

“Estimating the Secondary Cosmic Ray Flux using two Pancake type Geiger Mueller Tubes” Mike Loughlin 11/17/2022

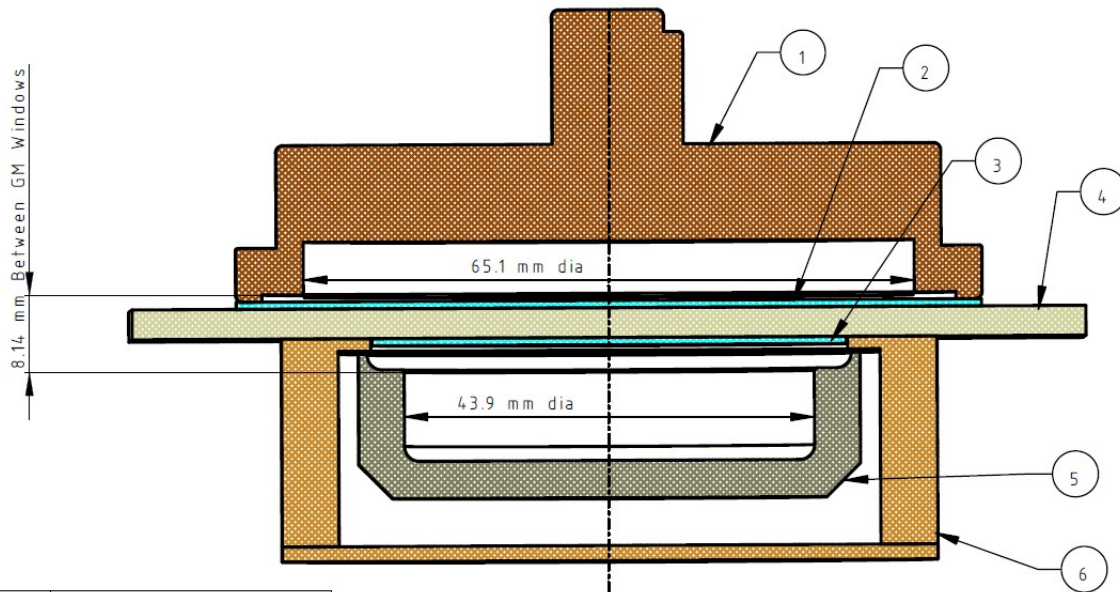
The goal was to build a simple, inexpensive Cosmic Ray detector with as large a view of the sky as possible. This would permit more rapid measurements, with a minimum of adjustments and corrections required to determine the local flux. This arrangement will not provide any information about the angular distribution of the flux.

The purpose was just to have fun and learn something by repurposing hardware I already possessed. The Dolleiser paper (see References Section) was very informative regarding sources of error and how to estimate the solid angle of the sky as seen from a lower GM tube through an upper GM tube.

Section 1 - Detector Arrangement, Bill of Material, and Electronics



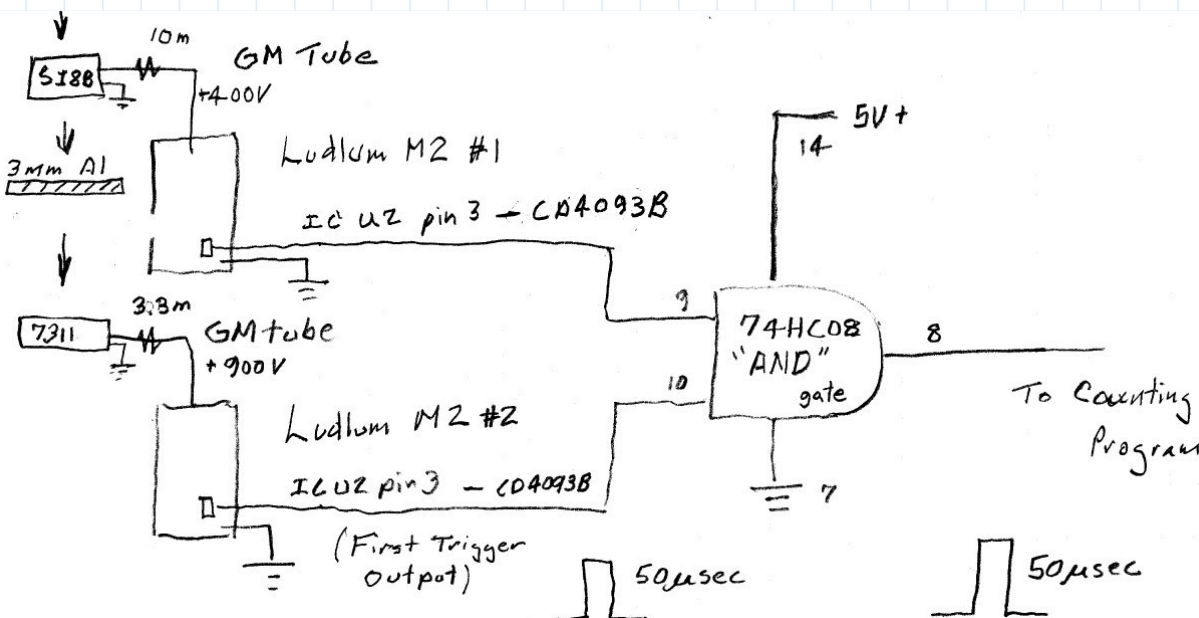
Detector Assembly
(Shown with Radium source for background "enhancement" coincidence test) Ref. Section 4



