROR}})\left(\frac{2}{\boxed{NA}}\right)}}$$

80 + 80

```
##              Df Sum Sq Mean Sq F value   Pr(>F)
## Group         2   1223   611.3   12.52 6.77e-06 ***
## Residuals   237  11571    48.8
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

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Planned comparisons – 1. Run t-tests

```
##   Group variable         n  mean    sd   min   max
##   <chr> <chr>        <dbl> <dbl> <dbl> <dbl> <dbl>
## 1 A     Likeability     80   2.5 0.928     1     4
## 2 A     Score           80  58.1  6.45    44    72
## 3 B     Likeability     80   4.5  1.01     2     7
## 4 B     Score           80  60.4  7.27    40    74
## 5 O     Likeability     79  2.11 0.847     1     4
## 6 O     Score           80  63.6  7.22    47    79
```

$$t = \frac{\boxed{58.1} - \boxed{60.4}}{\sqrt{\boxed{(48.8)}\left(\frac{2}{160}\right)}}$$

```
##              Df Sum Sq Mean Sq F value   Pr(>F)
## Group         2   1223   611.3   12.52 6.77e-06 ***
## Residuals   237  11571    48.8
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

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Planned comparisons – 1. Run t-tests

$$t = \frac{58.1 - 60.4}{\sqrt{(48.8)(0.0125)}}$$

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Planned comparisons – 1. Run t-tests

$$t = \frac{-2.3}{\sqrt{0.61}} \qquad t = \frac{-2.3}{0.78}$$

$$t = -2.94$$

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Planned comparisons – 1. Run t-tests

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$t = -2.94$, with 237 degrees of freedom
It's significant at $p = 0.05$ threshold

Degrees of Freedom	p=0.05	p=0.025	p=0.01	p=0.005
1	12.71	25.45	63.66	127.32
2	4.30	6.20	9.92	14.09
3	3.18	4.17	5.84	7.45
4	2.78	3.50	4.60	5.60
5	2.57	3.16	4.03	4.77
6	2.45	2.97	3.71	4.32
7	2.36	2.84	3.50	4.03
8	2.31	2.75	3.36	3.83
9	2.26	2.68	3.25	3.69
10	2.23	2.63	3.17	3.58
11	2.20	2.59	3.11	3.50
12	2.18	2.56	3.05	3.43
13	2.16	2.53	3.01	3.37
14	2.14	2.51	2.98	3.33
15	2.13	2.49	2.95	3.29
16	2.12	2.47	2.92	3.25
17	2.11	2.46	2.90	3.22
18	2.10	2.44	2.88	3.20
19	2.09	2.43	2.86	3.17
20	2.09	2.42	2.84	3.15
21	2.08	2.41	2.83	3.14
22	2.07	2.41	2.82	3.12
23	2.07	2.40	2.81	3.10
24	2.06	2.39	2.80	3.09
25	2.06	2.38	2.79	3.08
26	2.06	2.38	2.78	3.07
27	2.05	2.37	2.77	3.06
28	2.05	2.37	2.76	3.05
29	2.04	2.36	2.76	3.04
30	2.04	2.36	2.75	3.03
40	2.02	2.33	2.70	2.97
60	2.00	2.30	2.66	2.92
120	[illegible]	2.27	2.62	2.86
infinity	1.96	2.24	2.58	2.81

Planned comparisons – 2. Corrections

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- Continue to run t-tests, but adjust the *p* value to make it more conservative
- Only accept significant if below this threshold
- Bonferroni Correction:
  - A new p-value is generated from the prior significance level divided by the number of tests

$\underbrace{0.05}_{P\text{-value}} \div \underbrace{2}_{\text{Number of tests}} = \underbrace{0.025}_{\text{Bonferroni adjusted P-value}}$

Planned comparisons – 2. Corrections

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$t = -2.94$, with 237 degrees of freedom
It's significant at $p = 0.025$ threshold

$t = -2.14$, with 237 degrees of freedom
It's significant at $p = 0.05$ threshold

Degrees of Freedom	p=0.05	p=0.025	p=0.01	p=0.005
1	12.71	25.45	63.66	127.32
2	4.30	6.20	9.92	14.09
3	3.18	4.17	5.84	7.45
4	2.78	3.50	4.60	5.60
5	2.57	3.16	4.03	4.77
6	2.45	2.97	3.71	4.32
7	2.36	2.84	3.50	4.03
8	2.31	2.75	3.36	3.83
9	2.26	2.68	3.25	3.69
10	2.23	2.63	3.17	3.58
11	2.20	2.59	3.11	3.50
12	2.18	2.56	3.05	3.43
13	2.16	2.53	3.01	3.37
14	2.14	2.51	2.98	3.33
15	2.13	2.49	2.95	3.29
16	2.12	2.47	2.92	3.25
17	2.11	2.46	2.90	3.22
18	2.10	2.44	2.88	3.20
19	2.09	2.43	2.86	3.17
20	2.09	2.42	2.84	3.15
21	2.08	2.41	2.83	3.14
22	2.07	2.41	2.82	3.12
23	2.07	2.40	2.81	3.10
24	2.06	2.39	2.80	3.09
25	2.06	2.38	2.79	3.08
26	2.06	2.38	2.78	3.07
27	2.05	2.37	2.77	3.06
28	2.05	2.37	2.76	3.05
29	2.04	2.36	2.76	3.04
30	2.04	2.36	2.75	3.03
40	2.02	2.33	2.70	2.97
60	2.00	2.30	2.66	2.92
120	1.98	[illegible]	2.62	2.86
infinity	1.96	2.24	2.58	2.81

