Table 1: Mathematical definitions of complex network measures (see main text for an informal discussion). All binary and undirected measures are accompanied by their weighted and directed generalizations. Generalizations which have not been previously reported (to the best of our knowledge) are marked with a star (*). The Brain Connectivity Toolbox contains Matlab functions to compute most measures in this table.

Measure	Binary and undirected definitions	Weighted and directed definitions
Basic concepts and measures		
Basic concepts and notation	N is the set of all nodes in the network, and n is the number of nodes. L is the set of all links in the network, and l is number of links. (i,j) is a link between nodes i and j $(i,j \in N)$. a_{ij} is the connection status between i and j: $a_{ij} = 1$ when link (i,j) exists (when i and j are neighbors); $a_{ij} = 0$ otherwise ($a_{ii} = 0$ for all i). We compute the number of links as $l = \sum_{i,j \in N} a_{ij}$ (to avoid ambiguity with directed links we count each undirected link twice, as a_{ij} and as a_{ji}).	Links (i,j) are associated with connection weights w_{ij}. Henceforth we assume that weights are normalized, such that $0 \leq w_{ij} \leq 1$ for all i and j. l^{w} is the sum of all weights in the network, computed as $l^{\mathrm{w}} = \sum_{i,j \in N} w_{ij}$. Directed links (i,j) are ordered from i to j. Consequently, in directed networks a_{ij} does not necessarily equal a_{ji}.
Degree: number of links connected to a node	Degree of a node i, $$k_i = \sum_{j \in N} a_{ij}.$$	Weighted degree of i, $k_i^{\mathrm{w}} = \sum_{j \in N} w_{ij}$. (Directed) out-degree of i, $k_i^{\mathrm{out}} = \sum_{j \in N} a_{ij}$. (Directed) in-degree of i, $k_i^{\mathrm{in}} = \sum_{j \in N} a_{ji}$.
Shortest path length: a basis for measuring integration	Shortest path length (distance), between nodes i and j, $$d_{ij} = \sum_{a_{uv} \in g_{i \leftrightarrow j}} a_{uv},$$ where $g_{i \leftrightarrow j}$ is the shortest path (geodesic) between i and j. Note that $d_{ij} = \infty$ for all disconnected pairs i, j.	Shortest weighted path length between i and j, $d_{ij}^{\mathrm{w}} = \sum_{a_{uv} \in g_{i \leftrightarrow j}^{\mathrm{w}}} f(w_{uv})$, where f is a map (e.g. an inverse) from weight to length and $g_{i \leftrightarrow j}^{\mathrm{w}}$ is the shortest weighted path between i and j. Shortest directed path length from i to j, $d_{ij}^{\rightarrow} = \sum_{a_{ij} \in g_{i \rightarrow j}} a_{ij}$, where $g_{i \rightarrow j}$ is the directed shortest path from i to j.
Number of triangles: a basis for measuring segregation	Number of triangles around a node i, $$t_i = \frac{1}{2} \sum_{j,h \in N} a_{ij} a_{ih} a_{jh}.$$	(Weighted) geometric mean of triangles around i, $t_i^{\mathrm{w}} = \frac{1}{2} \sum_{j,h \in N} \left(w_{ij} w_{ih} w_{jh}\right)^{1/3}$. Number of directed triangles around i, $t_i^{\rightarrow} = \frac{1}{2} \sum_{j,h \in N} \left(a_{ij} + a_{ji}\right)\left(a_{ih} + a_{hi}\right)\left(a_{jh} + a_{hj}\right)$.
Measures of integration		
Characteristic path length	Characteristic path length of the network (e.g. Watts and Strogatz, 1998), $$L = \frac{1}{n} \sum_{i \in N} L_i = \frac{1}{n} \sum_{i \in N} \frac{\sum_{j \in N, j \neq i} d_{ij}}{n-1},$$ where L_i is the average distance between node i and all other nodes.	Weighted characteristic path length, $L^{\mathrm{w}} = \frac{1}{n} \sum_{i \in N} \frac{\sum_{j \in N, j \neq i} d_{ij}^{\mathrm{w}}}{n-1}$. Directed characteristic path length, $L^{\rightarrow} = \frac{1}{n} \sum_{i \in N} \frac{\sum_{j \in N, j \neq i} d_{ij}^{\rightarrow}}{n-1}$.

