MASALALAR

Tekislikda affin koordinatalar sistemasi

1. Affin koordinatalar sistemasiga nisbatan uchlarining $A(3;5), B(-4;6)\ va\ C(5;3;5)$ koordinatalari berilgan uchburchakni yasang.
   Izoh: Affin koordinatalar sistemasida o‘qlar orasidagi burchakni ixtiyoriy tartibda tanlab olishingiz mumkin. $(\alpha = 30^\circ, 60^\circ, 90^\circ)$
2. Tomoni $a=1$ bo‘lgan muntazam oltiburchak uchlarining koordinatalarini toping. Koordinatalar o‘qi qilib uning shunday ikki qo‘shni tomonlarini olingki, koordinatalar boshiga qarama-qarshi yotgan uchining koordinatalari musbat bo‘lsin.
3. Uchburchakning uchlari berilgan: $A(3,-7), B(5,2), C(-1,0)$. Har bir tomonning o‘rta nuqtasining koordinatalarini toping.
4. Uchburchak tomonlarining o‘rtalari $M_1(3,-2), M_2(1,6), M_3(-4,2)$ nuktalarda bo‘lsa, uning uchlarini aniqlang.
5. Uchlari $A(3,1), B(-1,4), va\ C(1,1)$ nuqtalarda bo‘lgan uchburchak medianalarining kesishish nuqtasini toping.
6. l to‘g’ri chiziqda $|A_1A_2| = |A_2A_3| = |A_3A_4| = |A_4A_5| = |A_5A_6|$ shartni qanoatlantiruvchi $A_1, A_2, A_3, A_4, A_5, A_6$ nuqtalar olingan. Agar $A_2(2,5)$ va $A_5(-1,7)$ bo‘lsa, qolgan nuqtalarning koordinatalarini toping.
7. $\beta = \left\{0, \vec{e}_1, \vec{e}_2, \vec{e}_3\right\}$ sistemasida $A(2,5,4); B(0,1,0); C(4,1,3); D(6,5,7)$ nuqtalar berilgan. $ABCD$ figura parallelogramm ekanini isbot qiling.
8. $\vec{a} = \{-2,1,5\}; \vec{b} = \{0,-2,6\}$ vektorlar berilgan. $\vec{a}+2\vec{b}; 3\vec{a}-4\vec{b}; -7\vec{a}+2\vec{b}$ vektorlarning koordinatalarini toping.
9. $OABC$ tetraedrda $\overrightarrow{OA}, \overrightarrow{OB}, \overrightarrow{OA}, \overrightarrow{OB}, \overrightarrow{OC}$ larni bazis vektorlar deb olib, ABC yoq medianalari kesishgan nuqtaning koordinatalarini toping.
10. $\beta = \left\{O, \vec{e}_1, \vec{e}_2\right\}$ affin reperga nisbatan $A(2,1)$ va $B(-\frac{3}{2},3)$ berilgan. Koordinatalar boshi $O'(0,1)$ nuqtada bo‘lgan shunday $\beta' = \left\{O', \vec{e}_1', \vec{e}_2'\right\}$ affin reperni topingki, unda $A(1,0)\ va\ B(0,1)$ bo‘lsin.
11. $\beta = \left\{O, \vec{e}_1, \vec{e}_2\right\}$ affin reperda A, B nuqtalar mos ravishda $(1,1)$ va $(2,2)$ koordinatalarga ega. A va B nuqtalar $(1,1)$ va $(1,-2)$ koordinatalarga ega bo‘ladigan $\beta' = \left\{O', \vec{e}_1', \vec{e}_2'\right\}$ affin reper mavjudmi?
12. $\vec{e}_1'(1,1), \vec{e}_2'(-3,1), O'(0,1)$ bo‘lsa, $\beta = \left\{O, \vec{e}_1, \vec{e}_2\right\}$ va $\beta' = \left\{O', \vec{e}_1', \vec{e}_2'\right\}$ affin reperlarda bir xil koordinatalarga ega bo‘lgan nuqtani toping.
13. Agar koordinatalarni almashtirish formulalari quyidagicha bo‘lsa, yangi koordinata vektorlarini va yangi koordinatalar boshining eski reperga nisbatan koordinatalarini toping:

a) $\begin{cases} x = x' - y' + 3 \\ y = -3y' - 2 \end{cases}$ b) $\begin{cases} x' = 3x - y \\ y' = 2x + 1 \end{cases}$

c) $\begin{cases} x = y' + 1 \\ y = x' + 2y' + 3 \end{cases}$ d) $\begin{cases} x = x' - 3y' + 4 \\ y = 3x' + \sqrt{2}y' - 1 \end{cases}$

14. Koordinatalarni almashtirish formulasi quyidagicha

$$\begin{cases} x = \frac{1}{2}x' - \frac{\sqrt{3}}{2}y' \\ y = \frac{\sqrt{3}}{2}x' + \frac{1}{2}y' \end{cases}$$

bo‘lsa, koordinata o‘qlari qanday burchakka burilgan?

15. $\beta = \{O, \vec{i}, \vec{j}\}$ dekart reperga nisbatan $A(\sqrt{8}, -\frac{1}{2})$ va $M(x, y)$ nuqtalar berilgan. Koordinata o‘qlari koordinatalar burchagi bissektrisalari bilan almashtirilganda, shu nuqtalarning koordinatalarini toping.

16. $\beta = \{O, \vec{i}, \vec{j}\}$ dekart reperda F figura $xy + 3x - 2y - 6 = 0$ tenglama bilan berilgan. Koordinatalar boshi $O'(2, -3)$ nuqtaga ko‘chirilgandan keyin F figuraning tenglamasi qanday bo‘ladi?

17. $\beta = \{O, \vec{e}_1, \vec{e}_2, \vec{e}_3\}$ sistemaga nisbatan $\vec{e}_1\,'(1, 0, 0)$, $\vec{e}_2\,'(0, 1, 0)$, $\vec{e}_3\,'(0, 0, 1)$, $O'(1, -3, 5)$ lar berilgan β dan $\beta' = \{O', \vec{e}_1\,', \vec{e}_2\,', \vec{e}_3\,'\}$ ga o‘tishdagi koordinatalarni almashtirish formulalarni yozing. β da berilgan $M(1, 1, 3)$ nuqtaning β' dagi koordinatalarini toping.

18. M nuqta $\beta = \{O, \vec{e}_1, \vec{e}_2, \vec{e}_3\}$ $M(0, 1, -3)$ ko‘rinishda, $\beta' = \{O', \vec{e}_1\,', \vec{e}_2\,', \vec{e}_3\,'\}$ da esa $M(2, -3, 5)$ ko‘rinishda berilgan bo‘lsa, koordinatalar boshi ko‘chirilgan O' nuqtaning β dagi koordinatalarini toping.

19. Biror $\beta = \{O, \vec{e}_1, \vec{e}_2, \vec{e}_3\}$ sistemaga nisbatan $\vec{e}_1\,'(1, -3, -1); \vec{e}_2\,'(0, 5, 1); \vec{e}_3\,'(0, 0, 3)$ vektorlar berilgan. $\vec{e}_1\,', \vec{e}_2\,', \vec{e}_3\,'$ lar bazis bo‘la olishini ko‘rsating va $\beta = \{O, \vec{e}_1, \vec{e}_2, \vec{e}_3\}$ dan $\beta' = \{O', \vec{e}_1\,', \vec{e}_2\,', \vec{e}_3\,'\}$ ga o‘tishdagi koordinatalarni almashtirish formulalarini yozing va $M(3, 1, -4)$ ning β' dagi koordinatalarini toping.

