# *MASALALAR*

## Tekislikda affin koordinatalar sistemasi

1. Affin koordinatalar sistemasiga nisbatan uchlarining $A(3;5), B(-4;6)$ *va* $C(5;3;5)$ koordinatalari berilgan uchburchakni yasang.
2. Tomoni $a=1$ bo‘lgan muntazam oltiburchak uchlarining koordinatalarini toping. Koordinatalar o‘qi qilib uning shunday ikki qo‘shni tomonlarini olingki, koordinatalar boshiga qarama-qarshi yotgan uchining koordinatalari musbat bo‘lsin.
3. Agar to‘rtburchakning uchlari $A(1,-3), B(8,0), C(4,8)$ va $D(-3,5)$ nuqtalarda bo‘lsa, $ABCD$ parallelogramm ekanligini ko‘rsating.
4. Agar to‘rtburchakning uchlari $A(1,1), B(2,3), C(5,0)$ va $D(7,-5)$ nuqtalarda bo‘lsa, $ABCD$ trapetsiya ekanligini isbot qiling.
5. Quyidagi uchta $A, B, C$ nuqtaning bir to‘g’ri chiziqda yotishini ko‘rsating:

   *a)* $A(2,1)$, $B(0,5)$, $C(4,-3)$

   *b)* $A(-1,0)$, $B(1,-2)$, $C(3,-4)$

   12. $A(2,1), B(0,5), C(4,-3)$ nuqtalar berilgan. $(AB,C), (BC,A), (AC,B)$ larni hisoblang.
13. Uchburchakning uchlari berilgan: $A(3,-7), B(5,2), C(-1,0)$. Har bir tomonning o‘rta nuqtasining koordinatalarini toping.
14. Uchburchak tomonlarining o‘rtalari $M_1(3,-2), M_2(1,6), M_3(-4,2)$ nuktalarda bo‘lsa, uning uchlarini aniqlang.
15. Parallelogrammning $A(-3,5)$ *va* $B(1,7)$ qo‘shni uchlari hamda diagonallari kesishgan $M(1,1)$ nuqta berilgan. Uning qolgan ikkita uchining koordinatalarini toping.
16. Uchlari $A(3,1), B(-1,4),$ *va* $C(1,1)$ nuqtalarda bo‘lgan uchburchak medianalarining kesishish nuqtasini toping.
17. $l$ to‘g’ri chiziqda $|A_1A_2|=|A_2A_3|=|A_3A_4|=|A_4A_5|=|A_5A_6|$ shartni qanoatlantiruvchi $A_1, A_2, A_3, A_4, A_5, A_6$ nuqtalar olingan. Agar $A_2(2,5)$ va $A_5(-1,7)$ bo‘lsa, qolgan nuqtalarning koordinatalarini toping.

## Fazoda affin koordinatalar sistemasi.

1. $\beta=\{0,\vec{e}_1,\vec{e}_2,\vec{e}_3\}$ sistemasida $A(2,5,4); B(0,1,0); C(4,1,3); D(6,5,7)$ nuqtalar berilgan. $ABCD$ figura parallelogramm ekanini isbot qiling.
2. $\overrightarrow{AB}=(-3,2,6)$ vektorning boshi $A(-1,0,4)$ nuqtada joylashgan. Uning oxiri bo‘lgan $B$ nuqtaning koordinatalarini toping.
3. Uchlari $A(2,0,-4); B(7,-15,16), C(-1,-1,11); D(-4,8,-1)$ nuqtalarda yotgan

to‘rtburchak trapetsiya ekanligini isbotlang.

4. $M_1(7,9,-8); M_2(-2,3,4); M(-5,1,8)$ nuqtalarning bir to‘g’ri chiziqda yotishini isbotlang.

5. $\vec{a}=\{-2,1,5\}; \vec{b}=\{0,-2,6\}$ vektorlar berilgan. $\vec{a}+2\vec{b}; 3\vec{a}-4\vec{b}; -7\vec{a}+2\vec{b}$ vektorlarning koordinatalarini toping.

