

We have got formula for Depth-limited search:

$$V_{\min\max}(s, d) = \begin{cases} \text{Utility}(s) & \text{IsEnd} \\ \text{Eval}(s) & d = 0 \\ \max_{a \in \text{Actions}(s)} V_{\min\max}(\text{Succ}(s, a), d) & \text{Player}(s) = \text{agent} \\ \min_{a \in \text{Actions}(s)} V_{\min\max}(\text{Succ}(s, a), d-1) & \text{Player}(s) = \text{opp} \end{cases}$$

We have $n+1$ agents so we need to modify minimax to get max for agent a_0 (pacman) and to minimize for agents a_1, \dots, a_n . To denote player and each one of n -opponents we are adding parameter "agent". For n -opponents we recursing through all of n states considering a depth of 1.

We adjusting $d-1$ to $\text{mod}(d)$ where

$$\text{mod}(d) = \begin{cases} d & \text{next_agent} < (n+1) \\ d-1 & \text{next_agent} = (n+1) \end{cases}$$

In this case we have:

$$U_{\text{minmax}}(s, \text{agent}, d) = \begin{cases} \text{Utility}(s) & \text{isEnd}(s) \\ \text{Eval}(s) & d=0 \\ \max_{a \in \text{Actions}(s)} U_{\text{minmax}}(\text{Succ}(s, a), \text{next_agent}, d) & \text{player}(s) = \text{player} \\ \min_{a \in \text{Actions}(s)} U_{\text{minmax}}(\text{Succ}(s, a), \text{next_agent}, \text{mod}(d)) & \text{player}(s) = \text{opponent} \end{cases}$$

Expectimax recurrence differs from minimax in the way that actions of our opponent (ghost) are random. so we take average move for the opponent and we will get:

$$\begin{aligned}
 & \text{Utility}(s) \\
 & \text{Eval}(s) \\
 V_{\text{exptminmax}}(s, \text{agent}, d) = & \begin{cases} \max_{a \in \text{Actions}(s)} V_{\text{exptminmax}}(\text{Succ}(s, a), \text{next_agent}, d) & \text{player}(s) = \text{player} \\ \frac{\sum_{a \in \text{Actions}(s)} V_{\text{exptminmax}}(\text{Succ}(s, a), \text{next_agent}, \text{mod}(d))}{|\text{Actions}(s)|} & \text{player}(s) = \text{opponent.} \end{cases}
 \end{aligned}$$