

Congestion Hot Spot Identification using Automated Pattern Recognition

Lisa Kessler¹, Barbara Karl¹ and Klaus Bogenberger¹

Abstract—This paper introduces a methodology which identifies congestion hot spots for individual congestion types. The proposed algorithm first isolates coherent congested clusters out of a spatio-temporally discretized speed matrix. Then, virtually driven trajectories which pass through the respective congestion area are calculated and their speed profiles are analyzed. A congestion type is assigned to each trajectory and thereafter, a congestion type for the overall cluster is determined. Considering the spatial and temporal start and end points of each cluster along with its assigned congestion type, accumulated occurrences of congestion can be determined. The methodology is applied to data derived from speed sensors along the Bavarian freeway A9 in Germany. The results show a high share of *Stop and Go* traffic in the Greater Munich Area. All over the considered stretch, *Jam Waves* occur frequently, limited to a few locations but widely spread in time.

I. MOTIVATION

Congestion accumulation points in both time and space can be experienced by anybody traveling regularly on roadways. There exist certain time slots (rush hours) where one may wish to avoid being on a freeway and also certain locations where the probability of congestion is high and an increase in the travel time is most likely. Several studies have been executed dealing with congestion patterns and congestion hot spots. In [1], congestion classification on alpine freeways is described. The authors of [2] discuss an advanced incident detection algorithm. In [3], freeway bottlenecks are identified. In addition, there exist studies published by public authorities, such as [4] and [5], which discuss spatial and temporal congestion statistics in general. One common open research question is the treatment of different congestion types and their accumulated occurrences.

The existence of different types of congestion sparks a need for a methodology for their differentiation, as traffic information and control measures should be adjusted based on the type of congestion. This research focuses on the differentiation of several congestion types and proposes a methodology to automatically determine their respective accumulated occurrences. An algorithm is presented which considers spatio-temporally discretized speeds and identifies coherent congestion clusters. Then, it determines the congestion type of each cluster using virtual trajectories driving through the congested regime. As a third step, the temporal and spatial occurrences of the respective congestion start and

end points are computed. While considering spatio-temporal areas of several similar time frames (e.g. days), congestion hot spots can be investigated.

The overall goal is to obtain more detailed dynamic traffic information and to be able to warn drivers early against traffic incidents. By identifying and localizing hot spots, freeway operators can control traffic in an optimized way.

This paper is structured as follows. Section II illustrates the definition of occurring congestion patterns. Four congestion types proposed in [6] are explained and additionally, lane-specific congestion is defined. In section III, the identification of separated congested regimes and the aggregation to clusters is presented. Furthermore, the assignment of a congestion type to each congested cluster is given. Section IV contains the application of the proposed methodology to data derived from the German autobahn A9. Section V gives a summary of the results and an outlook on future research.

II. DEFINITION OF CONGESTION PATTERNS

Emerging congestion can have many causes, each of which behaves individually in its spatial and temporal congested extensions. In order to optimize traffic control, each type of congestion needs a different treatment. This section aims at presenting congestion types according to their characteristics to receive a more accurate ground truth. Kerner et al. [7], [8], [9], [10] distinguished between three traffic phases (free-flow, synchronized flow, and wide moving jam), two of which concern congested areas. Helbing et al. [11] extended the prevailing traffic conditions in congestion to five phases. In [6], four congestion types were examined. In section II-B, these four types are explained in detail. Section II-C introduces a lane-specific congestion type in which speed propagation behaves differently on individual lanes.

A. Preliminaries

A spatio-temporally discretized speed matrix forms the basis of the analyses. Sensors (either by direct local speed measurements or by deriving speeds from travel time measurements) generate a speed distribution over time for certain locations or segments on the road. Let the speed matrix be available for a road stretch X and a time period T . A spatio-temporal speed function $v = v(x, t)$ returns the speed value for any location $x \in X$ and any time $t \in T$. Furthermore, let X be separated into a set of links $\{x_i\}_{i=1,\dots,m}$, the spatial resolution of the speed data. Similarly, let T be separated into a set of time intervals $\{t_j\}_{j=1,\dots,n}$. Sets $\{x_i\}$ and $\{t_j\}$ separate the spatio-temporal area $X \times T$ into a grid, and function v is constant inside each cell $x_i \times t_j$ of this

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¹The authors are with Chair of Traffic Engineering and Control, Technical University of Munich, Germany. lisa.kessler@tum.de

grid. Cells of interest for this analysis are only those which contain a speed value below a velocity threshold, the critical congestion speed v_{crit} .

