⇒ Problem 1: Perceptron

x_1	x_2	class
1	1	+1
-1	-1	-1
0	0.5	-1
0.1	0.5	-1
0.2	0.2	+1
0.9	0.5	+1

⇒ Weight vector $w = [1, 1]$

In Perceptron algorithm, we will calculate $w^T x$

* If $w^T x > 0$ and class is +1 then there is no change on w

* If $w^T x < 0$ and class is -1 then again there is no change on w.

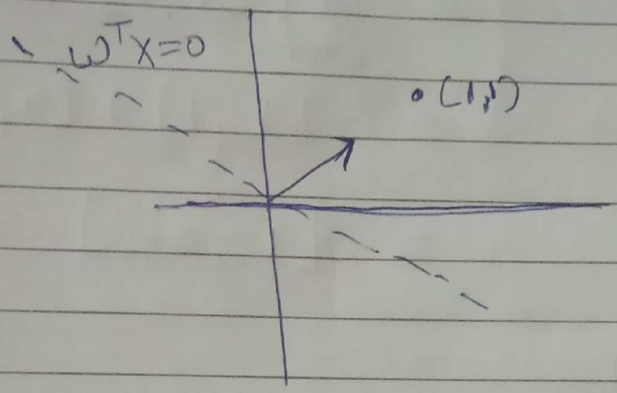
* If $w^T x < 0$ and class is +1 then w will be updated: $w = w + yx$
where $y = 1$

* If $w^T x > 0$ and class is -1
w will be updated: $w = w - yx$
where $y = 1$

1) $x_1 = 1$, $x_2 = 1$, class $= +1$

$$w^T x = [1\ 1]^T \cdot \begin{bmatrix} 1 \\ 1 \end{bmatrix}^T = 2 > 0 \text{ , class: } +1$$

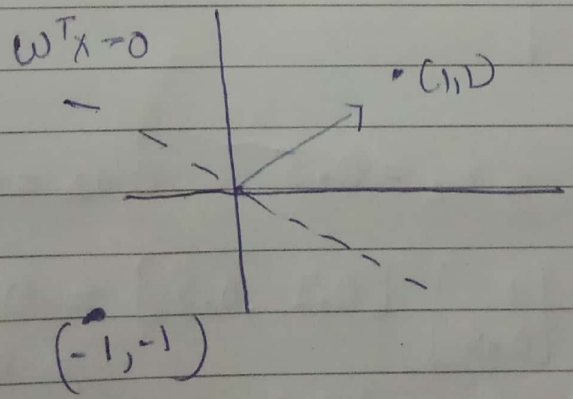
So, w remain unchanged.



2) $x_1 = -1$, $x_2 = -1$, class $= -1$

$$w^T x = [1\ 1]^T \begin{bmatrix} -1 \\ -1 \end{bmatrix}^T = -2 < 0 \text{ and class: } -1$$

So w remain unchanged.



3) $x_1 = 0, \quad x_2 = 0.5 \quad$ class $= -1$

$$w^T x = [1, 1]^T \begin{bmatrix} 0 \\ 0.5 \end{bmatrix}^T = 0.5 > 0$$ but class

