4MAVZU: TEKISLIKDA VA FAZODA AFFIN VA DEKART KOORDINATALAR SISTEMASI.

Tekislikda nuqtaning koordinatalari.

Tekislikda nuqtaning o‘rnini ma’lum sonlar yordamida aniqlashga imkon beradigan usul ko‘rsatilgan bo‘lsa, tekislikda koordinatalar sistemasi berilgan deyiladi.

Tekislikda koordinatalarning turli sistemalari mavjud bo‘lib, biz ularning soddalarini quramiz.

Tekislikda koordinatalarning affin sistemasi.

Tekislikda biror 0 nuqtaning qo‘yilgan nokalleniar ixtiyoriy ikkita $\vec{e}_1,\vec{e}_2$ vektorlar berilgan bo‘lsin. Bu vektorlar sistemasi $(\vec{e}_1,\vec{e}_2)$ vektorlar orqali o‘tuvchi a,b to‘g’ri chiziqlarni olamiz .

Ta’rif: *Musbat yo‘nalishlari mos ravishda* $\vec{e}_1,\vec{e}_2$ *vektorlar bilan aniqlanuvchi* a,b *to‘g’ri chiziqlardan tashkil topgan sistema tekilikda koordinatalarning affin sistemasi yoki affin reperi deb ataladi va u* $\vec{V}(0,\vec{e}_1,\vec{e}_2)$ *kabi belgilanadi.*

$0 = a \cap b$ nuqta koordinatalar boshi, $\vec{e}_1,\vec{e}_2$-vektorlar esa koordinata vektorlari deyiladi. Musbat yo‘nalishlari $\vec{e}_1,\vec{e}_2$ vektorlar bilan aniqlangan a,b to‘g’ri chiziqlar mos ravishda abstsissalar va ordinatalar o‘qlari deb ataladi. Tekislikda affin reperi berilgan bo‘lsin. Shu tekislikning M nuqtasi uchun $\overrightarrow{OM}$ vektor M nuqtaning radius vektorlari deyiladi va $\overrightarrow{OM}$ vektor quyidagicha ifodalanadi. $\overrightarrow{OM} = x_1\vec{e}_1 + y_1\vec{e}_2$

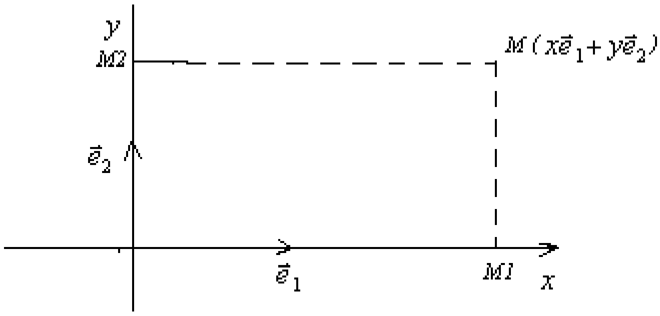
Ta’rif: $\overrightarrow{OM}$ *radius vektorning* x_1, y_1 *koordinatalari* M *nuqtaning* $(0;\vec{e}_1,\vec{e}_2)$ *affin reperidagi koordinatalari deyiladi va* $M(x_1, y_1)$ *bilan belgilanadi.*

Tekislikda affin sistemasi berilgan bo‘lsa, biror N nuqtaga koordinata bo‘lmish x_1, y_1 mos keladi va aksincha. Tekislikda $(0;\vec{e}_1,\vec{e}_2)$ affin reperda abtsissalar o‘qiga koordinatalar boshidan boshlab $OM_1 = x \cdot \vec{e}_1$ vektor ordinatalar o‘qiga $OM_2 = y \cdot \vec{e}_2$ vektorni ko‘yib xosil kilingan M_1, M_2 nuqtalardan mos ravishda abtsissalar va ordinatalar o‘qlariga parallel to‘g’ri chiziqlar o‘tkazsak, ularning kesishgan nuqtasi M nuqta bo‘ladi.

$$\overrightarrow{OM} = \overrightarrow{OM}_1 + \overrightarrow{OM}_2 = x\vec{e}_1 + y\vec{e}_2$$

shunday qilib reperga nisbatan

$$M(x, y) \Leftrightarrow \overrightarrow{OM} = x_1\vec{e}_1 + y_1\vec{e}_2 \qquad (1)$$



1-rasm

Shunday qilib affin reperiga nisbatan $M(x,y)$ nuqtalar $\Leftrightarrow \overrightarrow{OM}$ vektor bilan aniqlanadi va $\overrightarrow{OM} = x\vec{e}_1 + y\vec{e}_2$. Agar M nuqtaning abtsissasi nol bo'lsa $(x=0)$ unda (1) tenglamada shu ko'rinishda bo'ladi:

$$\overrightarrow{OM} = y\vec{e}_2$$

M nuqta Oy o'qda yotadi. Koordinatalar tekisligi butun tekislikni 4ta bo'lakka ajratadi. Agar M nuqta koordinatalari koordinata o'qida yotmasa uning koordinatalari ishorasiga qarab qaysi chorakda yotishi aniqlanadi. Agar M nuqta koordinatalari

$x>0, y>0$ bo'lsa 1-chorakda

$x<0, y>0$ bo'lsa 2-chorakda

$x<0, y<0$ bo'lsa 3-chorakda

$x>0, y<0$ bo'lsa 4-chorakda

$(0,\vec{e}_1,\vec{e}_2)$ reperga nisbatan $A(x_1,y_1); B(x_2,y_2)$ nuqtalarni olaylik. Bu nuqtalarning radiuslari vektorlari $\overrightarrow{OA} = x_1\vec{e}_1 + y_1\vec{e}_2$, $\overrightarrow{OB} = x_2\vec{e}_1 + y_2\vec{e}_2$ ko'rinishda bo'ladi. Bularni bilgan xolda $\overrightarrow{AB}$ vektorning koordinatalarini topamiz.

$$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = x_2\vec{e}_1 + y_2\vec{e}_2 - x_1\vec{e}_1 - y_1\vec{e}_2 = (x_2 - x_1)\vec{e}_1 + (y_1 - y_2)\vec{e}_2$$

Bundan $\overrightarrow{AB}$ vektorning koordinatalari $\overrightarrow{AB} = (x_2 - x_1; y_1 - y_2)$ xosil bo'ladi.

Ta'rif: *Affin reperining koordinata vektorlari* $\vec{e}_1, \vec{e}_2$ *ortanormallangan bazisni tashkil etsin, ya'ni* $\vec{e}_1 \perp \vec{e}_2, |\vec{e}_1| = |\vec{e}_2| = 1$ *bu xolda biz koordinatalarning to'g'ri burchakli sistemasi yoki Dekart reperi berilgan deymiz. So'ng reperni* $(0,i,j)$ *ko'rinishda belgilaymiz.* Bu erda $i^2 = j^2 = 1, ij = 0$

Ta'rif: M_1, M_2 *nuqtalar orasidagi masofa deb,* $\overrightarrow{M_1M_2}$ *yoki* $\overrightarrow{M_2M_1}$ *vektorlar uzunligiga aytiladi.*

Ta'rifga ko'ra $\rho(M_1M_2)=\left|\overrightarrow{M_1M_2}\right|$ $M_1(x_1,y_1);M_2(x_2,y_2)$ bo'lsin, u xolda $\overrightarrow{M_1M_2}=\overrightarrow{0M_2}-\overrightarrow{0M_1}$ bo'ladi.

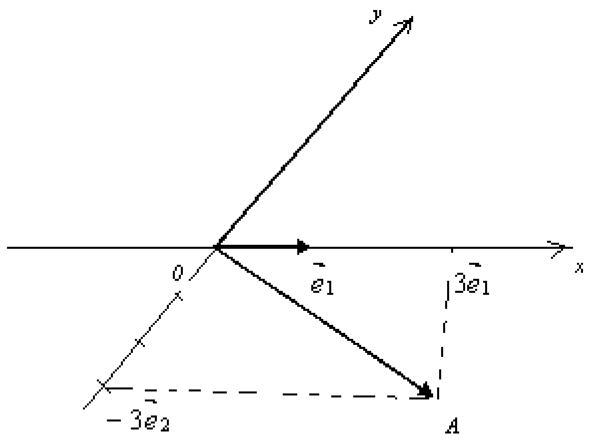
$\overrightarrow{M_1M_2}=\overrightarrow{0M_2}-\overrightarrow{0M_1}=(x_2-x_1;y_2-y_1)$ vektorning uzunligini topish formulasini bilgan xolda $\rho(M_1,M_2)$ vektorning uzunligini topamiz.

$$\left|\overrightarrow{M_1M_2}\right|=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2} \qquad (5)$$

Demak, berilgan M_1,M_2 nuqtalar orasidagi masofa (5) ga qarab topiladi.

