CSU 12002 Homework I

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Q1.

(a). ①. $P(x)$ = " x likes mathematics." (where x is every student taking this module)

②. $P(x)$ = " x takes this module " (where x is student)

$Q(x)$ = " x likes mathematics "

$\forall x \in P(x)$ that $Q(x)$ is true

(All student taking this module likes mathematics.)

Domain of Discourse of x : set of students

③. Let $P(x)$ be " x takes this module " (where x is student)

Let $Q(x)$ be " x likes mathematics "

$$\forall x (P(x) \rightarrow Q(x))$$

④. $P(x)$ = " x takes this module " (where x is student)

$Q(x)$ = " x likes mathematics "

$P(x) < Q(x)$

$\forall x \in P(x)$, $P(x) + Q(x)$

(+ All student taking this module like mathematics)

⑤. $P(x)$ = "There exists x that doesn't like mathematics"
(where x is student taking this module)
There exists student ~~that~~ taking this module that doesn't like mathematics.

⑥. $P(x)$ = "x takes this module." (where x is student)

$Q(x)$ = "x doesn't likes mathematics"

~~$\exists x \in P(x), Q(x)$~~ $\exists x \in P(x)$ that $Q(x)$ is true

There exists student taking this module that doesn't like mathematics.

Domain of discourse of x : set of student

⑦. Let $P(x)$ be "x takes this module" (where x is student)

Let $Q(x)$ be "x doesn't like mathematics"

$\exists x\, (P(x)) \to Q(x)$

There exists student taking this module that doesn't like mathematics.

⑧. $P(x)$ = "x takes ~~the~~ this module" (where x is student)

$Q(x)$ = "x doesn't like mathematics"

~~$P(x) \wedge Q(x)$~~ $Q(x) \wedge P(x)$

$\exists x \in P(x)$, $P(x) + Q(x)$

(+ There exists student taking this module that doesn't like mathematics).

(b). ①. $P(x)$ = " x ~~ta~~ has taken exactly two mathematics modules in Trinity".
(where x is every student taking this module)

②. $P(x)$ = " X takes this module" (where x is student)

$Q(x)$ = " x has taken ~~taken~~ exactly two mathematics modules in Trinity"

~~$\forall x \in P(x), Q(x)$~~ $\forall x \in P(x)$ that $Q(x)$ is true

(All student taking this module has taken exactly two mathematics modules in Trinity.)

③. Let $P(x)$ be " X takes this module" (where X is Student)

let $Q(x)$ be " X has taken exactly two mathematics modules in Trinity"

$$\forall x (P(x) \rightarrow Q(x))$$

Domain of discourse of X : set of student

④. let $P(x)$ be " X takes this module"

let $Q(x)$ be " x has taken exactly two mathematics modules in Trinity"

$P(x) \angle Q(x)$

$\forall x \in P(x), \ P(x) + Q(x)$

(+ All student taking this module has taken exactly two mathematics modules in Trinity)

⑤. $P(x)$ = "There exists x that has not taken exactly two mathematics modules in Trinity" (where X is student taking this module)

There exists student taking this module that has not taken exactly two mathematics modules in Trinity.

6. $P(x)$ = "x takes this module" (where x is student)

$Q(x)$ = " x has not taken exactly two mathematics modules in Trinity"

$\cancel{\exists x \in P(x), Q(x)}$ $\exists x \in P(x)$ that $Q(x)$ is true

There exists student taking this module that has not taken exactly two mathematics modules in Trinity.

Domain of discourse of x : set of students

7. $P(x)$ = "x takes this module" (where x is student)

$Q(x)$ = " x has not taken exactly two mathematics modules in Trinity"

$\cancel{\exists x}$ $\exists x (P(x)) \rightarrow Q(x)$

There exists student taking this module that has not taken exactly two mathematics modules in Trinity.

8. $P(x)$ = " x takes this module" (where x is student)

$Q(x)$ = " x has not taken exactly two mathematics modules in Trinity"

$\cancel{P(x) < Q(x))}$

$Q(x) < P(x)$

$\exists x \in P(x), \; P(x) + Q(x)$

(+ There exists student taking this module that has not taken exactly two mathematics modules in Trinity)

(c). ①. $P(x)$ = "~~There is a~~ x has never been on campus"
(where x is a student taking this module)

②. $P(x)$ = "x takes this module" (where x is student)

$Q(x)$ = "x has never been on compus"

~~$\exists x \in P(x), Q(x)$~~ $\exists x \in P(x)$ that $Q(x)$ is true

(There exists a student taking this module who has never been on compus)

Domain of discourse of x : set of student

③ . let $P(x)$ be "x takes this module" (where x is student)

let $Q(x)$ be "x has never been on compus"

$$\exists x \, (P(x)) \rightarrow Q(x)$$

④. $P(x)$ = "x takes this module" (where x is student)

$Q(x)$ = "x has never been on compus"

$$P(x) > Q(x)$$

$$\exists x \in P(x), \; P(x) + Q(x)$$

(+ There exists a student taking this module who has never been on compus)

⑤. $P(x)$ = "Every x has been on compus" (where x is student taking this module)

Every student taking this module has been on compus.

