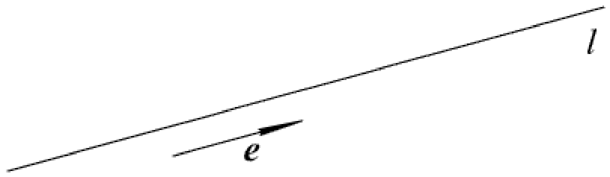
MAVZU5:

Vektorning o‘qqa proeksiyasi. Vektorlarning skalyar ko‘paytmasi.

Musbat yo‘nalishi tanlab olingan l to‘g‘ri chiziq o‘q deb ataladi. O‘qning yo‘nalishini odatda strelka bilan ko‘rsatiladi(1-chizma), bu strelkaning yo‘nalishi l to‘g‘ri chiziqdagi musbat yo‘nalishni aniqlovchi $\vec{e}$ vektor yo‘nalishi bilan bir xil bo‘ladi.



1-rasm. O‘q

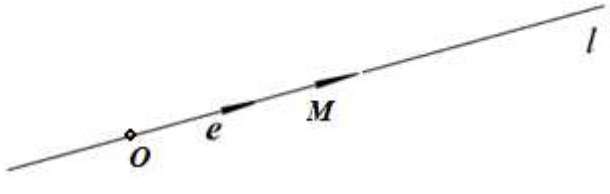
Yo‘nalishi o‘qdagi musbat yo‘nalishi bilan bir xil bo‘lgan hamda uzunligi birga teng bo‘lgan (ya’ni $|\vec{e}|=1$) $\vec{e}$ vektor o‘qning bazisi deyiladi.

Agar o‘qda biror bazis tanlangan bo‘lsa, u holda o‘qdagi har bir vektorga to‘la aniqlangan bitta son mos keltiriladi va bu son vektorning bazis bo‘yicha yoyilmasining koeffitsientidan iborat bo‘ladi.

l o‘qda yotgan $\overrightarrow{OM}$ vektor shu o‘qda tanlangan $\vec{e}$ bazis bilan kollinear bo‘ladi. Vektorlarning kollinear bo‘lish shartidan (2-chizma)

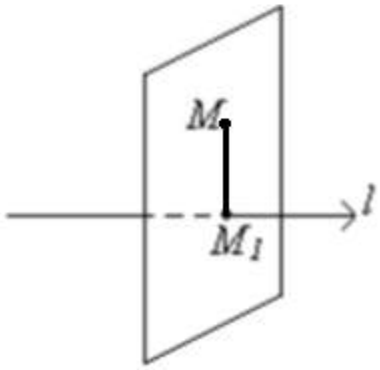
$$\overrightarrow{OM} = x\vec{e} \tag{1.1}$$

munosabatni yoza olamiz. (1.1) dagi x soni odatda $\overrightarrow{OM}$ vektorning koordinatasi deyiladi. Agar x son $\overrightarrow{OM}$ vektorning koordinatasi bo‘lsa, uning $M(x)$ ko‘rinishdagi yozuvi x son $\overrightarrow{OM}$ vektorning koordinatasi degan ma’noni anglatadi, shu bilan birga x son M nuqtaning koordinatasi degan ma’noni ham anglatadi.



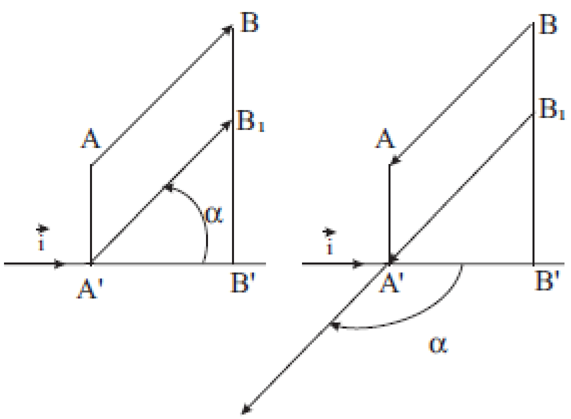
2-chizma.

Bizga fazoda l o‘q berilgan bo‘lsin. M nuqtaning l o‘qdagi proyeksiyasi berilgan nuqtadan o‘qqa tushurilgan $\overline{MM_1}$ perpendikulyarning M_1 asosidir (3-chizma).



3-chizma.

1-ta’rif. $\overrightarrow{AB}$ *vektorning* l *o‘qdagi proyeksiyasi deb, shunday* $\left|\overrightarrow{A_1B_1}\right|$ *vektorning uzunligiga aytiladiki, unda* A_1 *va* B_1 *mos ravishda* A *va* B *nuqtalarning* l *o‘qdagi ortogonal proyeksiyalari bo‘lib, bu uzunlik* l *o‘qni yo‘nalishi bilan bir xil bo‘lganda musbat ishora bilan, aks holda manfiy ishora bilan olinadi(4-chizma).*



4-chizma.

$\overrightarrow{AB}$ vektorning l o‘qdagi proyeksiyasini $pr_l\overrightarrow{AB}$ simvoli bilan belgilaymiz. Ta’rif bo‘yicha

$$pr_l\overrightarrow{AB} = \pm\left|\overrightarrow{A_1B_1}\right|.$$

(1.4.1) formulaga ko‘ra

$$\overrightarrow{A_1B_1} = x\vec{e} \qquad (1.4.2)$$

deb yoza olamiz. Bu tenglikdagi x son $\overrightarrow{AB}$ vektorning proyeksiyasidir, ya'ni

$$x = pr_l \overrightarrow{AB}.$$

Agar $\overrightarrow{AB} = \vec{0}$ yoki $\overrightarrow{AB} \perp l$ bo'lsa, u holda $pr_l \overrightarrow{AB} = 0$ bo'ladi.

