Statistical Assignment 1 Minjuan Luo

Question 1:

using the binomial distribution

$X \sim \text{Binomial}(4, \frac{13}{1471})$

$P(X=4) = (\frac{13}{1471})^4 \times (1-\frac{13}{1471})^{4-4} \times \binom{4}{4}$ formula: $P(X=x) = p^x \binom{n}{x} (1-p)^{n-x}$ $(0 \le x \in n)$

$= (0.1108)^4 = 0.0002$

Question 2:

using the bernoulli distribution

$X \sim \text{Bernoulli}(\frac{7}{1253})$

$pmf = P(X=1) = \frac{7}{1253} = 0.00558 \simeq 0.0056$

Question 3:

using the bernouli distribution

there are 3 situation:

situation 1: $X \sim \text{Bernoulli}(0.85) = 0.85$

situation 2: $(1-X \sim \text{Bernoulli}(0.85)) \times (X \sim \text{Bernoulli}(0.85)) = 0.15 \times 0.85 = 0.1275$

situation 3: $(1-X \sim \text{Bernoulli}(0.85)) \times (1-X \sim \text{Bernoulli}(0.85)) \times X \sim \text{Bernoulli}(0.85) = 0.15 \times 0.15 \times 0.85 = 0.0191$

total $= 0.85 + 0.1275 + 0.0191 = 0.9966$ Hence $E(\text{total}) = 0.9966 \simeq 0.997$

Question 4: using the bernoulli distribution

set $E(x)$ as missing the hoop

when $x=0$: $E(x) = (1-0.17)^5 = (0.83)^5 = 0.3939 \simeq 0.394$

when $x=1$: $E(x) = C_4^5 = 0.17 \times (1-0.17)^4 = 0.0806 \simeq 0.081$

$E(x=0) + E(x=1) = 0.4746 \simeq 0.475$

Question 5:

total member $= 4+1-1 = 4$

$X \sim \text{Bernoulli}(\frac{1}{4}) = 0.25 \simeq 0.250$

Question 6: using uniform distribution

total max $= 20$ pdf total $= 19$

total min $= 1$

current max $= 15$

current min $= 5$ pdf current $= 10$

$P = \frac{10}{19} = 1.9 = 0.5263 \simeq 0.526$

Question 7:

$P(\text{red duck}) = \frac{25}{100} = \frac{1}{4}$

$P(\text{NO.20}) = \frac{4}{100} - \frac{1}{100} = \frac{3}{100}$

$P(\text{red duck}) + P(\text{NO.20}) = 0.25 + 0.03 = 0.280$

Question 8: using the poisson distribution

formula: $P(X=x) = \frac{e^{-\mu}\mu^{x}}{x!}$

$P(X=8) = \frac{11^{8} \times e^{-11}}{8!}$ (9 minutes = 1 unit) $= 0.088794 \approx 0.089$

Question 9: using the poisson distribution

formula $P(X=x) = \frac{e^{-\mu}\mu^{x}}{x!}$

$P(X=10) = \frac{9^{10} \times e^{-9}}{10!} \times 2 = 0.237$

Question 10: using the poisson distribution

$P_1(X=5) = \frac{5^{5} \times e^{-5}}{5!} = 0.1725 \approx 0.173$

$P_2(X=5) = \frac{5^{5} \times e^{-5}}{5!} = 0.1725 \approx 0.173$

$P_1(X=5) + P_2(X=5) = 0.346$

Question 11: using the exponent distribution

$Pdf = \lambda e^{\lambda(-x)}$ $\lambda = 10 \div 5 = 2$

$P(X<3) = 1 - 2x e^{2x(-3)} = 1 - 2x \frac{1}{e^{6}} = 0.995$

Question 12: using the exponent distribution

$cdf = 1 - e^{\lambda(-x)}$ $\lambda = 1$ $x = \frac{5}{6} = 0.83$

$P(X>5) = 1 - (1 - e^{\lambda(-x)}) = e^{\lambda(-x)} = \frac{1}{e^{0.83}} = 0.436$

Question 13: using the exponent distribution

$cdf = 1 - e^{\lambda(-x)}$ $\lambda = 15$ $x = \frac{3}{15} = 0.2$

$P(X>3) = e^{\lambda(-x)} = e^{-0.2 \times 15} = 0.472$

Question 14:

Event A: is a staff $P(A) = \frac{245}{1026} = 0.239$

Event A': is a ticket holder $P(B) = 1 - 0.239 = 0.761$

Event B: allow to enter VIP $P(B|A) = 0.47$

Event B': not allow $P(B|A') = 0.06$

Bayes theorem: $P(A|B) = 0.47 \times 0.239 / (0.47 \times 0.239 + 0.06 \times 0.761)$

$= 0.11233 / 0.11233 + 0.4566$

$= 0.11233 / 0.56893 = 0.1974 \approx 0.198$

Question 15: using Z score

$z = \frac{x-\mu}{\sigma} = \frac{5.1-4.4}{2.9} = 0.241$

Question 16: using Z score

$z = \frac{x-\mu}{\sigma}$ $x = z\sigma + \mu = 0.29 \times 1.6 + 5.1 = 5.564 \approx 5.56$

Question 17: using Z score

$Z = \frac{x-\mu}{\sigma}$ $x = z\sigma + \mu = 0.1 \times 1.2 + 4.1 = 4.22$

Question 18:

$x \sim N(31, 1.7)$

$x \leq 31.4$

$Z = (x-\mu)/\sigma = (31.4-31)/1.7 = 0.24$

$P(X \leq 31.4) = 1-0.24$

$X \geq 33.1$

$P(X \geq 33.1) = 1 - P(X \leq 33.1)$