Using an interpolation method (e.g. *Adaptive Smoothing Method* [12], [13], [14] for local measurements or the methods presented in [15] and [16] for travel time measurements), any missing detections of speed values can be smoothed.

B. Congestion Types

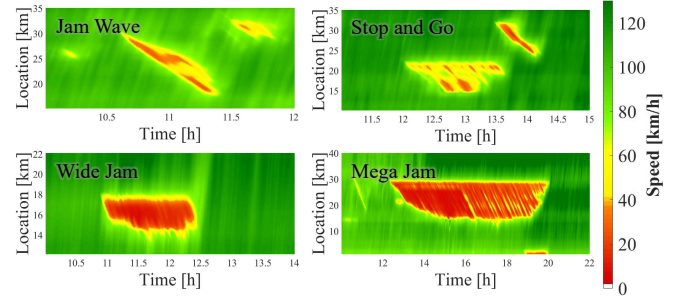
A project report [17] introduced the congestion types (a) *Jam Wave*, (b) *Stop and Go*, (c) *Wide Jam*, and (d) *Mega Jam*. A *Jam Wave* corresponds to a single congestion wave, a thin stripe implying a temporarily low velocity. *Stop and Go* waves are several narrow stripes representing congestion waves separated by free-flow sections. A *Wide Jam* is a broad area with predominant congestion velocity. An extensive area with the domination of speed values below the velocity threshold is called a *Mega Jam*. It represents a widely spread traffic breakdown (Fig. 1a).

Based on this subdivision, the authors of [6] introduced a methodology to automatically classify occurring congestion events which roughly comprises the following. Basically, virtual trajectories drive through the entire area starting from the spatial beginning of the considered road stretch and continue with the prevailing speed in each discretized cell. Whenever the trajectory reaches the bounds of a cell, it continues to the neighboring cell at the corresponding speed. The resulting speed profile is analyzed according to certain parameters. When a congested cell is traversed at time t_0 (Fig. 1b) before which there was no congestion for a duration of at least t_{break} , the time period spent in congestion is analyzed until the speed is above v_{crit} for at least t_{break} again. If it is a short incident ($\leq t_{JamWave}$), it is said to be a *Jam Wave*. If it is greater than time $t_{MegaJam}$, it is a *Mega Jam*. Otherwise, the number of speed drops below v_{crit} is calculated. If there are fewer than $n_{StopandGo}$ speed breakdowns, the congestion type is *Wide Jam*, otherwise *Stop and Go* traffic.

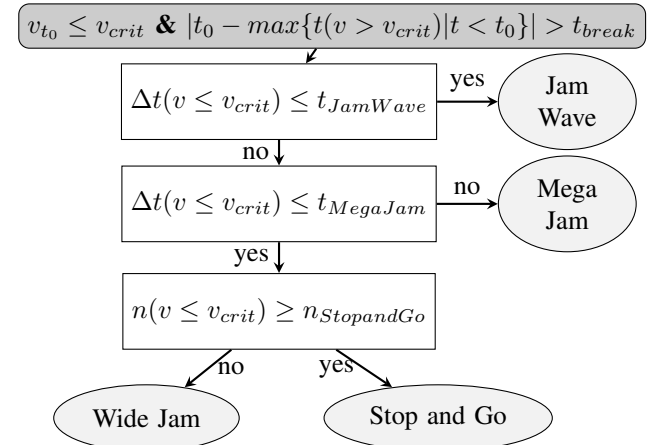
The authors conducted a sensitivity analysis on the parameter set and recommended optimal parameter values. One disadvantage of the method proposed in [6] is that it does not ensure that the considered spatio-temporal traffic region contains only one isolated congested area. However, in order to reliably determine the congestion type, each virtual trajectory should only traverse a single congested area. Section III presents a methodology to fill this gap.

C. Lane-Specific Congestion

In addition to these four congestion types, a further kind of jam can be experienced. For example, upstream of an interchange, vehicles could be changing their lanes to exit respectively to continue on the freeway. If the tail end of the congestion on the crossing freeway reaches approaching vehicles on the main freeway, the speeds of the lanes can oscillate individually. This type of congestion is called *Lane-Specific Congestion* since specific lanes are affected by reduced velocities. Besides congestion back-up, another



(a) Speed contour of congestion types



(b) Schematic representation of congestion classification

Fig. 1: Definition of four congestion types [6]

possible cause of this congestion type is heavy road grades, where significantly different speeds prevail on truck lanes and car lanes while going uphill [1], [18].