Misol: Berilgan $(0,\vec{e},\vec{e})$ Affin reperida $A(3;-3),B(0;3),C(-2;0)$ nuqtalarni yasang.

1) $\overrightarrow{OA}=3\vec{e}_1-3\vec{e}_2$



2-rasm

2) $(0,\vec{e},\vec{e})$ reperda $A(1;-2)$ $\overrightarrow{AB}(-1;3)$ B nuqtalarning koordinatasini toping.

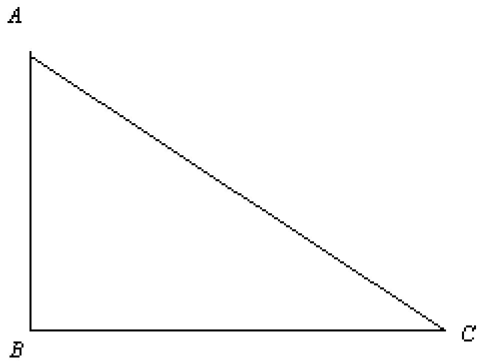
$OB(x,y)$ $\overrightarrow{AB}=\overrightarrow{OB}-\overrightarrow{OA}$ $-1=x-1,x=0$ $\overrightarrow{OA}=\vec{e}_1-2\vec{e}_2$ $3=y+2$ $y=1$ $OB=(0;1)$

3) $M_1(-1;0),M_2(2;3)$ nuqtalardagi masofani toping.

$\overrightarrow{M_1M_2}=\sqrt{(-1-2)^2+(0-3)^2}=\sqrt{9+9}=3\sqrt{2}$

4) Uchlari $A(3;2),B(6;5),C(1;10)$ nuqtalarda bo'lgan uchburchakning to'g'ri burchakli ekanligini isbotlang.

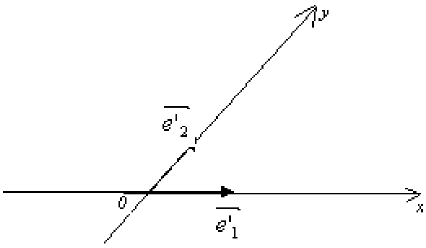
$$\rho(AB)=\sqrt{3^2+3^2}=3\sqrt{2}$$
$$\rho(AC)=\sqrt{2^2+8^2}=2\sqrt{17}$$
$$\rho(BC)=\sqrt{25+25}=5\sqrt{2}$$
$$\rho(AB)^2+\rho(BC)^2=\rho(AC)^2$$

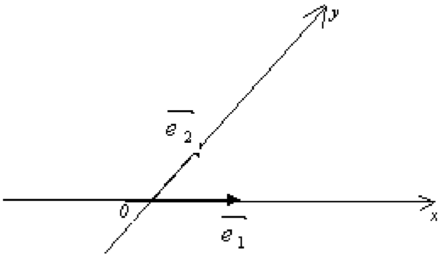


18-rasm

Affin koordinatalar sistemasini almashtirish.

Bizga ikkita $Q(0;\vec{e}_1;\vec{e}_2)$ va $(o';\vec{e_1'};\vec{e_2'})$ reper berilgan bo‘lsin.





4-rasm . 5-rasm.

Bizga qulayligi uchun (1) eski koordinata sistemasini (2) koordinata sistemasini yangi koordinata sistemasi deb ataymiz. Bundan tashkari bizga yangi koordinata sistemasining eski koordinata sistemasiga nisbatan vaziyati berilgan bo‘lsin.

$O'(c_1,c_2)$, $\vec{e_1'}(a_1,a_2)$, $\vec{e_2'}(b_1,b_2)\ldots$ Biz tekislikda M nuqta oldingi M nuqtaning eski koordinata sistemasiga nisbatan koordinatalari (x,y) bo‘lsin, ya’ni koordinata sistemasiga nisbatan koordinatalari (x',y') bo‘lsin.

$\begin{pmatrix} a_1 & a_2 \\ b_1 & b_2 \end{pmatrix} \neq 0$ deb shart qo‘yamiz.

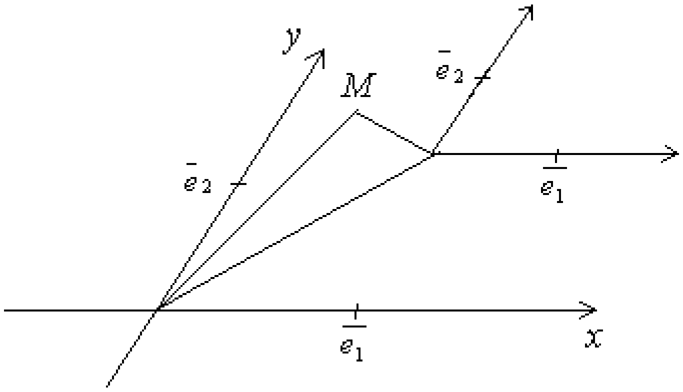
$\overrightarrow{OM} = x\vec{e}_1 + y\vec{e}_2$. $\overrightarrow{OM'} = x'\vec{e_1'} + y'\vec{e_2'}$; $\overrightarrow{OM} = \overrightarrow{OO'} + \overrightarrow{O'M} = c_1\vec{e_1} + c_2\vec{e_2} + x'\vec{e_1'} + y'\vec{e_2'}$

$x\vec{e_1} + y\vec{e_2} = c_1\vec{e_1} + c_2\vec{e_2} + x'(a_1\vec{e_1} + b\vec{e_2}) + y'(b_1\vec{e_1} + b_2\vec{e_2}) =$

$= (a_1x' + b_1y' + c_1)\vec{e_1} + (a_2x' + b_2y' + c_2)\vec{e_2}$

$(x - a_1x' - b_1y' - c_1)\vec{e_1} + (y - a_2x' - b_2y' - c_1)\vec{e_2} = 0$

$$\begin{aligned} x &= (a_1x' + b_1y' + c_1) \\ y &= (a_2x' + b_2y' + c_2) \end{aligned} \qquad (1)$$



rasm

Demak biz M nuqtaning eski koordinata sistemasining koordinatalari yangi koordinata sistemasidagi koordinatalar orqali ifodaladik.(1) formula bir koordinatalar sistemasida boshqa affin koordinatalar sistemasiga o‘tish formulalari deyiladi. 2 ta xususiy xolni qaraymiz.

1. xol: $0 \neq 0'$, $\vec{e_1} \neq \vec{e_1'}$, $\vec{e_2} \neq \vec{e_2'}$ (2)

(1) dan va (2) dan $a_1 = 1, a_2 = 0; b_1 = 0, b_2 = 0$

U xolda (1) formulani quyidagicha yozish mumkin.

$$\begin{cases} x = x' + c_1 \\ y = y' + c_2 \end{cases} \quad (3)$$

(1)formulaga parallel ko‘chirish formulasi deyiladi.

2. xol: Koordinatalar boshi ustma-ust tushgan xol, lekin bazis vektorlari teng bo‘lmasin. U xolda

$$0 = 0^1; c_1 = c_2 = 0$$

$$\begin{cases} x = a_1 x' + b_1 y' \\ y = a_2 x' + b_2 y' \end{cases} \quad (4)$$

Dekart koordinatalar sistemasini almashtiish.

Bizga ikkita dekart koordinatalar sistemasi berilgan bo‘lsin.

$\beta = (o, i, j), \beta = (o', i', j')$ $O' = (c_1; c_2)$, $i' = (a_1; a_2)$, $j' = (b_1; b_2)$,

$$\vec{i'} = a_1 \vec{i} + a_2 \vec{j}\,; \ \vec{j'} = b_1 \vec{i} + b_2 \vec{j} \quad (1)$$

$$(\vec{i}\,\hat{}\,\vec{i'}) = \alpha$$

1) Bu ikkita koordinata sistemasi bir xil aryentatsiyali bo‘lsin. U xolda $\vec{j'}$ orasidagi burchakni xisoblaymiz.