2-ta'rif. *l o'q bilan $\vec{a}$ vektor orasidagi burchak deb, l o'qning birlik vektori $\vec{e}$ bilan $\vec{a}$ vektor orasidagi burchakka aytiladi (4-rasm).*

Vektorning o'qdagi proyeksiyasi xossalari.

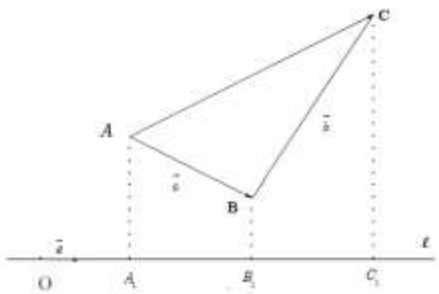
1^0. Vektorlarning yig'indisining biror o'qdagi proyeksiyasi qo'shiluvchi vektorlarning shu o'qdagi proyeksiyalari yig'indisiga teng, ya'ni

$$pr_l(\vec{a} + \vec{b} + \vec{c} + \ldots + \vec{d}) = pr_l\vec{a} + pr_l\vec{b} + pr_l\vec{c} + \ldots + pr_l\vec{d}$$

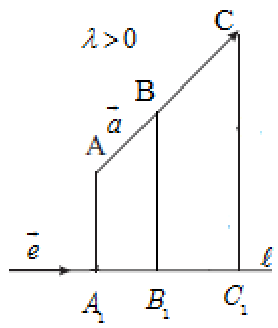
Isbot. Isbotni ikki vektor uchun keltiramiz,ya'ni

$$pr_l(\vec{a} + \vec{b}) = pr_l\vec{a} + pr_l\vec{b} \qquad (1.4.3)$$

ekanini isbot qilamiz. Agar $\vec{a} = \overrightarrow{AB},\ \vec{b} = \overrightarrow{BC}$ bo'lsin desak, u holda $\vec{a} + \vec{b} = \overrightarrow{AC}$ bo'ladi (5-chizma).



5-chizma.



6-chizma.

5-chizmadan, (1.4.2) ni e'tiborga olsak,

$$\overrightarrow{A_1B_1} = x_1\vec{e},\ \overrightarrow{B_1C_1} = x_2\vec{e},\ \overrightarrow{A_1C_1} = x_3\vec{e}$$

ni yoza olamiz. Bunda $x_1 = pr_l\vec{a},\ x_2 = pr_l\vec{b},\ x_3 = pr_l(\vec{a} + \vec{b})$. Endi $x_3 = x_1 + x_2$ ekanini ko'rsatamiz. Ravshanki, $x_3\vec{e} = pr_l\overrightarrow{A_1C_1} = \overrightarrow{A_1B_1} + \overrightarrow{B_1C_1} = x_1\vec{e} + x_2\vec{e} = (x_1 + x_2)\vec{e}$. Shunday qilib, (1.4.3) formula isbot qilindi.

Qo'shiluvchilar soni ikkitadan ortiq bo'lganda ham isbot shunga o'xshash olib boriladi.

2^0. Vektorning songa ko‘paytmasining proyeksiyasi shu vektor proyeksiyasini o‘sha songa ko‘paytmasiga teng, ya’ni

$$pr_l(\lambda\vec{a}) = \lambda\, pr_l\vec{a},\ \ \lambda \neq 0.$$

Isbot. $\overrightarrow{AB} = \vec{a},\ \overrightarrow{AC} = \lambda\vec{a}$ bo‘lsin deb faraz qilaylik. A, B, C nuqtalarning l o‘qdagi proyeksiyalari A_1, B_1, C_1 bo‘lsin (6-chizma). AA_1, BB_1, CC_1 kesmalar o‘zaro parallel, shuning uchun $\overrightarrow{A_1C_1} = \lambda\overrightarrow{A_1B_1}$. (1.4.2) formulaga ko‘ra

$$\overrightarrow{A_1B_1} = x\vec{e},\ \ \overrightarrow{A_1C_1} = x_1\vec{e},$$

ya’ni $x = pr_l\vec{a},\ x_1 = pr_l(\lambda\vec{a})$ deb belgilasa

$$x_1\vec{e} = \overrightarrow{A_1C_1} = \lambda\overrightarrow{A_1B_1} = \lambda(x\vec{e}) = (\lambda x)\vec{e},$$

bundan esa $x = \lambda x_1$ kelib chiqadi. Xossaning isboti tugadi.

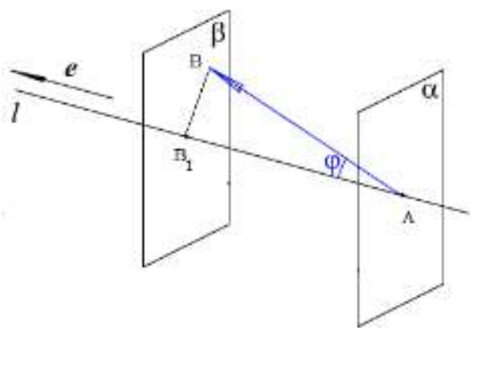
3^0. Teng vektorlarning bitta o‘qqa proyeksiyalari o‘zaro tengdir.

