

#### Path

- Sequence of edges connecting vertices
- $w_{v_1,v_n} = v_1e_1v_2 \dots v_{n-1}e_{n-1}v_n$  with  $e_i = (v_i,v_{i+1}) \in E$  ,  $1 \leq i < n$
- *n* is the length of the path

#### Simple Path

• 
$$p_{v_1,v_n} = v_1 e_1 v_2 \dots v_{n-1} e_{n-1} v_n$$
  
with  $\forall v_i, v_j : i \neq j \rightarrow v_i \neq v_j$  and  $e_i = (v_i, v_{i+1}) \in E, 1 \leq i < n$ 

#### Cvcle

- Walk with  $v_1 = v_n$  is a cycle
- Graph is acyclic if  $\exists v \in V : w_{v,v} \in G$

### Subgraph

- · A graph whose vertices and edges are fully part of a larger graph is a subgraph of that larger graph
- $G(V_G, E_G)$  is a subgraph of  $H(V_H, E_H)$  if  $V_G \subseteq V_H$  and  $E_G \subseteq E_H$
- G is a subgraph of H denoted as  $G \subseteq H$

### Vertex-/Edge-Induced Subgraph

- · Subgraph defined by a subset of vertices / edges of a given graphs
- A subgraph of  $H(V_H, E_H)$  induced by

### Cliques

- · Fully edge-connected set of vertices
- $C \subseteq V$  with  $\forall v_i, v_i \in C: (v_i, v_i) \in E$

### **Connected Components**

• Fully path-connected set of vertices, i.e. there exists a path between every pair of vertices

- $C\subseteq V$  with  $\forall v_i,v_j\in C\ \exists p\in G\colon p=v_i,\ldots,v_j$  With considering direction: strongly connected
- Without considering direction: weakly connected
  - Directed edges are treated like undirected ones

# **Basic Concepts: Measures**

### Degree (Valency)

### Degree of a vertex v

$$\deg_{in} = 2 \qquad \qquad \deg_{out} = 3$$

$$\deg(v) = \deg_{out}(v) + \deg_{in}(v)$$

### Degree of a graph G

$$\deg(G) = \max_{v \in V} \deg(v)$$

### Degree distribution

→ Probability distribution of vertex degree in a graph



### Distance

$$v \qquad d(v,u) = 2$$

→ Number of edges in a shortest path connecting two vertices

#### **Eccentricity**

$$v \qquad \qquad \epsilon(v) = 3$$

**Average Distance** → Average shortest path

→ Average eccentricity of any vertex in the graph

 $\rightarrow$  Longest shortest path starting from v

### Radius

→ Minimum eccentricity of any vertex in the graph

$$r(G) = 2$$

### Diameter

→ Maximum eccentricity of any vertex in the graph

d(G) = 4

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### Degree (Valency)

- In/out degree of a vertex: Number of incoming/outgoing edges of that vertex
  - $\deg_{out}(v) = |\{(v, u) \in E\}|$
  - $\deg_{in}(v) = |\{(u, v) \in E\}|$
  - $\deg(v) = \deg_{out}(v) + \deg_{in}(v)$
- Degree of a graph: Maximum vertex degree
  - $deg(G) = \Delta(G) = max_{v \in V} deg(v)$

### **Distance**

- Distance between two vertices in a graph is number of edges in a shortest path connecting them
- $d(v,u) = \mathsf{min}_{p_{v,u} \in G} |p_{v,u}|$

### **Eccentricity**

- Greatest distance of vertex to any other vertex
- $\epsilon(v) = \max_{u \in V} d(v, u)$

### Radius

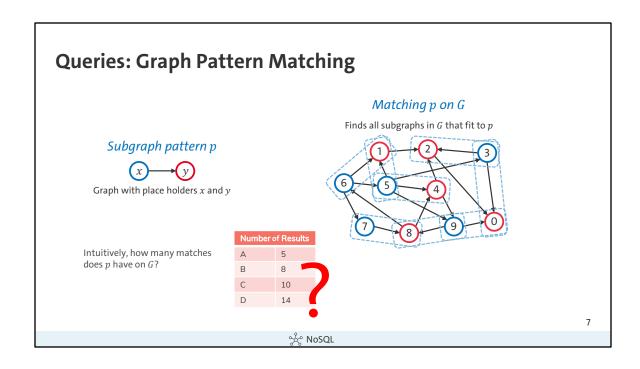
 $r(G) = \min_{v \in V} \epsilon(v) = \min_{v \in V} (\max_{u \in V} d(v, u))$ 