$(\vec{i}',\vec{j})=90^\circ+\alpha$, $(\vec{i}',\vec{j})=90^\circ-\alpha$, $(\vec{i},\vec{i}')=a_1(\vec{i},\vec{i})+a_2(\vec{j},\vec{i})$,

$|\vec{i}|\cdot|\vec{i}'|\cos\alpha=a_1|\vec{i}|^2+a_2|\vec{j}||\vec{i}|\cos 90^\circ$, $a_1=\cos\alpha$

$(\vec{i}',\vec{j})=a_1(\vec{i},\vec{j})+a_2(\vec{j},\vec{j})$, $(\vec{j}',\vec{i})=b_1(\vec{i},\vec{i})+b_2(\vec{j},\vec{i})$,

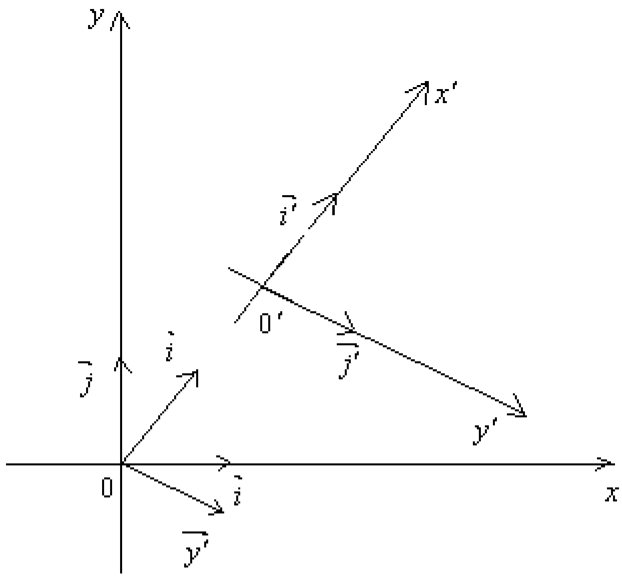
Bundan kelib chiqadi:

$b_1=\cos(90^\circ+\alpha)=-\sin\alpha$, $a_2=\cos(90^\circ-\alpha)=\sin\alpha$, $(\vec{j},\vec{j}')=b_1(\vec{i},\vec{j})+b_2(\vec{j},\vec{j})$, $b_2=\cos\alpha$

$$\begin{cases} x=x'\cos\alpha-y'\sin\alpha+c_1 \\ y=x'\sin\alpha+y'\cos\alpha+c_2 \end{cases} \qquad (2)$$

2)Endi koordinatalar sistemalari har xil aryentatsiyali bo‘lsin:

$(\vec{i},\vec{i}')=\alpha$, $(\vec{i}',\vec{j}')=270^\circ-\alpha$, $(\vec{i}',\vec{j})=90^\circ+\alpha$, $(\vec{j}',\vec{j}')=180^\circ+\alpha$.



-rasm .

Har xil aryentatsiyali: $(\vec{i}',\vec{j})=\alpha$, $(\vec{i}',\vec{j}')=270^\circ-\alpha$, $(\vec{i},\vec{i})=1$, $(\vec{i}',\vec{j})=90^\circ+\alpha$, $(\vec{i}',\vec{j})=90^\circ-\alpha$, $(\vec{i},\vec{j})=0$, $(\vec{j}',\vec{j}')=180^\circ+\alpha$, $(\vec{i},\vec{i}')=\cos\alpha$. $\vec{i}'=a_1\vec{i}+b_1\vec{j}$, $\vec{j}'=a_2\vec{i}+b_2\vec{j}$, $(\vec{i},\vec{j}')=\cos(270^\circ+\alpha)$, $(\vec{i},\vec{i}')=a_1(\vec{i},\vec{i})+b_1(\vec{i},\vec{j})=a_1$, $a_1=\cos\alpha$, $(\vec{i},\vec{j})=\sin\alpha$.

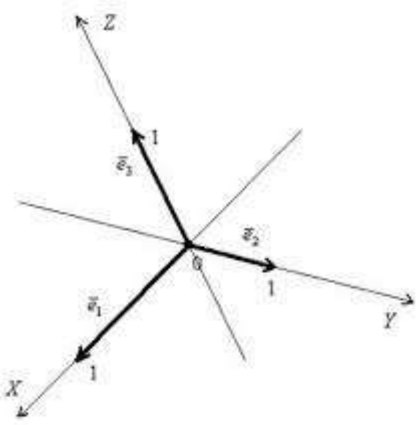
$$\begin{cases} x'=x\cos\alpha-y\sin\alpha+c_1 \\ y'=x\sin\alpha+y\cos\alpha+c_2 \end{cases}$$

$b_1=\sin\alpha$, $a_2=\sin\alpha$, $b_2=-\cos\alpha$.

Fazoda yoki tekislikda affin koordinatalar sistemasini kiritish uchun birorta bazis va bitta nuqta tanlanadi.

Ta’rif: Agar $(\vec{e}_1,\vec{e}_2,\vec{e}_3)$ bazis va O nuqta berilgan bo‘lsa, $\overline{OM}$ vektorning $(\vec{e}_1,\vec{e}_2,\vec{e}_3)$ bazisdagi koordinatalari M nuqtaning affin koordinatalari deyiladi.

O nuqtadan o‘tib, $\vec{e}_1, \vec{e}_2, \vec{e}_3$ vektorlar bilan aniqlanadigan to‘g‘ri chiziqlar mos ravishda Ox, Oy, Oz deb belgilab, ular koordinata o‘qlari, birinchisi abstsissalar (X) o‘qi, ikkinchisi ordinatalar (Y) o‘qi va, nihoyat, uchinchisi applikatalar (Z) o‘qi deb ataladi. Bu o‘qlarning har ikkitasi bilan aniqlanadigan uchta tekislik Oxy, Oxz, Oyz deb belgilab, ular koordinata tekisliklari deb ataladi (1.5.1-rasm).



1.5.1-rasm.

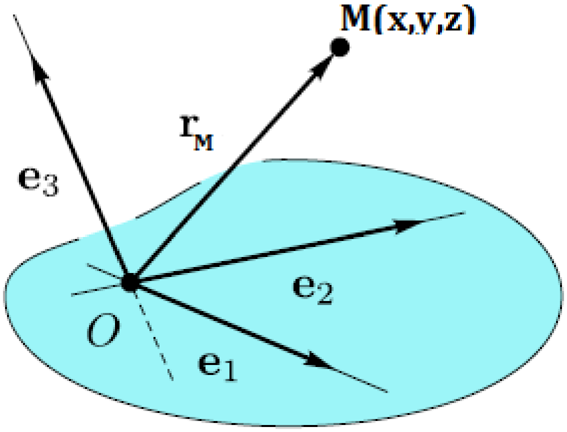
Fazodagi har bir M nuqtaga aniq bir $\overrightarrow{OM}$ vektorni doimo mos keltirish mumkin, ya’ni boshi koordinatalar boshida, oxiri esa berilgan M nuqtada bo‘lgan vektorni mos keltiradi.

$\overrightarrow{OM}$ vektorning koordinatalar (x, y, z) bo‘lsa, u holda bu uchta x, y, z son M nuqtaning affin sistemadagi koordinatalari bo‘ladi:

$$\overrightarrow{OM}(x,\ y,\ z) \Leftrightarrow M(x,\ y,\ z).$$

Demak, fazo nuqtalari to‘plami bilan ma’lum tartibda olingan haqiqiy sonlar uchliklari to‘plami orasida biektiv moslik mavjud.

Berilgan nuqtaning koordinatalarini topish uchun shu nuqta radius-vektorining koordinatalarini topish kifoya va aksincha (1.5.2-rasm).



1.5.2-rasm.

Umuman, $M(x,\ y,\ z)$ nuqtani yasash uchun, ya’ni

$$\overrightarrow{OM} = x\,\vec{e}_1 + y\,\vec{e}_2 + z\,\vec{e}_3 \qquad (1.5.1)$$

vektorning oxirini topish uchun quyidagi qoidadan foydalaniladi: koordinatalar boshidan Ox o‘q bo‘yicha $x\vec{e}_1$ vektor, uning oxiridan Oy o‘qqa parallel holda $y\vec{e}_2$ vektor qo‘yiladi, so‘ngra uning oxiridan $z\vec{e}_3$ vektor yasalsa, shu vektorning oxiri izlangan nuqta bo‘ladi.