Lane-Specific Congestion can be detected if the sensor technology on a road stretch is capable of measuring speeds on each lane individually (e.g. induction loops, radars, cameras). Then, for each lane, a separate spatio-temporal speed matrix is required and again, using virtual trajectories, travel time differences on a road stretch can be identified.

III. AUTOMATED DETECTION OF CONGESTION PATTERNS

This section describes a methodology to automatically detect congestion patterns and to assign a unique congestion type to each jam. The algorithm works in three steps. First, coherent, isolated congestion clusters are identified based on the method described in [19] and [20]. Second, the methodology defined in [6] and extended in section II is applied to each of the found clusters. If the algorithm is run for several similar days (3rd step), accumulated congestion occurrences in space and time can be determined.

As outlined in section II-A, the basis is a spatio-temporally discretized speed distribution. If high-resolution raw data are available, these can be used directly. Otherwise, raw data have to be interpolated and smoothed, for example using one of the interpolation methods mentioned in section II-A.

A. Identification of Separated Congested Regimes and Aggregation into Clusters

The authors of [19] and [20] proposed an algorithm to identify congestion clusters. In their papers, this serves as a basis for comparing speed matrices from different traffic detection technologies to identify which technology can detect congestion earlier. The first part of their algorithm consists of the clustering. Starting from the discretized speed matrix (Fig. 2a), all cells containing a velocity below the congestion threshold v_{crit} are considered (Fig. 2b).

Two such cells $x_{i_1} \times t_{j_1}$ and $x_{i_2} \times t_{j_2}$ are denoted as *connected* if the following two conditions hold:

$$i_1 \in \{i_2 - 1, i_2, i_2 + 1\} \quad (1)$$

$$j_1 \in \{j_2 - 1, j_2, j_2 + 1\} \quad (2)$$

This definition of congestion clusters leads to many very small clusters (Fig. 2c). Furthermore, it can be observed that these clusters are frequently located close to each other.

In order to achieve a smoothing effect, clusters which are located close to each other are merged into one single cluster. Here, *close to* means that the travel time of a virtual trajectory is not larger than $t_{merge} \in \mathbb{R}_{\geq 0}$ between one cluster and the other. To this end, all of the clusters located in $X \times T$ are iterated over, and a set of virtual driving trajectories is computed for each, with each trajectory starting from a different corner of a cell belonging to the current cluster. This can be done for some starting point $(x_S, t_S) \in X \times T$ by solving the ordinary differential equation

$$\frac{dx}{dt} = v(x(t), t) \quad (3)$$

with initial condition $x(t_S) = x_S$. For each virtual trajectory, the computation is terminated as soon as time $t_S + t_{merge}$ is reached (Fig. 2d, illustrated for cluster A). All clusters touched by one of the generated trajectories are assigned to the original cluster (Fig. 2e). This means that all congested cells belonging to one of these clusters are part of the same cluster [20].

Similarly to [19], only clusters of a relevant size are considered. If the spatio-temporal size of a cluster is smaller than an area A_{min} , it is eliminated.

The result is a set of isolated and coherent clusters throughout the entire speed matrix. This separation of congestion into individual clusters ensures the applicability of the congestion type classification algorithm.

It is also possible to consider locations with non-continuously available data, for example, if several segments on a road are not equipped with traffic sensors. Clusters are identified reliably if locations without any speed information are filled up with a free-flow speed value. The described algorithm then finds clusters in separate location sets.

In the case of *Lane-Specific Congestion*, clusters are identified using the same procedure but on speed matrices derived from individual lanes. The result is a set of congestion clusters per lane.

B. Assignment of One Congestion Type to Cluster

The parameter t_{merge} corresponds to the parameter t_{break} defined in section II-B. Both denote the travel time of a virtual trajectory traveling in free-flow after having crossed a congested regime. Therefore, this parameter does not need to be considered when assigning a congestion type to a cluster.

The second part of the proposed algorithm takes one cluster as input. A cluster is defined as convex hull of all congested cells belonging to the congested area identified by the first part of the algorithm (section III-A). This convex hull is embedded in an area of free-flow conditions to ensure that virtually driving trajectories do not cross speed cells of any further congested regime (Fig. 2f for cluster A).