Global efficiency	Global efficiency of the network (Latora and Marchiori, 2001), $E = \frac{1}{n}\sum_{i\in N} E_i = \frac{1}{n}\sum_{i\in N}\frac{\sum_{j\in N, j\neq i} d_{ij}^{-1}}{n-1}$, where E_i is the efficiency of node i.	Weighted global efficiency, $E^{\mathrm{w}} = \frac{1}{n}\sum_{i\in N}\frac{\sum_{j\in N,j\neq i}\left(d_{ij}^{\mathrm{w}}\right)^{-1}}{n-1}$. Directed global efficiency, $E^{\rightarrow} = \frac{1}{n}\sum_{i\in N}\frac{\sum_{j\in N,j\neq i}\left(d_{ij}^{\rightarrow}\right)^{-1}}{n-1}$.
Measures of segregation		
Clustering coefficient	Clustering coefficient of the network (Watts and Strogatz, 1998), $C = \frac{1}{n}\sum_{i\in N} C_i = \frac{1}{n}\sum_{i\in N}\frac{2t_i}{k_i(k_i-1)}$, where C_i is the clustering coefficient of node i ($C_i = 0$ for $k_i < 2$).	Weighted clustering coefficient (Onnela et al., 2005), $C^{\mathrm{w}} = \frac{1}{n}\sum_{i\in N}\frac{2t_i^{\mathrm{w}}}{k_i(k_i-1)}$. See Saramaki et al. (2007) for other variants. Directed clustering coefficient (Fagiolo, 2007), $C^{\rightarrow} = \frac{1}{n}\sum_{i\in N}\frac{t_i^{\rightarrow}}{(k_i^{\mathrm{out}}+k_i^{\mathrm{in}})(k_i^{\mathrm{out}}+k_i^{\mathrm{in}}-1)-2\sum_{j\in N} a_{ij}a_{ji}}$.
Transitivity	Transitivity of the network (e.g. Newman, 2003), $T = \frac{\sum_{i\in N} 2t_i}{\sum_{i\in N} k_i(k_i-1)}$. Note that transitivity is not defined for individual nodes.	Weighted transitivity*, $T^{\mathrm{w}} = \frac{\sum_{i\in N} 2t_i^{\mathrm{w}}}{\sum_{i\in N} k_i(k_i-1)}$. Directed transitivity*, $T^{\rightarrow} = \frac{\sum_{i\in N} t_i^{\rightarrow}}{\sum_{i\in N}\left[(k_i^{\mathrm{out}}+k_i^{\mathrm{in}})(k_i^{\mathrm{out}}+k_i^{\mathrm{in}}-1)-2\sum_{j\in N} a_{ij}a_{ji}\right]}$.
Local efficiency	Local efficiency of the network (Latora and Marchiori, 2001), $E_{\mathrm{loc}} = \frac{1}{n}\sum_{i\in N} E_{\mathrm{loc},i} = \frac{1}{n}\sum_{i\in N}\frac{\sum_{j,h\in N,j\neq i} a_{ij}a_{ih}\left[d_{jh}(N_i)\right]^{-1}}{k_i(k_i-1)}$, where $E_{\mathrm{loc},i}$ is the local efficiency of node i, and $d_{jh}(N_i)$ is the length of the shortest path between j and h, that contains only neighbors of i.	Weighted local efficiency*, $E_{\mathrm{loc}}^{\mathrm{w}} = \frac{1}{n}\sum_{i\in N}\frac{\sum_{j,h\in N,j\neq i}\left(w_{ij}w_{ih}\left[d_{jh}^{\mathrm{w}}(N_i)\right]^{-1}\right)^{1/3}}{k_i(k_i-1)}$. Directed local efficiency*, $E_{\mathrm{loc}}^{\rightarrow} = \frac{1}{2n}\sum_{i\in N}\frac{\sum_{j,h\in N,j\neq i}(a_{ij}+a_{ji})(a_{ih}+a_{hi})\left(\left[d_{jh}^{\rightarrow}(N_i)\right]^{-1}+\left[d_{hj}^{\rightarrow}(N_i)\right]^{-1}\right)}{(k_i^{\mathrm{out}}+k_i^{\mathrm{in}})(k_i^{\mathrm{out}}+k_i^{\mathrm{in}}-1)-2\sum_{j\in N} a_{ij}a_{ji}}$.