Kesmani berilgan nisbatda bo‘lish.

Biror affin sistemaida $M_1(x_1, y_1, z_1)$, $M_2(x_2, y_2, z_2)$ $(M_1 \neq M_2)$ nuqtalar va biror haqiqiy λ son berilgan bo‘lsin.

Ta’rif. *M nuqta uchun*

$$\overrightarrow{M_1M} = \lambda\overrightarrow{MM_2} \qquad (1)$$

shart bajarilsa, M nuqta $M_1\,M_2$ kesmani λ nisbatda bo‘ladi deyiladi.

M_1, M_2 nuqtalarning koordinatalari orqali M nuqtaning x, y, z koordinatalarini topaylik. (1.5.1) ga asosan

$$\overrightarrow{M_1M} = \overrightarrow{OM} - \overrightarrow{OM_1} = (x - x_1)\vec{e}_1 + (y - y_1)\vec{e}_2 + (z - z_1)\vec{e}_3,$$

$$\overrightarrow{M_1M}(x - x_1, y - y_1, z - z_1).$$

$$\overrightarrow{MM_2} = \overrightarrow{OM_2} - \overrightarrow{OM} = (x_2 - x)\vec{e}_1 + (y_2 - y)\vec{e}_2 + (z_2 - z)\vec{e}_3,$$

$$\overrightarrow{MM_2} = (x_2 - x, y_2 - y, z_2 - z).$$

Bu ifodalarni (1) ga qo‘yib va $\vec{e}_1, \vec{e}_2, \vec{e}_3$ ning chiziqli erkliligini e’tiborga olsak,

$$x - x_1 = \lambda(x_2 - x), \;\; y - y_1 = \lambda(y_2 - y), \;\; z - z_1 = \lambda(z_2 - z).$$

Bulardan, $1+\lambda \neq 0$ farazda quyidagi munosabatga ega bo‘lamiz

$$x = \frac{x_1 + \lambda x_2}{1+\lambda}, \quad y = \frac{y_1 + \lambda y_2}{1+\lambda}, \quad z = \frac{z_1 + \lambda z_2}{1+\lambda}. \qquad (1.5.3)$$

Berilgan kesmani berilgan nisbatda bo‘luvchi nuqtaning koordinatalarini topish formulalari shulardir. Bu yerda albatta $\lambda \neq -1; \lambda = -1,$ ya’ni $1+\lambda = 0$ bo‘lgan holni biz hozircha qaramaymiz. $\lambda = 1$ bo‘lganda M nuqta M_1M_2 kesmaning o‘rtasi bo‘lib, bu holda (1.5.3) formulalar quyidagi ko‘rinishni oladi:

$$x = \frac{x_1 + x_2}{2}, \quad y = \frac{y_1 + y_2}{2}, \quad z = \frac{z_1 + z_2}{2}.$$

Bu formulalar kesma o‘rtasining koordinatalarini topish formulalaridir.

Affin sistemasining xususiy hollaridan biri to‘g‘ri burchakli dekart sistemasidir.

Ta’rif. *Ortonormalgan bazis yordamida berilgan koordinatalar sistemasi to‘gri burchakli yoki Dekart koordinatalar sistemasi deb ataladi.*

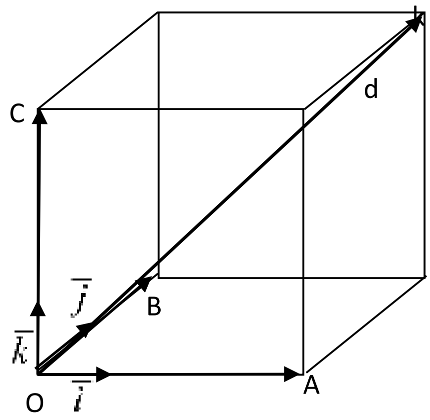
Affin sistemasidagi bazis vektorlar ortonormalangan bo‘lsa, ya’ni ularning har ikkitasi o‘zaro perpendikular bo‘lib, har biri birlik vektor bo‘lsa, $(O, \vec{i}, \vec{j}, \vec{k})$ dekart sistemai hosil qilinadi, bu yerda

$$\vec{i}^{\,2} = \vec{j}^{\,2} = \vec{k}^{\,2} = 1, \quad \vec{i}\,\vec{j} = \vec{j}\,\vec{k} = \vec{i}\,\vec{k} = 0. \qquad (1.5.4)$$

Bu sistemada metrik xarakterdagi masalalarni yechish ancha qulay.

1-teorema. *Dekart koordinatalar sistemasida vektorning berilgan bazisdagi koordinatalari, uning koordinatalar o‘qlariga tushirilgan proyeksiyalari bilan ustma-ust tushadi.*

Isbot. Bizga $\vec{i}, \vec{j}, \vec{k}$ ortonormal bazis berilgan bo‘lsa, bularning boshlarini O nuqtaga joylashtirib $Oxyz$ koordintalar sistemasini kiritaylik. Agar $\vec{a} = x\vec{i} + y\vec{j} + z\vec{k}$ bo‘lsa, $\vec{a}$ vektorning boshini koordinata boshiga joylashtirib, uning oxirini M bilan belgilaymiz. Agar M nuqtaning koordinata o‘qlariga ortogonal proyeksiyalarini A, B, C harflari bilan belgilasak: $\overrightarrow{OA} = x\vec{i}$, $\overrightarrow{OB} = y\vec{j}$, $\overrightarrow{OC} = z\vec{k}$ tengliklarni hosil qilamiz. Ikkinchi tomondan $\overrightarrow{OA}, \overrightarrow{OB}, \overrightarrow{OC}$ kesmalarning kattaliklari mos ravishda x, y, z sonlariga teng bo‘lganligi uchun $x = pr_{Ox}\vec{a},\ y = pr_{Oy}\vec{a},\ \ z = pr_{Oz}\vec{a}$ munosabatlarni hosil qilamiz (1.5.3-rasm).



1.5.3-rasm.

Xulosa. *Ushbu paragrafda fazoda affin va Dekart koordinatalar sistemasi va ularning xossalari, kesmani berilgan nisbatda bo‘lish batafsil bayon etildi.*

Fazoda affin va dekart koordinatalarni almashtirish.

Fazodagi biror nuqtaning tayin bir sistemadagi koordinatalaridan boshqa sistemadagi koordinatalariga o‘tishga to‘g‘ri keladi. Biz shu masalani ikkita affin sistema uchun hal qilamiz. $\beta = (O, \vec{e_1}, \vec{e_2}, \vec{e_3})$, $\beta' = (O', \vec{e'_1}, \vec{e'_2}, \vec{e'_3})$ affin sistemalar berilgan bo‘lsin.

I hol. Sistemalarning boshlari har xil bo‘lib, bazis vektorlari mos ravishda kollinear bo‘lsin, ya’ni $O \neq O'$, $\vec{e_1} \parallel \vec{e'_1}$, $\vec{e_2} \parallel \vec{e'_2}$, $\vec{e_3} \parallel \vec{e'_3}$ hamda O' ning β ga nisbatan koordinatalari a, b, c bo‘lsin (1.9.5-a chizma). U holda fazodagi ixtiyoriy M nuqtaning β va β' ga nisbatan koordinatalari mos ravishda x, y, z va x', y', z' bo‘lsa, shular orasidagi bog‘lanishni izlaymiz:

$$M(x, y, z) \Rightarrow \overrightarrow{OM}(x, y, z) \Rightarrow \overrightarrow{OM} = x\vec{e}_1 + y\vec{e}_2 + z\vec{e}_3$$

