INVESTIGATING THE BOTTOMONIUM SYSTEM

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In this program we solve the **Schrödinger equation** for the **Bottomonium system** and investigate the energies and spectra of its bound states mediated by the **strong interaction**.

QUANTUM PHYSICS

In the Macroscopic world all objects have a defined position, calculable with **Classical Mechanics**. Once we delve to the atomic scale it is not possible to measure exactly where a particle is and so we must use **Quantum Mechanics**. In Quantum Mechanics the position of a particle is expressed as a **function of probability**.

In a Quarkonium system a **quark and an anti-quark** are bonded together into bound states by the **strong force** to make a **Meson**. The **bottom (b)** and **charm (c)** quarks form particularly interesting bound states as they form energy levels and spectra similar to atoms.

Figure 1: Illustrations of the **Charmonium** system (left) and **Bottomonium** system (right) with arrows indicating the strong force.

The Quarkonium system must also obey the **Schrödinger equation** [1]:

$$-\frac{\hbar^2}{2\mu}\nabla^2\psi + V(r)\psi = E_{n,l}\psi \qquad (1)$$

Where in Quarkonium *V(r)* is the **potential**, μ is the **reduced mass** of the system, and $E_{n,l}$ is the **energy** of the system according to n, the principal quantum number and l the azimuthal quantum number. ψ , is the wavefunction.

METHOD

Firstly note all units are **natural units** such that c = ħ =1.

Second the wavefunction is split into its **radial**, $R_{n,l}(r)$ and **harmonic components**, $Y_{lm}(\theta,\phi)$ and define $u_{n,l}(r)$ as:

$$\psi = R_{n,l}(r)Y_{lm}(\theta,\phi) \quad (2) \qquad u_{n,l}(r) = rR_{n,l}(r) \quad (3)$$

This allows us to write the **differential equation** below [1]:

$$\frac{d^2u_{n,l}}{dr^2} - \frac{l(l+1)}{r^2}u_{n,l} + 2\mu(E_{n,l} - V(r))u_{n,l} \quad (4)$$

To solve the **second order** differential equation we first rewrite it as **two coupled first order** differential equations. In python we can solve coupled differential equations numerically using Odient a package contained within the scipy.integrate module. The wavefunctions are normalised in all plots using the **Simpson rule**.

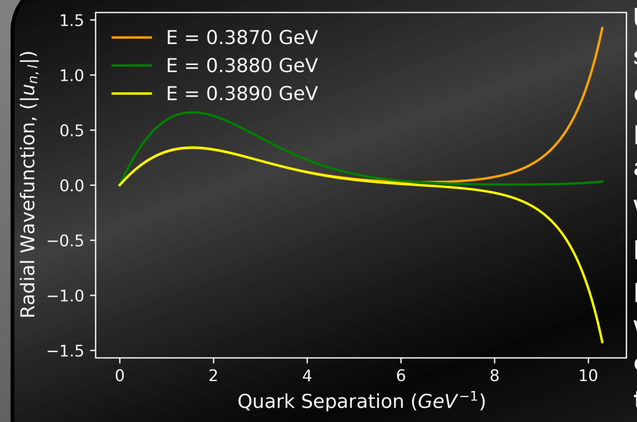
POTENTIAL CHOICE

The choice of potential can have a profound effect on the results of the program. The potential used in this program is [2]:

$$V(r) = br - \frac{4a}{3r} \qquad (5)$$

Where *a* and *b* are constants. The potential was chosen as it is the simplist potential which describes **quantum chromodynamic (QCD)** behaviour. The *a/r* term represents one **gluon exchange** as short distance where *a* is the **strong coupling constant** and the *br* term represents a **confining potential** which stops the quarks existing as free particles.

BISECTION ITERATION

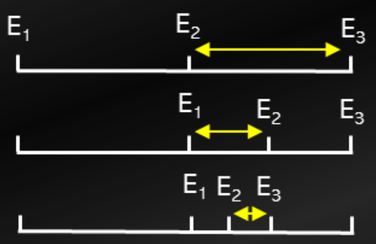


Using odeint we can solve equation (4) easily for a known potential and energy and obtain the wavefunction.

Figure 2: The green plot shows the wavefunction for the correct energy of the (1,0) state. The orange and yellow plots show energies just above and just below the correct energy.

- When the trial energy is too high or low the wavefunction will exhibit a different number of **turning points** and **nodes.**
- Utilising this behaviour and the **bisection method** we can **iterate** towards a correct energy.

The iteration goes as follows:



- Take two guess values E1 and E2 and define E3 as half of the sum of E1 and E2.
- Calculate the number of nodes and turning points for the wavefunction at E1, E2 and E3. If they change between E2 and E3 define E2 as E1 and repeat.
- The diagram the above visually represents the iteration process.
- For the bottomonium results the iteration was run **1000** times.

BOTTOMONIUM RESULTS

Using known values for the mass of the **Charmonium** ground state the **bisection method** was used to find *b* = 0.195 to 3 significant figures.

Using this value of *b*, *a* = 0.28 and the mass of the **Bottomonium**, the binding energy of states (1,0), (1,1) and (2,0) were found and their spectra plotted. The **mass of the state,** $M_{n,l}$ is related to the **binding energy,** $E_{n,l}$ using [2]:

$$M_{n,l} = m_1 + m_2 + E_{n,l} \qquad (6)$$

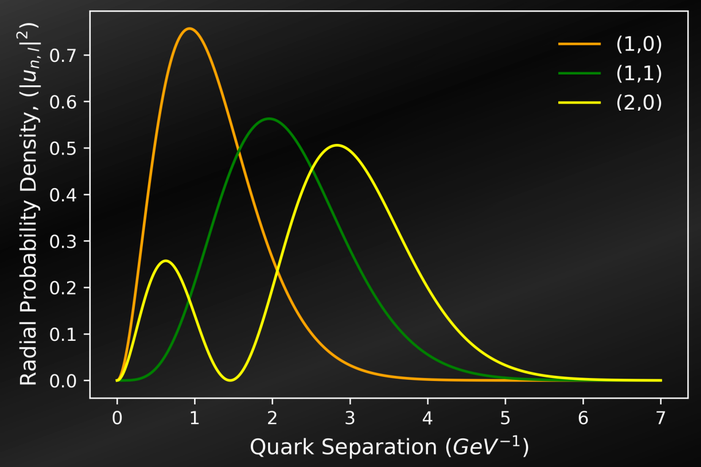


Figure 4: Plots of the **radial probability density** for the specified principal and azimuthal quantum numbers.

	Computed Results (GeV/c^2) (to 3 s.f)	Literature Results[4] (GeV/c^2)
(1,0)	9500	9402.4 ± 1.8
(1,1)	9890	9899.1 ± 1.0
(2,0)	10000	9974.6 ± 2.3

Table 1 [4]: Summary of the **computed** and **literature** results for the specified states.

Overall the results are agreeable with the literature as the (1,1) and (2,0) states are consistent to two significant figures. The (1,0) state is not consistent with the results but this could be improved using a more accurate potential which may work better at a lower energy level.

REFERENCES

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