1 Random Walk

We investigate the gamblers ruin problem on a random walk via the first step analysis, we define the probability of getting to given state conditioned by the current position, this is a function depending on the current position

$$f(k) = p \cdot f(k+1) + q \cdot f(k-1).$$

Then using the increment operator $\Delta f(k) = f(k+1) - f(k)$ we can rewrite the equation as

$$\Delta f(k) = (q/p)\Delta f(k-1).$$

Then by iterating it and using the boundaries $f(A) = 1$ and $f(-B) = -1$ then we get to

$$P(S_t \text{ hits A before -B} | S_0 = 0) = \frac{(q/p)^B - 1}{(q/p)^{A+B} - 1}$$

This same strategy can be appied to compute the expected time to hit the boundaries.

We also use the probability generating function and the first step analysis to find the expected time of the game on gamblers ruin.

$$\phi(z) = E[z^X] = \sum_{x=0}^{\infty} p(x) z^x$$