Item No.	Description
1	SI8B Geiger Mueller Tube
2	LD Polypropylene Centralizer
3	LD Polypropylene Centralizer
4	3mm Aluminum Beta Shield
5	LND Inc. 7311 Geiger Mueller Tube
6	Ludlum Model 44-9 Probe Housing

DESIGNED BY: Sparky		Title Secondary Cosmic Ray Detector		G	—
DATE: Oct 21, 2022				F	—
SIZE A4				E	—
				D	—
				C	—
				B	—
SCALE	WEIGHT (kg)	DRAWING NUMBER	SHEET		
Scale	Weight	Drawing number	Sheet		

Detector Bill of Assembly and Cross Section View



Electronics Sketch - (Ref. A)

Section 2 - Preliminary Data

Background rate SI8B GM tube	63.4 cpm	1798 counts
Background rate LND 7311 GM tube	37.0 cpm	1711 counts
Background Coincidence Rate	13.16 cpm	1685 counts
Background + Radium SI8B GM Tube rate	156.1 cpm	2809 counts
Background + Radium 7311 GM Tube rate	82.3 cpm	1892 counts
Background + Radium Coincidence Rate	13.47 cpm	1711 counts

Section 3 - Correction for the incomplete view of the sky by the Detector Assembly.

Looking at the Detector Assembly cross-section, it is evident that Secondary Cosmic Rays coming in at a low angle to the horizon will not intercept both GM tubes to generate coincident counts. The view of this sky by the Assembly is not complete, i.e. not equal to 2π steradians. This section attempts to estimate the missing fraction of sky and correct the measured flux to represent the full 2π rate. The missing rate is estimated to only be about 1% of the measured rate. This section details that estimate.

$D_1 := 6.61 \text{ cm}$ Upper GM tube window diameter

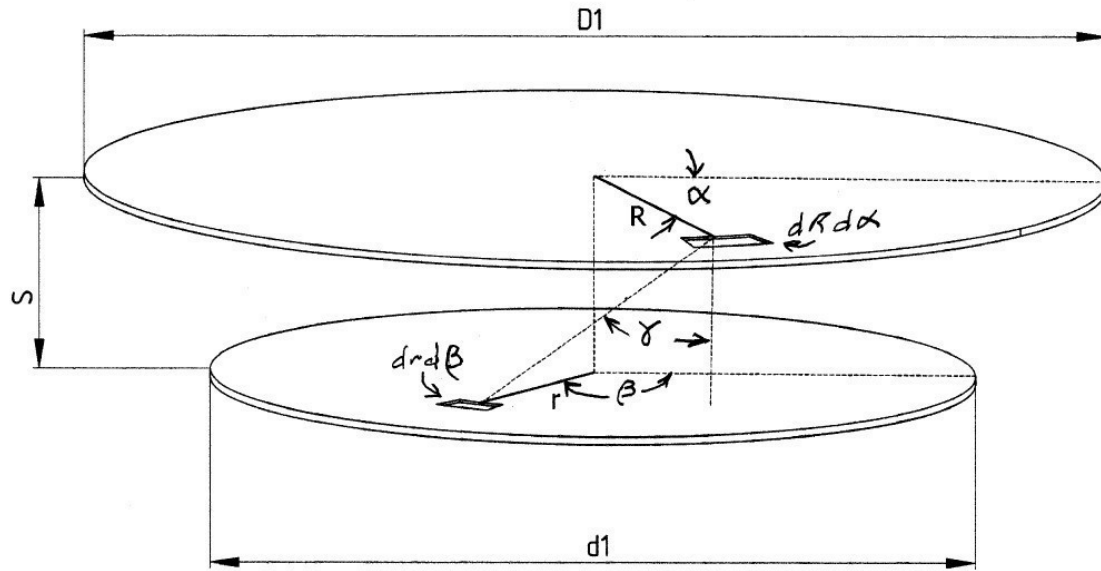
$d_1 := 4.39 \text{ cm}$ Lower GM tube window diameter

