Model:

$$y_t = \exp(x_t/2)z_t$$
$$x_t = \mu + \phi(x_{t-1} - \mu) + \sigma u_t$$

with $u_t, z_t \overset{iid}{\sim} N(0,1)$ for all t.
Priors:

$$\mu \sim N(0, 10)$$
$$\tau = 0.5(\phi + 1) \sim B(20, 1.5)$$
$$\sigma^2 \sim IG(2.5, 0.025)$$
$$X_0 \sim N\left(\mu, \frac{\sigma^2}{1-\phi^2}\right)$$

DGP Values:

$$\mu = -0.5$$
$$\phi = 0.8$$
$$\sigma^2 = 0.02$$
$$T = 500$$

Methods:

- PMMH (Andrieu, Doucet and Holenstein 2010): 20,000 draws, first 10,000 discarded. $\theta^{(i)} \sim MVN(\theta^{(i-1)}, 0.05)$ proposal distribution. Sample $\{\mu, \phi, \sigma^2, x_{0:T}\}$ jointly, then use all draws to estimate $\hat{p}(x_{T+1}) \approx \sum_{i=1}^{1} 0000 p(x_{T+1}|x_T^{(i)}, \mu^{(i)}, \phi^{(i)}, \sigma^{2,(i)})/10000$.

- VB: Multivariate normal approximation of $\{\mu, \phi, \ln(\sigma^2), x_{T+1}\}$ using

$$\begin{aligned}\ln(p(\mu, \phi, \sigma^2, x_{T+1}, y_{1:T})) \approx{}& \ln\left(p\left(\mu, \phi, \sigma^2\right)\right) + \ln\left(\hat{p}\left(y_{1:T}|\mu, \phi, \sigma^2\right)\right) \\ &+ \ln\left(\sum_{k=1}^{N} \pi_T^{(k)} p\left(x_{T+1}|\mu, \phi, \sigma^2, x_T^{(k)}\right)\right),\end{aligned}$$

  where $\hat{p}$ denotes a particle filter estimate and $\{x_T^{(k)}, \pi_T^{(k)}\}$ denote a point mass and associated weight of the particle estimate of $\hat{p}(x_T|\mu, \phi, \sigma^2, y_{1:T})$ for particle $k = 1, 2, \ldots, N$.

Results: Posterior Mean and Standard Deviation for PMMH and VB fit to twenty different datasets.