Averaged graph analysis and random failure resilience of a public transport network

Final project for the Data Analytics course for the MSc in Computer Science at University of Milano-Bicocca.

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Introduction

This project aims to analyze the structure of a city network using averaged graphs of the different means of public transport.

Average graphs represent the connectivity between regions in a city. Using this method allows to overcome the disjointed nature of the different networks of transport and possibly noisy datasets without having to expend significant resouces into data quality processing. Given different average graphs representing different types of connections (neighboring regions, transport direct connections), it is possible to analyze and compare each to understand how optimal a public transport network is. Simulation of random failures and attacks can also shed insight on the robustness of the network.

Project structure

The project has been developed to allow easy reusability/recomputation of the transit feed data for different times or cities. As each public transport regulator may implement the GTFS format differently, slight modifications might have to be made. The code provided in the project's GitHub repository is tested and works on Milan's Azienda Mobilità Ambiente e Territorio GTFS feed, the README containing the instructions to reproduce this work.

Data management

All data has been obtained by OpenMobilityData. The data consists of Milan's GTFS feed from the 4th of March 2019 to the 4th of April 2019. As the data is quite extensive, a PostgreSQL Docker container has been setup for ease of use to contain Milan's GTFS database, with automatized database population.

The GTFS (General Transit Feed Specification) is a collection of at least 6 and up to 13 CSV files, describing a transit system's scheduled operations. The necessary data to reconstruct a network is contained in the following tables:

- **Stops**: Defines stops where vehicles pick up or drop off riders.
- **Stop times**: Provides the times when a vehicle arrives at and departs from individual stops for each trip.
- **Trips**: Defines trips for each route. A trip is a sequence of two or more stops that occur during a specific time period.
- **Route**: Defines transit routes. A route is a group of trips that are displayed to riders as a single service.

Through a set of different queries and some data processing, tables containing every stop for every route for metro lines, buses and trams have been produced.

Following are some statistics for the resulting graphs:

- **Metro**:
  - Density: 0.02
  - Nodes: 106
  - Edges: 109
- **Bus**:
  - Density: 0.001
  - Nodes: 2110
  - Edges: 2163
- **Tram**:
  - Density: 0.004
  - Nodes: 449
  - Edges: 445

Data analytics methodology

The graphs of metro lines, buses and trams are disjoint: although different stops refer to the same street, they are located at different coordinates and have a different unique IDs. As such, metro stations, bus stops and tram stops never intersecate, although there are some exceptions for buses and trams sharing the same stop (same unique ID).

It's not sensible to conduct a network analysis and failure test on a disjointed network: although not directly connected, people can and will move from a metro station to its nearest bus or tram station, to move from point A to point B. In case of failures between stops, spatially close stops work as a back-off to allow paths A and B, with C failed, to be reached by a working stop D.

Graphs derived from large transit systems networks and how to represent mobility has been extensively studied. An approach to represent trajectory data of city-wide traffic dynamics in spatial and temporal domains has been proposed by Kim et al. (2016). An approach building on the previous, by Yildirimoglu and Kim (2018), analyzes different traffic flows on a shared spatial grid, allowing for multiple traffic flows to be analyzed as a real world network.

This project takes inspiration from both these approaches to analyze how Milan's transport network behaves in the spatial domain.

Neighbor graph

Partitioning a network in cells

The partitioning of a network in regions (hereon cells) according to the spatial relation of its nodes (the stops in this case) is based on a method developed by Adrienko and Adrienko (2011).

Let S be a set of seed points, where a point $p(lat, lon)$ represent a stop. Let γ be the radius specifying the radius of a cell. We'll have:

$$\forall p_n \in S \begin{cases} p_n \in C_i.m, C_i.c = avg(C_i.m) & : dist(p, C_i.c) < \gamma \\ p_n \in C_{i+1}.m, C_{i+1}.c = p_n & : otherwise \end{cases}$$

Where C_i in the presentated formula is a cell of points each inside the radius of the cell's centroid, $C_i.m$ represents the cell's members and $C_i.c$ is the cell's centroid. The centroid of each cell is estimated iteratively by finding the mean of each point as they get assigned to each cell. After all the points p have been grouped into cells G_i, the cells are emptied and the points redistributed to the closest cell.

The set of cells C represents the partitions into which the stops are grouped.

From C, it is possible to compute the Voronoi diagram of C, which partitions the space into geometrical cells with $C_i.centroid$ as a center, delineating the boundaries between each cell Its complementary graph, computed through the Delaunay triangulation of the centroids, represents the *graph of neighboring regions* (hereon neighbor graph), while the Voronoi diagram represents the *spatial subdivision of the area*. Since boundary regions can be neighbor to other distant regions through to-infinity borders (i.e. Voronoi Diagrams aren't bounded per se), a parameter ρ is given as a neighboring distance limit. The value chosen for this network is $\rho = 5km$.

