Result: Mean Field Approximation
Initialise λ randomly;
while *Not converged* **do**
  for $i = 1$ **to** 7 **do**
    Hold $\lambda_{j \neq i}$ fixed;
    Maximise λ_i ;
  end
end

Algorithm 1: Mean Field Variational Bayes

$$\begin{aligned} \nabla_\lambda L(\theta, \lambda) &= E_q \left[\nabla_\lambda [q_\theta (\log(p(\theta, y)) - \log(q(\theta)))] \right] \\ &= E_q \left[\nabla_\lambda [\log(q_\theta)] (\log(p(\theta, y)) - \log(q(\theta))) \right] \\ &\approx 1/S \sum_{s=1}^{S} \nabla_\lambda [\log(q_{\theta^s})] (\log(p(\theta^s, y)) - \log(q(\theta^s))) \end{aligned} \tag{1}$$

Result: Variational Approximation
Initialise λ to Methods of Moments Estimator;
while *Not converged* **do**
  Simulate θ_s for $s = 1, \dots S$ from $q(\theta^{t-1})$ **for** $i = 1$ **to** 7 **do**
    Hold $\lambda_{j \neq i}$ fixed;
    Calculate $\nabla^t_{\lambda_i} = 1/S \sum_{s=1}^{S} \nabla_\lambda [\log(q_{\theta^s})] (\log(p(\theta^s, y)) - \log(q(\theta^s)))$
  end
  Set $\lambda^t = \lambda^{t-1} + p_t \nabla^t_\lambda$ Set $t = t + 1$
end

Algorithm 2: Stochastic Gradient Ascent Algorithm 1

We can use $(\theta_1, \theta_2)' = \mu + L(\epsilon_1, \epsilon_2)'$ and $\theta_3 = Q^{-1}(\epsilon_3 | \alpha, \beta)$ where $(\epsilon_1, \epsilon_2) \sim N(0, I)$ and $\epsilon_3 \sim U(0, 1)$

Result: Variational Approximation
Initialise λ to Methods of Moments Estimator;
while *Not converged* **do**
  Simulate ϵ for $s = 1, \dots S$ from $N(0, I)$ and $U(0, 1)$ Transform $\theta^s = f(\epsilon^s, \lambda^{t-1})$ **for** $i = 1$ **to** 7 **do**
    Hold $\lambda_{j \neq i}$ fixed;
    Calculate $\nabla^t_{\lambda_i} = 1/S \sum_{s=1}^{S} \nabla_\lambda [(\log(p(\theta^s, y)) - \log(q(\theta^s)))]$
  end
  Set $\lambda^t = \lambda^{t-1} + p_t \nabla^t_\lambda$ Set $t = t + 1$
end

Algorithm 3: Stochastic Gradient Ascent Algorithm 2

```
library(knitr)
library(mvtnorm)
library(reshape)
summary = matrix( c(0.37, -0.30, 0.10, -0.03, 0.10, 49.2, 78.4, -77.5, 7.4,
```

```
                    0.37, -0.29, 0.09, 0, 0.10, 49.5, 79.8, -77.4, "1.3 x 10^{-6}",
                    0.32, -0.32, 0.11, -0.01, 0.12, 46.4, 78.7, -77.2, 59.5,
                    0.38, -0.30, -0.1, 0.03, -0.10, 48.7, 78.9, -77.4, 1.2) , byrow = TRUE,

colnames(summary) = c(paste0(expression(mu), 1), paste0(expression(mu), 2), "L11", "L21", "L
rownames(summary) = c("Method of Moments", "Mean Field", "Copula 1", "Copula 2")
kable(summary, format = "latex")
```

	mu1	mu2	L11	L21	L22	alpha	beta	L(theta)	time (seconds)*
Method of Moments	0.37	-0.3	0.1	-0.03	0.1	49.2	78.4	-77.5	7.4
Mean Field	0.37	-0.29	0.09	0	0.1	49.5	79.8	-77.4	1.3 x 10^{-6}
Copula 1	0.32	-0.32	0.11	-0.01	0.12	46.4	78.7	-77.2	59.5
Copula 2	0.38	-0.3	-0.1	0.03	-0.1	48.7	78.9	-77.4	1.2