Then, virtual trajectories are created traversing the entire spatio-temporal area of the speed matrix (Fig. 2g). Each trajectory starts with an temporal offset of δt_{traj} . Again, each trajectory travels with the piecewise constant speed defined in (3). The result is a speed trend per trajectory inside the considered cluster, an example of which is shown in Fig. 2h, which depicts the speed profile of the bold line from Fig. 2g. This speed progress is then analyzed according to the flow chart given in Fig. 1.

In order to determine one congestion type k per cluster, all trajectories trj_1, \dots, trj_n crossing that cluster are computed. The assigned congestion types k_1, \dots, k_n of each trajectory specify the overall congestion type. If all trajectories received the same congestion type z , then also k is set to z . If k_1, \dots, k_n result in two different congestion types (for example a congestion which behaves like *Wide Jam* in its interior and like *Jam Waves* at its temporal bounds), then k is set to the type which occurs more often than the percentage $n_{2types} > 0.5$. Analogously, if three congestion types are assigned, then k is set to the type of which the share is larger than $n_{3types} > 0.33$. In a rare case that one congestion is crossed by trajectories of all four congestion types, among which is the case of *Mega Jam*, then the overall congestion type of the cluster is also set to *Mega Jam*. In all other cases, k can not be determined uniquely and is set to *Mixed*.

In the case of *Lane-Specific Congestion*, the union of the convex hulls of the individual clusters is taken and virtual trajectories are computed which travel through the (possibly partly) congested area. If the pairwise travel time difference is larger than $t_{lanejam} \in \mathbb{R}_{\geq 0}$ for trajectories starting at the same time, the type of the considered congestion cluster is set to *Lane-Specific Congestion*.

C. Identification of Relevant Spatial and Temporal Congestion Type Hot Spots

The algorithm calculates the congestion type k as well as the locations and the timestamps of the start (x_s, t_s) and the end (x_e, t_e) of each congestion cluster. An analysis of several similar spatio-temporal areas leads to the identification of hot spots (e.g. several days of a month). The outcome is a distribution of the congestion types over the considered road stretch throughout the given times.

Analogously, the accumulations of *Lane-Specific Congestion* can be analyzed. Whenever congestion occurs concern-