20. $\beta = \{O, \vec{e}_1, \vec{e}_2, \vec{e}_3\}$ da $\vec{e}_1\,'(1, 0, 2); \vec{e}_2\,'(1, 0, -2); \vec{e}_3\,'(1, 1, 1)$ vektorlar berilgan. $\vec{e}_1\,', \vec{e}_2\,', \vec{e}_3\,'$ sistema bazis ekanligini ko‘rsating va $\beta' = \{O', \vec{e}_1\,', \vec{e}_2\,', \vec{e}_3\,'\}$ dagi o‘tishdagi koordinatalarni almashtirish formulalarini yozib $M(3, 1, -4)$ ning β' dagi $\vec{e}_1, \vec{e}_2, \vec{e}_3$ larning koordinatalarini toping.

21. $OABC$ tetraedr berilgan, $\overrightarrow{OA} = \vec{e}_1$, $\overrightarrow{OB} = \vec{e}_2$, $\overrightarrow{OC} = \vec{e}_3$ deb olib, $\beta = \{O, \vec{e}_1, \vec{e}_2, \vec{e}_3\}$ affin

sistemasidan $O' = A, \vec{e}_1' = \overrightarrow{AO}, \vec{e}_2' = \overrightarrow{AB}, \vec{e}_3' = \overrightarrow{AC}$ bo‘lgan $\beta' = \{O', \vec{e}_1', \vec{e}_2', \vec{e}_3'\}$ sistemaga o‘tishdagi koordinatalarni almashtirish formulalarini yozing.

22. $\beta = \{O, \vec{e}_1, \vec{e}_2, \vec{e}_3\}$ sistemadan $\beta' = \{O', \vec{e}_1', \vec{e}_2', \vec{e}_3'\}$ sistemaga o‘tishdagi ixtiyoriy nuqtaning bu ikki sistemaga nisbatan koordinatalari orasidagi bog’lanish ushbu $x = x' - 2y' + 3z' - 4$, $y = 5x' - y' - z'$, $z = z' + 1$ formulalar bilan berilgan. O' nuqtaning va $\vec{e}_1', \vec{e}_2', \vec{e}_3'$ vektorlarning β dagi koordinatalarini toping.

23. $\beta = \{O, \vec{e}_1, \vec{e}_2, \vec{e}_3\}$ da $\vec{e}_1'(-1,1,0); \vec{e}_2'(2,-1,0); \vec{e}_3'(0,0,5); O'(5,0,-2)$ lar berilgan. β dan $\beta' = \{O', \vec{e}_1', \vec{e}_2', \vec{e}_3'\}$ ga o‘tishdagi koordinatalarni almashtirish formulalarni yozing va β' da berilgan va $M(1,-3,4)$ ning β dagi koordinatalarini toping.

24. $\beta = \{O, \vec{e}_1, \vec{e}_2, \vec{e}_3\}$ dan $\beta' = \{O', \vec{e}_1', \vec{e}_2', \vec{e}_3'\}$ ga o‘tishda $\vec{e}_i, i = 1,2,3$ lar β da quyidagicha berilgan bo‘lsin: $\vec{e}_1'(4,3,-2); \vec{e}_2'(0,1,5); \vec{e}_3'(-1,0,5)$. $A(-1,0,27)$ va $B(1,0,-1)$ nuqtalarning yangi sistemadagi koordinatalarini toping.

25. $\beta = \{O, \vec{i}, \vec{j}, \vec{k}\}$ ni Oy o‘q atrofida α burchakka soat strelkasiga teskari yo‘nalishda borib, $\beta' = \{O', \vec{i}', \vec{j}', \vec{k}'\}$ sistemaga o‘tilgan. Koordinatalarni almashtirish formulalarini yozing, $\alpha = 45^\circ$ bo‘lganda $M(0,1,-\sqrt{2})$ uchun M ning β dagi koordinatalarini toping.

26. To‘g’ri burchakli dekart koordinatalar sistemasini shunday almashtiringki, unda $O = O', Oz = Oz'$ bo‘lsin va $[Ox'], [Oy']$ nurlar esa $(xOz), (yOz)$ koordinata burchaklarining bissektrisalaridan iborat bo‘lib, yangi bazis sistemasining bazis vektorlari birlik vektorlar bo‘lsin.