6. $M_1(1,-2,5); M_2(4,-2,2)$ nuqtalar berilgan. $[\overline{M_1M_2}]$ kesmani $\lambda=1:2$ nisbatda bo‘luvchi $M(x,y)$ nuqtani toping.

7. $OABC$ tetraedrda $\overrightarrow{OA},\overrightarrow{OB},\overrightarrow{OA},\overrightarrow{OB},\overrightarrow{OC}$ larni bazis vektorlar deb olib, $ABC$ yoq medianalari kesishgan nuqtaning koordinatalarini toping.

## Tekislikda affin va dekart koordinatalar sistemasini almashtirish

4. $\beta=\{O,\vec{e}_1,\vec{e}_2\}$ affin reperga nisbatan $A(2,1)$ va $B(-\frac{3}{2},3)$ berilgan. Koordinatalar boshi $O'(0,1)$ nuqtada bo‘lgan shunday $\beta'=\{O',\vec{e}_1',\vec{e}_2'\}$ affin reperni topingki, unda $A(1,0)\ va\ B(0,1)$ bo‘lsin.

5. $\beta=\{O,\vec{e}_1,\vec{e}_2\}$ affin reperda $A,B$ nuqtalar mos ravishda $(1,1)$ va $(2,2)$ koordinatalarga ega. $A$ va $B$ nuqtalar $(1,1)$ va $(1,-2)$ koordinatalarga ega bo‘ladigan $\beta'=\{O',\vec{e}_1',\vec{e}_2'\}$ affin reper mavjudmi?

6. $\vec{e}_1'(1,1), \vec{e}_2'(-3,1), O'(0,1)$ bo‘lsa, $\beta=\{O,\vec{e}_1,\vec{e}_2\}$ va $\beta'=\{O',\vec{e}_1',\vec{e}_2'\}$ affin reperlarda bir xil koordinatalarga ega bo‘lgan nuqtani toping.

7. Agar koordinatalarni almashtirish formulalari quyidagicha bo‘lsa, yangi koordinata vektorlarini va yangi koordinatalar boshining eski reperga nisbatan koordinatalarini toping:

a) $\begin{cases} x=x'-y'+3 \\ y=-3y'-2 \end{cases}$ b) $\begin{cases} x'=3x-y \\ y'=2x+1 \end{cases}$

c) $\begin{cases} x=y'+1 \\ y=x'+2y'+3 \end{cases}$ e) $\begin{cases} x=x'-3y'+4 \\ y=3x'+\sqrt{2}y'-1 \end{cases}$

d) $\begin{cases} x'=x+2y+1 \\ y'=x+y-6 \end{cases}$

9. Koordinatalarni almashtirish formulasi quyidagicha

$$\begin{cases} x=\frac{1}{2}x'-\frac{\sqrt{3}}{2}y' \\ y=\frac{\sqrt{3}}{2}x'+\frac{1}{2}y' \end{cases}$$

bo‘lsa, koordinata o‘qlari qanday burchakka burilgan?

10. $\beta=\{O,\vec{i},\vec{j}\}$ dekart reperga nisbatan $A(\sqrt{8},-\frac{1}{2})$ va $M(x,y)$ nuqtalar berilgan. Koordinata o‘qlari koordinatalar burchagi bissektrisalari bilan almashtirilganda, shu nuqtalarning koordinatalarini toping.

11. $\beta=\{O,\vec{i},\vec{j}\}$ dekart reperda $F$ figura $xy+3x-2y-6=0$ tenglama bilan berilgan. Koordinatalar boshi $O'(2,-3)$ nuqtaga ko‘chirilgandan keyin $F$ figuraning tenglamasi qanday bo‘ladi?

**Fazoda affin va dekart koordinatalar sistemasini almashtirish**

1. $\beta=\{O,\vec{e}_1,\vec{e}_2,\vec{e}_3\}$ sistemaga nisbatan $\vec{e}_1'(1,0,0)$, $\vec{e}_2'(0,1,0)$, $\vec{e}_3'(0,0,1)$, $O'(1,-3,5)$ lar berilgan $\beta$ dan $\beta'=\{O',\vec{e}_1',\vec{e}_2',\vec{e}_3'\}$ ga o‘tishdagi koordinatalarni almashtirish formulalarni yozing. $\beta$ da berilgan $M(1,1,3)$ nuqtaning $\beta'$ dagi koordinatalarini toping.

