

ST2004 Applied Probability I

Group assignment

See instructions at the end of this document.

Question 1 (14 marks)

At the Stratosphere Casino in Las Vegas, roulette wheels have 18 slots coloured red, 18 slots coloured black, and 1 slot (numbered 0) coloured green. The red and black slots are also numbered from 1 to 36.

You can play various ‘games’ or ‘systems’ in roulette. Four possible games are:

- g1) *Betting on Red*. This game involves just one bet. You bet 1\$ on red. If the ball lands on red you win 1\$ (that is you get back 2\$: the one you bet and the one you win), otherwise you lose.
- g2) *Betting on a Number*. This game involves only one bet. You bet 1\$ on a particular number, say 23; if the ball lands on that number you win 35\$, otherwise you lose.
- g3) *Martingale System*. This game involves more than one bet. In this game you start by betting 1\$ on red. If you lose, you double your previous bet; if you win, you bet 1\$ again. You continue to play until you have won 10\$, or the bet exceeds 100\$.
- g4) *Labouchere System*. This game involves more than one bet. In this game you start with the list of numbers (1, 2, 3, 4). You bet the sum of the first and last numbers on red (initially, thus, 5\$). If you win you delete the first and the last numbers from the list (so if you win your first bet, the list becomes (2, 3)), otherwise you add the sum to the end of your list (so if you lose your first bet, the list becomes (1, 2, 3, 4, 5)). You repeat this process until your list is empty, or the bet exceeds 100\$. If only one number is left on the list, you bet that number.

Different games offer different playing experiences; for example, some allow you to win more often than you lose, some let you play longer, some cost more to play, and some risk greater losses. The aim of this assignment is to compare the four games above by using the following criteria:

- c1. The expected winnings per game;
- c2. The proportion of games you win (note that a game is *won* if you make money and *lost* if you lose money);
- c3. The expected playing time per game, measured by the number of bets made;
- c4. The maximum amount you can lose;
- c5. The maximum amount you can win.

Tasks

- a) For each game run a study that estimates c_1 , c_2 , c_3 , c_4 and c_5 by simulating 10 000 repetitions of the game. Compare the values you obtain.
- b) For games g_1 and g_2 , check your estimates for c_1 and c_2 by calculating the exact answers. What is the percentage error in your estimates for 10 000 repetitions?
- c) As for c_1 and c_2 , plot running average together running variance for the five games. Based on visual investigation of such plots, do you think the estimates you get are reliable? If not, try to increase the number of repetitions of your study.
- d) Modify the study you ran for point a) so that, in addition to estimating the expected winning, expected proportion of wins, and expected playing time, it also estimates the variance of each of these values. For which game is the amount won most variable? For which game is the expected playing time most variable?

Question 2 (6 marks)

A deck of 100 cards - numbered $1, 2, \dots, 100$ - is shuffled and then turned over one card at a time. We say that a “hit” occurs whenever card i is the i th card to be turned over, $i = 1, \dots, 100$. Simulate 10 000 repetitions of the game to estimate the expectation and variance of the total number of hits.

Instructions

Submission

Submission date: **Friday 3rd of December 2021**. Late submissions will not be accepted. You can submit your project by uploading to Blackboard.

Assessment

The project will carry 20% of the course marks. 80% of the marks will be awarded for doing a competent job; an extra 20% will be awarded for flair, imagination, thoroughness (good projects will go well beyond the lectures).

Project and report

To complete this project you can use Excel or any software or resource that you think might help. The report should comprise no more than 10 pages¹ (7 pages for question 1, 3 pages for question 2), including tables, plots and diagrams, a good use of which is encouraged and will be rewarded, as long as they provide useful information. Details on the computations should be provided concisely. The report should be prepared **professionally** using any appropriate software (e.g. Word or Latex), with tables and diagrams being imported from Excel or equivalent.

Teamwork

A group project is proposed mainly because group learning can be valuable, but also because team-working is a useful skill. **All members of each team must make significant contribution** to the group project. Clearly the project work does naturally break into sections which can be conducted in parallel before being brought together. The team will receive a mark and all members will receive by default the same mark. In case particular situations arise I might decide to allocate the group mark to the individual team members in a different way.

Plagiarism and sources

While learning from each other and from other groups is encouraged, plagiarism is remarkably easy to spot, especially with online resources. This includes projects that were submitted for the same course in the last years. The same work submitted by two teams will receive a single mark which will be then divided between the teams involved, following discussion. Sources must be cited.

¹this page limit is only indicative but reports that are too long will be penalised