27. $\beta = \{O, \vec{i}, \vec{j}, \vec{k}\}$ ni Oz o‘q atrofida soat strelkasiga teskari yo‘nalishda α burchakka burishdan $\beta' = \{O', \vec{i}', \vec{j}', \vec{k}'\}$ sistema hosil bo‘lgan. β dan β' ga o‘tishdagi koordinatalarni almashtirish formulalarini toping.

28. To‘g’ri burchakli $ABCD$ trapetsiya berilgan. Asoslari $AD = 4, BC = 2$ va D burchagi 45° ga teng. $\overrightarrow{CD}$ vektorni bir o‘q deb, $\overrightarrow{AD}, \overrightarrow{AB}, \overrightarrow{BC}, \overrightarrow{AC}$ vektorlarning shu bir o‘qdagi proektsiyalarini toping.

29. $\beta = \{O, \vec{e}_1, \vec{e}_2, \vec{e}_3\}$ sistemadan $\beta' = \{O', \vec{e}_1', \vec{e}_2', \vec{e}_3'\}$ sistemaga o‘tishdagi ikki sistemaga nisbatan koordinatalar o‘rasidagi bog‘lanish ushbu $x = x' + 3y' - 2z' + 1$, $y = -2x' + y' + z'$, $z = z' - 1$ formulalar bilan berilgan. O nuqtaning va $\vec{e}_1, \vec{e}_2, \vec{e}_3$ vektorlarning dagi koordinatalarini toping. β \beta β da berilgan M(2,−1,3) M(2, -1, 3) M(2,−1,3) ning β \beta β dagi koordinatalarini toping.

30. Quyidagi uch A, B, C nuqtalarning bir to‘g‘ri chiziqda yotishini ko‘rsating:

a) $A(-3,2);\ B(-3,0);\ C(-10,-7)$

b) $A(7,-2);\ B(2,5);\ C(-3,-4)$

31. Agar $A(5,-5);\ B(-1,0);\ C(-8,2);\ D(-9,7)$ nuqtalari berilgan bo‘lsa, $ABCD$ parallelogramm ekanligini ko‘rsating.
32. Agar $A(5,-6);\ B(-10,2);\ C(-5,-5);\ D(5,-9)$ nuqtalari berilgan bo‘lsa, $ABCD$ trapetsiya ekanligini ko‘rsating.
33. Quyidagi uch nuqtaning bir to‘g‘ri chiziqda yotishini ko‘rsating:

a) $A(10,-2);\ B(-10,7);\ C(7,-2)$

b) $A(10,8);\ B(6,-2);\ C(-10,1)$

34. Agar $A(10,-1);\ B(9,-3);\ C(1,-10);\ D(-1,-10)$ nuqtalari berilgan bo‘lsa, $ABCD$ parallelogramm ekanligini ko‘rsating.
35. Agar $A(10,5);\ B(6,-5);\ C(7,1);\ D(-10,1)$ nuqtalari berilgan bo‘lsa, $ABCD$ trapetsiya ekanligini ko‘rsating.
36. Quyidagi uch nuqtaniнг bir to‘g‘ri chiziqda yotishini ko‘rsating:

a) $A(-8,-8);\ B(-9,7);\ C(1,7)$

b) $A(-3,-7);\ B(-7,7);\ C(2,-6)$

37. Agar $A(-6,-1);\ B(6,-3);\ C(5,3);\ D(-3,-7)$ nuqtalari berilgan bo‘lsa, $ABCD$ parallelogramm ekanligini ko‘rsating.
38. Agar $A(10,2);\ B(-9,-9);\ C(-10,7);\ D(9,5)$ nuqtalari berilgan bo‘lsa, $ABCD$ trapetsiya ekanligini ko‘rsating.
39. Quyidagi uch nuqtaniнг bir to‘g‘ri chiziqda yotishini ko‘rsating:

a) $A(-6,-8);\ B(0,-10);\ C(4,7)$

b) $A(-9,4);\ B(-1,-2);\ C(-4,-6)$