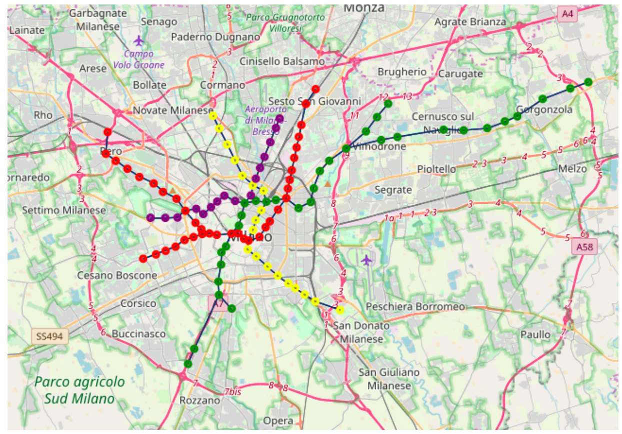


Figure 1: Metro network, stops colored according to the line they belong to

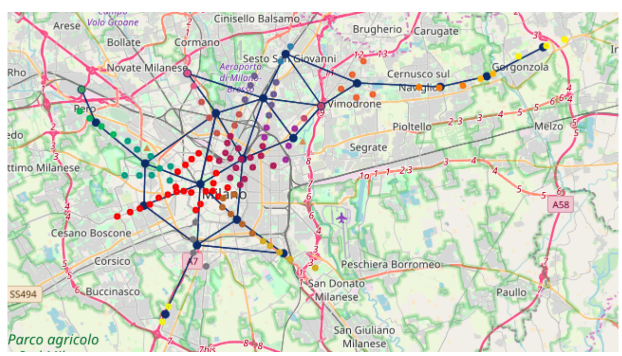


Figure 2: Partitioned metro network with $\gamma = 2km$, $\rho = 5km$, stops colored according to the C_i region they belong to

The neighbor graph represents the spatial connections between regions of a city. While connected spatially, neighboring regions might not be served by public transport. To obtain a graph representing the actual transit connections between regions of a city, we'll check the cell-to-cell flow between regions.

Cell-to-cell flow graph

The cell-to-cell flow takes into account the number of effective *direct links* between the single stops inside the regions. While the node of each regions remains the same, the edges vary according to the number of direct between the stops in the regions. Two regions having many stops directly connected between them will have a higher weight to their edges, and the opposite holds true as well. Two neighboring regions not connected by a direct link of their stops will not be connected in the cell-to-cell graph.

The cell-to-cell flow graph represent the actual connections served by the public transport.

Comparison - Network Analytics

Analyzing the difference between the neighbor graph and the cell-to-cell graph can give interesting insights into how well-connected are different city regions and how resilient each connection is. Centrality measures can help measuring these differences by computing the importance of a node in the graph according to different parameters. Some of the measures used are explained below:

The degree centrality represent the strength of the connection of a node to the network - lower degree regions might cause more inconveniences to the passengers were any of the stops to fail or close.

$$C_D(G) = \frac{\sum_{i=1}^{|V|}[deg(v*) - deg(v_i)]}{|V|^2 - 3|V| + 2}$$

The closeness centrality represent how much a node can influence other nodes, or spread information, in the network. Highly connected networks tend to have similar closeness for every node. For transit systems this represents a measure of how much a node can offer access to other nodes and spread passengers quickly to any destination of the network.

$$C_C(G) = \frac{1}{\sum_{x \in V} l(x, y)}$$

The betweenness centrality represent how much a node has influence over flows in a graph. For transit systems this is a measure of how much a region acts