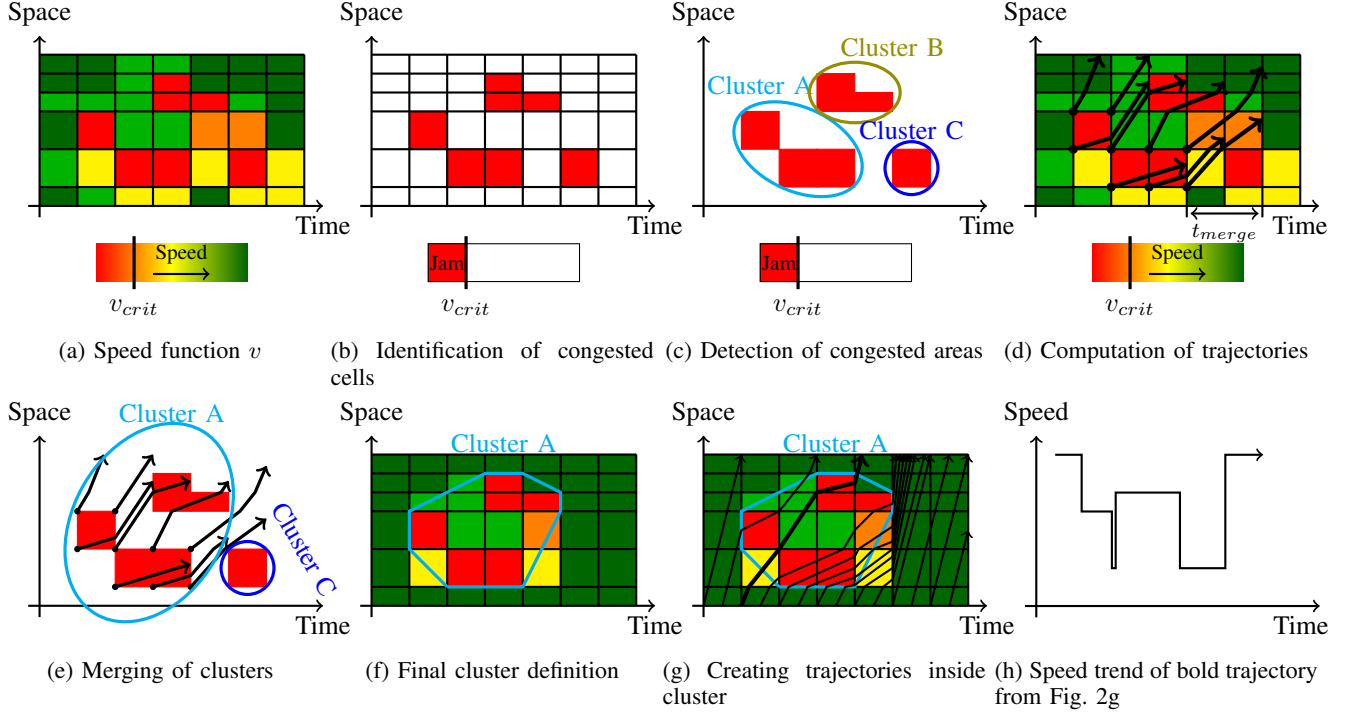


Fig. 2: Computation of congestion clusters and classification of congestion type [20], [6]

ing not all lanes of a freeway stretch, the locations and the times of the occurrences are stored and compared with other time windows on the same road segments.

IV. APPLICATION AND RESULTS

The hybrid methodology, composed from the algorithms presented in [19], [20], and [6] and extended by the algorithm introduced in section III, is applied to a data set derived from autobahn A9 in Bavaria, Germany.

A. Data Basis

The considered road stretch between Munich and Nuremberg covers a length of 157 km where data are available for both southbound (SB) and northbound (NB) driving direction. Data were measured in April 2019 (30 days). Stationary road sensors (induction loops) measure local speeds per lane minute-by-minute. The sensors are distributed within two different segments. Between km 372 and km 400, the SB and NB directions have 20 and 14 sensors, respectively, with an average spacing of 1.3 km; between km 475 and km 529, 45 SB and 39 NB sensors are located with an average spacing of 1.2 km. Fig. 3 shows a map of the road.

Spaces in between (km 400-475) are not equipped with traffic detectors which yields to empty speed cells. As described in section III-A, these locations without data can be accepted as part of the input speed matrix. To apply the algorithm, all empty cells are set to a free-flow speed $v_{freeflow}$. During the assignment of a congestion type to an identified congestion cluster, all other areas except the cluster are set to free-flow conditions. Similarly, locations with empty speed values can be filled with free-flow defaults.

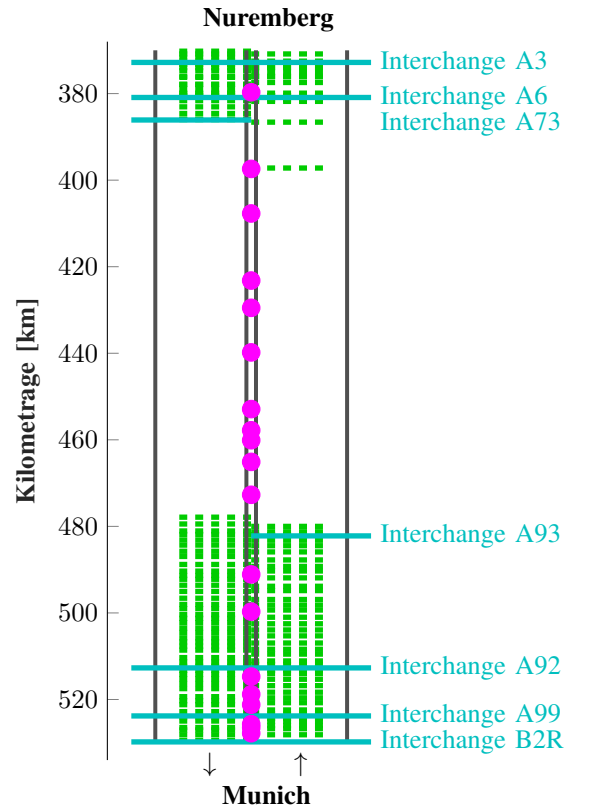


Fig. 3: Sketch of autobahn A9 stretch with stationary detection locations: interchanges (cyan), ramps (magenta), induction loops (dashed green)

Speed data are available both per lane and as a flow-weighted mean over all car and truck lanes. The congestion types *Jam Wave*, *Stop and Go*, *Wide Jam*, and *Mega Jam* are found based on the averaged speed values whereas *Lane-Specific Congestion* is determined based on lane-specific speed values.

B. Data Preparation

To ensure the recognition of various congested situations on the autobahn, some preprocessing steps are necessary. Spatio-temporally discretized speed matrices have to be created out of the measured speeds. To obviate missing detections at certain minutes or locations, the *Adaptive Smoothing Method* (ASM) [12], [14] is applied to derive smoothed speed values. The ASM parameters used are as shown in Tab. I.