### Average distance

• 
$$L(G) = \frac{\sum_{v \in V} \epsilon(v)}{|V|}$$

### Diameter

 $d(G) = \max_{v \in V} \epsilon(v) = \max_{v \in V} (\max_{u \in V} d(v, u))$ 



### Pattern Syntax in neo4j: Vertex Pattern

• () unidentified vertex

(matrix) vertex identified by variable matrix
 (:Movie) unidentified vertex with label Movie

• (matrix:Movie:Action) vertex with labels *Movie* and *Action* identified by

variable *matrix* 

• (matrix:Movie {title: "The Matrix"}) + property title equal the string "The Matrix"

• (matrix:Movie {title: "The Matrix", released: 1997})

+ property *released* equal the integer 1997



These lists are not exhaustive, refer to the manual for more options. http://neo4j.com/docs/developer-manual/current/cypher/syntax/patterns/

### Pattern Syntax in neo4j: Edge Pattern

-- unidentified undirected edge (matches either direction)

--> unidentified directed edge

• -[role]-> directed edge identified by variable *role* 

• -[:ACTED\_IN]-> unidentified directed edge with label ACTED\_IN

• -[role:ACTED\_IN]-> directed edge with label ACTED\_IN identified by variable

role

• -[role:ACTED\_IN {roles: ["Neo"]}]-> + property roles contains the string "Neo"

• (a)-->(b)<--(c), (b)-->(d) branches



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### Pattern Syntax: Path pattern and Graph pattern

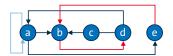
### Path patterns

- (a)-->(b)<--(c)--(d)-->(a)-->(e)
- (keanu:Person:Actor {name: "Keanu Reeves"})-[role:ACTED\_IN {roles: ["Neo"]}]-> (matrix:Movie {title: "The Matrix"})



### **Graph patterns**

- Should have at least one shared variable
- (a)-->(b)<--(c)--(d)-->(a)-->(e), (e)-->(b)-->(d), (a)-->(a)





### Path patterns

- · String of alternating vertex pattern and edge pattern
- Starting and ending with a vertex pattern

### **Graph Patterns**

- One or multiple path patterns
- · Path patterns should have at least one shared variable
- · Without shared variable graph pattern is disconnected
  - Results in a cross-product of the results for connected sub patterns
  - · Quadrating blow up in result size and computational complexity

# **Exercise: Pattern Syntax I**

Which of these are valid patterns in Neo4j?

```
A: (a,b:Movie)-[:SHOWN_IN]->(e),(f)
```

B: (a:Movie)-[:SHOWN\_IN]->()

C: (:Movie)-[:SHOWN\_IN]->

D: ()<--(a:Movie)

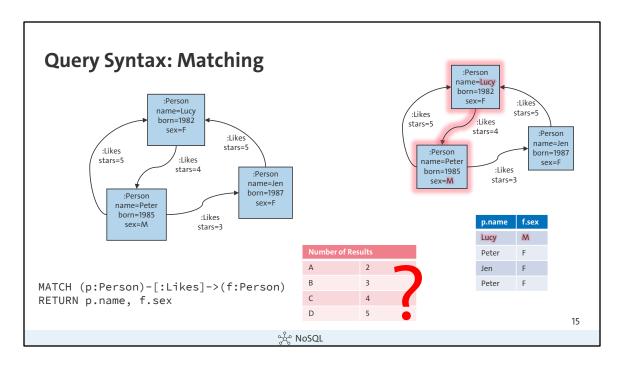


## **Exercise: Pattern Syntax II**

Which pattern specifies a loop?

```
A: (a:Movie {name: "Matrix"})-->(a)
B: (a:Movie {name: "Matrix"})-->(b:Movie {name: "Matrix"})
C: (a:Movie {name: "Matrix"})-->(a:Movie {name: "Matrix"})
D: (a:Movie {name: "Matrix"})-->({name: "Matrix"})
```

