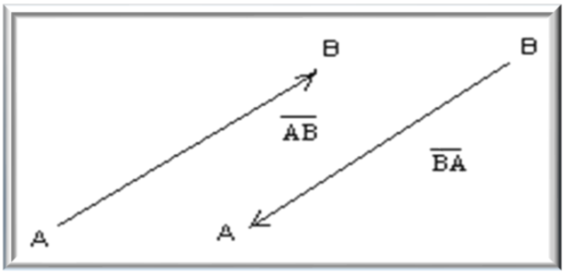
1-MA’RUZA

MAVZU: VEKTOR XAQIDA TUSHUNCHA

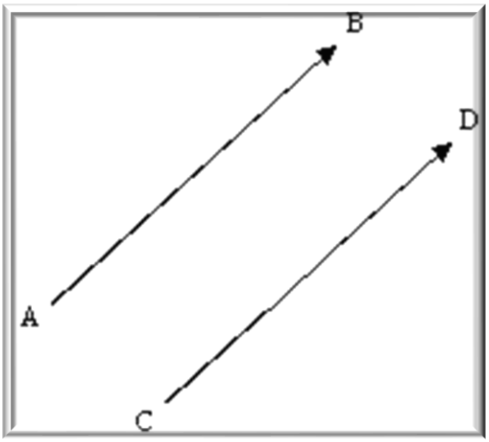
1-Ta’rif: *Berilgan kesmaning qaysi uchi birinchiligi va qaysi uchi ikkinchiligi aniqlangan bo’lsa bunday kesmaga yo’nalgan kesma deyiladi.*

1.1-Ta’rif: *Yo’nalishga ega bo’lgan kesma vector deb nom berilgan.*



1-rasm

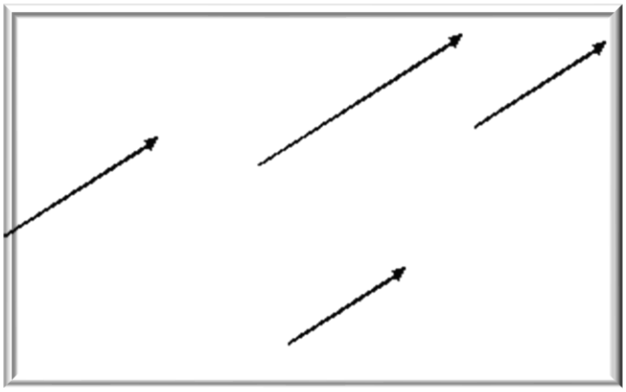
AB kesma uzunligi yo’nalgan kesmaning uzunligi deyiladi. (chiziqcha AB) (uzunligi $|AB|$)



2-rasm

2-Ta’rif: *Agar AB va CD nurlar bir xil yo’nalishli bo’lsa AB va CD yo’nalgan kesmalar (vektorlar) bir xil yo’nalishli deyiladi* $\overline{AB}\uparrow\uparrow\overline{CD}$ *aks xolda qarama-qarshi yo’nalishli deyiladi.* $\overline{AB}\uparrow\downarrow\overline{CD}$

3-Ta’rif: *Uzunliklari teng va bir xil yo’nalishli barcha kesmalar to’plami ozod vektor yoki qisqacha vektor deb ataladi.*



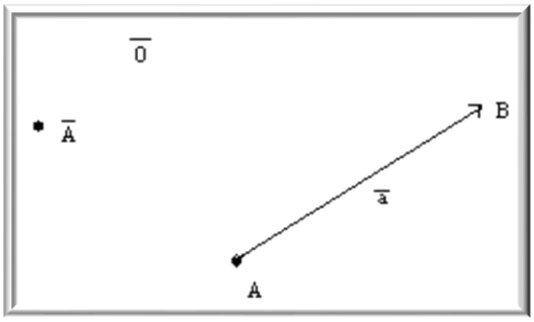
3-rasm

Vektorlar lotin harflarning kichkina harflari bilan belgilanadi va ustiga chiziqcha qo'yiladi yoki $\vec{a}, \vec{b}, \vec{c}, \overrightarrow{BA}, \overrightarrow{DC}$ katta harflar bilan belgilanadi. $\overrightarrow{AB}$ vektorda A-uchi boshi B-uchi oxiri.