Modularity	Modularity of the network (Newman, 2004b), $Q = \sum_{u\in M}\left[e_{uu} - \left(\sum_{v\in M} e_{uv}\right)^2\right]$, where the network is fully subdivided into a set of nonoverlapping modules M, and e_{uv} is the proportion of all links that connect nodes in module u with nodes in module v. An equivalent alternative formulation of the modularity (Newman, 2006) is given by $Q = \frac{1}{l}\sum_{i,j\in N}\left(a_{ij} - \frac{k_ik_j}{l}\right)\delta_{m_i,m_j}$, where m_i is the module containing node i, and $\delta_{m_i,m_j} = 1$ if $m_i = m_j$, and 0 otherwise.	Weighted modularity (Newman, 2004), $Q^{\mathrm{w}} = \frac{1}{l^{\mathrm{w}}}\sum_{i,j\in N}\left[w_{ij} - \frac{k_i^{\mathrm{w}}k_j^{\mathrm{w}}}{l^{\mathrm{w}}}\right]\delta_{m_i,m_j}$. Directed modularity (Leicht and Newman, 2008), $Q^{\rightarrow} = \frac{1}{l}\sum_{i,j\in N}\left[a_{ij} - \frac{k_i^{\mathrm{out}}k_i^{\mathrm{in}}}{l}\right]\delta_{m_i,m_j}$.

Measures of centrality		
Closeness centrality	Closeness centrality of node i (e.g. Freeman, 1978), $L_i^{-1} = \frac{n-1}{\sum_{j\in N, j\neq i} d_{ij}}$.	Weighted closeness centrality, $(L_i^{\mathrm{w}})^{-1} = \frac{n-1}{\sum_{j\in N, j\neq i} d_{ij}^{\mathrm{w}}}$. Directed closeness centrality, $(L_i^{\rightarrow})^{-1} = \frac{n-1}{\sum_{j\in N, j\neq i} d_{ij}^{\rightarrow}}$.
Betweenness centrality	Betweenness centrality of node i (e.g. Freeman, 1978), $b_i = \frac{1}{(n-1)(n-2)} \sum_{\substack{h,j\in N \\ h\neq j, h\neq i, j\neq i}} \frac{\rho_{hj}(i)}{\rho_{hj}}$, where ρ_{hj} is the number of shortest paths between h and j, and $\rho_{hj}(i)$ is the number of shortest paths between h and j that pass through i.	Betweenness centrality is computed equivalently on weighted and directed networks, provided that path lengths are computed on respective weighted or directed paths.
Within-module degree z-score	Within-module degree z-score of node i (Guimera and Amaral, 2005), $z_i = \frac{k_i(m_i) - \bar{k}(m_i)}{\sigma^{k(m_i)}}$, where m_i is the module containing node i, $k_i(m_i)$ is the within-module degree of i (the number of links between i and all other nodes in m_i), and $\bar{k}(m_i)$ and $\sigma^{k(m_i)}$ are the respective mean and standard deviation of the within-module m_i degree distribution.	Weighted within-module degree z-score, $z_i^{\mathrm{w}} = \frac{k_i^{\mathrm{w}}(m_i) - \bar{k}^{\mathrm{w}}(m_i)}{\sigma^{k^{\mathrm{w}}(m_i)}}$. Within-module out-degree z-score, $z_i^{\mathrm{out}} = \frac{k_i^{\mathrm{out}}(m_i) - \bar{k}^{\mathrm{out}}(m_i)}{\sigma^{k^{\mathrm{out}}(m_i)}}$. Within-module in-degree z-score, $z_i^{\mathrm{in}} = \frac{k_i^{\mathrm{in}}(m_i) - \bar{k}^{\mathrm{in}}(m_i)}{\sigma^{k^{\mathrm{in}}(m_i)}}$.