PSYC214: Statistics
Lecture 3 – Assumptions of ANOVA and follow-up procedures – Part IV

Michaelmas Term
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Pairwise comparisons

There are two strategies for following-up significant ANOVAs

- Planned comparisons
  - T-tests
  - Bonferroni corrections
- Post-hoc comparisons

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Post hoc tests

- Post hoc comes from Latin for "after the event"
- Post hoc tests assess all possible combinations of differences between groups by comparing each mean with the other
- Make adjustments to *p* value, but more conservative than Bonferroni correction

Group	$\bar{A}_1$	$\bar{A}_2$	$\bar{A}_3$	$\bar{A}_4$	$\bar{A}_5$
$\bar{A}_1$	-	-	-	-	-
$\bar{A}_2$	●	-	-	-	-
$\bar{A}_3$	●	●	-	-	-
$\bar{A}_4$	●	●	●	-	-
$\bar{A}_5$	●	●	●	●	-

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Post hoc tests

Method	Equal N F	Normality	Use	Error control	Protection
Fisher PLSD	Yes	Yes	Yes	All	Most sensitive to Type 1
Tukey-Kramer HSD	No	Yes	Yes	All	Less sensitive to Type 1 than Fisher PLSD
Spjotvoll-Stoline	No	Yes	Yes	All	As Tukey-Kramer
Student-Newman Keuls (SNK)	Yes	Yes	Yes	All	Sensitive to Type 2
Tukey-Compromise	No	Yes	Yes	All	Average of Tukey and SNK
Duncan's Multiple Range	No	Yes	Yes	All	More sensitive to Type 1 than SNK
Scheffé's S	Yes	No	No	All	Most conservative
Games/Howell	Yes	No	No	All	More conservative than majority
Dunnett's test	No	No	No	T/C	More conservative than majority
Bonferroni	No	Yes	Yes	All, TC	Conservative

https://www.researchgate.net/profile/Cyril-Iaconelli/post/The_choice_of_post-hoc_test/

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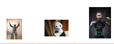
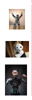
Post hoc tests – Tukey-Kramer HSD

Table IX: Tukey α = 0.05





Group	$\bar{A}_1$	$\bar{A}_2$	$\bar{A}_3$
$\bar{A}_1$	-	-	-
$\bar{A}_2$	●	-	-
$\bar{A}_3$	●	●	-

Studentized range statistic [num means, df]

$$W = \boxed{q_{(r, df_{ERROR})}} \frac{\sqrt{\boxed{Mean\ Square_{ERROR}}}}{\boxed{N_A}}$$

Within group variance from ANOVA output

Number of participants

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Table IX: Tukey α = 0.05

Table IX(a) Studentized range critical values (α = .05)