TABLE I: ASM Parameter Values Used for Smoothing the Sensor Data

Parameter	Value
Spatial grid distance	500 m
Temporal grid distance	1 min
Speed in congestion	-18 km/h
Free-flow speed	80 km/h
Crossover from free to congested traffic	80 km/h
Width of the transition region	10 km/h

C. Application of the Methodology

The previously described methodology is applied to these data. First, congestion clusters are identified based on the algorithm described in section III-A. The parameter values used to create the clusters are as defined in Tab. II. They are chosen based on expert knowledge relevant for German freeways though they are generalizable.

TABLE II: Parameter Values Used for Creating Separate Congestion Clusters [20]

Parameter	Value
Velocity threshold v_{crit}	40 km/h
Free-flow speed $v_{freeflow}$	120 km/h
Minimal free-flow time between congested areas t_{merge}	4 min
Minimal size of congested areas A_{min}	2 km · min

Second, a congestion type is assigned to each individual cluster according to the algorithm described in section III-B. Virtual trajectories drive through the entire congested area and whenever they hit a certain cluster, they change their speed from free-flow to the velocities inside the cluster. The parameter values used for the assignments are presented in Tab. III. In order to minimize the number of assignments to *Mixed*, n_{2types} and n_{3types} are set relatively low in an empirical way.

Fig. 4 illustrates the application of the methodology to data measured on the southbound driving direction from April 13, 2019. Fig. 4a shows the spatio-temporal area with congested regions and the identified separated clusters

TABLE III: Parameter Values Used for Assigning Congestion Types

Parameter	Value
Maximal duration of jam wave $t_{JamWave}$	3 min
Minimal duration mega jam $t_{MegaJam}$	30 min
Minimal number of speed drops $n_{StopandGo}$	2
Temporal offset of starting trajectories δt_{traj}	5 min
Minimal share of the cluster's congestion type if two different types occur in trajectories n_{2types}	0.51
Minimal share of the cluster's congestion type if three different types occur in trajectories n_{3types}	0.41
Minimal travel time increase in case of lane-specific congestion $t_{lanejam}$	50%

in blue. Virtual trajectories cross the congestion clusters in Fig. 4b, identifying two types of congestion: *Stop and Go* waves (magenta) and *Jam Wave* (black). Since 57% of the trajectories are classified as *Stop and Go* traffic, the overall cluster type is set to *Stop and Go*. In Fig. 4c, another congestion cluster is crossed by several *Wide Jam* trajectories (cyan). Just at the boundaries, two trajectories are classified as *Jam Wave* (black). They touch the congested regime barely. Four trajectories (magenta) are assigned to the type *Stop and Go* since the speed trends drop below v_{crit} several times. The overall congestion type classification of all clusters can be seen in Fig. 4d. It results in one *Stop and Go* cluster, two *Wide Jam* clusters, and 9 *Jam Wave* clusters. One *Mixed* cluster occurs: Applying an offset of 5 min, four trajectories cross this cluster, two of which are classified as a *Jam Wave* and two as a *Stop and Go* congestion. Therefore, the threshold n_{2types} is not fulfilled and the congestion type cannot be determined reliably.

Although the broad area around km 520 is congested for several hours, no *Mega Jam* is assigned but *Wide Jam* and *Jam Wave* clusters because trajectories pass through the stationary bottleneck in a short time interval (< 30 min).

D. Congestion Accumulation Points

Lastly, congestion type hot spots are determined. In southbound direction, 226 clusters were found in April 2019. Hereby, the share of congestion type *Jam Wave* amounts to 64.6%, *Stop and Go* to 17.3%, *Wide Jam* to 7.5%, and *Mega Jam* to 0.4%. Out of all detected clusters, 10.2% could not be assigned to a unique congestion type and are set to *Mixed*. In northbound direction, 145 clusters were identified, of which 55.9% belong to *Jam Wave*, 31.7% to *Stop and Go*, 5.5% to *Wide Jam*, and 0.7% to *Mega Jam*. *Mixed* was assigned in 6.2% cases.

The spatial hot spots in the SB direction are as depicted in Fig. 5a, aggregated to intervals of 2 km for a better illustration. Upstream of large interchanges, congestion occurs frequently. Specifically, the tail ends caused by the autobahn A99 and by the Munich city highway (at km 529 where the A9 terminates) are often reasons for the formation of congestion.

