1. What is Probability

The study of probability is the study of randomness
How they could use mathematics to describe random events
probability is a number between 0 and 1
probability of 1: event is certain to happen
probability of 0: event is possible, definitely will not happen
The higher the number the more likely it is to happen
Can be calculated mathematically or estimated by carrying out an experiment

2. Prosecutors Fallacy

- A murderer leaves blood at the scene of crime
- This blood type occurs in 1 out of 5000 people
- 100,000 people in the area at the time of crime
- person found with matching type
- Evidence is given at trial, that says the probability that an innocent person has the matching blood is 0.001
- Prosecutor states that therefore, the probability that the suspect is innocent is 0.001

- Let Event A be innocent and Event B be matching blood type

- P(A|B) Probability of being innocent given that the blood type matches
  - Sample space for B = 20 people
  - Only one murderer, so $P(A|B) = 1 - \frac{1}{20} = 0.95$

- So what was the value 0.001 discussed at trial
  - P(B|A) Probability of matching blood type given that the person is innocent

3. Bayes Theorem

- A firm buys computer chips from two companies X and Z
- They buy 400 chips from company X and 200 from company Z
- The rate of defects is 10% (Company X) and 20% (Company Z)
- We sample a chip at random
- The probability it comes from Company Z is: $P(Z) = 200/600 = 0.33$

4. What is probability distribution

A probability distribution is a mathematical function that describes the possible outcomes of a random variable along with their corresponding probability values

5. Continuous vs Discrete

- Discrete distribution: the random variable can assume one of a countable number of values
- Continuous distribution: the random variable can assume one of an infinite number of values

6. Continuous distribution

the p.d.f is the derivative of the c.d.f
the c.d.f is the integral of the p.d.f

pdf: probability density function
cdf: cumulative density function

$\because P(X=k) = 0$
$\therefore P(X \leq k) = P(X < k)$

7. Discrete Uniform Distribution

- Describes an random value which can take one of a countable number of values, each outcome equally

likely
- Two parameters a and b which refer to the lowest possible value and the highest possible value
- n=b-a+1 is the number of distinct possible values that X can take

8. Bernoulli Distribution
- Describes an random variable which is the result of a single experiment and can take values "success = 1" or "failure = 0".
- Single parameter 'p' which refers to the probability of success
- q=1-p is the probability of failure

9. Poisson Distribution
- Describes an random value which is the number of events that take place per unit time
  - Assumptions: Events have a constant rate. Events occur independently of the time since the last event
- One parameter: 'λ' which refers to the rate of events

10. Poisson Approximation to Binomial
- Binomial (n, p) ≈ Poisson(λ) (where λ = np)
- Appropriate where:
  - n ≥ 30 / n ≥ 100
  - P ≤ 0.05 / P ≤ 0.01
  - np ≤ 20 / np ≤ 5

11. Exponent Distribution
- How to find probabilities for time (T) between events of random variable X ~ poisson
- If λ = event rate, mean time between events (decay parameter) = 1/λ

12. Normal Distribution
- Lots of variables have this shape
- Appears in statistical theory quite a lot
- Symmetrical (左右[illegible])
- "Bell Shaped"
- Mean = Median = Mode
- Location is determined by the mean, μ
- Spread is determined by the standard deviation, σ
- The variable has an infinite theoretical range: +∞ to -∞

13. Why do we care about Z scores
- We have tables which tell us the probabilities for values under the standard normal distribution
- Differing formats, but in general, Column and Row Headings give probability for values < x

14. The t-distribution
- Unlike the normal distribution, it has a different shape for different sample size
- An important parameter is the Degrees of Freedom — sample size - 1

15. What is a sample
- A subset of a larger population
- Probability sampling: Every element has a known non-zero chance of being selected
- Non-probability sampling: Choice of selection of sampling units depends entirely on the discretion or judgment of the sampler