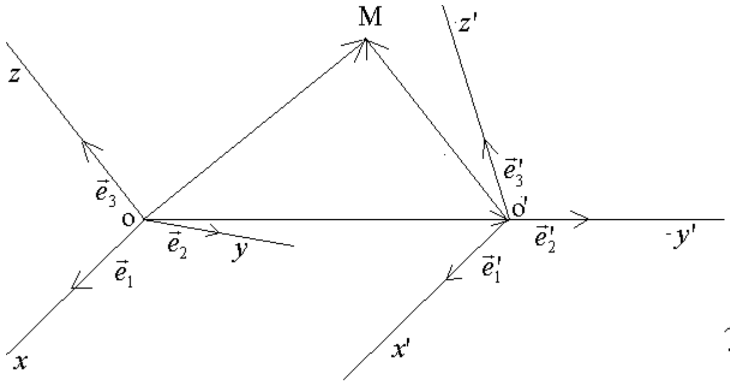
$$M(x', y', z') \Rightarrow \overrightarrow{O'M}(x', y', z') \Rightarrow \overrightarrow{O'M} = x'\vec{e_1} + y'\vec{e_2} + z'\vec{e_3}$$

$$\overrightarrow{OO'}(a, b, c) \Rightarrow \overrightarrow{OO'} = a\vec{e_1} + b\vec{e_2} + c\vec{e_3}\ .$$

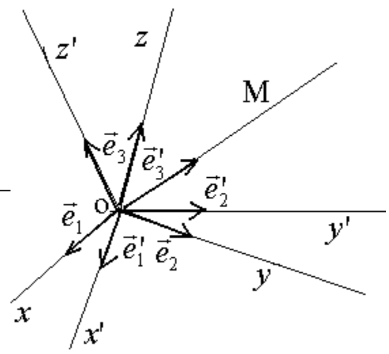
Lekin $\overrightarrow{OM}=\overrightarrow{OO'}+\overrightarrow{O'M}$ bo‘lgani uchun: $x\vec{e}_1+y\vec{e}_2+z\vec{e}_3=a\vec{e}_1+b\vec{e}_2+c\vec{e}_3+x'\vec{e}_1{}'+y'\vec{e}_2{}'+z'\vec{e}_3{}'$.

Bundan tashqari, bazis vektorlar mos ravishda kollinear bo‘lgani sabab –

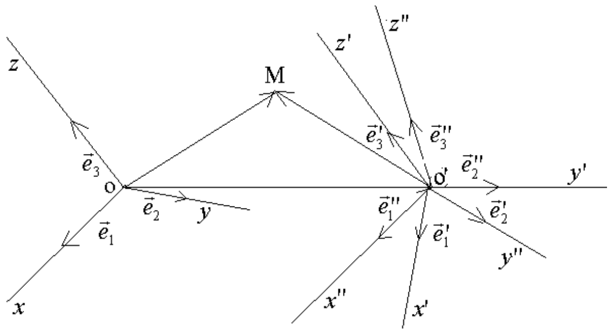
$$\vec{e}_1{}'=\lambda_1\vec{e}_1,\ \vec{e}_2{}'=\lambda_2\vec{e}_2,\ \vec{e}_3{}'=\lambda_3\vec{e}_3$$



a)



b)



c)

chizma.

demak,

$$x\vec{e}_1+y\vec{e}_2+z\vec{e}_3=(\lambda_1x'+a)\vec{e}_1+(\lambda_2y'+b)\vec{e}_2+(\lambda_3z'+c)\vec{e}_3 \quad (1.9.12)$$

$$x=\lambda_1x'+a,\ y=\lambda_2y'+b,\ z=\lambda_3z'+c \quad (1.9.13)$$

$\lambda_1=\lambda_2=\lambda_3=1$ bo‘lsa, ya’ni bazis vektorlar mos ravishda o‘zaro teng bo‘lsa, (1.9.13) quyidagi ko‘rinishni oladi: $x=x'+a,\ y=y'+b,\ z=z'+c$ (1.9.14)

Bu formulalar ba’zan koordinatalar sistemasini parallel ko‘chirish formulalari deb yuritiladi.

II hol. Sistemalarning boshlari bir xil, bazis vektorlarning yo‘nalishlari esa har xil bo‘lsin, u holda (b chizma): $O=O'$, $\vec{e}_1{}'=a_{11}\vec{e}_1+a_{21}\vec{e}_2+a_{31}\vec{e}_3$, $\vec{e}_2{}'=a_{12}\vec{e}_1+a_{22}\vec{e}_2+a_{32}\vec{e}_3$, $\vec{e}_3{}'=a_{13}\vec{e}_1+a_{23}\vec{e}_2+a_{33}\vec{e}_3$ bo‘lsin. Endi

$$A=\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \qquad (1.9.15)$$

matritsani tuzamiz. Bu matritsani bir bazisdan ikkinchi bazisga o‘tish matritsasi deb ataymiz, $\vec{e_1'}, \vec{e_2'}, \vec{e_3'}$ bazis vektorlar bo‘lgani uchun (1.9.15) matritsaning determinanti noldan farqlidir.

$$\begin{vmatrix} a_{11} & a_{21} & a_{31} \\ a_{12} & a_{22} & a_{32} \\ a_{13} & a_{23} & a_{33} \end{vmatrix} \neq 0. \qquad (1.9.16)$$

Aks holda, determinantning bir satri qolgan ikki satrining chiziqli kombinatsiyasidan iborat bo‘lib, $\vec{e_1'}, \vec{e_2'}, \vec{e_3'}$ ham chiziqli bog‘liq bo‘lar edi.

Fazoda ixtiyoriy M nuqtaning β va β' sistemaga nisbatan koordinatalarini mos ravishda x, y, z va x', y', z' deb olsak, $\overrightarrow{OM} = x\vec{e}_1 + y\vec{e}_2 + z\vec{e}_3$

$$\overrightarrow{OM} = x'\vec{e_1'} + y'\vec{e_2'} + z'\vec{e_3'}$$

ya’ni

$$x\vec{e}_1 + y\vec{e}_2 + z\vec{e}_3 = x'\vec{e_1'} + y'\vec{e_2'} + z'\vec{e_3'}.$$

Endi bu tenglikka $\vec{e_1'}, \vec{e_2'}, \vec{e_3'}$ ning qiymatlarini qo‘yib, $\vec{e}_1, \vec{e}_2, \vec{e}_3$ ga nisbatan guruhlasak,

$$x\vec{e}_1 + y\vec{e}_2 + z\vec{e}_3 = (a_{11}x' + a_{12}y' + a_{13}z')\vec{e}_1 + (a_{21}x' + a_{22}y' + a_{23}z')\vec{e}_2 + (a_{31}x' + a_{32}y' + a_{33}z')\vec{e}_3$$

bundan shu hosil bo‘ladi:

$$\begin{aligned} x &= a_{11}x' + a_{12}y' + a_{13}z' \\ y &= a_{21}x' + a_{22}y' + a_{23}z' \\ z &= a_{31}x' + a_{32}y' + a_{33}z' \end{aligned} \qquad (1.9.17)$$

Ushbu

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \qquad (1.9.18)$$

matritsa almashtirish matritsasi deb ataladi. (1.9.18) va (1.9.15) matritsalar o‘zaro transponirlangan matritsalardir. Bu matritsalar kvadrat matritsalar bo‘lgani uchun ularning uchinchi tartibli determinantlari o‘zaro teng bo‘lib, (1.9.16) ga asosan (1.9.18) ning determinanti noldan farqlidir, demak, (1.9.17) ni x', y', z' ga nisbatan yechsak,

$$\begin{aligned} x' &= a'_{11}x + a'_{12}y + a'_{13}z \\ y' &= a'_{21}x + a'_{22}y + a'_{23}z \\ z' &= a'_{31}x + a'_{32}y + a'_{33}z \end{aligned} \qquad (1.9.19)$$

hosil bo‘lib, bunda

$$a'_{ik} = \frac{A_{ik}}{\det A}; \ (i,k = 1,2,3)$$

A_{ik} esa A matritsa a_{ik} elementining algebraik to‘ldiruvchisidir.