4^0. Vektorning o‘qdagi proyeksiyasining kattaligi shu vektor uzunligini vektor va o‘qning musbat yo‘nalishi orasidagi φ burchak kosinusiga ko‘paytmasiga teng, ya’ni

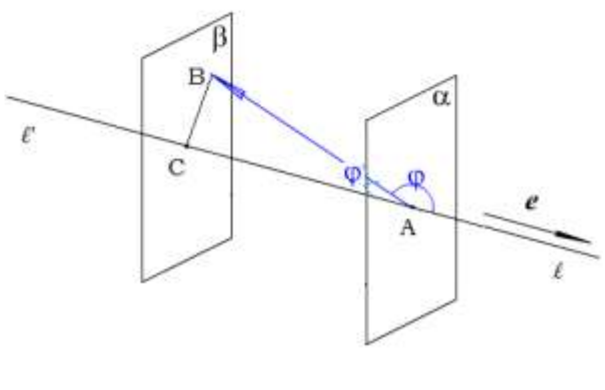
$$pr_l\,\vec{a} = |\vec{a}|cos\varphi. \tag{1.4.4}$$

Isbot. Aytaylik, $A \in l,\ \overrightarrow{AB} = \vec{a},\ B_1$ nuqta B ning l o‘qdagi proyeksiyasi bo‘lsin. $\overrightarrow{AB}$ vektor bilan o‘q orasidagi burchak o‘tkir bo‘lsa (7-chizma), proyeksiya ta’rifiga ko‘ra $pr_l\overrightarrow{AB} = +\left|\overrightarrow{AB_1}\right|$ bo‘ladi. ABB_1 uchburchakdan $\left|\overrightarrow{AB_1}\right| = \left|\overrightarrow{AB}\right|\cos\varphi = |\vec{a}|\cos\varphi$ ya’ni

$$pr_l\,\vec{a} = |\vec{a}|cos\varphi.$$



7-chizma.



8-chizma.

Agar φ burchak o‘tmas bo‘lsa, u holda

$$pr_l\overrightarrow{AB} = -\left|\overrightarrow{AC}\right|. \qquad (1.4.5)$$

8-chizmadagi ABC uchburchakdan

$$\left|\overrightarrow{AC}\right| = |\vec{a}|\,cos\varphi', \qquad (1.4.6)$$

bu yerda $\varphi' = \pi - \varphi$. Bundan

$$cos\varphi' = -cos\varphi \qquad (1.4.7)$$

(1.4.5)-(1.4.7) formulalardan izlangan (1.4.4) formula kelib chiqadi.

$\vec{a}$ va $\vec{b}$ vektorlar L_3 uch o‘lchovli chiziqli fazoning ixtiyoriy ikki vektori bo‘lsin.

Ta’rif*. Ikkita $\vec{a}$ va $\vec{b}$ vektorlarning skalyar ko‘paytmasi deb ushbu*

$$(\vec{a}\ \vec{b}) = |\vec{a}|\left|\vec{b}\right|\cos\varphi$$

tenglik bilan aniqlanadigan songa aytiladi. Bu yerda φ- $\vec{a}$ va $\vec{b}$ vektorlar orasidagi burchak.

Ikki vektorni skalyar ko‘paytirish amali quyidagi xossalarga ega:

1. Skalyar ko‘paytirish o‘rin almashtirish qonuniga bo‘ysunadi:

$$(\vec{a}\vec{b}) = (\vec{b}\vec{a})$$

Isbot. Ta’rifga ko‘ra

$$(\vec{a}\vec{b}) = |\vec{a}|\left|\vec{b}\right|\cos(\vec{a}\,{}^\wedge\vec{b}),$$

$$(\vec{b}\vec{a}) = \left|\vec{b}\right||\vec{a}|\cos(\vec{b}\,{}^\wedge\vec{a});$$

Kosinus juft funksiya ekanligini e’tiborga olsak, $\cos(\vec{a}\,{}^\wedge\vec{b}) = \cos(\vec{b}\,{}^\wedge\vec{a})$ bundan kelib chiqadi: $(\vec{a}\vec{b}) = (\vec{b}\vec{a})$.

2. Har qanday vektorning o‘z-o‘ziga skalyar ko‘paytmasi bu vektor uzunligining kvadratiga teng:

$$(\vec{a}\,\vec{a}) = |\vec{a}|^2.$$

Isbot. Skalyar ko‘paytma ta’rifidan,

$$(\vec{a}\vec{a}) = |\vec{a}||\vec{a}|\cos(\vec{a}\,{}^\wedge\vec{a}) = |\vec{a}|^2\cos 0^\circ = |\vec{a}|^2.$$

$(\vec{a}\vec{a})$ ifoda $\vec{a}^2$ bilan belgilanadi va $\vec{a}$ vektorning skalyar kvadrati deb ataladi.

U holda $(\vec{a}\vec{a}) = |\vec{a}|^2$ tenglikdan $\vec{a}$ vektorning uzunligi:

$$|\vec{a}| = \sqrt{\vec{a}^2}.$$

3. Ikki vektorning skalyar ko‘paytmasi ularning birining uzunligi bilan ikkinchisining birinchisi yo‘nalishiga tushirilgan proyeksiyasi ko‘paytmasiga teng, ya’ni

$$(\vec{a}\vec{b}) = |\vec{a}|\, pr_{\vec{a}}\vec{b} = |\vec{b}|\, pr_{\vec{b}}\vec{a} \ (\vec{a} \neq 0,\ \vec{b} \neq 0).$$

Isbot.

$$(\vec{a}\vec{b}) = |\vec{a}||\vec{b}|\cos(\vec{a} \,\hat{}\, \vec{b}) = |\vec{a}|\, pr_{\vec{a}}\vec{b},$$

$$(\vec{a}\vec{b}) = |\vec{b}||\vec{a}|\cos(\vec{a} \,\hat{}\, \vec{b}) = |\vec{b}|\, pr_{\vec{b}}\vec{a}.$$

(bu yerda ortogonal proyeksiya ko‘zda tutilgan).

