Tekislikda va fazoda affin va dekart koordinatalar sistemasi

Tekislikda nuqtaning koordinatalari.

Tekislikda nuqtaning o‘rnini ma’lum sonlar yordamida aniqlashga imkon beradigan usul ko‘rsatilgan bo‘lsa, tekislikda koordinatalar sistemasi berilgan deyiladi.Tekislikda koordinatalarning turli sistemalari mavjud bo‘lib, biz ularning soddalarini quramiz.

Tekislikda koordinatalarning affin sistemasi.

Tekislikda biror O nuqtaga qo‘yilgan nokalleniar ixtiyoriy ikkita $\vec{e}_1, \vec{e}_2$ vektorlar berilgan bo‘lsin. Bu vektorlar sistemasi $(\vec{e}_1, \vec{e}_2)$ vektorlar orqali o‘tuvchi a,b to‘g’ri chiziqlarni olamiz .

1-Ta’rif: *Musbat yo‘nalishlari mos ravishda* $\vec{e}_1, \vec{e}_2$ *vektorlar bilan aniqlanuvchi* a,b *to‘g’ri chiziqlardan tashkil topgan sistema tekilikda koordinatalarning affin sistemasi yoki affin reperi deb ataladi va u* $\vec{V}(0, \vec{e}_1, \vec{e}_2)$ *kabi belgilanadi.*

$0 = a \cap b$ nuqta koordinatalar boshi, $\vec{e}_1, \vec{e}_2$-vektorlar esa koordinata vektorlari deyiladi. Musbat yo‘nalishlari $\vec{e}_1, \vec{e}_2$ vektorlar bilan aniqlangan a,b to‘g’ri chiziqlar mos ravishda abssissalar va ordinatalar o‘qlari deb ataladi. Tekislikda affin reperi berilgan bo‘lsin. Shu tekislikning M nuqtasi uchun $\overrightarrow{OM}$ vektor M nuqtaning radius vektorlari deyiladi va $\overrightarrow{OM}$ vektor quyidagicha ifodalanadi:

$$\overrightarrow{OM} = x_1\vec{e}_1 + y_1\vec{e}_2$$

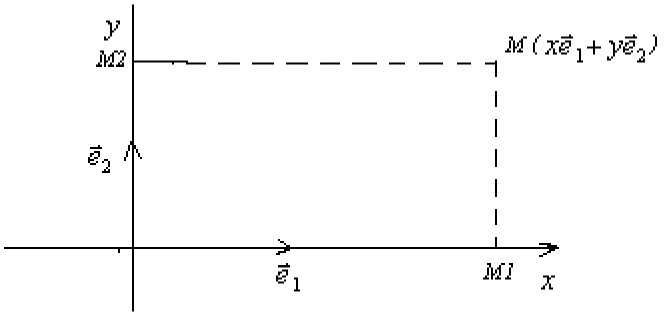
2-Ta’rif: $\overrightarrow{OM}$ *radius vektorning* x_1, y_1 *koordinatalari* M *nuqtaning* $(0; \vec{e}_1, \vec{e}_2)$ *affin reperidagi koordinatalari deyiladi va* $M(x_1, y_1)$ *bilan belgilanadi.*

Tekislikda affin sistemasi berilgan bo‘lsa, biror M nuqtaga koordinata bo‘lmish x_1, y_1 mos keladi va aksincha. Tekislikda $(0; \vec{e}_1, \vec{e}_2)$ affin reperda abtsissalar o‘qiga koordinatalar boshidan boshlab $OM_1 = x \cdot \vec{e}_1$ vektor ordinatalar o‘qiga $OM_2 = y \cdot \vec{e}_2$ vektorni qo‘yib hosil qilingan M_1, M_2 nuqtalardan mos ravishda abtsissalar va ordinatalar o‘qlariga parallel to‘g’ri chiziqlar o‘tkazsak, ularning kesishgan nuqtasi M nuqta bo‘ladi.

$$\overrightarrow{OM} = \overrightarrow{OM}_1 + \overrightarrow{OM}_2 = x\vec{e}_1 + y\vec{e}_2$$

shunday qilib reperga nisbatan

$$M(x, y) \Leftrightarrow \overrightarrow{OM} = x_1\vec{e}_1 + y_1\vec{e}_2 \qquad (1)$$