The temporal hot spots in the SB direction are illustrated in Fig. 5b, aggregated to intervals of 30 min. Two typical peaks

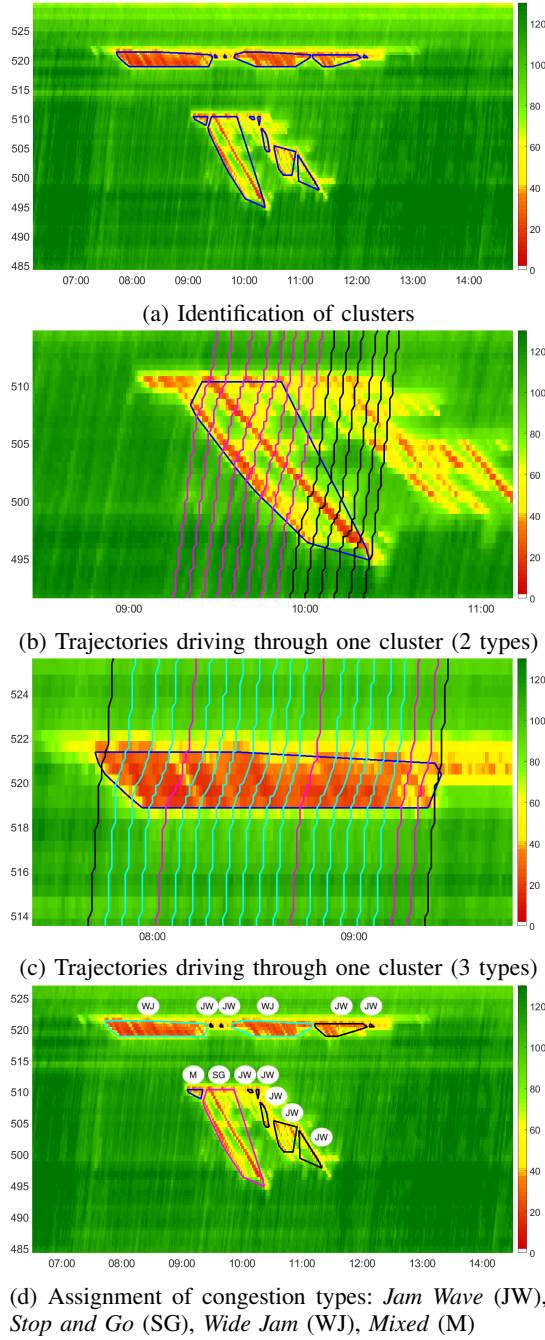


Fig. 4: Application of algorithm to data from April 13, 2019. Speeds in [km/h], *Jam Wave* in black, *Stop and Go* in magenta, *Wide Jam* in cyan, *Mixed* in blue

(AM and PM peak) occur, essentially due to the detected *Jam Waves*. The share of *Stop and Go* traffic is significantly increased between 6:00 and 8:00. *Wide Jams* are evenly distributed throughout the day, only during the rush hours (9:00-10:00 and 16:00-18:00) they are more distinctive.

Both information is united and Fig. 5c presents spatio-temporal congestion hot spots in the SB direction without aggregation. A *Stop and Go* conglomeration is detected between 6:00 and 8:00 around km 520. It comes from the

back-up of autobahn A99, onto which two out of five lanes exit from autobahn A9. *Jam Waves* are not distributed all throughout the grid, but restricted to a few locations (km 375-380, km 520, and km 525). Especially in the Nuremberg area (km 375-380), they are not limited to peak hours, but occur during all times of day.