4- Ta'rif: $\overrightarrow{AB}$ *vektorning uzunligi deb* $\left|\overline{AB}\right|$ *kesmaning uzunligiga teng.*

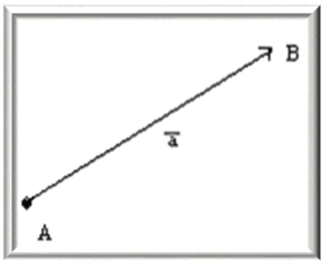
5- Ta'rif: *Uzunligi birga teng bo'lgan vektor- Birlik vektor deyiladi.*

6- Ta'rif: *Boshi va oxiri ustma-ust tushgan vektor- Nol vektor deyiladi.*



4-rasm

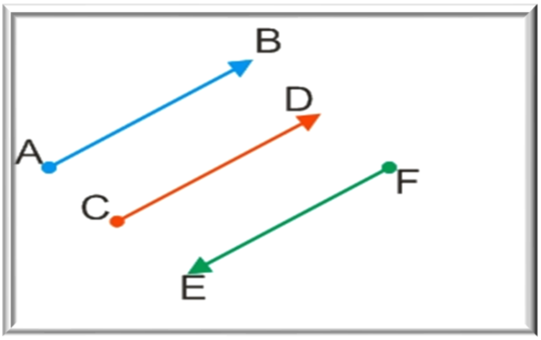
7- Ta'rif: *Agar,* $\overrightarrow{AB}$ *va* $\overrightarrow{CD}$ *yo'nalgan kesmalar bir xil yo'nalgan bo'lsa,* $\vec{A}$ *va* $\vec{B}$ *vektorlar bir xil yo'nalishli deyiladi va* $\vec{a}\uparrow\uparrow\vec{в}$



5-rasm

$\overrightarrow{AB}$ *va* $\overrightarrow{CD}$ *qarama-qarshi yo'nalishli bo'lsa* $\vec{a}$ *va* $\vec{в}$ *vektorlar qarama-qarshi bo'ladi.*

8- Ta'rif: *Bita to'g'ri chiziqqa parallel bo'lgan vektorlar koleniyar vektorlar deyiladi.*



6-rasm

Bu erda $\overrightarrow{AB}, \overrightarrow{CD} va \overrightarrow{FE}$ vektorlar koleniar hisoblanadi.

VEKTORLAR USTIDA CHIZIQLI AMALLAR

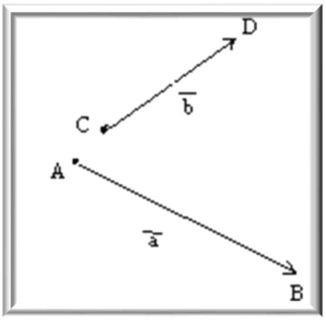
Vektorlar ustida bajariladigan amallar:

1) qo’shish
2) ayirish
3) vektorni songa ko’paytirish.

Vektorlarni qo’shish.

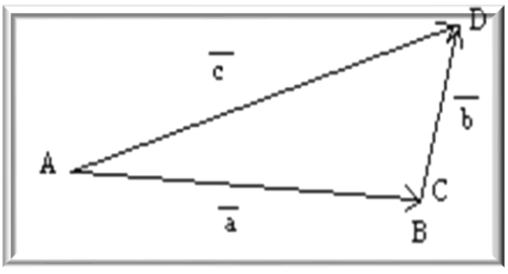
1)uchburchak
2)paralellogram

1) $\vec{a} = \overrightarrow{AB}$ $\vec{в} = \overrightarrow{CD}$



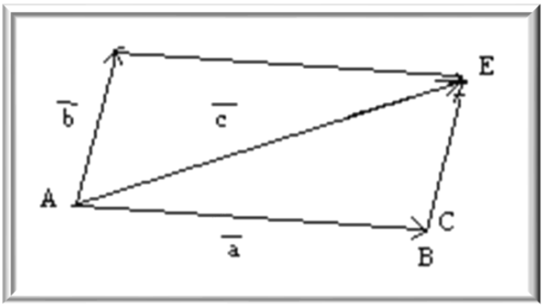
5-rasm

$\vec{a}$ va $\vec{в}$ vektorlarni qo’shish uchun $\vec{a}$ vektor oxiriga $\vec{в}$vektorni boshi olib kelib qo’yiladi va $\vec{a}$ vektorning boshidan $\vec{в}$ vektorning oxiriga qarata $\vec{c}$ vektor xosil bo’ladi. $\vec{c} = \vec{a} + \vec{в}$