1-rasm

$\overrightarrow{OM} = x\vec{e}_1 + y\vec{e}_2$. Agar M nuqtaning abssissasi nol bo‘lsa $(x=0)$ unda (1) tenglamada shu ko‘rinishda bo‘ladi:

$$\overrightarrow{OM} = y\vec{e}_2$$

M nuqta Oy o‘qda yotadi. Koordinatalar tekisligi butun tekislikni 4 ta bo‘lakka ajratadi. Agar M nuqta koordinatalari koordinata o‘qida yotmasa uning koordinatalari ishorasiga qarab qaysi chorakda yotishi aniqlanadi. Agar M nuqta koordinatalari

$x>0, y>0$ bo‘lsa 1-chorakda

$x<0, y>0$ bo‘lsa 2-chorakda

$x<0, y<0$ bo‘lsa 3-chorakda

$x>0, y<0$ bo‘lsa 4-chorakda

$(0,\vec{e}_1,\vec{e}_2)$ reperga nisbatan $A(x_1,y_1);B(x_2,y_2)$ nuqtalarni olaylik. Bu nuqtalarning radiuslari vektorlari $\overrightarrow{OA} = x_1\vec{e}_1 + y_1\vec{e}_2$, $\overrightarrow{OB} = x_2\vec{e}_1 + y_2\vec{e}_2$ ko‘rinishda bo‘ladi. Bularni bilgan holda $\overrightarrow{AB}$ vektorning koordinatalarini topamiz.

$$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = x_2\vec{e}_1 + y_2\vec{e}_2 - x_1\vec{e}_1 - y_1\vec{e}_2 = (x_2 - x_1)\vec{e}_1 + (y_1 - y_2)\vec{e}_2$$

Bundan $\overrightarrow{AB}$ vektorning koordinatalari $\overrightarrow{AB} = (x_2 - x_1; y_1 - y_2)$ hosil bo‘ladi.

3-Ta’rif: *Affin reperining koordinata vektorlari* $\vec{e}_1,\vec{e}_2$ *ortanormallangan bazisni tashkil etsin, ya’ni* $\vec{e}_1 \perp \vec{e}_2, |\vec{e}_1| = |\vec{e}_2| = 1$ *bu holda biz koordinatalarning to‘g’ri burchakli sistemasi yoki Dekart reperi berilgan deymiz. So‘ng reperni* $(0,i,j)$ *ko‘rinishda belgilaymiz.* Bu yerda $\vec{i}^2 = \vec{j}^2 = 1, \quad \vec{i} \times \vec{j} = 0$

4-Ta’rif: M_1, M_2 *nuqtalar orasidagi masofa deb,* $\overrightarrow{M_1M_2}$ *yoki* $\overrightarrow{M_2M_1}$ *vektorlar uzunligiga aytiladi.*

Ta’rifga ko‘ra $\rho(M_1M_2)=\left|\overrightarrow{M_1M_2}\right|$ $M_1(x_1,y_1);M_2(x_2,y_2)$ bo‘lsin, u holda $\overrightarrow{M_1M_2}=\overrightarrow{0M_2}-\overrightarrow{0M_1}$ bo‘ladi.

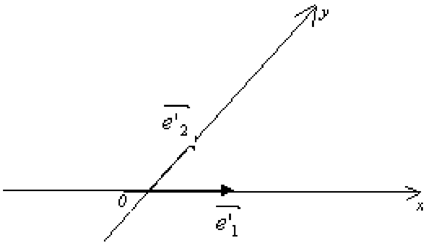
$\overrightarrow{M_1M_2}=\overrightarrow{0M_2}-\overrightarrow{0M_1}=(x_2-x_1;y_2-y_1)$ vektorning uzunligini topish formulasini bilgan holda $\rho(M_1,M_2)$ vektorning uzunligini topamiz.

$$\left|\overrightarrow{M_1M_2}\right|=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2} \qquad (2)$$

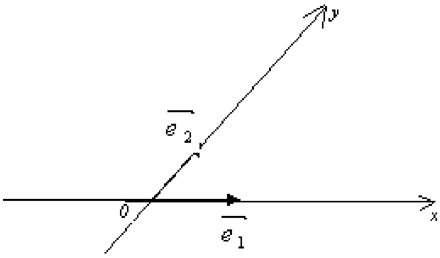
Demak, berilgan M_1,M_2 nuqtalar orasidagi masofa (2) ga qarab topiladi.