4. Skalyar ko‘paytirish skalyar ko‘paytuvchiga nisbatan guruhlanish qonuniga bo‘ysunadi, ya’ni

$$((m\vec{a})\vec{b}) = m(\vec{a}\vec{b}), \text{ bu yerda } m \in R.$$

Isbot. Yuqoridagi 1va 3 xossalarga ko‘ra

$$((m\vec{a})\vec{b}) = (\vec{b}(m\vec{a})) = |\vec{b}|\, pr_{\vec{b}}(m\vec{a}) = |\vec{b}|\, m\, pr_{\vec{b}}\vec{a} = m(\vec{b}\vec{a}) = m(\vec{a}\vec{b}).$$

5. Ikkita vektorning skalyar ko‘paytmasi nolga teng bo‘lishi uchun ularning o‘zaro perpendikulyar bo‘lishi zarur va yetarlidir.

$$(\vec{a}\vec{b}) = 0\ ,\ \vec{a} \perp \vec{b}$$

Isbot. $\vec{a} \perp \vec{b}$ Bu holda $(\vec{a} \,\hat{}\, \vec{b}) = \dfrac{\pi}{2} \Rightarrow (\vec{a}\vec{b}) = |\vec{a}||\vec{b}| cos\dfrac{\pi}{2} = 0.$

6. Skalyar ko‘paytirish taqsimot qonuniga bo‘ysunadi, ya’ni har qanday $\vec{a},\vec{b},\vec{c}$ vektorlar uchun

$$(\vec{a} + \vec{b})\vec{c} = (\vec{a}\vec{c}) + (\vec{b}\vec{c}). \qquad (1)$$

Isbot. (1) munosabatning $\vec{c} = \vec{0}$ hol uchun o‘rinli ekanligi ravshan. $\vec{c} \neq \vec{0}$ bo‘lsin. Yuqoridagi 1 va 3 xossalarga ko‘ra

$$(\vec{a} + \vec{b})\vec{c} = \vec{c}(\vec{a} + \vec{b}) = |\vec{c}|\, pr_{\vec{c}}(\vec{a} + \vec{b}) = |\vec{c}|(pr_{\vec{c}}\vec{a} + pr_{\vec{c}}\vec{b}) = (\vec{c}\vec{a}) + (\vec{c}\vec{b}) = (\vec{a}\vec{c}) + (\vec{b}\vec{c}).$$

Ta'rif. *Ikkita vektor skalyar ko'paytmasi nolga teng bo'lsa, bunday vektorlar ortogonal deyiladi.*

Yuqoridagi ta'rifdan ikkita vektor ortogonal vektorlar bo'lsa ular orasidagi burchak 90° gradusga teng bo'ladi.

Skalyar ko'paytmaning koordinatalardagi ifodasi.

L_3 chiziqli fazoda ortonormallangan $(\vec{e}_1, \vec{e}_2, \vec{e}_3)$ bazisni olaylik. $\vec{a}, \vec{b}$ vektorlar bu bazisga nisbatan (x_1, y_1, z_1) va (x_2, y_2, z_2) koordinatalarga ega bo'lsin:

$$\vec{a} = x_1\,\vec{e}_1 + y_1\,\vec{e}_2 + z_1\,\vec{e}_3,$$
$$\vec{b} = x_2\,\vec{e}_1 + y_2\,\vec{e}_2 + z_2\,\vec{e}_3.$$

Yuqoridagi 4 va 6 xossalarga asoslanib

$$\begin{aligned}(\vec{a}\vec{b}) &= (x_1\,\vec{e}_1 + y_1\,\vec{e}_2 + z_1\,\vec{e}_3)(x_2\,\vec{e}_1 + y_2\,\vec{e}_2 + z_2\,\vec{e}_3) = \\ &= x_1\,x_2\,\vec{e}_1^{\,2} + y_1 y_2\,\vec{e}_2^{\,2} + z_1\,z_2\vec{e}_3^{\,2} + (x_2\,y_1 + x_1 y_2)\,\vec{e}_1\,\vec{e}_2 + \\ &\quad + (z_1\,y_2 + y_1 z_2)\vec{e}_2\vec{e}_3 + (x_1 z_2 + z_1 x_2\,)\,\vec{e}_1\,\vec{e}_3\end{aligned}$$

munosabatni yoza olamiz, bazislarni ortonormalligini e'tiborga olsak,

$$(\vec{a}\vec{b}) = x_1\,x_2 + y_1 y_2 + z_1\,z_2. \tag{1.6.2}$$

Demak, koordinatalari bilan berilgan ikki vektorning skalyar ko'paytmasi bu vektorlar mos koordinatalari ko'paytmalarining yig'indisiga teng.

Skalyar ko'paytmadan kelib chiqadigan ba'zi natijalar.

1) **Vektorlar orasidagi burchak**. Ikki $\vec{a}, \vec{b}$ vektor orasidagi burchak ushbu formula bo'yicha hisoblanadi:

$$\cos\varphi = \frac{(\vec{a}\vec{b})}{|\vec{a}||\vec{b}|},$$

bu yerda φ - $\vec{a}$ va $\vec{b}$ vektorlar orasidagi burchak.