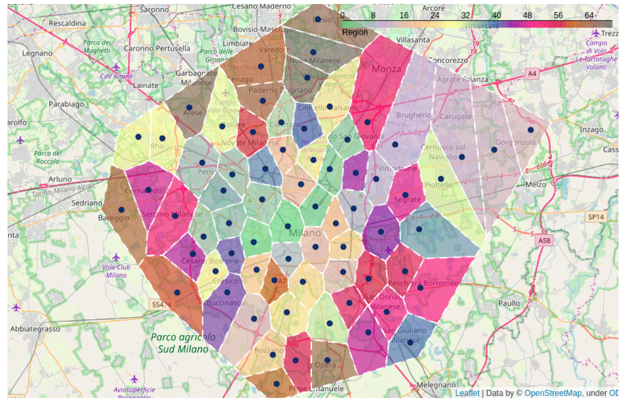


Figure 3: Voronoi diagram of the public transport network

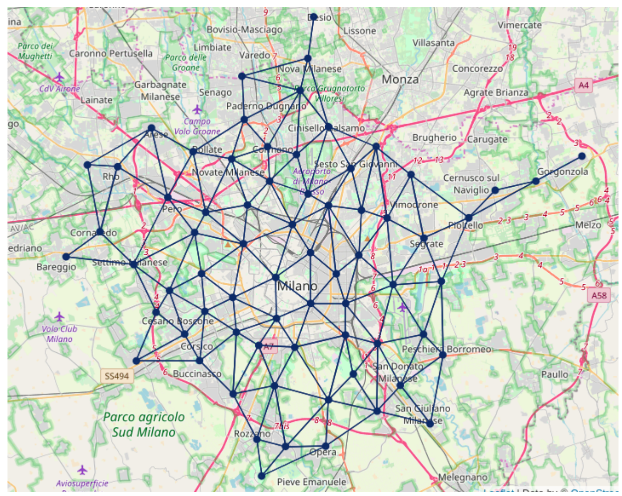


Figure 4: Partitions of the public transport network with $\gamma = 2km$, $\rho = 5km$, corresponds to the Delaunay triangulation of the Voronoi diagram

as a bridge between different clusters of regions in the graph.

$$C_B(G) = \sum_{s \neq v \neq t \in V} \frac{\sigma_{st}(v)}{\sigma_{st}}$$

The comparison of these distributions seems to denote that, for the clustering parameters chosen, Milan's transport network connects efficiently the center and the nort-western regions of the city, while it is still somewhat lacking for the regions outside the main Milan province.

An assortativity analysis was conducted on both networks, finding how both of them tend to be mostly assortative - hubs connecting to hubs. The center of the city seems to show a number of hubs helping to redistribute the flow of traffic along most regions.

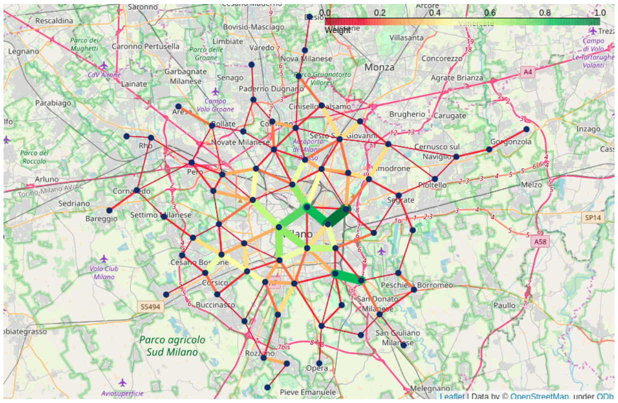


Figure 5: Cell-to-cell flow graph of the public transport network

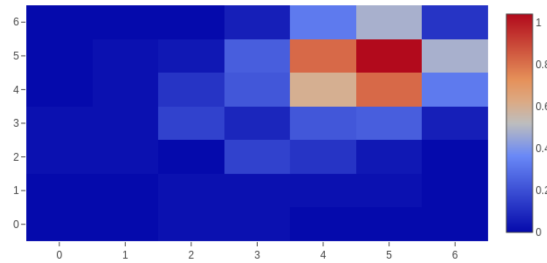


Figure 7: Degree correlation matrix of the neighbor graph

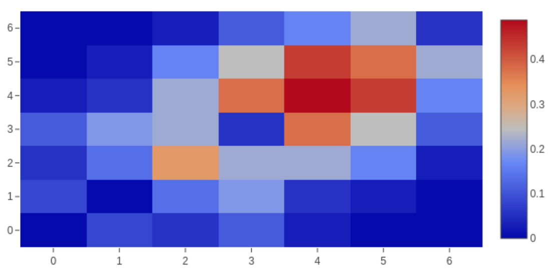


Figure 6: Degree correlation matrix of the C2C graph

The C2C graph is more assortative than the nieghbor graph as it presents a large number of nodes of the same degree connecting with each other.

Resilience to attacks and failures

In the resilience analysis we consider how the cell-to-cell flow graph reacts to different attack strategy. Clearly, this won't give a detailed insight into how each single stop reacts to failure, being an averaged graph, but on other hand, it allows us to observe how **area of effect** attack might influence the network. Such attacks might be, terrorist attacks or natural disasters, influencing a region with a radius γ. Moreover, being the γ a simple parameter, it's possible to play with it to make it fit different scenarios - it is possible to create microregions that roughly represent a single stop, although we suggest to select a γ value that's more than the minimum distance between two stops in the network, to avoid keeping disjointed routes.