6-rasm

2) $\vec{a}$ *va* $\vec{в}$ vektorlarning boshlari bir nuqtaga olib kelib qo'yiladi D uchdan $\vec{a}$ vektor parallel qilib paralellogramm xosil bo'ladi.



7-rasm

$\vec{a}$ *va* $\vec{в}$ vektorlarning yig'indisi uning diaganallari bo'yicha o'tadi.

Vektorlarni qo'shish amallari quyidagi xossalarga ega:

1^0. Vektorlarning qo'shishning guruxlanish (assotsiativlik)

$$\vec{a} + (\vec{в} + \vec{c}) = (\vec{a} + \vec{в}) + \vec{c}$$

2^0. Vektorlarning qo'shishning urin almashishligi (komutativlik).

$$\vec{a} + \vec{в} = \vec{в} + \vec{a}$$

3^0.Har qanday $\vec{a}$ vektorga 0 vektori qo'shilsa, $\vec{a}$ vektor hosil bo'ladi.

$$\vec{a} + 0 = \vec{a}$$

4^0.Har qanday $\vec{a}$ vektor uchun shunday $\vec{a}'$ vektor mavjudki:

$$\vec{a} + \vec{a}' = 0$$

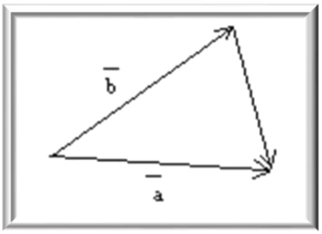
Isbot. $\vec{a} = \overrightarrow{OA}$ deb yozsak $\overrightarrow{OA} + \overrightarrow{AO} = \vec{0}$ $\vec{a} + \overrightarrow{AO} = \vec{0}$ $\vec{a}' = \overrightarrow{AO}$ $\vec{a}'$ vektorga $\vec{a}$ vektor qarama-qarshi vektor deyiladi.

Vektorlarni ayirish

$\vec{a}$ *va* $\vec{в}$ vektorlarning ayirmasi deb, $\vec{a}$ va $-\vec{в}$ vektorlarning yig'indisiga aytiladi.

$\vec{a} - \vec{в}$ ko'rinishda yoziladi.

$\vec{a}$ *va* $\vec{в}$ vektorlarning ayirmasini topish uchun ularning boshlarini bir nuqtaga olib kelamiz va ularning oxirlarini tutashtiramiz.



8-rasm

$\vec{a}$ *va* $\vec{в}$ vektorlarni bir joyga olib kelamiz. Xosil bo'lgan vektor $\vec{a} - \vec{в}$ deb yoziladi.

Vektorni songa ko'paytirish.

Ta'rif: $\vec{a}\neq 0$ *vektorning* $\alpha\in R$ *ko'paytmasi deb, shunday vektorga aytiladiki, bu vektorning yo'nalishi* $\alpha>0$ *bo'lsa* $\vec{a}$ *vektorning yo'nalishi bilan bir xil,* $\alpha<0$ *bo'lsa* $\vec{a}$ *vektorning yo'nalishiga qarama-qarshi bo'lib, uzunligi* α *ning moduliga* $\vec{a}$ *vektorning uzunligining ko'paytmasiga teng bo'ladi va* $\alpha\cdot\vec{a}$ *ko'rinishda yoziladi.*

9-rasm

bu ta'rifdan quyidagi mulohazalar kelib chiqadi.