Koordinatalari bilan berilgan $\vec{a}(x_1, y_1, z_1), \vec{b}(x_2, y_2, z_2)$ vektorlar uchun

$$\cos\varphi = \frac{(\vec{a}\vec{b})}{|\vec{a}||\vec{b}|} = \frac{x_1 x_2 + y_1 y_2 + z_1 z_2}{\sqrt{x_1^2 + y_1^2 + z_1^2}\,\sqrt{x_2^2 + y_2^2 + z_2^2}}.$$

I. **Vektor uzunligi**. $\vec{a}(x,\,y,\,z)$ vektorning uzunligi uning koordinatalari kvadratlarining yig'indisidan olingan arifmetik kvadrat ildizga teng:

$$|\vec{a}| = \sqrt{x^2 + y^2 + z^2}.$$

2) **Vektorlarning perpendikulyarlik sharti**. $\vec{a}(x_1, y_1, z_1), \vec{b}(x_2, y_2, z_2)$ vektorlarning perpendikularlik sharti quyidagicha bo‘ladi:

$$x_1x_2 + y_1y_2 + z_1z_2 = 0$$

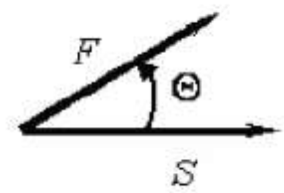
Haqiqatan, $\vec{a} \perp \vec{b} \Rightarrow (\vec{a}\vec{b}) = 0.$ (1.6.2) dan

$$x_1x_2 + y_1y_2 + z_1z_2 = 0$$

3) **Berilgan yo‘nalishda vektorning proyeksiyasi**. Berilgan $\vec{b}$ vektor yo‘nalish bo‘yicha $\vec{a}$ vektorining proyektsiyasi quyidagi formula bo‘yicha hisoblanadi

$$pr_{\vec{b}}\vec{a} = \frac{\left(\vec{a}\cdot\vec{b}\right)}{\left|\vec{b}\right|} = \frac{x_1 x_2 + y_1y_2 + z_1 z_2}{\sqrt{x_2^2 + y_2^2 + z_2^2}}.$$

O‘zgarmas kuchning ishi. Agar $\vec{F}$ - to‘g‘ri chiziqli $\vec{S}$ siljishga ta’sir etuvchi o‘zgarmas kuch bo‘lsa, u holda bu kuchning belgilangan siljishdagi A ishi $A = \vec{F}\left|\vec{S}\right|\cos(\vec{F}\,^\wedge\,\vec{S})$ ga teng, ya’ni $A = (\vec{F}\cdot\vec{S})$ (1.6.1-rasm).



1.6.1-rasm.

Ikki vektorning skalyar ko‘paytmasi.

Bizga V_3 vektor fazoda ixtiyoriy ikkita $\vec{a}\,va\,\vec{b}$ vektorlar berilgan bo‘lsin.

Ta’rif: $\vec{a}\,va\,\vec{b}$ vektorlarning skalyar ko‘paytmasi deb ular uzunliklarining ular orasidagi burchak kosinusining ko‘paytmasiga aytiladi. $\vec{a}\vec{b}$ yoki $(\vec{a},\vec{b})$ ko‘rinishda yoziladi.

Demak $(\vec{a},\vec{b}) = \left|\vec{a}\right|\left|\vec{b}\right|\cos\varphi \quad \varphi = (\vec{a}\,^\wedge\,\vec{b})$

Skalyar ko‘paytma xossalari:

1. Skalyar ko‘paytma o‘rin almashinish konuniga bo‘ysunadi.(skalyar ko‘paytmasi kamutativdir).

$$\vec{a}\vec{b} = \vec{b}\vec{a}$$

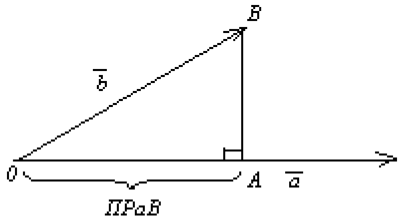
2. Har qanday vektorning o‘zini-o‘ziga skalyar ko‘paytmasi bu vektor uzunligining kvadratiga tengdir.

$$\vec{a}\cdot\vec{a}=\left|\vec{a}\right|^2$$

Isbot. $\vec{a}\cdot\vec{a}=\left|\vec{a}\right|\cdot\left|\vec{a}\right|\cos(\vec{a}\,\hat{}\,\vec{a})=\left|\vec{a}\right|^2\quad\cos 0=\left|\vec{a}\right|^2$

3. $\vec{a}\cdot\vec{b}=\left|\vec{a}\right|pr_{\vec{a}}\vec{b}=\left|\vec{b}\right|pr_{\vec{b}}\vec{a}$

Isbot:



Rasm

$\triangle OAB$ to‘g’ri burchakli uchburchak bo‘lib- $\cos\varphi=\dfrac{pr_{\vec{a}}\vec{b}}{\left|\vec{b}\right|}$, $pr_{\vec{a}}\vec{b}=\left|\vec{b}\right|\cos\varphi$

$pr_{\vec{a}}\vec{b}=\left|\vec{a}\right|\cos\varphi$, $\vec{a}\cdot\vec{b}=\left|\vec{a}\right|\cdot\left|\vec{b}\right|\cos\varphi=\left|\vec{a}\right|pr_{\vec{a}}\vec{b}$, $\vec{a}\cdot\vec{b}=\left|\vec{a}\right|pr_{\vec{a}}\vec{b}$.