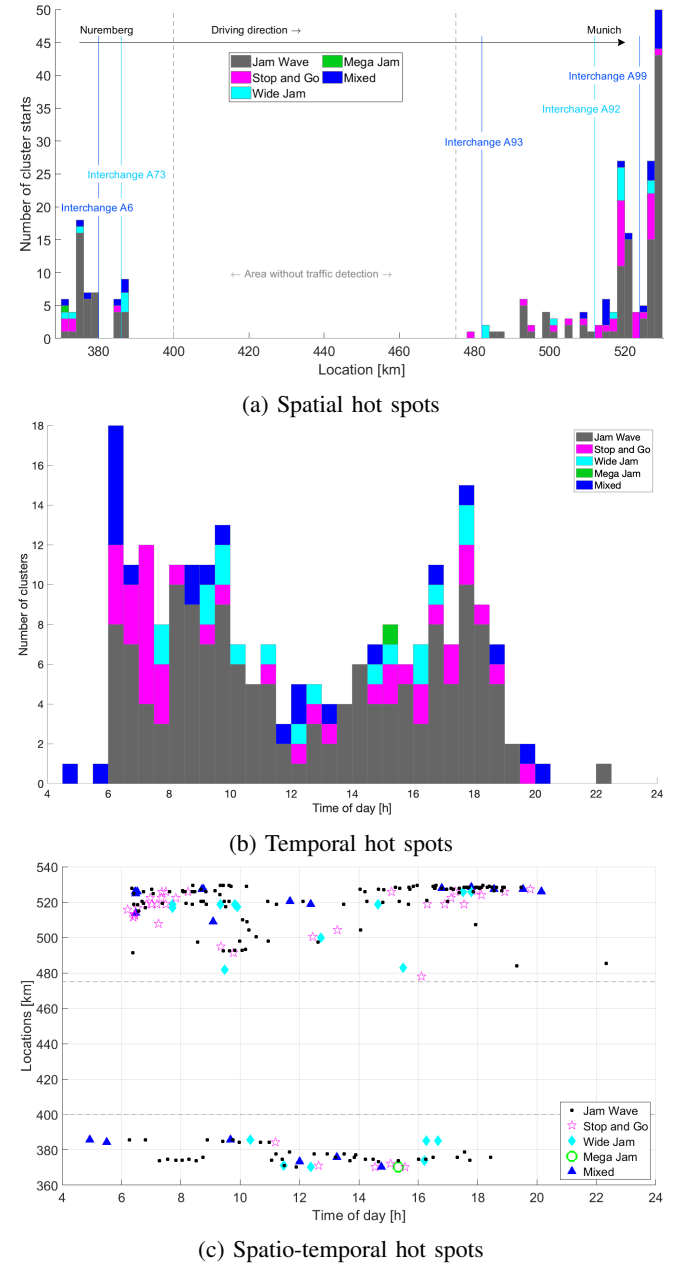


Fig. 5: Results of the accumulation study (SB direction, April 2019)

Lane-Specific Congestion was mainly detected upstream of exit ramps, specifically in the area of interchange A99 in the southbound direction (km 522). Due to a construction site, the lane number decreased from two to one on the ramp and caused a wide tail end. This congestion type occurs on all times of the day.

V. CONCLUSION AND OUTLOOK

This paper presented a novel approach for identifying congestion type hot spots. Based on an algorithm which detects congestion clusters on a spatio-temporally discretized grid, a methodology to automatically assign one of five congestion types (*Jam Wave*, *Stop and Go*, *Wide Jam*, *Mega Jam*, and *Lane-Specific Congestion*) was introduced. The program calculates virtual trajectories driving through the entire spatio-temporal area and analyzes their velocity profiles. Depending on the number of trajectories assigned to a certain congestion type, the cluster's overall type is determined. The spatial and temporal extensions of the clusters determine accumulated congestion type occurrences. The methodology was applied to data from the German autobahn A9 near Munich. It shows a significantly high share of *Stop and Go* traffic in the Greater Munich Area in the morning hours. Furthermore, the analysis determined that *Jam Waves* are not distributed all over the grid, but instead restricted to a few locations. In the Nuremberg area, these *Jam Waves* occur throughout the day, as opposed to other areas, in which they are limited to only certain times of day.

A congestion-type dependent control helps freeway operators to optimize traffic flow. Knowing the hot spots, the controller can estimate based on the beginning of (recurrent) congestion which detour recommendations are useful. Furthermore, traffic planning is supported, e.g. where to place sensors along a road to reconstruct the ground truth more exactly. *Stop and Go* traffic needs more dense detection in order that free-flow sections which should be considered part of the *Stop and Go* congestion are not misidentified. *Jam Waves* require small-scale detection, e.g. via local measurements, otherwise the averaged speeds (over a larger road segment) could be above the velocity threshold and no congestion is recognized at all. Traffic flow with *Lane-Specific Congestion* can be optimized if the lanes are handled dynamically and controlled individually. The recognition of hot spots leads to more detailed traffic information in advance or even to the ability to make predictions, in which case warnings can be sent, e.g. that drivers should change their current lanes early to increase traffic safety.

The work will be continued to investigate speed data from sources other than stationary sensors, e.g. travel time measurements. The algorithm will be extended to analyze which traffic detection technology can resolve the spatio-temporal distribution of individual congestion types best. Another point is to ensure the applicability of the (currently offline available) methodology also in online control.

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