1) $\vec{a}\cdot 0=\vec{0}$

2) $0\cdot\alpha=\vec{0}$

3) $1\cdot\vec{a}=\vec{a}$

4) $-1\cdot\vec{a}=-\vec{a}$

5) $\vec{a}$ va $\alpha\ \vec{a}$ vektorlar kollenear.

Teorema: *Agar* $\vec{a}\,//\,\vec{b}$ $(\vec{a}\neq 0)$ *bo'lsa, u xolda shunday* α *son mavjud bo'ladiki* $\vec{b}=\alpha\cdot\vec{a}$ *tenglik o'rinli bo'ladi.*

Bu erda uchta xol mavjud bo'lishi mumkin.

Isbot. 1) $\vec{a}\uparrow\uparrow\vec{b}$ bo'lsin $\dfrac{\vec{a}}{|\vec{a}|}$ bu vektor yo'nalishi $\vec{a}$ bilan bir xil bo'lgan birlik vektordir. $\dfrac{\vec{a}}{|\vec{a}|}=\dfrac{\vec{b}}{|\vec{b}|}$ $\quad\vec{b}=\dfrac{\vec{a}|\vec{b}|}{|\vec{a}|}=\left|\dfrac{\vec{b}}{\vec{a}}\right|\cdot\vec{a}$ bu erda $\alpha=\dfrac{|\vec{b}|}{|\vec{a}|}$ deb olsak $\vec{b}=\alpha\cdot\vec{a}$ bo'ladi. Yo'nalishda bo'lsa teorema isbotlandi.

2) $\vec{a}\uparrow\downarrow\vec{b}$ bo'lsin. $\dfrac{\vec{a}}{|\vec{a}|}\uparrow\uparrow\vec{a}$ birlik vektor. $\dfrac{\vec{b}}{|\vec{b}|}\uparrow\uparrow\vec{b}$ birlik vektor. $\vec{a}\uparrow\downarrow\vec{b}$ edi.

$-\dfrac{\vec{a}}{|\vec{a}|}=\dfrac{\vec{b}}{|\vec{b}|}$ bo'ladi. $\vec{b}=-\left|\dfrac{\vec{b}}{\vec{a}}\right|\cdot\vec{a}$ $\quad\alpha=-\dfrac{|\vec{b}|}{|\vec{a}|}$ $\quad\vec{b}=\alpha\cdot\vec{a}$ bo'ladi.

3) $\vec{b}=0$ bo'lsin u xolda $\alpha=0$ deb olamiz. $\vec{b}=\alpha\cdot\vec{a}$ bo'ladi.

Teorema isbot bo'ladi.

Demak, vektorni songa ko'paytirishning bu teoremadan bu kelib chiqadi.

$$\vec{a}\,//\,\vec{b}\Leftrightarrow\vec{b}=\alpha\cdot\vec{a}$$

Demak, $\vec{a}$ va $\vec{b}$ vektorlar kollenear bo'lishi $\vec{b}=\alpha\cdot\vec{a}$ bo'lishi zarur va etarli.

Vektorni songa ko'paytirish quyidagi:

1) $\forall\vec{a}$ va har qanday $\alpha,\beta \in R$ sonlar uchun $\alpha\cdot\left(\vec{\beta}\cdot\vec{a}\right)=(\alpha\cdot\beta)\cdot\vec{a}$ tenglik o'rinli.
2) har qanday($\forall\vec{a}$) va har qanday $\alpha,\beta \in R$ sonlar uchun $(\alpha+\beta)\cdot\vec{a}=\alpha\cdot\vec{a}+\vec{a}\cdot\beta$
3) $\forall\vec{a}$ va $\vec{b}$ vektorlar va har qanday $\alpha \in R$ son uchun $\alpha\cdot\left(\vec{a}+\vec{b}\right)=\alpha\cdot\vec{a}+\alpha\cdot\vec{b}$ o'rinli.

Shunday qilib barcha vektorlar to'plami V da aniqlangan vektorlarni ko'shish va vektorni songa ko'paytirish amallari quyidagi xossalarni qanoatlantiradi.