Participation coefficient	Participation coefficient of node i (Guimera and Amaral, 2005), $y_i = 1 - \sum_{m\in M} \left(\frac{k_i(m)}{k_i}\right)^2$, where M is the set of modules (see modularity), and $k_i(m)$ is the number of links between i and all nodes in module m.	Weighted participation coefficient, $y_i^{\mathrm{w}} = 1 - \sum_{m\in M} \left(\frac{k_i^{\mathrm{w}}(m)}{k_i^{\mathrm{w}}}\right)^2$ Out-degree participation coefficient, $y_i^{\mathrm{out}} = 1 - \sum_{m\in M} \left(\frac{k_i^{\mathrm{out}}(m)}{k_i^{\mathrm{out}}}\right)^2$. In-degree participation coefficient, $y_i^{\mathrm{in}} = 1 - \sum_{m\in M} \left(\frac{k_i^{\mathrm{in}}(m)}{k_i^{\mathrm{in}}}\right)^2$.

Network motifs		
Anatomical and functional motifs	J_h is the number of occurrences of motif h in all subsets of the network (subnetworks). h is an n_h node, l_h link, directed connected pattern. h will occur as an anatomical motif in an n_h node, l_h link subnetwork, if links in the subnetwork match links in h (Milo et al., 2002). h will occur (possibly more than once) as a functional motif in an n_h node, $l_h' \geq l_h$ link subnetwork, if at least one combination of l_h links in the subnetwork matches links in h (Sporns and Kötter, 2004).	(Weighted) intensity of h (Onnela et al., 2005), $I_h = \sum_u \left(\prod_{(i,j)\in L_{h^u}} w_{ij}\right)^{\frac{1}{l_h}}$, where the sum is over all occurrences of h in the network, and L_{h^u} is the set of links in the uth occurrence of h. Note that motifs are directed by definition.
Motif z-score	z-score of motif h (Milo, 2002), $z_h = \frac{J_h - \langle J_{\mathrm{rand},h}\rangle}{\sigma^{J_{\mathrm{rand},h}}}$, where $\langle J_{\mathrm{rand},h}\rangle$ and $\sigma^{J_{\mathrm{rand},h}}$ are the respective mean and standard deviation for the number of occurrences of h in an ensemble of random networks.	Intensity z-score of motif h (Onnela et al., 2005), $z_h^I = \frac{I_h - \langle I_{\mathrm{rand},h}\rangle}{\sigma^{I_{\mathrm{rand},h}}}$, where $\langle I_{\mathrm{rand},h}\rangle$ and $\sigma^{I_{\mathrm{rand},h}}$ are the respective mean and standard deviation for the intensity of h in an ensemble of random networks.

Motif fingerprint	n_h node motif fingerprint of the network (Sporns and Kötter, 2004), $F_{n_h}(h') = \sum_{i\in N} F_{n_h,i}(h') = \sum_{i\in N} J_{h',i}$, where h' is any n_h node motif, $F_{n_h,i}(h')$ is the n_h node motif fingerprint for node i, and $J_{h',i}$ is the number of occurrences of motif h' around node i.	n_h node motif intensity fingerprint of the network, $F^I_{n_h}(h') = \sum_{i\in N} F^I_{n_h,i}(h') = \sum_{i\in N} I_{h',i}$, where h' is any n_h node motif, $F^I_{n_h,i}(h')$ is the n_h node motif intensity fingerprint for node i, and $I_{h',i}$ is the intensity of motif h' around node i.
Measures of resilience		
Degree distribution	Cumulative degree distribution of the network (Barabasi and Albert, 1999), $P(k) = \sum_{k' \geq k} p(k')$, where $p(k')$ is the probability of a node having degree k'.	Cumulative weighted degree distribution, $P(k^{\mathrm{w}}) = \sum_{k' \geq k^{\mathrm{w}}} p(k')$, Cumulative out-degree distribution, $P(k^{\mathrm{out}}) = \sum_{k' \geq k^{\mathrm{out}}} p(k')$. Cumulative in-degree distribution, $P(k^{\mathrm{in}}) = \sum_{k' \geq k^{\mathrm{in}}} p(k')$.
Average neighbor degree	Average degree of neighbors of node i (Pastor-Sattoras et al., 2001), $k_{\mathrm{nn},i} = \frac{\sum_{j\in N} a_{ij} k_j}{k_i}$.	Average weighted neighbor degree (modified from Barrat et al., 2004), $k^{\mathrm{w}}_{\mathrm{nn},i} = \frac{\sum_{j\in N} w_{ij} k^{\mathrm{w}}_j}{k^{\mathrm{w}}_i}$. Average directed neighbor degree*, $k^{\rightarrow}_{\mathrm{nn},i} = \frac{\sum_{j\in N}(a_{ij}+a_{ji})(k_i^{\mathrm{out}}+k_i^{\mathrm{in}})}{2(k_i^{\mathrm{out}}+k_i^{\mathrm{in}})}$.