III hol. Sistemalar fazoda ixtiyoriy vaziyatda joylashgan bo‘lsin. β sistema berilgan bo‘lib, shu sistemaga nisbatan β' sistema elementlarining koordinatalari quyidagicha bo‘lsin: $O'(a,b,c)$,

$$\begin{aligned} \vec{e_1'} &= a_{11}\vec{e}_1 + a_{21}\vec{e}_2 + a_{31}\vec{e}_3 \\ \vec{e_2'} &= a_{12}\vec{e}_1 + a_{22}\vec{e}_2 + a_{32}\vec{e}_3 \\ \vec{e_3'} &= a_{13}\vec{e}_1 + a_{23}\vec{e}_2 + a_{33}\vec{e}_3 \end{aligned} \quad \begin{vmatrix} a_{11} & a_{21} & a_{31} \\ a_{12} & a_{22} & a_{32} \\ a_{13} & a_{23} & a_{33} \end{vmatrix} \neq 0 \qquad (1.9.20)$$

β dan β' ga o‘tish uchun biz yana shunday uchinchi $\beta'' = (O'', \vec{e_1''}, \vec{e_2''}, \vec{e_3''})$ affin sistemani qaraymizki, u β ni $\overrightarrow{OO'}$ vektor qadar parallel ko‘chirishdan hosil bo‘lsin. U holda fazodagi ixtiyoriy M nuqtaning koordinatalarini bu sistemalarga nisbatan mos ravishda x, y, z; x', y', z'; x'', y'', z'' deb belgilasak (1.9.5-c chizma).

β dan β'' orasidagi bog‘lanish (1.9.14) ga asoslanib

$$x = x'' + a, \ y = y'' + b, \ z = z'' + c \qquad (1.9.21)$$

β' bilan β'' orasidagi bog‘lanish esa (1.9.7) ga asosan

$$\begin{aligned} x'' &= a_{11}x' + a_{12}y' + a_{13}z' \\ y'' &= a_{21}x' + a_{22}y' + a_{23}z' \\ z'' &= a_{31}x' + a_{32}y' + a_{33}z' \end{aligned}$$

buni (1.9.21) ga qo‘ysak, izlanayotgan quyidagi ifoda hosil qilinadi:

$$x = a_{11}x' + a_{12}y' + a_{13}z' + a$$
$$y = a_{21}x' + a_{22}y' + a_{23}z' + b$$
$$z = a_{31}x' + a_{32}y' + a_{33}z' + c \qquad (1.9.22)$$

(1.9.22) ni x', y', z' ga ((1.9.20) shart o‘rinli bo‘lgani uchun) nisbatan ham echish mumkin, demak, M nuqtaning β ga nisbatan koordinatalari ma’lum bo‘lsa, shu nuqtaning koordinatalarini β' ga nisbatan ham topish mumkin.

Bir affin sistemasidan ikkinchi affin sistemasiga o‘tish 12 ta parametrga bog‘liqdir, chunki (1.9.22) almashtirishni aniqlaydigan ushbu 12 ta parametr kiradi. Bular $a, b, c, a_{11}, a_{12}, a_{13}, a_{21}, a_{22}, a_{23}, a_{31}, a_{32}, a_{33}$

Agar β, β' dekart sistemalari bo‘lsa, ularni almashtirish 12 ta parametrga emas, balki eng ko‘pi bilan 6 ta parametrga bog‘liq bo‘lib qoladi. Haqiqatan ham, $\vec{e}_1 = \vec{i}, \vec{e}_2 = \vec{j}, \vec{e}_3 = \vec{k}$ va $\vec{e}_1' = \vec{i}', \vec{e}_2' = \vec{j}', \vec{e}_3' = \vec{k}'$ bo‘lsa, 3-§ paragrefdagi (1.3.1) ni e’tiborga olsak,

$$\begin{aligned} a_{11}^2 + a_{21}^2 + a_{31}^2 &= 1 \\ a_{12}^2 + a_{22}^2 + a_{32}^2 &= 1 \\ a_{13}^2 + a_{23}^2 + a_{33}^2 &= 1 \end{aligned} \qquad (1.9.23)$$

$$\begin{aligned} a_{11}a_{12} + a_{21}a_{22} + a_{31}a_{32} &= 0 \\ a_{11}a_{13} + a_{21}a_{23} + a_{31}a_{33} &= 0 \\ a_{12}a_{13} + a_{22}a_{23} + a_{32}a_{33} &= 0 \end{aligned} \qquad (1.9.24)$$

Demak, (1.9.22) dagi 12 ta parametr (1.9.23) va (1.9.24) dagi 6 ta shartni qanoatlantirishi kerak, u holda jami 6 ta parametr qoladi. «Algebra va sonlar nazariyasi» kursidan ma’lumki, (1.9.18) ko‘rinishdagi kvadrat matritsaning elementlari (1.9.23) va (1.9.24) shartlarning barchasini qanoatlantirsa, bunday matritsa ortogonal matritsa deb ataladi. Bundan quyidagi xulosa kelib chiqadi: bir dekart sistemaidan ikkinchi dekart sistemasiga o‘tish matritsasi ortogonal matritsadan iborat.

Xulosa. *Ushbu paragrafda oriyentasiya, tekislikda affin va Dekart koordinatalari sistemasini almashtirish, fazoda affin va Dekart koordinatalar sitemasini almashtirish batafsil bayon etildi.*

MASALALAR

Tekislikda affin koordinatalar sistemasi

1. Affin koordinatalar sistemasiga nisbatan uchlarining $A(3;5), B(-4;6)\ va\ C(5;3;5)$ koordinatalari berilgan uchburchakni yasang.

2. Quyidagi nuqtalarga (Ox) o‘qqa nisbatan simmetrik bo‘lgan nuqtalarning koordinatalarini toping $(w = \vec{e}_1, \vec{e}_2 = 30^\circ)$:

a) $A(2,3)$ *b)* $B(-3,2)$ *c)* $C(-1,1)$ *d)* $D(-2,5)$ *e)* $E(-4,6)$

3. Quyidagi nuqtalarga Oy o‘qqa nisbatan simmetrik bo‘lgan nuqtalarning koordinatalarini toping $(w = \vec{e}_1, \vec{e}_2 = 30^\circ)$:

a) $A(3,3)$ *b)* $B(-2,-4)$ *c)* $C(2,-1)$ *d)* $D(5,-4)$ *e)* $E(-1,1)$

4. Quyidagi nuqtalarga koordinatalar boshiga nisbatan simmetrik bo‘lgan nuqtalarning koordinatalarini toping :

a) $A(-1,2)$ *b)* $B(3,-1)$ *c)* $C(-2,2)$ *d)* $D(-2,5)$ *e)* $E(-3,-5)$

5. Quyida berilgan shartlarga asoslanib, $M(x, y)$ nuqta koordinatalar sistemasining qaysi choragida yotishi mumkinligini ayting.

a) $xy > 0$ *b)* $xy < 0$ *c)* $xy = 0$ *d)* $x - y = 0$

6. Tomoni $a = 1$ bo‘lgan muntazam oltiburchak uchlarining koordinatalarini toping. Koordinatalar o‘qi qilib uning shunday ikki qo‘shni tomonlarini olingki, koordinatalar boshiga qarama-qarshi yotgan uchining koordinatalari musbat bo‘lsin.

7. Quyidagi Vektorlarning boshlari $M(-1,2)$ nuqtada bo‘lsa, ular oxirlarining koordinatalarini toping:

1) $\vec{a}_1(3,0)$ 2) $\vec{a}_2(-5,3)$ 3) $\vec{a}_3(3,-2)$ 4) $\vec{a}_4(-1,-2)$

8. Parallelogrammning uchta A, B, C uchining koordinatalari bo‘yicha to‘rtinchi uchining koordinatalarini toping:

a) $A(1,4),\quad B(3,-1),\quad C(0,2)$

b) $A(-1,0),\quad B(2,1),\quad C(4,-1)$

9. Agar to‘rtburchakning uchlari $A(1,-3), B(8,0), C(4,8)$ va $D(-3,5)$ nuqtalarda bo‘lsa, $ABCD$ parallelogramm ekanligini ko‘rsating.

10. Agar to‘rtburchakning uchlari $A(1,1), B(2,3), C(5,0)$ va $D(7,-5)$ nuqtalarda bo‘lsa, $ABCD$ trapetsiya ekanligini isbot qiling.

11. Quyidagi uchta A, B, C nuqtaning bir to‘g’ri chiziqda yotishini ko‘rsating:

a) $A(2,1),\quad B(0,5),\quad C(4,-3)$

b) $A(-1,0),\quad B(1,-2),\quad C(3,-4)$

12. $A(2,1), B(0,5), C(4,-3)$ nuqtalar berilgan. $(AB,C), (BC,A), (AC,B)$ larni hisoblang.