Affin koordinatalar sistemasini almashtirish.

Bizga ikkita $Q(0;\vec{e}_1;\vec{e}_2)$ va $(o';\overrightarrow{e_1'};\overrightarrow{e_2'})$ reper berilgan bo‘lsin.



2-rasm .



3-rasm.

$O'(c_1,c_2)$, $\overrightarrow{e_1'}(a_1,a_2)$, $\overrightarrow{e_2'}(b_1,b_2)$ Biz tekislikda M koordinatalari (x,y) bo‘lsin, ya’ni koordinata sistemasiga nisbatan koordinatalari (x',y') bo‘lsin. $\begin{pmatrix} a_1 & a_2 \\ b_1 & b_2 \end{pmatrix}\neq 0$ deb shart qo‘yamiz.

$$\overrightarrow{OM}=x\vec{e}_1+y\vec{e}_2 .$$

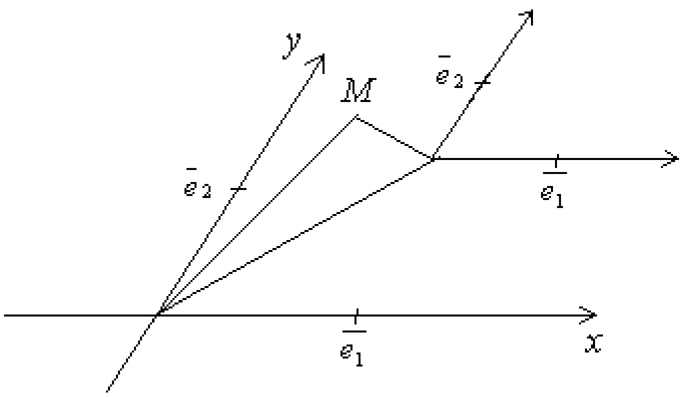
$$\overrightarrow{OM'}=x'\overrightarrow{e_1'}+y'\overrightarrow{e_2'};\overrightarrow{OM}=\overrightarrow{OO'}+\overrightarrow{O'M}=c_1\overrightarrow{e_1}+c_2\overrightarrow{e_2}+x'\overrightarrow{e_1'}+y'\overrightarrow{e_2'}$$

$$x\overrightarrow{e_1}+y\overrightarrow{e_2}=c_1\overrightarrow{e_1}+c_2\overrightarrow{e_2}+x'(a_1\overrightarrow{e_1}+b\overrightarrow{e_2})+y'(b_1\overrightarrow{e_1}+b_2\overrightarrow{e_2})=$$

$$=(a_1x'+b_1y'+c_1)\overrightarrow{e_1}+(a_2x'+b_2y'+c_2)\overrightarrow{e_2}$$

$$(x-a_1x'-b_1y'-c_1)\overrightarrow{e_1}+(y-a_2x'-b_2y'-c_1)\overrightarrow{e_2}=0$$

$$\begin{aligned} x&=(a_1x'+b_1y'+c_1)\\ y&=(a_2x'+b_2y'+c_2)\end{aligned} \qquad (3)$$



4-rasm

Dekart koordinatalar sistemasini almashtirish.

Bizga ikkita dekart koordinatalar sistemasi berilgan bo‘lsin.

$\beta=(o,i,j),\beta'=(o',i',j')$ $O'=(c_1;c_2)$, $i'=(a_1;a_2)$, $j'=(b_1;b_2)$,

$$\vec{i}'=a_1\vec{i}+a_2\vec{j}\,;\ \vec{j}'=b_1\vec{i}+b_2\vec{j} \qquad (4)$$

$$(\vec{i}\,\hat{}\,\vec{i}')=\alpha$$

1) Bu ikkita koordinata sistemasi bir xil aryentatsiyali bo‘lsin. U holda $\vec{j}'$ orasidagi burchakni hisoblaymiz.

