Numerical investigations for Mixed-ICP-EM scheme

Jiashen Guan

Here, four numerical investigations are investigated to show excellent performances of our newly devised scheme. We start our studies with a numerical limit test and three dynamic examples. Ogden model and generalized $Q2 \times Q1$ Taylor-Hood elements based on NURBS are utilized in the following. The pressure function space is generated by the k refinement to achieve the highest possible continuity. In the dynamic cases, p+a+1 Gauss quadrature points are adopted in each direction.

1 Numerical investigations

1.1 Numerical limit test

An effective approach for obtaining principal stretches is an important procedure in simulating elastodynamics formulated in principal stretches, such as a model of Ogden-type used in this study. Typically, a so-called perturbation method is used to deal with the case of identical principal stretches. In [2], however, the author pointed out that the numerical perturbation method may lead to potential pitfalls when combined with the discrete gradient method time-stepping scheme for elastodynamics formulated in principal stretches. To prevent such potential pitfalls, we choose a direct spectral decomposition of the right Cauchy-Green deformation tensor to get principal stretches, referred to a robust algorithm [3]. Test cases from [2] are stated as the following.

We consider a fixed initial deformation tensor F_1 and an updating deformation gradient via

$$F_2 := F_1 + \xi D,$$

where D is is a pre-defined deformation tensor and ξ is a parameter controlling the slight differences between the initial and the current configuration. It is noted that, two-dimension deformation cases were analysed in [2]. In our study we simply modify original two-dimension cases into a three-dimension cases. The initial deformation tensor F_1 is set as

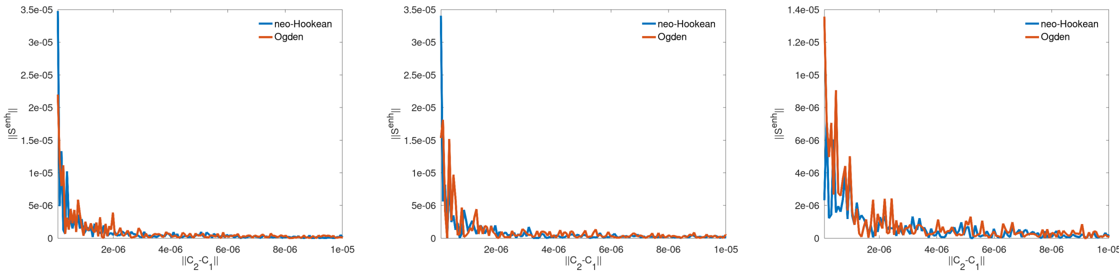
$$F_1 = \begin{bmatrix} 1.5 & 0.0 & 0.0 \\ 0.1 & 0.8 & 0.0 \\ 0.0 & 0.0 & 1.0 \end{bmatrix}.$$

Three basis deformation types: compression, simple shear and a mixed deformation are considered as D,

$$D_{\text{comp}} = \begin{bmatrix} 0.0 & 0.0 & 0.0 \\ 0.0 & -1.0 & 0.0 \\ 0.0 & 0.0 & 1.0 \end{bmatrix}, \quad D_{\text{shear}} = \begin{bmatrix} 0.0 & 1.0 & 0.0 \\ 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 1.0 \end{bmatrix}, \quad D_{\text{mix}} = \begin{bmatrix} 0.0 & 1.0 & 0.0 \\ 0.0 & -1.0 & 0.0 \\ 0.0 & 0.0 & 1.0 \end{bmatrix}.$$

We analyze the numerical limit behavior between the classical neo-Hookean model formulated in invariants and Ogden model in terms of principal stretches. Due to the fully incompressible limit, the volumetric contribution vanishes and only isochoric part is considered in these computations. The material parameters for Ogden model are $N = 1$, $\alpha_p = 2$, $\mu_p = 5000$ Pa, equivalent to case that $\mu = 5000$ Pa for classical neo-Hookean model. Numerical limit behaviors under three type deformations are showed in the Fig 1.

In the Fig 1, The oscillations of $||\boldsymbol{S}^{\text{enh}}||$ for three type deformations is of magnitude $\mathcal{O}(10^{-5})$, which in general do not lead to further numerical difficulties. Moreover, the Fig 1 (a) and (b) show that the Ogden model may even exhibit better performance than the classical neo-Hookean model using our method.



(a) (b) (c)

Figure 1: Numerical limit behavior of the norm of the 'stress enhancement' $||\boldsymbol{S}^{\text{enh}}||$ for different deformation types: (a) compression, (b) shear, (c) mix

1.2 tumbling L-shaped block

A L-shaped configuration is plotted in the Fig 2. This classical benchmark problem was firstly introduced by Simo and Tarnow [4] to verify the algorithmic conservation properties. We refer to [1] for the setting of this benchmark problem. The corresponding material parameters and loads condition are summarized in the Fig 2. We simulate the case for 30 s. In the Fig 3 a sequence of deformed configuration with pressure distribution is showed at $t = \{0, 3.7, 10.7, 15.7, 20, 26.8\}s$. The time step size $\Delta t = 0.01s$.