4. Skalyar ko‘paytma guruxlash qonuniga bo‘y sunadi.

$$m\vec{a}\cdot m\vec{b}=m(\vec{a}\cdot\vec{b})$$

5. Skalyar ko‘paytma 0ga teng bo‘lsa, orasidagi burchak 90° bo‘ladi:

$$\vec{a}\cdot\vec{b}=0\Leftrightarrow\varphi=90^\circ\quad\vec{a},\vec{b}\neq 0$$

6. Skalyar ko‘paytma taqsimot qonuniga bo‘y sunadi (distribyutiv), ya’ni

$$(\vec{a}+\vec{b})\vec{c}=\vec{a}\cdot\vec{c}+\vec{b}\cdot\vec{c}$$

Isbot:
$$(\vec{a}+\vec{b})\vec{c}=\vec{c}(\vec{a}+\vec{b})=\left|\vec{c}\right|pr_{\vec{c}}(\vec{a}+\vec{b})=\left|\vec{c}\right|(pr_{\vec{c}}\vec{a}+pr_{\vec{c}}\vec{b})=$$
$$=\left|\vec{c}\right|pr_{\vec{c}}\vec{a}+\left|\vec{c}\right|pr_{\vec{c}}\vec{b}=(\vec{c}\cdot\vec{a})+(\vec{c}\cdot\vec{b})=(\vec{a}\vec{c})+(\vec{b}\vec{c})$$

7. $\{\vec{e}_1\vec{e}_2\vec{e}_3\}$ vektorlar V_3 vektor fazoning bazis vektorlari bo‘lsin. U xolda

$$(\vec{e}_i\vec{e}_j)=\begin{cases}1,\ agar\ i=j\\0,\ agar\ i\neq j\end{cases}$$

Ta’rif: *Ikkita vektor ortaganal deyiladi agar ular orasidagi burchak* 90° *bo‘lsa.*

Skalyar ko‘paytmaning koordinatalarda ifodalanishi.

Bizga $\vec{a}(x_1y_1z_1)$ va $\vec{b}(x_2y_2z_2)$ vektorlar berilgan bo‘lsin.

$$\vec{a}=x_1\vec{e}_1+y_1\vec{e}_2+z_1\vec{e}_3$$
$$\vec{b}=x_2\vec{e}_1+y_2\vec{e}_2+z_2\vec{e}_3$$

bo‘ladi.

$$\vec{a}\cdot\vec{b}=(x_1\vec{e}_1+y_1\vec{e}_2+z_1\vec{e}_3)(x_2\vec{e}_1+y_2\vec{e}_2+z_2\vec{e}_3)=x_1x_2\vec{e}_1^{\,2}+x_1\vec{e}_1y_2\vec{e}_2+x_1\vec{e}_1y_2\vec{e}_3+$$
$$+y_1\vec{e}_2x_2\vec{e}_1+y_1\vec{e}_2^{\,2}y_2+y_1\vec{e}_2z_2\vec{e}_3+z_1\vec{e}_3x_2\vec{e}_1+z_1\vec{e}_3y_2\vec{e}_2+z_1\vec{e}_3^{\,2}z_2=x_1x_2(\vec{e}_1\vec{e}_1)+x_1y_2(\vec{e}_2\vec{e}_1)+x_1y_2(\vec{e}_3\vec{e}_1)+$$
$$+y_1x_2(\vec{e}_1\vec{e}_2)+y_1y_2(\vec{e}_2\vec{e}_2)+y_1z_2(\vec{e}_3\vec{e}_2)+z_1x_2(\vec{e}_1\vec{e}_3)+z_1y_2(\vec{e}_2\vec{e}_3)+z_1z_2(\vec{e}_3\vec{e}_3)=x_1x_2+y_1y_2+z_1z_2$$
$$\vec{a}\cdot\vec{b}=x_1x_2+y_1y_2+z_1z_2,$$
$$|\vec{a}|^2=(\vec{a}\cdot\vec{a})=x_1^2+y_1^2+z_1^2,$$
$$|\vec{a}|=\sqrt{x_1^2+y_1^2+z_1^2}\,.$$

Ikki vektor orasidagi burchak $\cos\varphi=\dfrac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|}=\dfrac{x_1x_2+y_1y_2+z_1z_2}{\sqrt{x_1^2+y_1^2+z_1^2}\sqrt{x_2^2+y_2^2+z_2^2}}$

Misol: Burchakni toping. $\vec{a}(3;-4;0),\vec{b}(4;3;0)$

Yechish:

$$\cos\varphi=\frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|}=\frac{3\cdot4-4\cdot3+0}{\sqrt{9+16+0}\sqrt{16+9+0}}=\frac{12-12+0}{5\cdot5}=0$$
$$\cos\varphi=0$$
$$\varphi=90^\circ$$

MISOLLAR

Ikki vektorning skalyar ko‘paytmasi. Vektorlarning uzunligi va ikki vektor orasidagi burchak

1. $\vec{a},\vec{b}$ vektorlar orasidagi burchak $\varphi=\dfrac{2\pi}{3}$ va $|\vec{a}|=4,|\vec{b}|=4$ ga teng bo‘lsa, $\vec{a}^{\,2},\vec{b}^{\,2},(\vec{a}+\vec{b})^2,(\vec{a}-\vec{b})^2,(3\vec{a}-2\vec{b})(\vec{a}+2\vec{b})$ larni hisoblang.

2. $\overrightarrow{OA}=\vec{a},\overrightarrow{OB}=\vec{b}$ vektorlar va $|\vec{a}|=2,|\vec{b}|=4$, $\varphi=\dfrac{\pi}{3}$ lar berilgan. AOB uchburchakning OA tomoni va OM medianasi orasidagi α burchakni hisoblang.

3. $ABCD$ to‘g’ri to‘rtburchakning $A(1,-2,2);B(1,4,0);C(-4,1,1);D(-5,-5,3)$ uchlari berilgan. Uning diagonallari orasidagi α burchakni hisoblang.

4. $\vec{a}=\{4,-2,-4\};\vec{b}=\{6,-3,2\}$ vektorlar berilgan. $\vec{a}\vec{b},\ \vec{a}^{\,2},\ \vec{b}^{\,2},\ (\vec{a}+\vec{b})^2,\ (\vec{a}-\vec{b})^2$ $(2\vec{a}-3\vec{b})(\vec{a}+2\vec{b})$ larni hisoblang.

