A framework for the construction of the fiber directions in vessels

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1. Centerline computation

Centerline can be defined as the line drawn between two sections of a tubular structure which maximize the distance from the boundary. The problem of centerline computation inside an object $\Omega \subset \Re^3$ can therefore be formulated as looking for a path $\mathbf{C} = \mathbf{C}(s)$ traced between two points $\mathbf{p}_1$ and $\mathbf{p}_2$ for which the following functional

$$E_{\text{centerline}}(\mathbf{C}) = \int_{0=\mathbf{C}^{-1}(\mathbf{p}_0)}^{L=\mathbf{C}^{-1}(\mathbf{p}_1)} F(\mathbf{C}(s))ds$$

is minimal, where $F(\mathbf{x})$ is a scalar field which is lower for more internal points, for example a decreasing function of the distance transform associated with Ω , defined as

$$\mathrm{DT}(\mathbf{x}) = \min_{\mathbf{y}\in\partial\Omega}\{|\mathbf{x}, \mathbf{y}|\}$$

where $|,|$ denotes the Euclidean distance, and $\partial\Omega$ the boundary of Ω . It is possible to demonstrate [7] that by choosing $F(\mathbf{x}) = \mathrm{DT}^{-1}(\mathbf{x})$, centerlines defined as in Equation 1 lie on the medial axis of Ω, $\mathrm{MA}(\Omega)$, defined as the locus of centers of the maximal inscribed balls in Ω , where an inscribed ball is maximal if it is not strictly contained in any other inscribed ball. Dealing with piecewise linear approximations of $\partial\Omega$, a method to obtain an approximation of the medial axis of Ω is to compute the embedded Voronoi diagram of a point set P densely sampling $\partial\Omega$ [2]. The Voronoi diagram of P is defined as

$$\mathrm{Vor}(P) = \bigcup_{\mathbf{p}\in P} \partial V(\mathbf{p})$$

where $V(\mathbf{p})$ is the Voronoi region associated with point $\mathbf{p}$, defined as

$$V(\mathbf{p}) = \left\{\mathbf{x} \in \Re^3 : |\mathbf{p}, \mathbf{x}| \leq |\mathbf{q}, \mathbf{x}| \forall \mathbf{q} \in P\right\}$$

In 3D, the Voronoi diagram is a non-manifold surface made up of convex polygons, whose vertices are the centers of the maximal emtpy balls with respect to point set P , whose radius is indicated by $R(\mathbf{x})$. Computation of the embedded Voronoi diagram was performed by first computing the Delaunay tessellation of $P, \mathrm{Del}(P)$, removing the tetrahedra whose circumcenter falls outside the object (using outward surface normals) and then constructing only those Voronoi polygons whose vertex loops are complete.

We then solved the problem in Equation 1 on the embedded Voronoi diagram, taking $F(\mathbf{x}) = R^{-1}(\mathbf{x})$, with an approach similar to that presented in [4] for the computation of centerlines in 3D images. As shown in [3], the strong formulation of Equation 1 is the Eikonal equation

$$|\nabla T(\mathbf{x})| = F(\mathbf{x})$$

with boundary condition $T(\mathbf{p}_0) = 0$. Equation 5 is a nonlinear partial hyperbolic equation that models first arrival times of a wavefront propagating over the domain with speed $F^{-1}(\mathbf{x})$. A very effecient method for the solution of the Eikonal equation is the Fast Marching Method ([5]), based on upwind finitedifference approximation, originally developed for orthogonal grids and successively extended to triangulated manifolds ([6]). In order to solve the problem on the Voronoi diagram, we extended the Fast Marching Method to polygonal non-manifolds [7], in which more than two polygons can share a point or an edge. Once the Eikonal equation is solved over the whole Voronoi diagram with boundary condition $T(\mathbf{p}_0) = 0$, centerlines are obtained by backtracing a path from $\mathbf{p}_1$ along the direction of maximum descent of $T(\mathbf{x})$. The resulting centerline is a piecewise linear line defined on $\mathrm{Vor}_E(P)$, whose vertices lie on Voronoi polygon boundaries. Moreover, values of Voronoi sphere radius $R(\mathbf{x})$ are defined on centerlines, so that centerline points are associated with maximal inscribed spheres.

2. Local reference system

Two geometrical descriptors ([8]) are computed from the centerlines $\mathcal{C}$:

- The local radius.(not correct)
- The local reference system.(not correct)

The local radius $r(\mathbf{x}_c)$ of the vessel is the distance from a point on the centerline $\mathbf{x}_c$ to the tubular geometry. This local radius is computed efficiently by first storing all the mesh vertices of the tubular geometry in a search tree (approximate nearest neighbor, ANN) [17,18] and by using the search tree to compute the closest point $\mathbf{x}_p \in \mathcal{X}$. The local radius $r(\mathbf{x}_c)$ is the distance between $\mathbf{x}_p$ and $\mathbf{x}_c$.

The local reference system is a set of three axes defined for every point in the tubular volume $\mathbf{x} \in \mathbb{R}^3$. The local axes are the abscissa (unit vector tangent to the centerlines), the normal (a vector perpendicular to the centerlines), and the binormal that can be calculated as the cross product of the abscissa and the normal: $(\mathbf{e}_t(\mathbf{x}), \mathbf{e}_n(\mathbf{x}), \mathbf{e}_{bn}(\mathbf{x}))$. The local reference system is computed as follows: for a point $\mathbf{x}$ in the volume to be remeshed (i) compute, by using a fast search tree such as ANN, the two closest points on the centerlines $\mathbf{x}_{c1}$ and $\mathbf{x}_{c2}$ so that $\mathbf{e}_t = \mathbf{x}_{c2} - \mathbf{x}_{c1}$; (ii) compute the normal $\mathbf{e}_n = \mathbf{x} - \mathbf{x}_{c1}$; and (iii) compute the binormal as the cross product of the abscissa and the normal. Figure 2 shows an example of the local radius $r(\mathbf{x}_c)$ for a point on the centerline and of a local reference system for a point $\mathbf{x}$ in the volume of an aortic arch. The local reference system is uniquely defined. However, in the neighborhood of bifurcations, there will be abrupt changes in the orientations of the axes. As will be explained in the next sections, the local reference system enables us to define an anisotropic mesh metric to produce anisotropic meshes. The fact that there exist abrupt changes at bifurcations is not really an issue for the anisotropic mesh generator.

References

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