1^0. $\left(\vec{a}+\vec{b}\right)+\vec{c}=\left(\vec{b}+\vec{c}\right)+\vec{a}$ (assotsativlik)

2^0. $\vec{a}+\vec{b}=\vec{b}+\vec{a}$ (komutativlik)

3^0. Shunday $\vec{0}$ vektor mavjudki $\vec{a}+\vec{0}=\vec{a}$

4^0. Shunday $\vec{a}'$ vektor mavjud mavjudki $\vec{a}+\vec{a}'=\vec{0}$

5^0. $\alpha\left(\vec{a}\cdot\vec{\beta}\right)=(\alpha\cdot\beta)\cdot\vec{a}$

6^0. $(\alpha+\beta)\cdot\vec{a}=\alpha\cdot\vec{a}+\beta\cdot\vec{a}$
7^0. $\alpha\left(\vec{a}+\vec{b}\right)=\alpha\cdot\vec{a}+\alpha\cdot\vec{b}$
8^0. $1\cdot\vec{a}=\vec{a}$

bu sakkiz xossani qanoatlantiruvchi vektorlar to'plami V vektor fazo deyiladi. (chiziqli fazo deb yuritiladi).

MISOL VA MASALALAR

1.*ABC* uchburchakka *AD* mediana o'tkazilgan. $\overline{AD}$ vektorni $\overline{AB}$, $\overline{AC}$ vektorlar orqali ifodalang.

2. $\vec{a}$ *va* $\vec{в}$ vektorlarning berilishiga ko'ra ularning quyidagi chiziqli kombinatsiyalarini yasang:

1) $3\vec{a}+\frac{1}{2}\vec{b}$ 2) $\vec{a}-\frac{1}{2}\vec{b}+\vec{c}$ 3) $\vec{a}-\frac{2}{5}\vec{b}$

3. Quyidagi vektorlarni modulini hisoblang:

$\vec{a}=\{3;2;1\}$ $\vec{b}=\{4;2;1\}$ $\vec{c}=\{5;0;7\}$

4. *A(3;2;1)* va *B(4;3;5)* nuqtalar berilgan. $\overrightarrow{AB}$ va $\overrightarrow{BA}$ vektorlarning koordinatalarini toping.

5. Oxiri (1;-1;2) nuqtada bo'lgan $\vec{a}=\{2;-3;-1\}$ vector boshiining koordinatalarini toping

6. Agar $\vec{a}\{4;-7;3\}$ va $\vec{b}=\left\{-5;9;\frac{1}{2}\right\}$ bo'lsa $\vec{c}=3\vec{a}+2\vec{b}$ vektorning koordinatalarini toping.

7. $\bar{a}$ va $\bar{b}$ vektorlarning berilishiga ko'ra ularning quyidagi chiziqli kombinatsiyalarini yasang:

$3\overline{a}+\frac{1}{2}\overline{b}$; $\frac{1}{2}\overline{a}+\frac{1}{3}\overline{b}$; $\overline{a}-3\overline{b}$; $-3\overline{a}-\frac{1}{2}\overline{b}+\overline{c}$.

8. $\overline{AB}=\bar{c}$, $\overline{BC}=\bar{a}$, $\overline{CA}=\bar{b}$ vektorlar ABC uchburchakning tomonlari. ABC uchburchakning $\overline{AQ}$, $\overline{BN}$, $\overline{CP}$ medianalaridan iborat vektorlarni $\bar{a}$, $\bar{b}$, $\bar{c}$ vektorlar orqali ifodalang. $\bar{a}=\{-3,-2,6\}$ va $\bar{b}=\{-2,1,10\}$ vektorlar berilgan. Quyidagi vektorlarning koordinatalarini toping:

$2\overline{a}-\frac{1}{3}\overline{b}$; $\overline{a}+2\overline{b}$; $\overline{4a}-5\overline{b}$; $\frac{1}{3}\overline{a}+2\overline{b}$.

9. $B(4,-2,0)$ nuqta $\bar{a}(2,-3,-1)$ vektorning oxiri bo'lsa, bu vektor boshining koordinatalarini toping.

10. $\bar{a}(2,-3)$ va $\bar{b}(-3,4)$ vektorlar berilgan. Agar $\bar{a}=-\bar{c}+3\bar{b}$ bo'lsa, $\bar{c}$ vektorning koordinatalarini toping.

