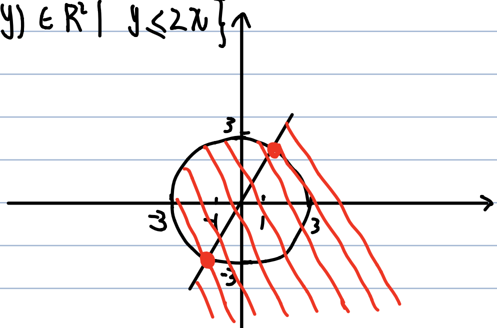
Minjuan Luo   mathematics 2   20313326

Q1.

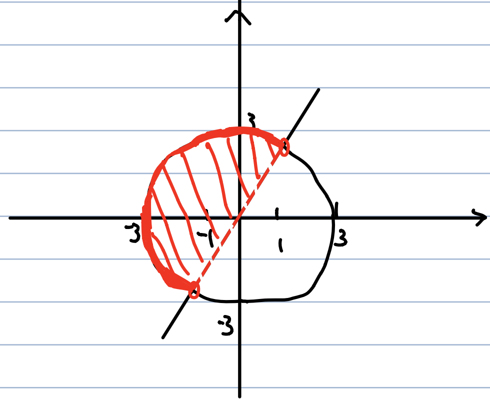
(a) $L=\{(x,y)\in R^2 \mid x^2+y^2\leq 9\}$

$M=\{(x,y)\in R^2 \mid y\leq 2x\}$

(i) $L\cup M$



(ii) $L\setminus M$



(b) $A=\{(t^2,3t^2) \mid t\in R\}$

$B=\{(x,y)\in R^2 \mid y=3x^2, x\geq 0\}$

If $A=B$, then $A\subseteq B$ and $B\subseteq A$

first prove $A\subseteq B$:

Let (x,y) be an arbitrary element of A

then $(x,y)\in R^2$, and for some $t\in R$

we have $x=t^2$, $y=3t^4$

Hence $y=3x^2 \Rightarrow y-3x^2=0$

and $3t^4-3t^4=0$

So $(x,y)\in B$, and $A\subseteq B$

then prove $B\subseteq A$:

Let $(x,y)\in B$, then $y=3x^2$

we must show there is $t\in R$ such that $x=t^2$ and $y=3t^4$, so $(x,y)\in A$

Let $y=3t^4$, then $t^4=\frac{y}{3}$

Then since $y=3x^2$

$x^2=\frac{y}{3}=t^4=(t^2)^2$

So $(x,y)=(t^2,3t^4)\in A$, so $B\subseteq A$

So $A=B$

Q2:

set $A \begin{array}{|c|c|}\hline 0 & 0 \\ \hline 1 & 1 \\ \hline\end{array}$ $B \begin{array}{|c|c|}\hline 0 & 1 \\ \hline 0 & 1 \\ \hline\end{array}$ $\bar{A} \begin{array}{|c|c|}\hline 1 & 1 \\ \hline 0 & 0 \\ \hline\end{array}$ $\bar{B} \begin{array}{|c|c|}\hline 1 & 0 \\ \hline 1 & 0 \\ \hline\end{array}$

(a) $A \ll B = A\cup\bar{B}$

that is $A\cup\bar{B}=\overline{\bar{A}\cap B}$

$A\ll B=A\cup\bar{B}=$ $\begin{array}{c|c|c|}\multicolumn{1}{c}{} & \multicolumn{2}{c}{B} \\ \cline{2-3} & 1 & 0 \\ \cline{2-3} A & 1 & 1 \\ \cline{2-3}\end{array}$ $\bar{A}\cap B=$ $\begin{array}{c|c|c|}\multicolumn{1}{c}{} & \multicolumn{2}{c}{B} \\ \cline{2-3} & 0 & 1 \\ \cline{2-3} A & 0 & 0 \\ \cline{2-3}\end{array}$ $\overline{\bar{A}\cap B}=$ $\begin{array}{c|c|c|}\multicolumn{1}{c}{} & \multicolumn{2}{c}{B} \\ \cline{2-3} & 1 & 0 \\ \cline{2-3} A & 1 & 1 \\ \cline{2-3}\end{array}$

(b) $A \ll (B \cap C) = A \cup \overline{(B \cap C)}$

$(A \ll B) \ll C = (A \cup \bar{B}) \ll C = (A \cup \bar{B}) \cup \bar{C}$

set A

0	0	0	0
1	1	1	1

B

0	0	1	1
0	0	1	1

C

0	1	1	0
0	1	1	0

$A \ll (B \cap C)$:

$\overline{B \cap C} =$

1	1	0	1
1	1	0	1

$A \cup (\overline{B \cap C}) =$

1	1	0	1
1	1	1	1

$(A \ll B) \ll C$:

$A \cup \bar{B} =$

1	1	0	0
1	1	1	1

$(A \cup \bar{B}) \cup \bar{C} =$

1	1	0	1
1	1	1	1

(c) $A \ll (B \cup C) = A \cup (\overline{B \cup C})$

$(A \ll B) \cap (A \ll C) = (A \cup \bar{B}) \cap (A \cup \bar{C})$

$A \ll (B \cup C)$:

$\overline{B \cup C} =$

1	0	0	0
1	0	0	0

$A \cup (\overline{B \cup C}) =$

1	0	0	0
1	1	1	1

$(A \ll B) \cap (A \ll C)$

$A \cup \bar{B} =$

1	1	0	0
1	1	1	1

$A \cup \bar{C} =$

1	0	0	1
1	1	1	1

$(A \cup \bar{B}) \cap (A \cup \bar{C}) =$

1	0	0	0
1	1	1	1

Q3: Let 1: "study" and 0: "not study"