2. $M$ nuqta $\beta=\{O,\vec{e}_1,\vec{e}_2,\vec{e}_3\}$ $M(0,1,-3)$ ko‘rinishda, $\beta'=\{O',\vec{e}_1',\vec{e}_2',\vec{e}_3'\}$ da esa $M(2,-3,5)$ ko‘rinishda berilgan bo‘lsa, koordinatalar boshi ko‘chirilgan $O'$ nuqtaning $\beta$ dagi koordinatalarini toping.

3. Biror $\beta=\{O,\vec{e}_1,\vec{e}_2,\vec{e}_3\}$ sistemaga nisbatan $\vec{e}_1'(1,-3,-1);\vec{e}_2'(0,5,1);\vec{e}_3'(0,0,3)$ vektorlar berilgan. $\vec{e}_1',\vec{e}_2',\vec{e}_3'$ lar bazis bo‘la olishini ko‘rsating va $\beta=\{O,\vec{e}_1,\vec{e}_2,\vec{e}_3\}$ dan $\beta'=\{O',\vec{e}_1',\vec{e}_2',\vec{e}_3'\}$ ga o‘tishdagi koordinatalarni almashtirish formulalarini yozing va $M(3,1,-4)$ ning $\beta'$ dagi koordinatalarini toping.

4. $\beta=\{O,\vec{e}_1,\vec{e}_2,\vec{e}_3\}$ da $\vec{e}_1'(1,0,2);\vec{e}_2'(1,0,-2);\vec{e}_3'(1,1,1)$ vektorlar berilgan. $\vec{e}_1',\vec{e}_2',\vec{e}_3'$ sistema bazis ekanligini ko‘rsating va $\beta'=\{O',\vec{e}_1',\vec{e}_2',\vec{e}_3'\}$ dagi o‘tishdagi koordinatalarni almashtirish formulalarini yozib $M(3,1,-4)$ ning $\beta'$ dagi $\vec{e}_1,\vec{e}_2,\vec{e}_3$ larning koordinatalarini toping.

5. $OABC$ tetraedr berilgan, $\overrightarrow{OA}=\vec{e}_1$, $\overrightarrow{OB}=\vec{e}_2$, $\overrightarrow{OC}=\vec{e}_3$ deb olib, $\beta=\{O,\vec{e}_1,\vec{e}_2,\vec{e}_3\}$ affin sistemasidan $O'=A,\vec{e}_1'=\overrightarrow{AO},\vec{e}_2'=\overrightarrow{AB},\vec{e}_3'=\overrightarrow{AC}$ bo‘lgan $\beta'=\{O',\vec{e}_1',\vec{e}_2',\vec{e}_3'\}$ sistemaga o‘tishdagi koordinatalarni almashtirish formulalarini yozing.

6. $\beta=\{O,\vec{e}_1,\vec{e}_2,\vec{e}_3\}$ sistemadan $\beta'=\{O',\vec{e}_1',\vec{e}_2',\vec{e}_3'\}$ sistemaga o‘tishdagi ixtiyoriy nuqtaning bu ikki sistemaga nisbatan koordinatalari orasidagi bog’lanish ushbu $x=x'-2y'+3z'-4$, $y=5x'-y'-z'$, $z=z'+1$ formulalar bilan berilgan. $O'$ nuqtaning va $\vec{e}_1',\vec{e}_2',\vec{e}_3'$ vektorlarning $\beta$ dagi koordinatalarini toping.