$(\vec{i}',\vec{j})=90^\circ+\alpha$, $(\vec{i}',\vec{j})=90^\circ-\alpha$, $(\vec{i},\vec{i}')=a_1(\vec{i},\vec{i})+a_2(\vec{j},\vec{i})$,

$|\vec{i}|\cdot|\vec{i}'|\cos\alpha=a_1|\vec{i}|^2+a_2|\vec{j}||\vec{i}|\cos 90^\circ$, $a_1=\cos\alpha$

$(\vec{i}',\vec{j})=a_1(\vec{i},\vec{j})+a_2(\vec{j},\vec{j})$, $(\vec{j}',\vec{i})=b_1(\vec{i},\vec{i})+b_2(\vec{j},\vec{i})$,

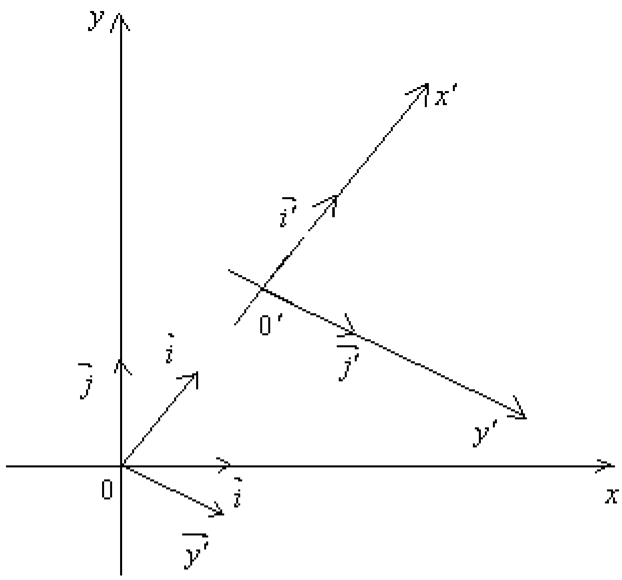
Bundan kelib chiqadi:

$b_1=\cos(90^\circ+\alpha)=-\sin\alpha$, $a_2=\cos(90^\circ-\alpha)=\sin\alpha$, $(\vec{j},\vec{j}')=b_1(\vec{i},\vec{j})+b_2(\vec{j},\vec{j})$, $b_2=\cos\alpha$

$$\begin{cases} x=x'\cos\alpha-y'\sin\alpha+c_1 \\ y=x'\sin\alpha+y'\cos\alpha+c_2 \end{cases} \qquad (5)$$

2)Endi koordinatalar sistemalari har xil aryentatsiyali bo‘lsin:

$(\vec{i},\vec{i}')=\alpha$, $(\vec{i}',\vec{j}')=270^\circ-\alpha$, $(\vec{i}',\vec{j})=90^\circ+\alpha$, $(\vec{j}',\vec{j}')=180^\circ+\alpha$.



5-rasm .

Har xil aryentatsiyali: $(\vec{i'},\vec{j})=\alpha$, $(\vec{i'},\vec{j'})=270^\circ-\alpha$, $(\vec{i},\vec{i})=1$, $(\vec{i'},\vec{j})=90^\circ+\alpha$, $(\vec{i'},\vec{j})=90^\circ-\alpha$, $(\vec{i},\vec{j})=0$, $(\vec{j'},\vec{j'})=180^\circ+\alpha$, $(\vec{i},\vec{i'})=\cos\alpha$. $\vec{i'}=a_1\vec{i}+b_1\vec{j}$, $\vec{j'}=a_2\vec{i}+b_2\vec{j}$, $(\vec{i},\vec{j'})=\cos(270^\circ+\alpha)$, $(\vec{i},\vec{i'})=a_1(\vec{i},\vec{i})+b_1(\vec{i},\vec{j})=a_1$, $a_1=\cos\alpha$, $(\vec{i},\vec{j})=\sin\alpha$.

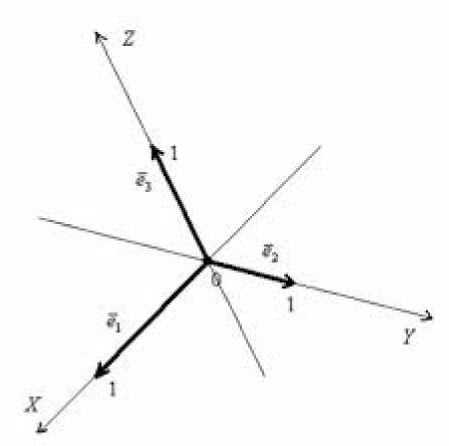
$$\begin{cases} x' = x\cos\alpha - y\sin\alpha + c_1 \\ y' = x\sin\alpha + y\cos\alpha + c_2 \end{cases}$$

$$b_1=\sin\alpha,\ a_2=\sin\alpha,\ b_2=-\cos\alpha.$$

Fazoda yoki tekislikda affin koordinatalar sistemasini kiritish uchun birorta bazis va bitta nuqta tanlanadi.