The origin and scenario of an attack or cascading failures may vary wildly. It can be due to an eccessive load of the network, or be topological in nature (a node, or stop, becomes inaccessible, cutting the link between other nodes). We chose to inspect the second type of attacks, analyzing different strategies that go from random nodes elimination to eliminating nodes with the highest centrality of a certain kind.

- **Random vertex** (RV): vertices are removed in random order
- **Random neighbor** (RN): one by one, a random neighbor of a random node is removed, until the node has no neighbors
- **Degree** (C_D): vertices are removed in the graph according to their highest degree, recomputed after every elimination
- **Initial degree** (C_{Di}): vertices are removed in the graph according to their highest degree, computed once at the beginning
- **Closeness** (C_D): vertices are removed in the graph according to their highest closeness, recomputed after every elimination
- **Initial closeness** (C_{Ci}): vertices are removed in the graph according to their highest closeness, computed once at the beginning
- **Betweenness** (C_B): vertices are removed in the graph according to their highest betweenness, recomputed after every elimination
- **Initial betweenness** (C_{Bi}): vertices are removed in the graph according to their highest betweenness, computed once at the beginning
- **Clustering coefficient** (C_C): vertices are removed in the graph according to their highest clustering coefficient, recomputed after every elimination
- **Initial clustering coefficient** (C_{CLi}): vertices are removed in the graph according to their high-

est clustering coefficient, computed once at the beginning

- **Eigenvalue** (C_E): vertices are removed in the graph according to their highest eigenvalue, recomputed after every elimination
- **Initial eigenvalue** (C_{Ei}): vertices are removed in the graph according to their highest eigenvalue, computed once at the beginning
- **PageRank** (C_P): vertices are removed in the graph according to their highest pagerank, recomputed after every elimination
- **Initial PageRank** (C_{Pi}): vertices are removed in the graph according to their highest pagerank, computed once at the beginning
- **Edge betweenness** (C_{BE}): edges are removed in the graph according to their highest edge betweenness, ecomputed after every elimination
- **Initial edge betweenness** (C_{BEi}): edges are removed in the graph according to their highest edge betweenness, computed once at the beginning

Results

The results are measured by assessing the size of the giant connected component GCC, representing the largest connected component of a given network. The measure of the GCC will be called S, and be such that:

$$S = \frac{N_{GCC}}{N} * 100$$

Where N_{GCC} is the number of vertices in the GCC and N is the number of nodes in the graph. The effectiveness of the attack scenario will be judged by their impact on the value of S. In particular, we asses when the GCC reaches its percolation threshold and dissolves.

We assessed that the RV and RN scenarios are the least effective attacks conducted against the C2C graph. Different tests show how it always takes over 30 attacks for RV, and even more for RN to make the network inadoperable. Being casual in nature, we analyzed the dispersion of the RV attack. This is done to assure the effectiveness of the attack itself - being casual in nature, it's not possible to assure that one single attack represents the whole scenario. In 10 different attack scenarios, we found out that the effectiveness of the RV scenario is compressed in the interval [30, 50], securing it as the least effective scenario.

The scenarios regarding the initial list of centrality measure fared somewhat better. We obtained that by eliminating nodes with higher initial betweenness we remove nodes with a high percentage of transit in

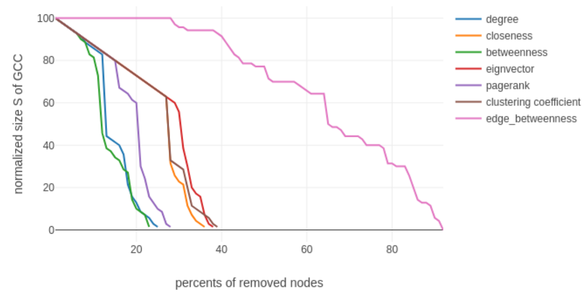


Figure 8: Graph of the centrality attack scenarios from the initial measurements

shortest paths, breaking up the network the fastest. It being a global measure plays a big role in it. Another measure with a high level of effectiveness is degree. Eigenvector, clustering and closeness are the least effective in relative terms, as they take a couple more attacks to render the network inadoperable. The edge betweenness attack is by far the least effective, taking way more attacks to break up the network. This is due to the edge removal being less impactful than the removal of a whole node.

The scenarios regarding updated centrality measure is by far the most effective. The most effective attacks are closeness and betweenness. Both are global indicators, and the recomputation allows the choosing of the most impactful node in every iteration.