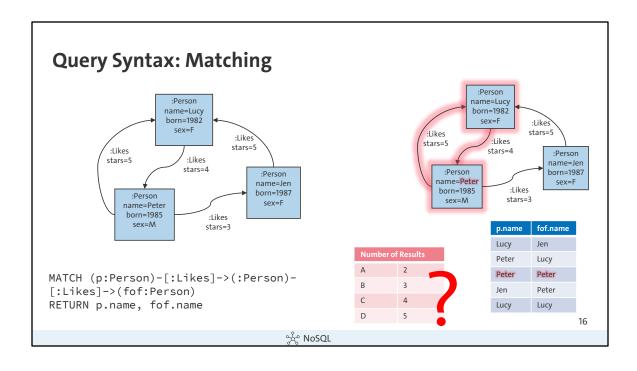
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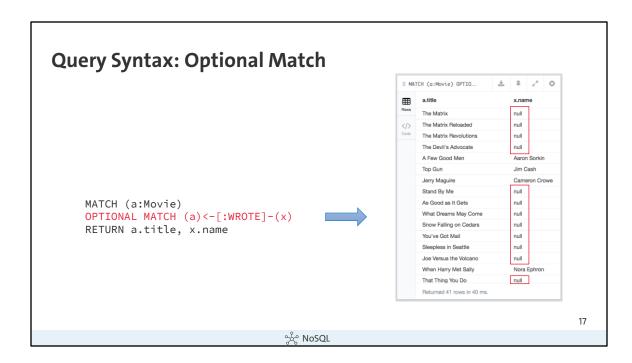


http://neo4j.com/docs/developer-manual/current/cypher/clauses/match/http://neo4j.com/docs/developer-manual/current/cypher/clauses/return/

### **MATCH Clause**

- Primary way of getting data from a Neo4j database
- Allows you to specify the patterns
- Named pattern element, e.g. (p:Person), will be bound to the match instance
- Query can have multiple MATCH clauses

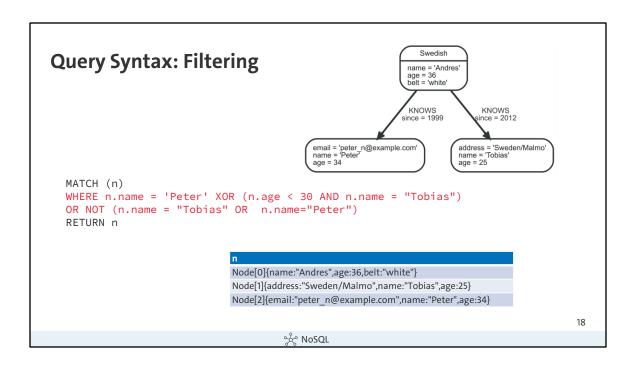




http://neo4j.com/docs/developer-manual/current/cypher/clauses/optional-match/

### **OPTIONAL MATCH clause**

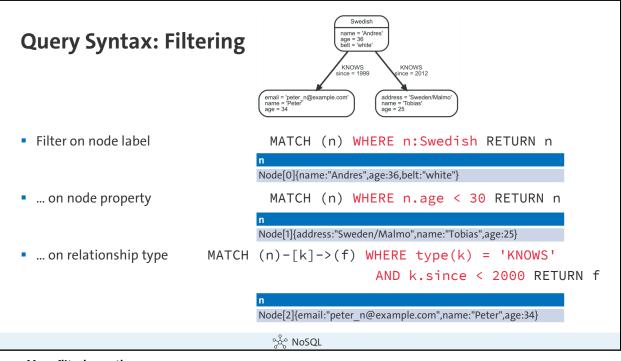
- Matches patterns against your graph database, just like MATCH
- Matches the complete pattern or not
- If no matches are found, OPTIONAL MATCH will use NULLs as bindings
- Like relational outer join



http://neo4j.com/docs/developer-manual/current/cypher/clauses/where/

### **WHERE** clause

- After an (OPTIONAL) MATCH, it adds constraints to the (optional) match
- · WHERE becomes part of the pattern
- After a WITH, it just filters the result
- Syntax: WHERE <expression>