$R_1 := \frac{D_1}{2}$ Upper GM tube window radius

$r_1 := \frac{d_1}{2}$ Lower GM tube window radius

$s := .814 \text{ cm}$ Distance between GM tube windows

$CR := \frac{13.16}{min}$ GM tubes measured Coincidence Rate, cpm



Distance Between Differential elements =

$$\sqrt{(R \cdot \cos(\alpha) - r \cdot \cos(\beta))^2 + (R \cdot \sin(\alpha) - r \cdot \sin(\beta))^2 + s^2}$$

$$\cos(\gamma) = S / \sqrt{(R \cdot \cos(\alpha) - r \cdot \cos(\beta))^2 + (R \cdot \sin(\alpha) - r \cdot \sin(\beta))^2 + s^2}$$

$S\Omega$ is the product of the lower (LND 7311) GM tube window area and the average solid angle visible to the upper (SI8B) GM tube window area. The expression in the integral below represents the product of the area of the lower disk, and the average of the inverse square of the distance and the cosine of the angle of between differential area elements.

$$S\Omega := \int_0^{2 \cdot \pi} \int_0^{R_1} \int_0^{2 \cdot \pi} \int_0^{r_1} s \cdot R \cdot r \cdot ((R \cdot \cos(\alpha) - r \cdot \cos(\beta))^2 + (R \cdot \sin(\alpha) - r \cdot \sin(\beta))^2 + s^2)^{-1.5} dr d\beta dR d\alpha$$

(Eq. 1)

$$S\Omega = 68.038 \text{ (cm}^2 \cdot \text{sr)}$$

(Ref B)

$$A_1 := \pi \cdot r_1^2 \quad A_1 = 15.136 \text{ cm}^2$$

Area of Lower GM tube window

$$SA := \frac{S\Omega}{A_1}$$

$$SA = 4.495$$

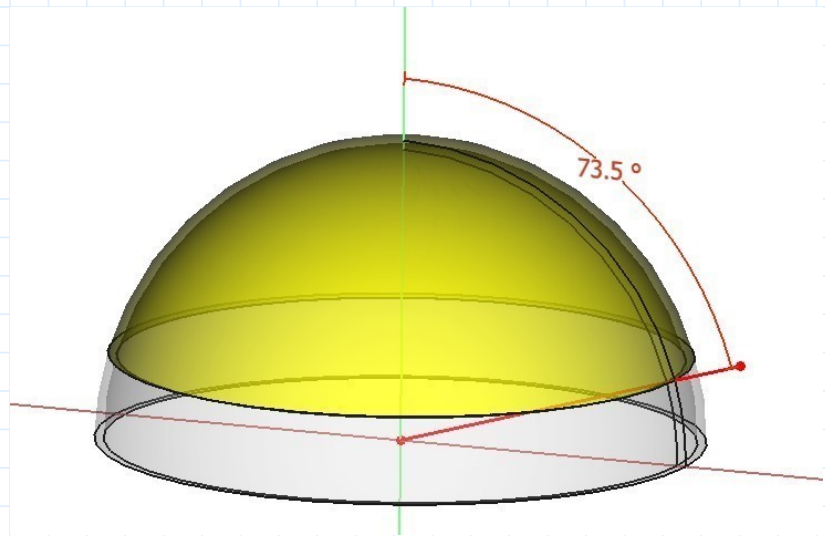
$$\Theta_1 := \arccos\left(1 - \frac{SA}{2 \cdot \pi}\right)$$

$$\Theta_1 = 1.282 \text{ rad}$$

$$\Theta_1 = 73.465^\circ$$

Average Solid Angle of the view to the sky of the the lower GM tube through the upper GM tube in steradians. (Eq.2)