In conclusion, the network is most vulnerable to **smart** strategies that try to maximize the damage for every attack, by choosing nodes that act as hubs or gateways.

The change in diameter and average shortest path was also analyzed for the most effective strategy and the least effective strategy. The information obtained from this test allows us to verify how paths change during an attack. We can assess how the diameter increases in size as the size between peripheral nodes increases, until a point into which the GCC is broken up, and the leftover bigger component decreases drastically in diameter. The average shortest path presents a similar behaviour. The more nodes get eliminated, the longer the shortest path between nodes has to become, as deviations in the shortest path must be introduced. These measures in other terms represent how long it would take, given a failure or an attack scenario, to move inside the city.

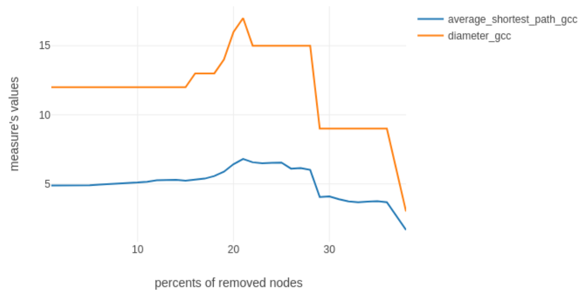


Figure 9: Diameter and average shortest path in the updating clustering coefficient scenario

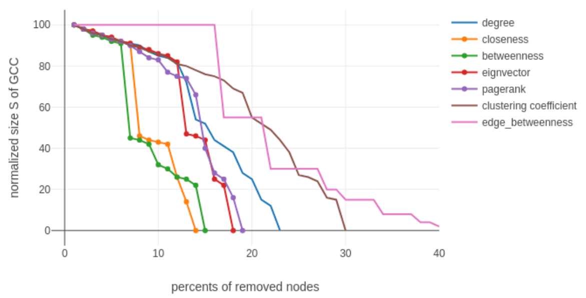
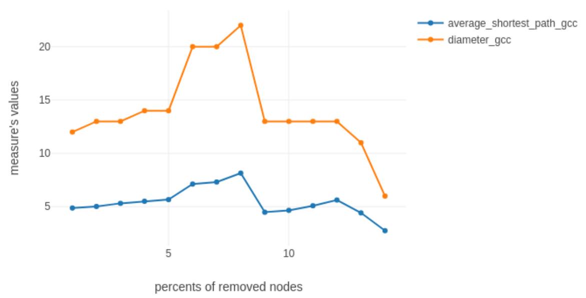


Figure 10: Graph of the centrality attack scenarios with update



Conclusions

This analysis drives two major themes:

- The usefulness of average graphs in the comparative analysis of transportation means in a city
- The resilience of Milan's public transportation network

c	N	k_avg	diameter	short_path	c_avg	b_avg	pr_avg	ev_avg	cluster	S
0	70	0.053	12	4.88	0.213	0.07	0.014	0.055	0.215	100
3	67	0.05	13	5.222	0.2	0.081	0.015	0.055	0.191	95.714
7	63	0.047	13	5.561	0.189	0.094	0.017	0.054	0.154	90.0
10	59	0.045	16	6.176	0.172	0.119	0.018	0.056	0.063	84.286
12	51	0.051	17	6.567	0.165	0.159	0.022	0.073	0.09	72.857
13	38	0.071	13	5.313	0.212	0.176	0.03	0.113	0.058	54.286
17	27	0.087	14	5.698	0.194	0.232	0.046	0.17	0.0	38.571
18	20	0.123	10	4.258	0.259	0.182	0.065	0.267	0.0	28.571
20	11	0.2	8	3.382	0.256	0.2	0.136	0.703	0.0	15.714

Figure 11: Variation of the centrality measures for the updated degree measure

For the first point, it can be said that average graphs offer great utility for analists at a lower cost than the alternative - costly data quality assessments - and can also allow for different kind of analytic explorations. This project explored strictly the topology of the network, but it is possible to analyze the network from a temporal dimension, by creating an average network for each time slice of a day, to assess the frequency of the transportations. (Adrienko and Adrienko (2011))

The second points builds on the first one, as it made possible to analyze what was initially a disjointed network in a complete fashion, avoiding erroneus assumptions that could lead to false conclusions - as we mentioned in the beginning, passengers do travel between disjointed stops.

Future works could include:

- Topological analysis of regions at different σ sizes
- Temporal analysis of traffic flows in the city
- Attack scenarios in a temporal domain
- Different city networks comparisons

References

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