Assortativity coefficient	Assortativity coefficient of the network (Newman, 2002), $r = \frac{l^{-1}\sum_{(i,j)\in L} k_i k_j - \left[l^{-1}\sum_{(i,j)\in L}\frac{1}{2}(k_i+k_j)\right]^2}{l^{-1}\sum_{(i,j)\in L}\frac{1}{2}(k_i^2+k_j^2) - \left[l^{-1}\sum_{(i,j)\in L}\frac{1}{2}(k_i+k_j)\right]^2}$.	Weighted assortativity coefficient (modified from Leung and Chau, 2007), $r^{\mathrm{w}} = \frac{l^{-1}\sum_{(i,j)\in L} w_{ij} k_i^{\mathrm{w}} k_j^{\mathrm{w}} - \left[l^{-1}\sum_{(i,j)\in L}\frac{1}{2} w_{ij}(k_i^{\mathrm{w}}+k_j^{\mathrm{w}})\right]^2}{l^{-1}\sum_{(i,j)\in L}\frac{1}{2} w_{ij}\left((k_i^{\mathrm{w}})^2+(k_j^{\mathrm{w}})^2\right) - \left[l^{-1}\sum_{(i,j)\in L}\frac{1}{2} w_{ij}(k_i^{\mathrm{w}}+k_j^{\mathrm{w}})\right]^2}$. Directed assortativity coefficient (Newman, 2002), $r^{\rightarrow} = \frac{l^{-1}\sum_{(i,j)\in L} k_i^{\mathrm{out}} k_j^{\mathrm{in}} - \left[l^{-1}\sum_{(i,j)\in L}\frac{1}{2}(k_i^{\mathrm{out}}+k_j^{\mathrm{in}})\right]^2}{l^{-1}\sum_{(i,j)\in L}\frac{1}{2}\left[(k_i^{\mathrm{out}})^2+(k_j^{\mathrm{in}})^2\right] - \left[l^{-1}\sum_{(i,j)\in L}\frac{1}{2}(k_i^{\mathrm{out}}+k_j^{\mathrm{in}})\right]^2}$.
Other concepts		
Degree distribution preserving network randomization.	Degree-distribution preserving randomization is implemented by iteratively choosing four distinct nodes $i_1, j_1, i_2, j_2 \in N$ at random, such that links $(i_1, j_1), (i_2, j_2) \in L$, while links $(i_1, j_2), (i_2, j_1) \notin L$. The links are then rewired such that $(i_1, j_2), (i_2, j_1) \in L$ and $(i_1, j_1), (i_2, j_2) \notin L$ (Maslov and Sneppen, 2002). "Latticization" (a lattice-like topology) results if an additional constraint is imposed, $\lvert i_1 + j_2\rvert + \lvert i_2 + j_1\rvert < \lvert i_1 + j_1\rvert + \lvert i_2 + j_2\rvert$ (Sporns and Kötter, 2004).	The algorithm is equivalent for weighted and directed networks. In weighted networks, weights may be switched together with links; in this case the weighted degree distribution is not preserved, but may be subsequently approximated on the topologically randomized graph with a heuristic weight reshuffling scheme.
Measure of network small-worldness.	Network small-worldness (Humphries et al., 2008), $S = \frac{C/C_{\mathrm{rand}}}{L/L_{\mathrm{rand}}}$, where C and C_{rand} are the clustering coefficients, and L and L_{rand} are the characteristic path lengths of the respective tested network and a random network. Small-world networks often have $S \gg 1$.	Weighted network small-worldness, $S^{\mathrm{w}} = \frac{C^{\mathrm{w}}/C^{\mathrm{w}}_{\mathrm{rand}}}{L^{\mathrm{w}}/L^{\mathrm{w}}_{\mathrm{rand}}}$. Directed network small-worldness, $S^{\rightarrow} = \frac{C^{\rightarrow}/C^{\rightarrow}_{\mathrm{rand}}}{L^{\rightarrow}/L^{\rightarrow}_{\mathrm{rand}}}$. In both cases, small-world networks often have $S \gg 1$.