5. ABC uchburchakning $A(-1,-2,4);B(-4,-2,0);C(3,-2,1)$ uchlari berilgan. B uchining tashqi burchagini toping.

6. α ning qanday qiymatida $\vec{a}=\alpha\vec{i}-3\vec{j}+2\vec{k}$ va $\vec{b}=\vec{i}+2\vec{j}-\alpha\vec{k}$ vektorlar o‘zaro perpendikular bo‘ladi.

7. $\vec{a}\vec{b}=3$ shartni qanoatlantiruvchi $\vec{a}=\{2,1,-1\}$ vektorga kollinear bo‘lgan $\vec{b}$ vektorning koordinatalarini toping.

8. Yoyilmalarda $|\vec{p}|=2\sqrt{2}$, $|\vec{q}|=4$ va $\varphi=\dfrac{\pi}{2}$ ekanligi ma’lum bo‘lsa, $\vec{a}=5\vec{p}+2\vec{q}$

va $\vec{b}=\vec{p}-3\vec{q}$ vektorlarga yasalgan parallelogram diagonallarining uzunligi hisoblansin.

9. $\vec{a}(3,1)$ va $\vec{b}(1,3)$ vektorlarga qurilgan parallelogram diagonallarining uzunliklari yig'indisini toping.

10. Agar $|\vec{a}|=6, |\vec{a}+\vec{b}|=11$ va $|\vec{a}-\vec{b}|=7$ bo'lsa, $|\vec{b}|$ ning qiymatini toping.

11. $\overrightarrow{AB}(-3,0,2)$ va $\overrightarrow{AC}(7,-2,2)$ vektorlar ABC uchburchakning tomonlaridir. Shu uchburchakning AN medianasi uzunligini toping.

12. $|\vec{a}|=3, |\vec{b}|=4$. $\vec{a}$ *va* $\vec{b}$ vektorlar orasidagi burchak 60° ga teng. λ ning qanday qiymatida $(\vec{a}-\lambda\vec{b})$ vektor $\vec{a}$ vektorga perpendikulyar bo'ladi?

13. Agar $M(1,1); N(2,3)$ va $K(-1,2)$ bo'lsa, MNK uchburchakning eng katta burchagini toping.

14. $\vec{a}=2\vec{i}+\vec{j}$ va $\vec{b}=-2\vec{j}+\vec{k}$ vektorlarga yasalgan parallelogrammning diagonallari orasidagi burchakni toping.

15. $\vec{i},\vec{j},\vec{k}$ koordinata o'qlari bo'ylab yo'nalgan birlik vektorlar va $\vec{a}=5\vec{i}+\sqrt{2}\vec{j}-3\vec{k}$ bo'lsa, $\vec{a}$ va $\vec{i}$ vektorlar orasidagi burchakning kosinusini toping.

16. Uchta $\vec{a}=\{2,4\}, \vec{b}=\{-3,1\}, \vec{c}=\{5,-2\}$ vektor berilgan. 1) $2\vec{a}+3\vec{b}-5\vec{c}$ 2) $\vec{a}+24\vec{b}+14\vec{c}$ vektorlar topilsin.

17. Uchta $\vec{a}=\{5,3\}; \vec{b}=\{2,0\}; \vec{c}=\{4,2\}$ vektor berilgan. $\vec{b}$ vektorning boshini $\vec{a}$ vektorning oxiri bilan $\vec{b}$ vektorning oxiri bilan $\vec{c}$ vektorning boshini tutashtirdirganda $\vec{a}$, $\vec{b}$, $\vec{c}$ vektorlar uchburchak hosil qilinsin.

18. Quyidagi hollarning har birida $\vec{C}$ vektorni $\vec{a}$ *va* $\vec{b}$ vektorlarning chiziqli kombinatsiyasi shaklida ifodalang:

1) $\vec{a}=\{4,-2\}; \vec{b}=\{3,5\}; \vec{c}=\{1,-7\}$

2) $\vec{a}=\{5,4\}; \vec{b}=\{-3,0\}; \vec{c}=\{19,8\}$

3) $\vec{a}=\{i-6,2\}; \vec{b}=\{4,7\}; \vec{c}=\{9,-3\}$

19. $\vec{a}=\{6,-8\}$ vektor berilgan. $\vec{a}$ ga kolinear va: 1) $\vec{a}$ bilan bir xil yo'nalgan; 2) $\vec{a}$ bilan qarama-qarshi yo'nalgan birlik vektor topilsin.

20. Uchta $\vec{a}=\{5,7,2\}; \vec{b}=\{3,0,4\}; \vec{c}=\{-6,1,-1\}$ vektor berilgan.

1) $3\vec{a}-2\vec{b}+\vec{c}$

2) $5\vec{a}+6\vec{b}+4\vec{c}$ vektorlar topilsin.

21. Quyidagi hollarning har birida $\vec{d}$ vektorni $\vec{a}$, $\vec{b}$, $\vec{c}$ vektorlarning chiziqli

kombinatsiyasi shaklida ifodalang:

1) $\vec{a}=\{2,3,1\};\vec{b}=\{5,7,0\};\vec{c}=\{3,-2,4\};\vec{d}=\{4,12,-3\}$

2) $\vec{a}=\{5,-2,0\};\vec{b}=\{0,-3,4\};\vec{c}=\{-6,0,1\};\vec{d}=\{25,-22,16\}$

3) $\vec{a}=\{3,5,6\};\vec{b}=\{2,-7,1\};\vec{c}=\{12,0,6\};\vec{d}=\{0,20,18\}$

22. $\vec{a}(3,5,7);\vec{b}(-2,6,1)$ va $\vec{c}(2,-4,0)$ vektorlar uchun: $1)\vec{a}\vec{b}$, $2)\vec{a}\vec{c}$, $3)\vec{b}\vec{c}$, $4)(2\vec{a}-\vec{b})(3\vec{b}+\vec{c})$, $(3\vec{a}+2\vec{c})(2\vec{b}-c)$ skalyar ko‘paytmasini hisoblang.