⑥. $P(x)$ = " x takes this module" (where x is student)

$Q(x)$ = " x has been on compus "

$\cancel{\forall x \in P(x),\ Q(x)}$ $\quad$ $\forall x \in P(x)$ that $Q(x)$ is true

Every student taking this module has been on compus.

Domain of discourse of x : set of student

⑦. let $P(x)$ be " x takes this module" (where x is student)

let $Q(x)$ be " x has been on compus"

$$\forall x\,(P(x) \rightarrow Q(x))$$

Every student taking this module has been on compus.

⑧. $P(x)$ = " x takes this module" (where x is student)

$Q(x)$ = " x has been on compus"

$\cancel{P(x) \rightarrow Q(x)}$

$P(x) < Q(x)$

$\forall x \in P(x),\ P(x) + Q(x)$

(+ Every student taking this module has been on compus)

Q2. If x is an even integer then x^2+x is an even integer.

If x is even, then $x=2k$ for some integer k

$$x^2+x = (2k)^2+2k$$
$$= 4k^2+2k$$
$$= 2(2k^2+k)$$
$$= 2n \text{ where } n=2k^2+k \text{ is an integer}$$

This shows that x^2+x is even.

Q3. (a). For all integer n, if n^2+2n+7 is odd, then n is even.

$P=$"n^2+2n+7 is odd" $\quad q=$"n is even"

$\neg P=$"n^2+2n+7 is even" $\quad \neg q=$"n is odd"

$P\to q$ true $=\neg q\to\neg P$ true

$\neg q\to\neg P=$"If n is odd, then n^2+2n+7 is even."

If n is odd, then $n=2k+1$ for some integer k

So $n^2+2n+7=(2k+1)^2+2(2k+1)+7$
$$= 4k^2+4k+1+4k+2+7$$
$$= 4k^2+8k+10$$
$$= 2(2k^2+4k+5)$$
$$= 2l \text{ where } l=2k^2+4k+5 \text{ is an integer}$$

So n^2+2n+7 is even, which means $\neg q \to \neg p$ is true

Then $p \to q$ is also true.

If n^2+2n+7 is odd, then n is even.

(b). To prove $p \leftrightarrow q$ is true, we need to prove that $p \to q$ and $q \to p$ are true.

$p \to q$ is already proved in (a).

$q \to p$ = "If n is even, n^2+2n+7 is odd."

If n is even, $n = 2k$ for some integer k

So
$$\begin{aligned} n^2+2n+7 &= (2k)^2 + 2\times(2k)+7 \\ &= 4k^2+4k+7 \\ &= 2(2k^2+2k)+7 \\ &= 2L+7 \text{ where } L = 2k^2+2k \text{ is an integer} \end{aligned}$$

So n^2+2n+7 is odd, which means $q \to p$ is true.

We can say that for all integers, n^2+2n+7 is odd if and only if n is even.

Q4.

(a). The following is false.

For all real number, if $\overset{P}{\underline{f(-x)=f(x)}}$, the real function $\overset{q}{\underline{f(x)=\frac{x}{x+3}}}$ is an even function. ~~(P⇒q)~~

$$f(x)=\frac{x}{x+3} \qquad f(-x)=\frac{-x}{-x+3}=\frac{x}{x-3}$$

Since $\frac{x}{x+3} \neq \frac{x}{x-3}$

$f(x) \neq f(-x) \Rightarrow P$ is not true

so q is not true, which means the real function $f(x)=\frac{x}{x+3}$ is not an even function.

(b). prove $x^4 \geq \frac{(2x-1)(2x+1)}{4}$ for all real number x

$$x^4 \geq \frac{(2x-1)(2x+1)}{4}$$

$$\Leftrightarrow \quad 4x^4 \geq (2x-1)(2x+1)$$

$$\Leftrightarrow \quad 4x^4 \geq 4x^2-1$$

$$\Leftrightarrow \quad 4x^4-4x^2+1 \geq 0$$

$$\Leftrightarrow \quad (2x^2-1)^2 \geq 0$$

Since $(2x^2-1)^2$ is a square. It is always positive and so the last statement is true. Hence $x^4 \geq \frac{(2x-1)(2x+1)}{4}$ for all real number x.