Average Angle of the view to the sky of the the lower GM tube through the upper GM tube in degrees from vertical



$$J_m := \frac{CR}{A_1}$$

$$J_m = 0.869 \frac{1}{\text{cm}^2 \cdot \text{min}}$$

$$J_0 := \frac{J_m}{\left(\int_{-\Theta_1}^{\Theta_1} (\cos(\theta))^2 d\theta \right)}$$

$$J_0 = 93.185 \frac{1}{\text{m}^2 \cdot \text{s}}$$

$$J_0 = 0.559 \frac{1}{\text{cm}^2 \cdot \text{min}}$$

CR Flux as measured, i.e. Coincidence rate/lower GM area, i.e. uncorrected Secondary Cosmic Ray Flux with less than 2π viewing angle.

J_0 is the vertical flux. J_{0s} is the vertical flux for 1 steradian. Both calculations assume that intensity varies with Cosine² of the zenith angle. (Ref. C)

$$J_{0s} := J_0 \cdot \int_{-32.771^\circ}^{32.771^\circ} (\cos(\theta))^2 d\theta \quad (\text{Eq 3})$$

$$J_{0s} = 95.709 \frac{1}{\text{m}^2 \cdot \text{s} \cdot \text{sr}}$$

$$J_2\pi := \int_0^{\pi} J_0 \cdot (\cos(\theta))^2 d\theta$$

Correction of measured flux for full 2π view, assuming that intensity varies with Cosine² of the zenith angle.

(Eq. 4) (Ref B and C)

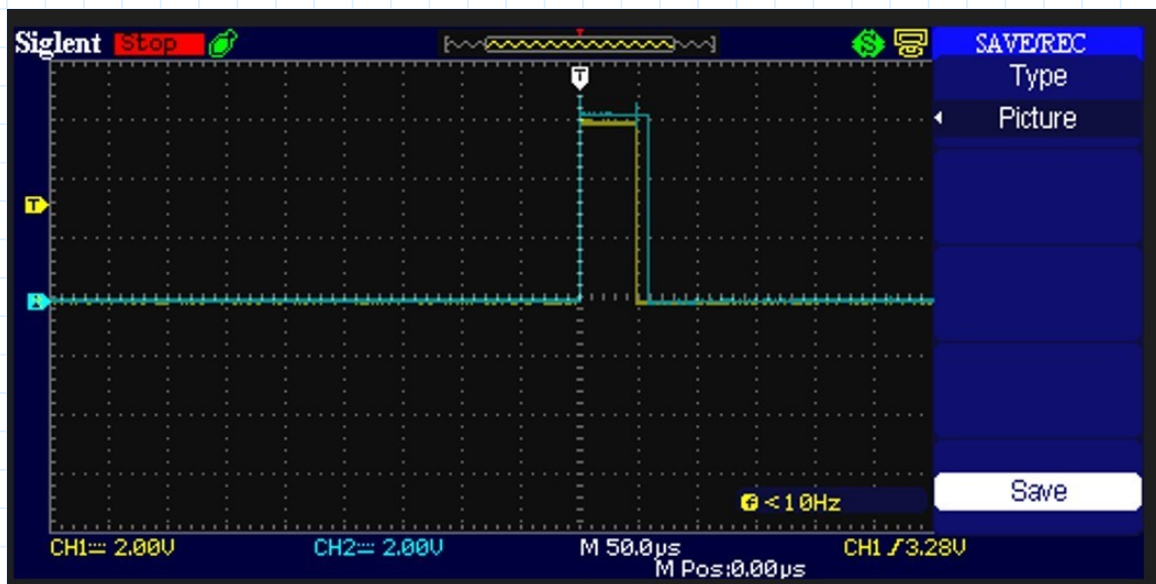
$$J_2\pi = 0.878 \frac{1}{\text{cm}^2 \cdot \text{min}}$$

$$J_2\pi = 146.374 \frac{1}{\text{m}^2 \cdot \text{s}}$$

Section 4 - Error Estimates in measured Coincidence rate from Background Terrestrial Radiation

This section considers two sources of error; (4.1) Random coincident counts between the two GM tubes of the assembly (4.2) Coincident counts caused by a single gamma ray interacting with both GM tubes.