### More filtering options

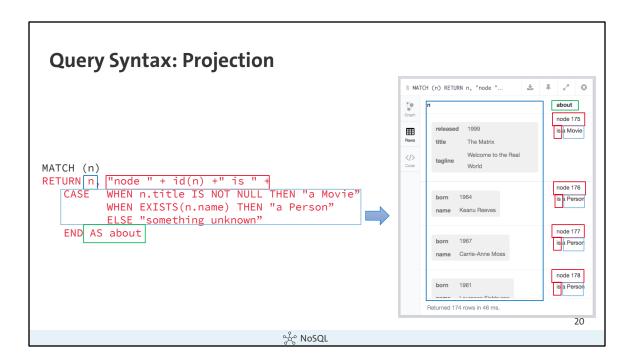
```
On Lists
MATCH (n) WHERE a.name IN ["Peter", "Tobias"] RETURN n
Node[1]{address:"Sweden/Malmo",name:"Tobias",age:25}
Node[2]{email:"peter_n@example.com",name:"Peter",age:34}
On string properties
MATCH (n) WHERE n.name = 'Peter' RETURN n
- prefixes
                      MATCH (n) WHERE n.name STARTS WITH 'Pet' RETURN n

    suffixes

                      MATCH (n) WHERE n.name ENDS WITH 'ter' RETURN n
                      MATCH (n) WHERE n.name CONTAINS 'ete' RETURN n
- infixes
                      MATCH (n) WHERE n.name =~ 'P[et]+r?' RETURN n
-regular expressions
Node[2]{email:"peter_n@example.com",name:"Peter",age:34}
On property (non-)existence
MATCH (n) WHERE exists(n.belt) RETURN n
                                 MATCH (n) WHERE NOT n.belt = 'white' RETURN n
<u>f</u>alse is default for missing values
Node[0]{name:"Andres",age:36,belt:"white"}
MATCH (n) WHERE NOT exists(n.belt) RETURN n
MATCH (n) WHERE n.belt IS NULL RETURN n
```

```
Node[1]{address:"Sweden/Malmo",name:"Tobias",age:25}

Using patterns for filtering
MATCH (tobias { name: 'Tobias' }),(others)
WHERE others.age > 30 AND (tobias)<--(others)
RETURN others
Node[0]{name:"Andres",age:36,belt:"white"}
```



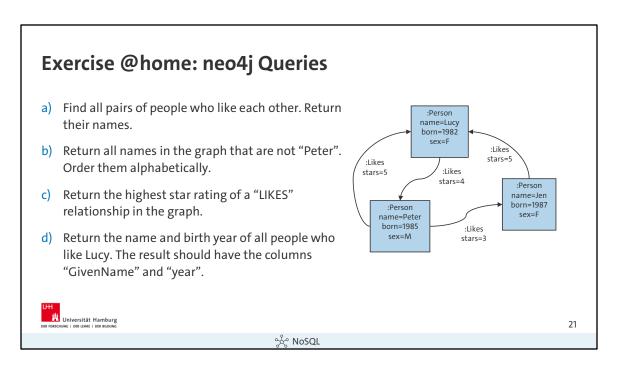
http://neo4j.com/docs/developer-manual/current/cypher/clauses/return/http://neo4j.com/docs/developer-manual/current/cypher/syntax/expressions/

### **RETURN** clause

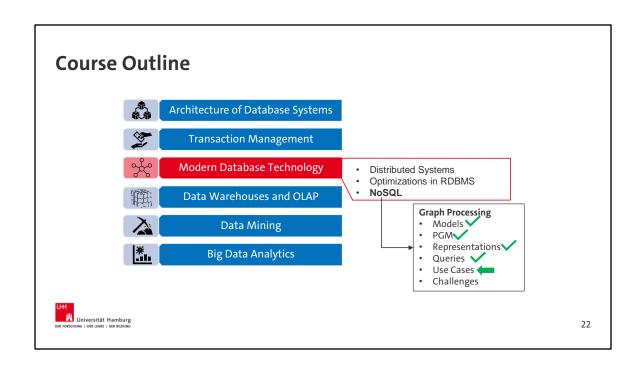
- · Defines what to include in the query result set
- Comparable with relational projection
- · Only once per query
- · Allows to return nodes, edges, properties, or any expressions
- · Column can be rename using AS < new name >