13. Uchburchakning uchlari berilgan: $A(3,-7), B(5,2), C(-1,0)$. Har bir tomonning o‘rta nuqtasining koordinatalarini toping.

14. Uchburchak tomonlarining o‘rtalari $M_1(3,-2), M_2(1,6), M_3(-4,2)$ nuktalarda bo‘lsa, uning uchlarini aniqlang.

15. Parallelogrammning $A(-3,5)\, va\, B(1,7)$ qo‘shni uchlari hamda diagonallari kesishgan $M(1,1)$ nuqta berilgan. Uning qolgan ikkita uchining koordinatalarini toping.

16. Uchlari $A(3,1), B(-1,4), va\ C(1,1)$ nuqtalarda bo‘lgan uchburchak medianalarining kesishish nuqtasini toping.

17. l to‘g’ri chiziqda $|A_1A_2| = |A_2A_3| = |A_3A_4| = |A_4A_5| = |A_5A_6|$ shartni qanoatlantiruvchi $A_1, A_2, A_3, A_4, A_5, A_6$ nuqtalar olingan. Agar $A_2(2,5)$ va $A_5(-1,7)$ bo‘lsa, qolgan nuqtalarning koordinatalarini toping.

Fazoda affin koordinatalar sistemasi.

1. $\beta = \{0, \vec{e}_1, \vec{e}_2, \vec{e}_3\}$ sistemasida $A(2,5,4); B(0,1,0); C(4,1,3); D(6,5,7)$ nuqtalar berilgan. $ABCD$ figura parallelogramm ekanini isbot qiling.

2. $\overrightarrow{AB} = (-3,2,6)$ vektorning boshi $A(-1,0,4)$ nuqtada joylashgan. Uning oxiri bo‘lgan B nuqtaning koordinatalarini toping.

3. Uchlari $A(2,0,-4); B(7,-15,16), C(-1,-1,11); D(-4,8,-1)$ nuqtalarda yotgan to‘rtburchak trapetsiya ekanligini isbotlang.

4. $M_1(7,9,-8); M_2(-2,3,4); M(-5,1,8)$ nuqtalarning bir to‘g’ri chiziqda yotishini isbotlang.

5. $\vec{a} = \{-2,1,5\}; \vec{b} = \{0,-2,6\}$ vektorlar berilgan. $\vec{a} + 2\vec{b};\ 3\vec{a} - 4\vec{b};\ -7\vec{a} + 2\vec{b}$ vektorlarning koordinatalarini toping.

6. $M_1(1,-2,5); M_2(4,-2,2)$ nuqtalar berilgan. $[\overline{M_1M_2}]$ kesmani $\lambda = 1:2$ nisbatda bo‘luvchi $M(x,y)$ nuqtani toping.

7. $OABC$ tetraedrda $\overrightarrow{OA}, \overrightarrow{OB}, \overrightarrow{OA}, \overrightarrow{OB}, \overrightarrow{OC}$ larni bazis vektorlar deb olib, ABC yoq medianalari kesishgan nuqtaning koordinatalarini toping.

Tekislikda affin va dekart koordinatalar sistemasini almashtirish

1. M nuqta biror koordinatalar sistemasiga nisbatan $x = -6, y = 3$ koordinatalarga ega. Koordinatalar boshi ushbu:

a) $O_1(-3,0)$ *b)* $O_2(-4,3)$ *c)* $O_3(5,-8)$

nuqtalardan biriga ko‘chirilsa, shu nuqtaning koordinatalari qanday bo‘ladi?

2. Quyidagi hollar uchun $\beta=\{O,\vec{e}_1,\vec{e}_2\}$ affin reperdan $\beta'=\{O',\vec{e'_1},\vec{e'_2}\}$ affin reperga o‘tish formulalarini yozing:

$a)\ \vec{e'_1}(2,1), \vec{e'_2}(-2,1)$ $b)\ \vec{e'_1}(1,1), \vec{e'_2}(0,1)$

$c)\ \vec{e'_1}(1,0), \vec{e'_2}(1,1)$ $d)\ \vec{e'_1}(1,0), \vec{e'_2}(0,1)$

3. Quyidagi berilganlarga asosan $\beta=\{O,\vec{e}_1,\vec{e}_2\}$ affin reperdan $\beta'=\{O',\vec{e'_1},\vec{e'_2}\}$ affin reperga o‘tish formulalarini yozing:

$a)\ \vec{e'_1}(-3,0), \vec{e'_2}(1,2), O'(-3,5)$

$b)\ \vec{e'_1}(1,0), \vec{e'_2}(0,1), O'(2,0)$

$c)\ \vec{e'_1}(1,1), \vec{e'_2}(1,0), O'(0,-5)$

4. $\beta=\{O,\vec{e}_1,\vec{e}_2\}$ affin reperga nisbatan $A(2,1)$ va $B(-\frac{3}{2},3)$ berilgan. Koordinatalar boshi $O'(0,1)$ nuqtada bo‘lgan shunday $\beta'=\{O',\vec{e'_1},\vec{e'_2}\}$ affin reperni topingki, unda $A(1,0)\, va\, B(0,1)$ bo‘lsin.

5. $\beta=\{O,\vec{e}_1,\vec{e}_2\}$ affin reperda A,B nuqtalar mos ravishda $(1,1)$ va $(2,2)$ koordinatalarga ega. A va B nuqtalar $(1,1)$ va $(1,-2)$ koordinatalarga ega bo‘ladigan $\beta'=\{O',\vec{e'_1},\vec{e'_2}\}$ affin reper mavjudmi?

6. $\vec{e'_1}(1,1), \vec{e'_2}(-3,1), O'(0,1)$ bo‘lsa, $\beta=\{O,\vec{e}_1,\vec{e}_2\}$ va $\beta'=\{O',\vec{e'_1},\vec{e'_2}\}$ affin reperlarda bir xil koordinatalarga ega bo‘lgan nuqtani toping.

7. Agar koordinatalarni almashtirish formulalari quyidagicha bo‘lsa, yangi koordinata vektorlarini va yangi koordinatalar boshining eski reperga nisbatan koordinatalarini toping:

a) $\begin{cases} x = x' - y' + 3 \\ y = -3y' - 2 \end{cases}$ b) $\begin{cases} x' = 3x - y \\ y' = 2x + 1 \end{cases}$

c) $\begin{cases} x = y' + 1 \\ y = x' + 2y' + 3 \end{cases}$ e) $\begin{cases} x = x' - 3y' + 4 \\ y = 3x' + \sqrt{2}y' - 1 \end{cases}$

d) $\begin{cases} x' = x + 2y + 1 \\ y' = x + y - 6 \end{cases}$

8. $\beta=\{O,\vec{i},\vec{j}\}$ dekart reper berilgan. Koordinatalar o‘qini quyidagi burchaklardan biriga burishdagi koordinatalarni almashtirish formulalarini yozing:

$a) 30^\circ$ $b) 45^\circ$ $c) 120^\circ$ $d) -60^\circ$ $e) 75^\circ$

9. Koordinatalarni almashtirish formulasi quyidagicha

$$\begin{cases} x = \frac{1}{2}x' - \frac{\sqrt{3}}{2}y' \\ y = \frac{\sqrt{3}}{2}x' + \frac{1}{2}y' \end{cases}$$

bo‘lsa, koordinata o‘qlari qanday burchakka burilgan?

10. $\beta = \{O, \vec{i}, \vec{j}\}$ dekart reperga nisbatan $A(\sqrt{8}, -\frac{1}{2})$ va $M(x, y)$ nuqtalar berilgan. Koordinata o‘qlari koordinatalar burchagi bissektrisalari bilan almashtirilganda, shu nuqtalarning koordinatalarini toping.

11. $\beta = \{O, \vec{i}, \vec{j}\}$ dekart reperda F figura $xy + 3x - 2y - 6 = 0$ tenglama bilan berilgan. Koordinatalar boshi $O'(2, -3)$ nuqtaga ko‘chirilgandan keyin F figuraning tenglamasi qanday bo‘ladi?