Section 4.1 Random coincident counts between the two GM tubes of the assembly



The oscilloscope trace above shows a coincident event of the discharge of both GM tubes as seen by the AND gate of the electronics. The pulse length of the LND 7311 signal is about 60 useconds (blue). The pulse length of the SI8B signal is about 50 useconds (yellow). As a simplifying assumption, both pulse lengths will be 60 microseconds in the calculations below.

$$BG_d := \frac{37}{1 \text{ min}}$$

Background Rate of the LND 7311 GM Tube

$$dt := 60 \text{ } \mu\text{s}$$

Deadtime of LND 7311 and SI8B GM Tube

$$BG_D := \frac{63.4}{1 \text{ min}}$$

Background Rate of the SI8B GM Tube

$$Prob_{dD} := 2 \cdot BG_d \cdot BG_D \cdot dt$$

Rate of 2-tier random coincident count of both GM tubes (Ref. C)

$$Prob_{dD} = 0.00469 \frac{1}{\text{min}}$$

This Coincident rate is very small relative to measured flux rate (13.2 cpm).

Section 4.2 Coincident counts caused by single gamma rays interacting with both GM tubes.

This is a more complicated estimate. The basic idea was to increase the "Background rate" with a Radium source (watch hand) and observe the increase in coincident counts. Then use proportionality to estimate the coincident count rate fraction due to the original background rate.

Simplifying Assumptions:

(1) Assume approximately 13 cpm of the measured background rate (with or without Radium enhancement) is from secondary cosmic rays. (2) Assume the balance is gamma radiation in the case of either Background or "Enhanced Background". (3) Assume that the energy response (GM efficiency) of the Radium enhancement is similar to that of Background Gamma radiation.

$$BG_{\text{gamma}} := BG_d - \frac{13}{\text{min}}$$

Original Background gamma count rate with LND 7311 GM tube

$$BG_{\text{gamma}} = 24 \frac{1}{\text{min}}$$

$$BG_{\text{Radium}} := \left(\frac{82.3}{\text{min}} - \frac{13}{\text{min}} \right)$$

Enhanced Background gamma count rate with Radium and LND 7311 GM tube

$$BG_{\text{Radium}} = 69.3 \frac{1}{\text{min}}$$

$$CR = 13.16 \frac{1}{\text{min}}$$

Original measured Coincident Rate

$$CR_{\text{Radium}} := \frac{13.47}{\text{min}}$$

Coincident Rate with Radium enhanced "Background".

$$\Delta CR := (CR_{\text{Radium}} - CR)$$

Change in Coincidence rate with Radium enhancement.

$$\Delta BG := BG_{\text{Radium}} - BG_{\text{gamma}}$$

Change in "Background" gamma rate with Radium enhancement.

$$CR_{\text{gamma}} := \frac{\Delta CR}{\Delta BG} \cdot BG_{\text{gamma}}$$

Estimated Coincident count rate caused by background Terrestrial gamma discharging both tubes. This error represents about 1.25% of Total measured Cosmic Ray flux. - No Correction was made.

$$CR_{\text{gamma}} = 0.164 \frac{1}{\text{min}}$$

Section 5 - Conclusions

The local Secondary Cosmic Ray Flux, as measured by this apparatus and adjusted for its restricted view of the sky, is about **.88 counts cm⁻² min⁻¹**. Analysis of the solid angle viewed by the detector gave a result of 4.495 steradians (1.43 π), but the % of missed counts is very small because the angular distribution is more intense near the zenith.

An estimate of random coincident events from background counts yielded a negligible rate with respect to the measured coincidence rate. A (rough) estimate of coincidence events caused by background gamma rays intersecting and discharging both GM tubes yielded a value of 1.25% of the total measured coincident rate. This estimate required a number of "simplifying assumptions" that are logical, but unproven.

Section 6 - References

Ref A - Ludlum Model 2A Survey Meter Manual, Revised Feb. 1990

Ref B - Secondary Cosmic Rays, Marek Dolleiser, August 3, 2022
Equation #5

<https://www.gammaspectacular.com/phpBB3/viewtopic.php?f=16&t=1025>

Ref C - http://courses.washington.edu/phys433/muon_counting/counting_telescope.pdf