We observe data $y_1, y_2, \ldots, y_T$ from the AR(1) process:

$$
\begin{aligned}
y_t &= \mu + \phi(y_{t-1} - mu) + \epsilon_t \\
\epsilon_t &\sim \mathcal{N}\left(0, \sigma^2\right) \text{ for t = 2, \ldots, T} \\
y_1 &\sim \mathcal{N}\left(\mu, \frac{\sigma^2}{1-\phi^2}\right).
\end{aligned}
$$

The following priors are chosen, with a stretched beta distribution for ϕ to ensure stationarity.

$$
\begin{aligned}
p(\mu) &\sim \mathcal{N}\left(\bar{\mu}, \lambda^2\right) \\
p(\sigma^2) &\sim \mathcal{IG}\left(\text{shape} = a, \text{scale} = b\right) \\
p(\phi) &\propto \left\{\frac{1+\phi}{2}\right\}^{p_1 - 1} \left\{\frac{1-\phi}{2}\right\}^{p_2 - 1}
\end{aligned}
$$

The posterior distribution for $\theta = (\mu, \phi, \sigma^2) is then$

$$
\begin{aligned}
p(\theta | y_{1:T}) &\propto p(y_1|\theta) \prod_{t=2}^{T} p(y_t|\theta, y_{t-1}) p(\mu) p(\sigma^2) p(\phi) \\
&\propto \frac{1-\phi^2}{\sigma^2}^{1/2} \exp\left\{\frac{-(1-\phi^2)(y_1-\mu)^2}{2\sigma^2}\right\} \sigma^{T-1} \exp\left\{-\frac{\sum_{t=2}^{T}(y_t - \mu - \phi(y_{t-1}-\mu))^2}{2\sigma^2}\right\} \\
&\times \sigma^{-2(a+1)} \exp\left\{\frac{-b}{\sigma^2}\right\} \exp\left\{\frac{(\mu-\bar{\mu})^2}{2\lambda^2}\right\} \left\{\frac{1+\phi}{2}\right\}^{p_1-1} \left\{\frac{1-\phi}{2}\right\}^{p_2-1}
\end{aligned}
$$