5-Ta’rif: Agar $(\vec{e}_1,\vec{e}_2,\vec{e}_3)$ bazis va O nuqta berilgan bo‘lsa, $\overline{OM}$ vektorning $(\vec{e}_1,\vec{e}_2,\vec{e}_3)$ bazisdagi koordinatalari M nuqtaning affin koordinatalari deyiladi.

O nuqtadan o‘tib, $\vec{e}_1,\vec{e}_2,\vec{e}_3$ vektorlar bilan aniqlanadigan to‘g‘ri chiziqlar mos ravishda Ox, Oy, Oz deb belgilab, ular koordinata o‘qlari, birinchisi abssissalar (X) o‘qi, ikkinchisi ordinatalar (Y) o‘qi va, nihoyat, uchinchisi applikatalar (Z) o‘qi deb ataladi. Bu o‘qlarning har ikkitasi bilan aniqlanadigan uchta tekislik *Oxy, Oxz, Oyz* deb belgilab, ular koordinata tekisliklari deb ataladi (6-rasm).



6-rasm.

Fazodagi har bir M nuqtaga aniq bir $\overrightarrow{OM}$ vektorni doimo mos keltirish mumkin, ya’ni boshi koordinatalar boshida, oxiri esa berilgan M nuqtada bo‘lgan vektorni mos keltiradi.

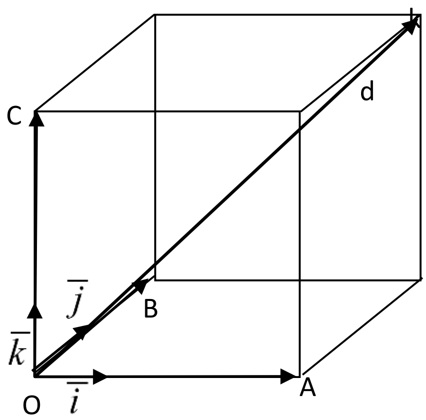
$\overrightarrow{OM}$ vektorning koordinatalar (x, y, z) bo‘lsa, u holda bu uchta x, y, z son M nuqtaning affin sistemadagi koordinatalari bo‘ladi:

$$\overrightarrow{OM}(x,\ y,\ z) \Leftrightarrow M(x,\ y,\ z).$$

Demak, fazo nuqtalari to‘plami bilan ma’lum tartibda olingan haqiqiy sonlar uchliklari to‘plami orasida biektiv moslik mavjud.

1-teorema. *Dekart koordinatalar sistemasida vektorning berilgan bazisdagi koordinatalari, uning koordinatalar o‘qlariga tushirilgan proyeksiyalari bilan ustma-ust tushadi.*

$\triangle$ ***Isbot***. Bizga $\vec{i}, \vec{j}, \vec{k}$ ortonormal bazis berilgan bo‘lsa, bularning boshlarini O nuqtaga joylashtirib $Oxyz$ koordintalar sistemasini kiritaylik. Agar $\vec{a} = x\vec{i} + y\vec{j} + z\vec{k}$ bo‘lsa, $\vec{a}$ vektorning boshini koordinata boshiga joylashtirib, uning oxirini M bilan belgilaymiz. Agar M nuqtaning koordinata o‘qlariga ortogonal proyeksiyalarini A, B, C harflari bilan belgilasak: $\overrightarrow{OA} = x\vec{i}$, $\overrightarrow{OB} = y\vec{j}$, $\overrightarrow{OC} = z\vec{k}$ tengliklarni hosil qilamiz. Ikkinchi tomondan $\overrightarrow{OA}, \overrightarrow{OB}, \overrightarrow{OC}$ kesmalarning kattaliklari mos ravishda x, y, z sonlariga teng bo‘lganligi uchun $x = pr_{Ox}\vec{a},\ y = pr_{Oy}\vec{a},\ \ z = pr_{Oz}\vec{a}$ munosabatlarni hosil qilamiz (7-rasm) $\blacktriangle$.



7-rasm.