Error df	k = 2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	18.0	27.0	32.8	37.1	40.4	43.1	45.4	47.4	49.1	50.6	52.0	53.2	54.3	55.4	56.3	57.2	58.0	58.8	59.6
2	6.08	8.33	9.80	10.9	11.7	12.4	13.0	13.5	14.0	14.4	14.7	15.1	15.4	15.7	15.9	16.1	16.4	16.6	16.8
3	4.50	5.91	6.82	7.50	8.04	8.48	8.85	9.18	9.46	9.72	9.95	10.2	10.3	10.5	10.7	10.8	11.0	11.1	11.2
4	3.93	5.04	5.76	6.29	6.71	7.05	7.35	7.60	7.83	8.03	8.21	8.37	8.52	8.66	8.79	8.91	9.03	9.13	9.23
5	3.64	4.60	5.22	5.67	6.03	6.33	6.58	6.80	6.99	7.17	7.32	7.47	7.60	7.72	7.83	7.93	8.03	8.12	8.21
6	3.46	4.34	4.90	5.30	5.63	5.90	6.12	6.32	6.49	6.65	6.79	6.92	7.03	7.14	7.24	7.34	7.43	7.51	7.59
7	3.34	4.16	4.68	5.06	5.36	5.61	5.82	6.00	6.16	6.30	6.43	6.55	6.66	6.76	6.85	6.94	7.02	7.10	7.17
8	3.26	4.04	4.53	4.89	5.17	5.40	5.60	5.77	5.92	6.05	6.18	6.29	6.39	6.48	6.57	6.65	6.73	6.80	6.87
9	3.20	3.95	4.41	4.76	5.02	5.24	5.43	5.59	5.74	5.87	5.98	6.09	6.19	6.28	6.36	6.44	6.51	6.58	6.64
10	3.15	3.88	4.33	4.65	4.91	5.12	5.30	5.46	5.60	5.72	5.83	5.93	6.03	6.11	6.19	6.27	6.34	6.40	6.47
11	3.11	3.82	4.26	4.57	4.82	5.03	5.20	5.35	5.49	5.61	5.71	5.81	5.90	5.98	6.06	6.13	6.20	6.27	6.33
12	3.08	3.77	4.20	4.51	4.75	4.95	5.12	5.27	5.39	5.51	5.61	5.71	5.80	5.88	5.95	6.02	6.09	6.15	6.21
13	3.06	3.73	4.15	4.45	4.69	4.88	5.05	5.19	5.32	5.43	5.53	5.63	5.71	5.79	5.86	5.93	5.99	6.05	6.11
14	3.03	3.70	4.11	4.41	4.64	4.83	4.99	5.13	5.25	5.36	5.46	5.55	5.64	5.71	5.79	5.85	5.91	5.97	6.03
15	3.01	3.67	4.08	4.37	4.59	4.78	4.94	5.08	5.20	5.31	5.40	5.49	5.57	5.65	5.72	5.78	5.85	5.90	5.96
16	3.00	3.65	4.05	4.33	4.56	4.74	4.90	5.03	5.15	5.26	5.35	5.44	5.52	5.59	5.66	5.73	5.79	5.84	5.90
17	2.98	3.63	4.02	4.30	4.52	4.70	4.86	4.99	5.11	5.21	5.31	5.39	5.47	5.54	5.61	5.67	5.73	5.79	5.84
18	2.97	3.61	4.00	4.28	4.49	4.67	4.82	4.96	5.07	5.17	5.27	5.35	5.43	5.50	5.57	5.63	5.69	5.74	5.79
19	2.96	3.59	3.98	4.25	4.47	4.65	4.79	4.92	5.04	5.14	5.23	5.31	5.39	5.46	5.53	5.59	5.65	5.70	5.75
20	2.95	3.58	3.96	4.23	4.45	4.62	4.77	4.90	5.01	5.11	5.20	5.28	5.36	5.43	5.49	5.55	5.61	5.66	5.71
24	2.92	3.53	3.90	4.17	4.37	4.54	4.68	4.81	4.92	5.01	5.10	5.18	5.25	5.32	5.38	5.44	5.49	5.55	5.59
30	2.89	3.49	3.85	4.10	4.30	4.46	4.60	4.72	4.82	4.92	5.00	5.08	5.15	5.21	5.27	5.33	5.38	5.43	5.47
40	2.86	3.44	3.79	4.04	4.23	4.39	4.52	4.63	4.73	4.82	4.90	4.98	5.04	5.11	5.16	5.22	5.27	5.31	5.36
60	2.83	3.40	3.74	3.98	4.16	4.31	4.44	4.55	4.65	4.73	4.81	4.88	4.94	5.00	5.06	5.11	5.15	5.20	5.24
120	2.80	3.36	3.68	3.92	4.10	4.24	4.36	4.47	4.56	4.64	4.71	4.78	4.84	4.90	4.95	5.00	5.04	5.09	5.13
∞	2.77	3.31	3.63	3.86	4.03	4.17	4.29	4.39	4.47	4.55	4.62	4.68	4.74	4.80	4.85	4.89	4.93	4.97	5.01

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Post hoc tests – Tukey-Kramer HSD

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Group	$\bar{A}_1$	$\bar{A}_2$	$\bar{A}_3$
$\bar{A}_1$	-	-	-
$\bar{A}_2$	●	-	-
$\bar{A}_3$	●	●	-

Studentized range statistic

$$W = \boxed{3.31}\sqrt{\frac{\boxed{48.8}}{\boxed{239}}}$$

Within group variance from ANOVA output

Number of participants

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Post hoc tests – Tukey-Kramer HSD

Lancaster University

Group	$\bar{A}_1$	$\bar{A}_2$	$\bar{A}_3$
$\bar{A}_1$	-	-	-
$\bar{A}_2$	●	-	-
$\bar{A}_3$	●	●	-

$$W = 3.31\sqrt{0.20}$$

$$W = 1.48$$

Means that differ over 1.48 will be statistically significant

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Post hoc tests – Tukey-Kramer HSD

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- Take home message
- As you add more and more mean comparisons, you require larger critical values (q) in the standardized table to find a statistical difference!
- As such, test what you need, not what you don't!

Group	$\bar{A}_1$	$\bar{A}_2$	$\bar{A}_3$	$\bar{A}_4$	$\bar{A}_5$
$\bar{A}_1$	-	-	-	-	-
$\bar{A}_2$	●	-	-	-	-
$\bar{A}_3$	●	●	-	-	-
$\bar{A}_4$	●	●	●	-	-
$\bar{A}_5$	●	●	●	●	-

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Lecture 3 – Assumptions of ANOVA and follow-up procedures

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Review of Lecture 3

- Assumptions of ANOVA
  - Assumption of independence
  - Assumption of normality
  - Assumption of homogeneity of variance
- Data transformations
- Pairwise between-level comparisons
  - Planned comparisons
  - Post-hoc tests

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Thank you for attention! Questions?

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