11. To'rtburchakning uchta *M(2,-4), N(-4,0), P(2,-2)* uchlari berilgan. Agar $\overline{MN}=4\overline{QP}$ bo'lsa, *Q* uchining koordinatalarini toping.

12. $\bar{a}(2,3)$, $\bar{b}(3,-2)$, $\bar{c}(4,19)$ vektorlar uchun $\bar{c}=m\bar{a}+n\bar{b}$ tenglik o'rinli bo'lsa, mn ko'paytmaning qiymatini toping.

13. Quyidagi ifodani soddalashtiring:

$$\overline{OP}-\overline{EP}+\overline{KD}-\overline{KA},$$
$$\overline{AD}+\overline{MP}+\overline{EK}-\overline{EP}-\overline{MD},$$
$$\overline{AC}-\overline{BC}-\overline{PM}-\overline{AP}+\overline{BM},$$
$$\overline{AB}+\overline{BC}+\overline{CD}-\overline{AD}+\overline{MN}.$$

14. $ABCDA_1B_1C_1D_1$ parallelepiped berilgan bo'lsa, $\overline{AB}+\overline{AD}+\overline{AA_1}$ yig'indi vektorni toping.

15. $ABCD$ parallelogrammning ikkita qo'shni tomoni $\overline{AB}=\bar{a}$, $\overline{AD}=\bar{b}$, diagonal-larining kesishish nuqtasi esa *M* bo'lsa, $\overline{MA}$, $\overline{MB}$, $\overline{MC}$, $\overline{MD}$ vektorlarni $\bar{a}$ va $\bar{b}$ vektorlar orqali ifodalang.

16. *M* nuqta ABC uchburchak medianalarining kesishgan nuqtasi bo'lsa, $\overline{MA}+\overline{MB}+\overline{MC}=\bar{0}$ ekanligini isbotlang.

17. ABC uchburchak va uning og'irlik markazi *G* berilgan bo'lsa, u holda ixtiyoriy *M* nuqta uchun $\overline{MG}=\frac{1}{3}\left(\overline{MA}+\overline{MB}+\overline{MC}\right)$ ekanligini isbotlang.

18. $\overline{AB}=2\overline{BC}$ shartni qanoatlantiruvchi A, B, C nuqtalar berilgan. Ixtiyoriy *O* nuqta uchun

$$\overline{OB}=\frac{1}{3}\overline{OA}+\frac{2}{3}\overline{OC}$$

tenglik o’rinli ekanligini isbotlang.
19. Muntazam $ABCDEF$ oltiburchakda $\overline{AB}=\overline{p}$ va $\overline{BC}=\overline{q}$ ekani ma’lum: Quyidagi $\overline{CD}, \overline{DE}, \overline{EF}, \overline{FA}, \overline{AC}, \overline{AD}$ va $\overline{AE}$ vektorlar $\overline{p}$ va $\overline{q}$ vektorlar orqali ifodalansin.
20. Muntazam $ABCDEF$ oltiburchakda $\overline{AB}=\overline{p}$ va $\overline{BC}=\overline{q}$ ekani ma’lum: Vektorlarning quyidagi nisbatlari topilsin:

$$\frac{\overline{BC}}{\overline{\overline{AD}}}, \frac{\overline{BC}}{\overline{\overline{EF}}}, \frac{\overline{CF}}{\overline{\overline{AB}}}, \frac{\overline{AB}}{\overline{\overline{BC}}}.$$

TESTLAR

1. $\overline{a}=\{1;5\}$, $\overline{b}=\{3;-1\}$, $\overline{c}=\{0;1\}$ **векторлар берилган.** α **нинг кандай кийматларида** $\overline{p}=\overline{a}+\alpha\overline{b}$, $\overline{q}=\overline{a}-\overline{c}$ **векторлар коллинеар булади?**