7. $\beta=\{O,\vec{e}_1,\vec{e}_2,\vec{e}_3\}$ da $\vec{e}_1'(-1,1,0);\vec{e}_2'(2,-1,0);\vec{e}_3'(0,0,5);O'(5,0,-2)$ lar berilgan. $\beta$ dan

$\beta' = \left\{O', \vec{e}_1', \vec{e}_2', \vec{e}_3'\right\}$ ga o‘tishdagi koordinatalarni almashtirish formulalarni yozing va $\beta'$ da berilgan va $M(1,-3,4)$ ning $\beta$ dagi koordinatalarini toping.

8. $\beta = \left\{O, \vec{e}_1, \vec{e}_2, \vec{e}_3\right\}$ dan $\beta' = \left\{O', \vec{e}_1', \vec{e}_2', \vec{e}_3'\right\}$ ga o‘tishda $\vec{e}_i, i = 1,2,3$ lar $\beta$ da quyidagicha berilgan bo‘lsin: $\vec{e}_1'(4,3,-2); \vec{e}_2'(0,1,5); \vec{e}_3'(-1,0,5)$. $A(-1,0,27)$ va $B(1,0,-1)$ nuqtalarning yangi sistemadagi koordinatalarini toping.

9. $\beta = \left\{O, \vec{i}, \vec{j}, \vec{k}\right\}$ ni $Oy$ o‘q atrofida $\alpha$ burchakka soat strelkasiga teskari yo‘nalishda borib, $\beta' = \left\{O', \vec{i}', \vec{j}', \vec{k}'\right\}$ sistemaga o‘tilgan. Koordinatalarni almashtirish formulalarini yozing, $\alpha = 45^\circ$ bo‘lganda $M(0,1,-\sqrt{2})$ uchun $M$ ning $\beta$ dagi koordinatalarini toping.

10. To‘g’ri burchakli dekart koordinatalar sistemasini shunday almashtiringki, unda $O = O', Oz = Oz'$ bo‘lsin va $\left[Ox'\right), \left[Oy'\right)$ nurlar esa $(xOz), (yOz)$ koordinata burchaklarining bissektrisalaridan iborat bo‘lib, yangi bazis sistemasining bazis vektorlari birlik vektorlar bo‘lsin.

11. $\beta = \left\{O, \vec{i}, \vec{j}, \vec{k}\right\}$ ni $Oz$ o‘q atrofida soat strelkasiga teskari yo‘nalishda $\alpha$ burchakka burishdan $\beta' = \left\{O', \vec{i}', \vec{j}', \vec{k}'\right\}$ sistema hosil bo‘lgan. $\beta$ dan $\beta'$ ga o‘tishdagi koordinatalarni almashtirish formulalarini toping.

12. To‘g’ri burchakli $ABCD$ trapetsiya berilgan. Asoslari $AD = 4, BC = 2$ va $D$ burchagi $45^\circ$ ga teng. $\overrightarrow{CD}$ vektorni bir o‘q deb, $\overrightarrow{AD}, \overrightarrow{AB}, \overrightarrow{BC}, \overrightarrow{AC}$ vektorlarning shu bir o‘qdagi proektsiyalarini toping.

13. $\vec{a}$ vektor $Ox$ va $Oy$ o‘qlari bilan mos ravishda $\alpha = \frac{\pi}{3}, \beta = \frac{2\pi}{3}$ li burchaklar tashkil etadi. Agar $\left|\vec{a}\right| = 2$ bo‘lsa, uning koordinatalarini hisoblang.

14. Kesmaning uchlari $M(3,-2)$ va $N(10,-9)$ nuqtalarda yotadi. $C$ nuqta kesmani $\lambda = \frac{2}{5}$ nisbatda bo‘lsa, shu nuqtaning koordinatalarini toping.

15. $B(-3,4)$ nuqta $AC$ kesmani $\lambda = \frac{2}{3}$ nisbatda bo‘lsa, $A(1,2)$ ni bilgan holda $C(x, y)$ ni koordinatalarini toping.

16. $C(-5,4)$ nuqta $AB$ kesmani $\lambda = \frac{3}{4}$ nisbatda, $D(6,-5)$ nuqta esa $\mu = \frac{2}{3}$ bo‘lsa, $A$ va $B$ nuqtalarning koordinatalari topilsin.