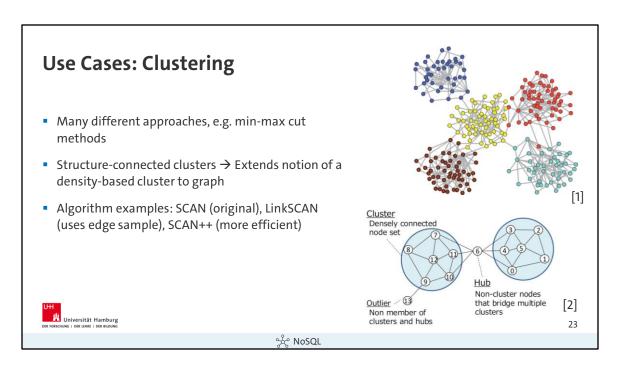
### More clauses adapted from SQL

- ORDER BY, e.g. MATCH (n) RETURN n ORDER BY n.name DESC
- LIMIT, e.g. MATCH (n) RETURN n ORDER BY n.name LIMIT 3
- SKIP, e.g. MATCH (n) RETURN n ORDER BY n.name SKIP 3
- Unique results, e.g. MATCH (a { name: "A" })-->(b) RETURN DISTINCT b
- Aggregations, e.g. MATCH (n) RETURN count(\*)



Solution: White on white





### Clustering

Division of a graph into a set of subgraphs such that there are a dense set of edges within every cluster and few edges between clusters

### Sources:

- [1] https://www.ncbi.nlm.nih.gov/pmc/articles/PMC2542420/
- [2] https://www.slideshare.net/LazyShion/scan-efficient-algorithm-for-finding-clusters-hubs-and-outliers-on-largescale-graphs-vldb-2015

The SCAN paper: Xu et al. SCAN: A Structural Clustering Algorithm for Networks. KDD. 2007

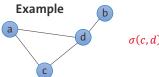
### A very simplified description of SCAN

- Neigborhood of a vertex: Each vertex that is connected via an edge and the vertex itself
- Distance (specific to SCAN and different from our general definition on slide 6): Intersection
  of the neigborhoods of two vertices
- Normalized distance: Distance normalized by the geometric mean of the size of the two neighborhoods

 $\sigma(v,w) = \frac{|\Gamma(v) \cap \Gamma(w)|}{\sqrt{|\Gamma(v)| \cdot |\Gamma(w)|}}$ 

Normalized distance between v and w Neighborhood intersection between v and w Sizes of the neighborhoods

- Structural Similarity: If  $\sigma(v,w) > \epsilon$ , v and w are considered structurally similar
- Core: Vertices with more than μ structural similar neighbors are core vertices

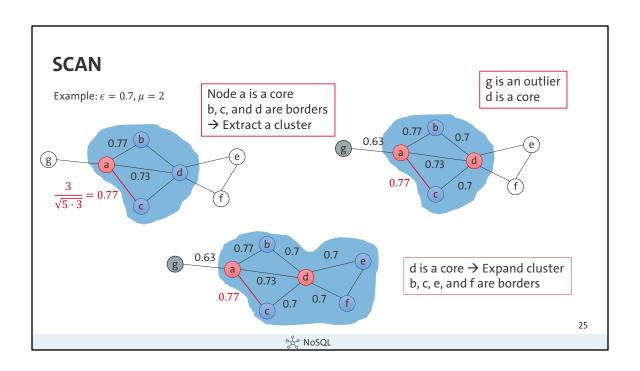


 $\sigma(c,d) = \frac{3}{\sqrt{3\cdot 4}} = 0.866$ 

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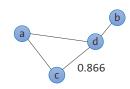
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- If a core is found, the cluster is expanded
- · All structurally similar vertices that are reachable from a core, are initially a border
- · If a vertex is not part of a cluster and all it's neighbors either belong to one or no cluster, it is an outlier
- If a vertex is not part of a cluster but has neighbors in two or more clusters, it is a hub



# **Exercise Clustering**

$$\epsilon=0.8$$
,  $\mu=2$ 



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