CSU11001 Assignment 4 Minjuan Luo ID: 20313326

Q1 (a) indefinite intergral

(i) $h(y) = y(8-5y^2) \longrightarrow \int y(8-5y^2)\,dy$

set $u = 8-5y^2$

$\frac{du}{dy} = -10y \Rightarrow du = -10y\,dy \Rightarrow dy = \left(\frac{-1}{10y}\right) du$

Hence $\int y u \left(-\frac{1}{10y}\right) du = \int -\frac{1}{10} u\,du = -\frac{1}{10}\int u\,du$

$= -\frac{1}{10} \times \frac{u^{1+1}}{1+1} = -\frac{u^2}{20} = -\frac{(8-5y^2)^2}{20} + C$

(ii) $g(x) = \frac{3+24x^3}{x}$ $(x>0)$ $\longrightarrow \int \frac{3+24x^3}{x}\,dx \rightarrow \int \left(\frac{3}{x} + 24x^2\right) dx$

Hence $\int \left(\frac{3}{x} + 24x^2\right) dx = \int \frac{3}{x}\,dx + \int 24x^2\,dx$

$\int \frac{3}{x}\,dx = 3\int \frac{1}{x}\,dx = 3\ln x \quad (x>0)$

$\int 24x^2\,dx = 8x^3$

result: $3\ln x + 8x^3 + C$

(iii) $f(t) = 12\sin(8t) + 3e^{-10t} \longrightarrow \int 12\sin(8t) + 3e^{-10t}\,dt$

Hence $\int 12\sin(8t) + 3e^{-10t}\,dt = \int 12\sin(8t)\,dt + \int 3e^{-10t}\,dt$

$\int 12\sin(8t)\,dt$: set $8t = u$ $\quad \frac{du}{dt} = 8 \quad dt = \frac{1}{8}du$

Hence $\int 12\sin u \times \frac{1}{8}du = \frac{12}{8}\int \sin u\,du = -\frac{3}{2}\cos u$

$= -\frac{3}{2}\cos(8t)$

$\int 3e^{-10t}\,dt$: set $-10t = u$ $\quad \frac{du}{dt} = -10 \quad dt = -\frac{1}{10}du$

Hence $\int 3e^u \cdot -\frac{1}{10}du = -\frac{3}{10}\int e^u\,du = -\frac{3}{10}e^u = -\frac{3}{10}e^{-10t}$

result: $-\frac{3}{2}\cos(8t) - \frac{3}{10}e^{-10t} + C$

(b) (i) $\int \frac{1}{2\sqrt{\theta}}\sin(\sqrt{\theta})\,d\theta$ set $u = \sqrt{\theta}$ $\quad \frac{du}{d\theta} = \frac{1}{2} \times \frac{1}{\sqrt{\theta}}$

$d\theta = 2\sqrt{\theta}\,du$ Hence : $\int \frac{1}{2u}\sin u\, 2\sqrt{\theta}\,du = \int \sin u \times \frac{1}{u} \times u\,du = \int \sin u\,du$

$= -\cos u$ put $u=\sqrt{\theta}$ back into the equation

Hence : $-\cos\sqrt{\theta}+C$

(ii) $\int \frac{2x}{\sqrt{1-x^4}}\,dx = 2\int \frac{x}{\sqrt{1-x^4}}\,dx$ set $u=x^2$

then $\frac{du}{dx}=2x$ $dx=\frac{1}{2x}\,du$

Hence $2\int \frac{x}{\sqrt{1-x^4}}\cdot\frac{1}{2x}\,du$

then calculate :

$$\frac{x}{\sqrt{1-x^4}}\times\frac{1}{2x} \rightarrow \frac{1}{2\sqrt{1-x^4}}$$

so $2\int\frac{1}{2\sqrt{1-x^4}}\,du = \int\frac{1}{\sqrt{1-u^2}}\,du$

$= \arcsin(u)$ and $u=x^2$

Hence : $\arcsin(x^2)+C$

Q2:

(a) $\frac{x}{(x-1)^2}+\frac{2}{(x-1)^2}-\frac{2}{3x-2} = \frac{x+2}{(x-1)^2}+\frac{-2}{3x-2}$

$= \frac{(x+2)(3x-2)-2(x-1)^2}{(x-1)^2(3x-2)} = \frac{3x^2+4x-4-2x^2+4x-2}{(x-1)^2(3x-2)} = \frac{x^2+8x-6}{(x-1)^2(3x-2)}$

(b) as $\frac{x^2+8x-6}{(x-1)^2(3x-2)} = \frac{x}{(x-1)^2}+\frac{2}{(x-1)^2}-\frac{2}{3x-2}$

Hence $\int\frac{x^2+8x-6}{(x-1)^2(3x-2)}\,dx = \int\frac{x}{(x-1)^2}+\frac{2}{(x-1)^2}-\frac{2}{(3x-2)}\,dx$

$= \int\frac{x}{(x-1)^2}\,dx+\int\frac{2}{(x-1)^2}\,dx-\frac{2}{(3x-2)}\,dx$

part 1:

$\int\frac{x}{(x-1)^2}\,dx \rightarrow$ set $x-1=u$ $\frac{du}{dx}=1$ $u+1=x$

Hence $\int\frac{u+1}{u^2}\,du = \int\frac{1}{u}\,du+\int\frac{1}{u^2}\,du$

$= \ln|u|-\frac{1}{u} \rightarrow \boxed{\ln|x-1|-\frac{1}{x-1}}$

part2:

$\int \frac{2}{(x-1)^2}\,dx = 2\int \frac{1}{(x-1)^2}\,dx$ set $x-1=u$ $\frac{du}{dx}=1$ $x=u+1$

Hence $2\int \frac{1}{u^2}\,du = 2\times \frac{u^{-1}}{-1} = \frac{-2}{u}$ $u=x-1$

so $\boxed{\frac{-2}{x-1}}$

part3:

$\int \frac{2}{3x-2}\,dx = 2\int \frac{1}{3x-2}\,dx$ set $3x-2=u$ $\frac{du}{dx}=3$ $x=\frac{u+2}{3}$

then: $2\int \frac{1}{3u}\,du = 2\times\frac{1}{3}\int \frac{1}{u}\,du = \frac{2}{3}\ln|u| = \boxed{\frac{2}{3}\ln|3x-2|}$

part1 + part2 + part3 : $\ln|x-1| - \frac{1}{x-1} - \frac{2}{x-1} - \frac{2}{3}\ln|3x-2|$

$= \ln|x-1| - \frac{3}{x-1} - \frac{2}{3}\ln|3x-2|$

result : $\ln|x-1| - \frac{3}{x-1} - \frac{2}{3}\ln|3x-2| + C$

Q3: $\int_0^1 xe^{4x}\,dx$

According to the 'integration by parts' formula : $\int u\,dv = uv - \int v\,du$

choose $u=x$ $dv=e^{4x}$

$\int_0^1 xe^{4x}\,dx = \int_0^1 u\,dv$ gives L.H.S of (IBP)

find terms of R.H.S of (IBP) we need : du and v

$u=x$ so $\frac{du}{dx}=1$ hence $du=dx$

$dv=e^{4x}\,dx$ so $v=\int_0^1 e^{4x}\,dx = \frac{1}{4}e^{4x}$

so: $\int_0^1 x\,e^{4x}\,dx = \int_0^1 u\,dv = uv - \int_0^1 v\,du$

$= x\cdot\frac{1}{4}e^{4x} - \int_0^1 \frac{1}{4}e^{4x}\,dx$

$= \frac{1}{4}xe^{4x} - \frac{1}{16}e^{4x} + C$

$x=1$ $x=1$ subtract $x=0$

$= \frac{1}{4}\times1\times e^4 - \left(\frac{1}{16}e^{4\times1} - \frac{1}{16}e^{4\times0}\right)$

$= \frac{4e^4}{16} - \frac{e^4}{16} + \frac{1}{16} = \frac{3e^4+1}{16}$