Fazoda affin va dekart koordinatalar sistemasini almashtirish

1. $\beta = \{O, \vec{e}_1, \vec{e}_2, \vec{e}_3\}$ sistemaga nisbatan $\vec{e}_1{}'(1,0,0)$, $\vec{e}_2{}'(0,1,0)$, $\vec{e}_3{}'(0,0,1)$, $O'(1,-3,5)$ lar berilgan β dan $\beta' = \{O', \vec{e}_1{}', \vec{e}_2{}', \vec{e}_3{}'\}$ ga o‘tishdagi koordinatalarni almashtirish formulalarni yozing. β da berilgan $M(1,1,3)$ nuqtaning β' dagi koordinatalarini toping.

2. M nuqta $\beta = \{O, \vec{e}_1, \vec{e}_2, \vec{e}_3\}$ $M(0,1,-3)$ ko‘rinishda, $\beta' = \{O', \vec{e}_1{}', \vec{e}_2{}', \vec{e}_3{}'\}$ da esa $M(2,-3,5)$ ko‘rinishda berilgan bo‘lsa, koordinatalar boshi ko‘chirilgan O' nuqtaning β dagi koordinatalarini toping.

3. Biror $\beta = \{O, \vec{e}_1, \vec{e}_2, \vec{e}_3\}$ sistemaga nisbatan $\vec{e}_1{}'(1,-3,-1); \vec{e}_2{}'(0,5,1); \vec{e}_3{}'(0,0,3)$ vektorlar berilgan. $\vec{e}_1{}', \vec{e}_2{}', \vec{e}_3{}'$ lar bazis bo‘la olishini ko‘rsating va $\beta = \{O, \vec{e}_1, \vec{e}_2, \vec{e}_3\}$ dan $\beta' = \{O', \vec{e}_1{}', \vec{e}_2{}', \vec{e}_3{}'\}$ ga o‘tishdagi koordinatalarni almashtirish formulalarini yozing va $M(3,1,-4)$ ning β' dagi koordinatalarini toping.

4. $\beta = \{O, \vec{e}_1, \vec{e}_2, \vec{e}_3\}$ da $\vec{e}_1{}'(1,0,2); \vec{e}_2{}'(1,0,-2); \vec{e}_3{}'(1,1,1)$ vektorlar berilgan. $\vec{e}_1{}', \vec{e}_2{}', \vec{e}_3{}'$ sistema bazis ekanligini ko‘rsating va $\beta' = \{O', \vec{e}_1{}', \vec{e}_2{}', \vec{e}_3{}'\}$ dagi o‘tishdagi koordinatalarni almashtirish formulalarini yozib $M(3,1,-4)$ ning β' dagi $\vec{e}_1, \vec{e}_2, \vec{e}_3$ larning koordinatalarini toping.

5. $OABC$ tetraedr berilgan, $\overrightarrow{OA} = \vec{e}_1, \overrightarrow{OB} = \vec{e}_2$, $\overrightarrow{OC} = \vec{e}_3$ deb olib, $\beta = \{O, \vec{e}_1, \vec{e}_2, \vec{e}_3\}$ affin sistemasidan $O' = A, \vec{e}_1{}' = \overrightarrow{AO}, \vec{e}_2{}' = \overrightarrow{AB}, \vec{e}_3{}' = \overrightarrow{AC}$ bo‘lgan $\beta' = \{O', \vec{e}_1{}', \vec{e}_2{}', \vec{e}_3{}'\}$ sistemaga o‘tishdagi koordinatalarni almashtirish formulalarini yozing.

6. $\beta = \{O, \vec{e}_1, \vec{e}_2, \vec{e}_3\}$ sistemadan $\beta' = \{O', \vec{e}_1{}', \vec{e}_2{}', \vec{e}_3{}'\}$ sistemaga o‘tishdagi ixtiyoriy nuqtaning bu ikki sistemaga nisbatan koordinatalari orasidagi bog’lanish

ushbu $x=x'-2y'+3z'-4$, $y=5x'-y'-z'$, $z=z'+1$ formulalar bilan berilgan. O' nuqtaning va $\vec{e}_1{}', \vec{e}_2{}', \vec{e}_3{}'$ vektorlarning β dagi koordinatalarini toping.

7. $\beta=\{O,\vec{e}_1,\vec{e}_2,\vec{e}_3\}$ da $\vec{e}_1{}'(-1,1,0); \vec{e}_2{}'(2,-1,0); \vec{e}_3{}'(0,0,5); O'(5,0,-2)$ lar berilgan. β dan $\beta'=\{O',\vec{e}_1{}',\vec{e}_2{}',\vec{e}_3{}'\}$ ga o‘tishdagi koordinatalarni almashtirish formulalarni yozing va β' da berilgan va $M(1,-3,4)$ ning β dagi koordinatalarini toping.

8. $\beta=\{O,\vec{e}_1,\vec{e}_2,\vec{e}_3\}$ dan $\beta'=\{O',\vec{e}_1{}',\vec{e}_2{}',\vec{e}_3{}'\}$ ga o‘tishda $\vec{e}_i, i=1,2,3$ lar β da quyidagicha berilgan bo‘lsin: $\vec{e}_1{}'(4,3,-2); \vec{e}_2{}'(0,1,5); \vec{e}_3{}'(-1,0,5)$. $A(-1,0,27)$ va $B(1,0,-1)$ nuqtalarning yangi sistemadagi koordinatalarini toping.

9. $\beta=\{O,\vec{i},\vec{j},\vec{k}\}$ ni Oy o‘q atrofida α burchakka soat strelkasiga teskari yo‘nalishda borib, $\beta'=\{O',\vec{i}',\vec{j}',\vec{k}'\}$ sistemaga o‘tilgan. Koordinatalarni almashtirish formulalarini yozing, $\alpha=45^\circ$ bo‘lganda $M(0,1,-\sqrt{2})$ uchun M ning β dagi koordinatalarini toping.

10. To‘g’ri burchakli dekart koordinatalar sistemasini shunday almashtiringki, unda $O=O', Oz=Oz'$ bo‘lsin va $[Ox'], [Oy']$ nurlar esa $(xOz), (yOz)$ koordinata burchaklarining bissektrisalaridan iborat bo‘lib, yangi bazis sistemasining bazis vektorlari birlik vektorlar bo‘lsin.

11. $\beta=\{O,\vec{i},\vec{j},\vec{k}\}$ ni Oz o‘q atrofida soat strelkasiga teskari yo‘nalishda α burchakka burishdan $\beta'=\{O',\vec{i}',\vec{j}',\vec{k}'\}$ sistema hosil bo‘lgan. β dan β' ga o‘tishdagi koordinatalarni almashtirish formulalarini toping.

12. To‘g’ri burchakli $ABCD$ trapetsiya berilgan. Asoslari $AD=4, BC=2$ va D burchagi 45° ga teng. $\overline{CD}$ vektorni bir o‘q deb, $\overline{AD}, \overline{AB}, \overline{BC}, \overline{AC}$ vektorlarning shu bir o‘qdagi proektsiyalarini toping.

13. $\vec{a}$ vektor Ox va Oy o‘qlari bilan mos ravishda $\alpha=\frac{\pi}{3}, \beta=\frac{2\pi}{3}$ li burchaklar tashkil etadi. Agar $|\vec{a}|=2$ bo‘lsa, uning koordinatalarini hisoblang.

14. $\vec{a}=\{-3,-2,6\}$ va $\vec{b}=\{-2,1,10\}$ vektorlar berilgan. Quyidagi vektorlarning koordinatalarini toping:

a) $2\vec{a}-\frac{1}{3}\vec{b}$ b) $\vec{a}+\vec{b}$ c) $4\vec{a}-3\vec{b}$ d) $\frac{1}{3}\vec{a}+3\vec{b}$ e) $\frac{5}{12}\vec{a}-\frac{2}{5}\vec{b}$

15. Kesmaning uchlari $M(3,-2)$ va $N(10,-9)$ nuqtalarda yotadi. C nuqta kesmani $\lambda=\frac{2}{5}$ nisbatda bo‘lsa, shu nuqtaning koordinatalarini toping.

16. $B(-3,4)$ nuqta AC kesmani $\lambda=\frac{2}{3}$ nisbatda bo‘lsa, $A(1,2)$ ni bilgan holda $C(x,y)$ ni koordinatalarini toping.

17. $C(-5,4)$ nuqta AB kesmani $\lambda = \frac{3}{4}$ nisbatda, $D(6,-5)$ nuqta esa $\mu = \frac{2}{3}$ bo'lsa, A *va* B nuqtalarning koordinatalari topilsin.