23.Koordinatalari bilan berilgan $\vec{a}(6,-8);\vec{b}(12,9);\vec{c}(2,-5);\vec{d}(3,7);\vec{m}(-2,6);\vec{n}(3,-9)$ vektorlar orasidagi $1)\vec{a}\,\hat{}\,\vec{b}$; $2)\vec{c}\,\hat{}\,\vec{d}$; $3)\vec{m}\,\hat{}\,\vec{n}$ ni toping.

24. Koordinatalari bilan berilgan $\vec{a}(8,4,1);\vec{b}(2,-2,1);\vec{c}(2,5,4);\vec{d}(6,0,-3)$ vektorlar orasidagi $1)\vec{a}\,\hat{}\,\vec{b}$; $2)\vec{c}\,\hat{}\,\vec{d}$ ni toping.

25.$|\vec{a}|=8,|\vec{b}|=5,\ (\vec{a}\,\hat{}\,\vec{b})=60^\circ$ berilgan bo‘lsa, $\vec{a}$ va $\vec{b}$ vektorlarning skalyar ko‘paytmasini toping.

26. $\vec{c}$ va $\vec{d}$ birlik vektor va $(\vec{c}\,\hat{}\,\vec{d})=135^\circ$ berilgan bo‘lsa, $\vec{c}$ va $\vec{d}$ vektorlarning skalyar ko‘paytmasini toping.

27. $|\vec{c}|=3,\ |\vec{d}|=7,\quad \vec{c}\parallel\vec{d}$ berilgan bo‘lsa, $\vec{c}$ va $\vec{d}$ vektorlarning skalyar ko‘paytmasini toping.

28. $\vec{a}$ va $\vec{b}$ vektorlar o‘zaro $\varphi=\dfrac{2\pi}{3}$ burchak tashkil qiladi. $|\vec{a}|=3,|\vec{b}|=4$ bo‘lsa, quyidagilarni hisoblang: $1)\vec{a}\vec{b}$; $2)\vec{a}^2$; $3)\vec{b}^2$; $4)(\vec{a}+\vec{b})^2$; $5)(\vec{a}-\vec{b})^2$; $6)(3\vec{a}+2\vec{b})^2$;

$7)(2\vec{a}-3\vec{b})^2$; $8)(3\vec{a}-2\vec{b})(\vec{a}+2\vec{b})$.

29.$\vec{a}$ va $\vec{b}$ vektorlar o‘zaro perpendikulyar, $\vec{c}$ vektor ularning har biri bilan $\varphi=\dfrac{\pi}{3}$ burchak hosil qilib, $|\vec{a}|=3,|\vec{b}|=5,|\vec{c}|=8$ ga teng bo‘lsa, quyidagilarni hisoblang: $1)(3\vec{a}-2\vec{b})(\vec{b}+3\vec{c})$; $2)(\vec{a}+\vec{b}+\vec{c})^2$; $3)(\vec{a}+2\vec{b}-3\vec{c})^2$; $4)(\vec{a}+\vec{b}-\vec{c})(\vec{a}+\vec{b}+\vec{c})$; $5)(2\vec{a}-\vec{b}+3\vec{c})(2\vec{a}+\vec{b}-3\vec{c})$.

30. $A(-1,3,-7);B(2,-1,5)\,va\,C(0,-1,5)$ nuqtalar berilgan bo‘lsa, $1)\sqrt{\overrightarrow{AB}^2}$; $2)\sqrt{\overrightarrow{AC}^2}$; $3)\sqrt{\overrightarrow{BC}^2}$; $4)(2\overrightarrow{AB}-\overrightarrow{CB})(2\overrightarrow{BC}+\overrightarrow{BA})$; $5)(3\overrightarrow{AB}-2\overrightarrow{CB})(3\overrightarrow{BC}+2\overrightarrow{AC})$ ifodalarni hisoblang.

31. ABC uchburchak tomonlarining uzunliklari berilgan: $|BC|=5,|CA|=6,|AB|=7$ bo‘lsa, $1)\overrightarrow{BA}\,va\,\overrightarrow{BC}$; $2)\overrightarrow{AB}\,va\,\overrightarrow{BC}$; $3)\overrightarrow{AB}\,va\,\overrightarrow{AC}$; $4)\overrightarrow{BA}\,va\,\overrightarrow{CA}$; $5)\overrightarrow{CA}\,va\,\overrightarrow{BC}$ vektorlarning skalyar ko‘paytmasi topilsin.

32. $\vec{a}$, $\vec{b}$ va $\vec{c}$ vektorlar, $\vec{a}+\vec{b}+\vec{c}=0$ shart bilan quyidagilar $|\vec{a}|=3, |\vec{b}|=1, |\vec{c}|=4$ berilgan bo‘lsa, $\vec{a}\vec{b}+\vec{b}\vec{c}+\vec{c}\vec{a}$ ni hisoblang.

33. $\vec{a}$, $\vec{b}$ va $\vec{c}$ vektorlar bir-birlari bilan 60° ga teng bo‘lgan burchak tashkil qilsa, hamda $|\vec{a}|=4, |\vec{b}|=2, |\vec{c}|=6$ berilgan bo‘lsa, $\vec{p}=\vec{a}+\vec{b}+\vec{c}$ vektorning modulini aniqlang.