A) 5
B) $-\frac{3}{11}$
C) $\frac{1}{13}$
D) $\frac{9}{11}$

2. $\overline{a}=\{1;5\}$, $\overline{b}=\{3;-1\}$, $\overline{c}=\{0;1\}$ **векторлар берилган.** α **нинг кандай кийматларида** $\overline{p}=\overline{a}+\alpha\overline{b}$, $\overline{q}=\overline{a}-\overline{c}$ **векторлар коллинеар булмайди?**

A) 5
B) $-\frac{3}{11}$
C) $\frac{1}{13}$
D) $\frac{9}{11}$

3. Vektorlarning ta’rifiga asoslanib ushbu tushunchalardan qaysi biri to’g’riligini aniqlang

a)Faqat son qiymati bilan aniqlangan kattalik vektor kattalikdir
b)Faqat yo’nalish bilan aniqlangan kattalik vektor kattalikdir
c)Xam son qiymati, xam yo’nalishi bilan aniqlangan kattalik vektor kattalikdir
d)Ixtiyoriy jismning xajmi vektor kattalikdir

4. Uzunliklari teng, bir xil yo’nalishga ega ikkita vektorlarga

a)birlik vektorlar deyiladi

b)Qarama-qarshi vektorlar deyiladi
c)ortogonal vektorlar deyiladi
d)o'zaro teng vektorlar deyiladi

5. Quyidagi vektorlarning qaysi biri birlik vektor $\vec{a}\left(-\frac{3}{5},\frac{4}{5}\right)$ $\vec{b}\left(\frac{2}{3},\frac{2}{3}\right)$ $\vec{c}(0,-1)$ $\vec{d}\left(\frac{3}{5},-\frac{4}{5}\right)$

a) A va B
b) A, C, D
c) C, D
d) birlik vektor yo'q

6. Quyidagi mulohazalardan qaysi biri to'g'ri?

a) $\vec{a}$ vektorni k sonigi ko'paytirish uchun uning uzunligi k soniga ko'paytiriladi

b) $\vec{a}$ va $\vec{b}$ vektorlarni yig'indisi deb, $\vec{a}$ vektorning boshidan $\vec{b}$ vektorning oxiriga yo'naltirilgan vektorga aytiladi

c) $\vec{a}$ va $\vec{b}$ vektorlarning skalyar ko'paymasi deb, mos koordinatalarini ko'paytirishga aytiladi

d) $\vec{a}$ va $\vec{b}$ vektorlarni ayirmasi deb, $\vec{b}$ vektor bilan yig'indisi $\vec{a}$ vektorga teng $\vec{c}$ vektorga aytiladi

7. $\vec{a}=\{2;-1;3\}$**,** $\vec{b}=\{-6;3;-9\}$ **vektorlar qanday o'zaro munosabatda bo'ladi?**
a) $\vec{a},\vec{b}$ - kolleniar vector
b) $\vec{a}\perp\vec{b}$
c) $\vec{a}\vec{b}=30^0$
d) $\vec{a},\vec{b}$ - kolleniar vector emas.

8. $\vec{a}=\{2;1;-3\}$**,** $\vec{b}=\{-6;-3;9\}$ **vektorlar qanday o'zaro munosabatda bo'ladi?**
a) $\vec{a},\vec{b}$ - kolleniar vector
b) $\vec{a}\perp\vec{b}$
c) $\vec{a}=\vec{b}$
d) $\vec{a},\vec{b}$ - kolleniar vector emas.

9. $\vec{P}=\{5;1;3\}$ **va** $Q=\{2;-1;4\}$ **nuqtalar berilgan.** $\overrightarrow{PQ}$ **vektorning koordinatasi topilsin**
a) $\{-3;-2;1\}$
b) {1;1;0}
c) {0;4;0}
d){0;0;4}

10. $\vec{a},\vec{b}$ **vektorlar o'zaro qanday joylashgan bo'lsa,** $\left|\vec{a}+\vec{b}\right|=\left|\vec{a}\right|+\left|\vec{b}\right|$ **tenglik o'rinli bo'ladi.**

a) $\vec{a}\uparrow\uparrow\vec{b}$

b) $\vec{a}\uparrow\downarrow\vec{b}$

c) $\vec{a}\perp\vec{b}$

d) $\vec{a}=-\vec{b}$