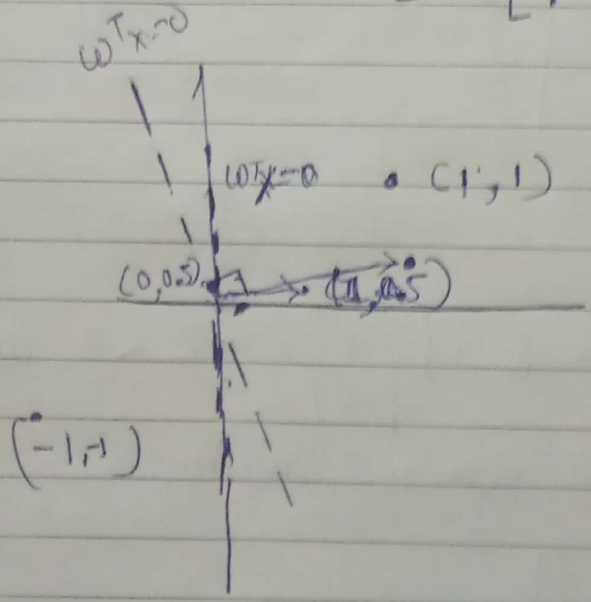
So new w will be

$$w_{new} = w_{old} + (-1)x$$

$$= [1, 1] + (-1)[0, 0.5]$$

$$= [1, 1] + [0, -0.5]$$

$$= [1, 0.5]$$



4) $x_1 = 0.1, \quad x_2 = 0.5 \quad$ class $= -1$

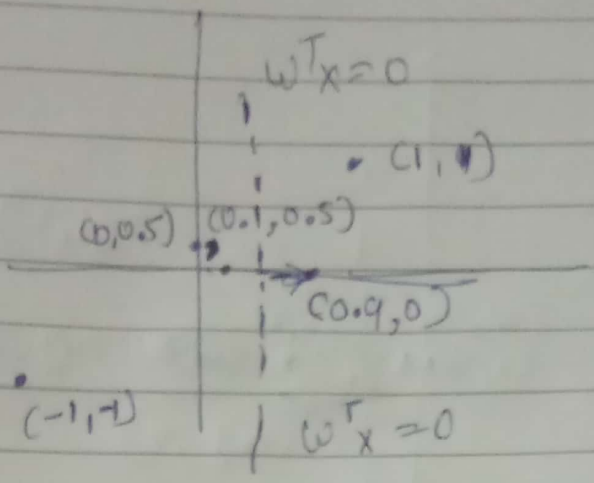
$$w^T x = \begin{bmatrix} 1 \\ 0.5 \end{bmatrix} [0.1, 0.5] = 0.1 + 0.25$$

$$= 0.35 > 0$$ but class is -1, so w will be updated

$$w_{new} = w_{old} + (-1)x$$

$$= [1, 0.5] + (-1)[0.1, 0.5]$$

$$w_{new} = [0.9, 0]$$

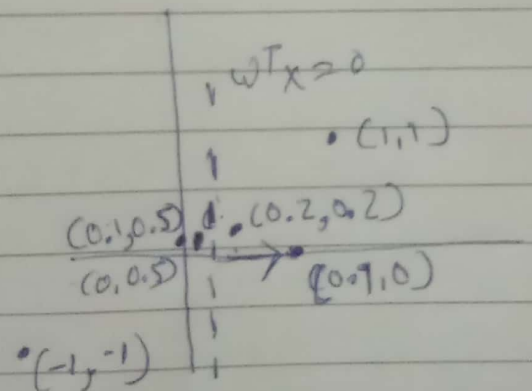


5) $x_1 = 0.2$, $x_2 = 0.2$ class $= +1$

$$w^T x = \begin{bmatrix} 0.9 \\ 0 \end{bmatrix} [0.2, 0.2] = 0.18 > 0$$

class is +1

So w is unchanged

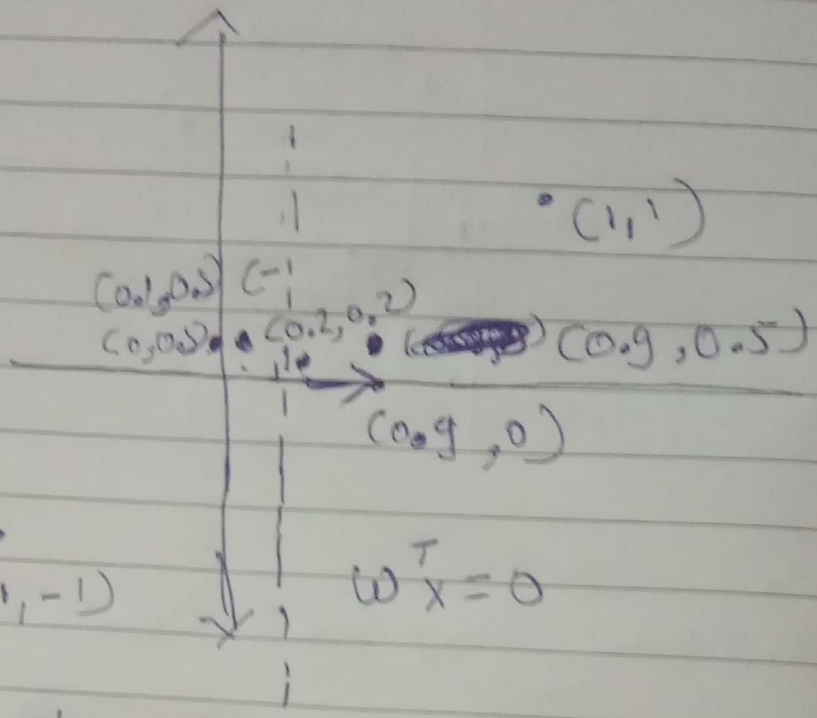


6) $x_1 = 0.9$, $x_2 = 0.5$) class $= +1$

$$w^T x = \begin{bmatrix} 0.9 \\ 0 \end{bmatrix} [0.9, 0.5)$$

$= 0.18 > 0$ and class $= +1$

So w is unchanged.



=> After step 4 W remain unchanged.
So, after 4 step perceptron algorithm converged.