1 studies all three subjects, set $111 \rightarrow 1$

3 studies programing and logic, set $110 \rightarrow 3-1=2$

4 studies programing and mathematics, set $101 \rightarrow 4-1=3$

5 studies logic and mathematics, set $011 \rightarrow 5-1=4$

7 study programing, set $100 \rightarrow 7-2-3-1=1$

10 study logic, set $010 \rightarrow 10-2-4-1=3$

10 study mathematics, set $001 \rightarrow 10-3-4-1=2$

(a) Hence study none of the 3 objects are

$18-(1+2+3+4+1+3+2)=2$ people

(b) Hence study only logic are 3 people

	00	01	11	10
0	2	2	4	3
1	1	3	1	2

Q4 :

(a) Find the least residue of 59^7 modulo 8

Now $59^2 = 3481 \equiv 1 \pmod 8$

So we get

$$
\begin{aligned}
59^7 &= (59^2)^3 \times 59 \\
&= (3481)^3 \times 59 \\
&\equiv 1^3 \times 59 \\
&\equiv 3 \pmod 8
\end{aligned}
$$

So the least residue is 3

(b) Find the least residue of 22^{57} modulo 19

since 19 is a prime number and 22 is not a multiple of 11

Fermat's Little Theorem tells us that :

$(22)^{19-1} \equiv 1 \pmod{19}$ that is $(22)^{18} \equiv 1 \pmod{19}$

$$
\begin{aligned}
(22)^{57} &= (22)^{54+3} \\
&= (22)^{54} \times (22)^3 \\
&= (22^{18})^3 \times (22)^3
\end{aligned}
$$

Using $(22)^{18} \equiv 1 \pmod{19}$ we get :

$$(22)^{57} \equiv 1^3 \times 22^3 \pmod{19}$$

Now $22 \equiv 3 \pmod{19}$

So we get :

$$(22)^{57} \equiv 3^3 \pmod{19}$$

$$(22)^{57} \equiv 8 \pmod{19}$$

so the least residue is 8

Q5.

(a) $17x \equiv 36 \pmod{40}$

Enclid's Algorithm :

$40 = 2 \times 17 + 6$

$17 = 1 \times 6 + 11$

$6 = 2 \times 3 + 0$

Bézout identity

$11 = 17 - 1 \times 6$

$11 = 17 - 1 \times (40 - 2 \times 17)$

$11 = 3 \times 17 - 40 \times 1$

$1 = 3 \times 17 - 10 - 40 \times 1$

Note that $40 \times 1 \equiv 0$ (modulo 40)

so we are left with $3 \times 17 - 10 \equiv 1 \pmod{40}$

Hence 3 is the multiplicative inverse of 17 modulo 40

$3\times 17x - 10x \equiv 36\times 3 - 10 \pmod{40}$
$41x = 98 \pmod{40}$
$1x = 18 \pmod{40}$

The solution set is the congruence class of $18 \pmod{40}$
$\{ \quad \ldots \quad , \; -22, \; 18, \; 58 \quad , \; \ldots \}$

Check: $18 \times 17 + 10 = 316$, $316 \equiv 36 \pmod{40}$

(b) $30x \equiv 16 \pmod{40}$

Encluid's Algorithm:
$40 = 1 \times 30 + 10$
$30 = 3 \times 10 + 0 \longrightarrow$ we don't have a 1 in this place

Bézout identity:

we can't use Bézout identity because
we don't have a 1 on the LHS
to form a congruence
the process will then led to

$0 = 30 - 3\times 10$
$0 = 30 - 3 \times (40 - 30)$
$0 = 30 - 3\times 10$
this is an infinite loop.

(c) $35x \equiv 20 \pmod{40} \longrightarrow$ The HCF of 35 and 40 is 5 and 20 is divisible by 5
$7x \equiv 4 \pmod{8}$ Try out values

$x=1$: $7\times 1 \equiv 7 \pmod{8}$
$x=2$: $7\times 2 \equiv 6 \pmod{8}$
$x=3$: $7\times 3 \equiv 5 \pmod{8}$
$x=4$: $7\times 4 \equiv 4 \pmod{8}$

So $x \equiv 4 \pmod{8}$ Note there are an infinite number of solutions
These are all congruent to 6 modulo 11 ie. they are contained in the set
$\{ \cdots , -12 , -4 , 4, 12, 20, \cdots \}$

(b) another prove
$30x \equiv 16 \pmod{40} \longrightarrow 15x \equiv 8 \pmod{20}$

$x=1$: $15\times 1 \equiv 15 \pmod{20}$ $x=6$: $15\times 6 \equiv 10 \pmod{20}$
$x=2$: $15\times 2 \equiv 10 \pmod{20}$ $x=7$: $15\times 7 \equiv 5 \pmod{20}$

$x=3:\quad 15\times 3 \equiv 5 \pmod{20}$ $\qquad$ $x=8:\quad 15\times 8 \equiv 0 \pmod{20}$

$x=4:\quad 15\times 4 \equiv 0 \pmod{20}$ $\qquad$ $x=9:\quad 15\times 9 \equiv 15 \pmod{20}$

$x=5:\quad 15\times 5 \equiv 15 \pmod{20}$ $\qquad$ $x=10:\quad 15\times 10 \equiv 10 \pmod{20}$

we can draw a conclusion that there is an infinite loop in every 4 x and getting the result : 15, 10, 5, 0
which means we will never get to 16
Hence this linear congruence has no solutions