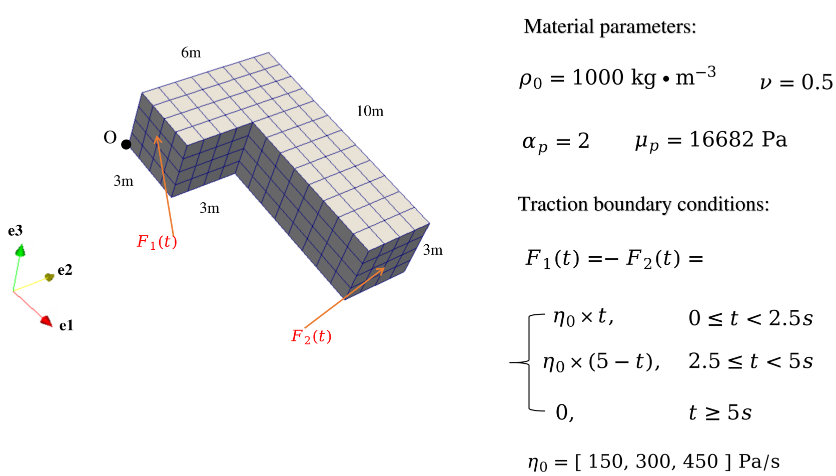


Figure 2: L-shaped block configuration, material parameters and traction boundary conditions

The evolution of total energy and total angular are plotted in the Fig 4. As expected the two important physical properties are preserved after loads vanish ($t > 5s$). In the Fig 5, we compare the absolute errors between our newly developed EM scheme and the mid-point rule with two time step size, $\Delta t = 0.01s$, and $\Delta t = 0.005s$. Fig 5 (a) and (b) shows that our new EM scheme excellently preserves the two physical constants with tiny absolute errors (magnitude of $\mathcal{O}(10^{-9})$ for total energy, magnitude of $\mathcal{O}(10^{-10})$ for total angular momentum norm). In addition, our EM scheme shows time step sizes independence. In contrast, Fig 5 (c) and (d) shows that the mid-point rule leads to large absolute errors (magnitude of $\mathcal{O}(10^{-2})$ for total energy, magnitude of $\mathcal{O}(10^{-6})$ for total angular momentum norm). And the mid-point rule shows time step sizes independence, which is conditional stable.

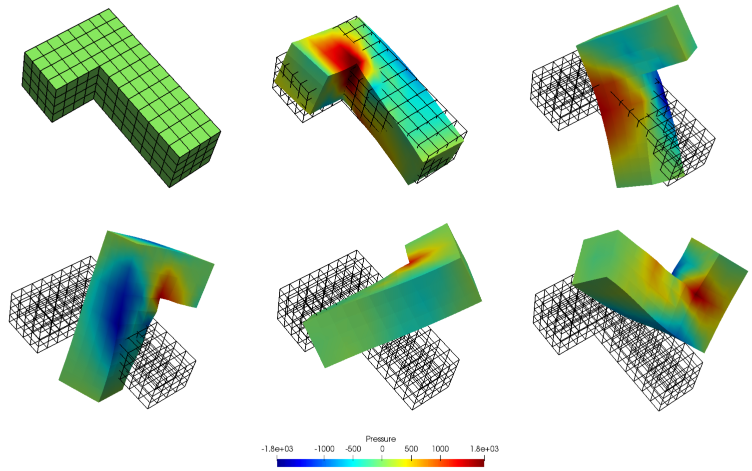
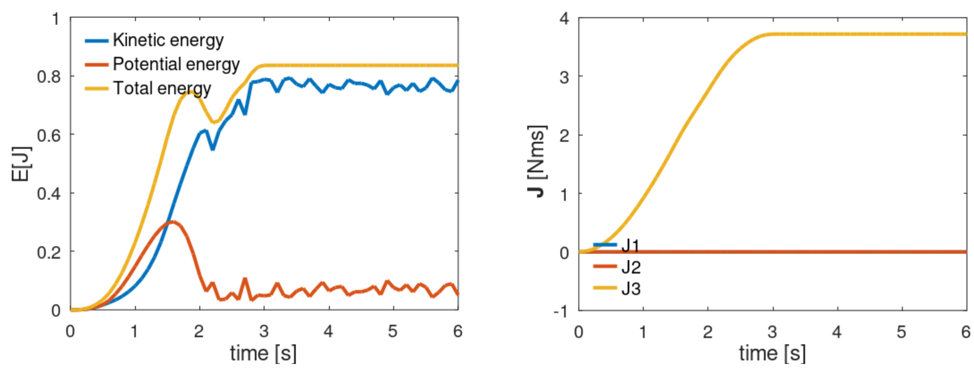


Figure 3: A sequence of deformed L-shaped block configuration with pressure contribution at $t = \{0, 3.7, 10.7, 15.7, 20, 26.8\}s$, $\Delta t = 0.01s$



(a) (b)

Figure 4: Evolution of total energy and total angular momentum within $[0 \sim 30]s$, $\Delta t = 0.01s$

1.3 twisting propeller

1.4 twisting column

References

[1] A.J. Gil, C.H. Lee, J. Bonet, and M.Aguirre. A stabilised Petrov-Galerkin formulation for linear tetrahedral elements in compressible, nearly incompressible and truly incompressible fast dynamics. *Computer Methods in Applied Mechanics and Engineering*, 276:659–690, 2014.

[2] R. Mohr, A. Menzel, and P. Steinmann. Galerkin-based mechanical integrators for finite elastodynamics formulated in principal stretches–pitfalls and remedies. *Computer methods in applied mechanics and engineering*, 197(49-50):4444–4466, 2008.

[3] W.M. Scherzinger and C.R. Dohrmann. A robust algorithm for finding the eigenvalues and eigenvectors of $3\times$ 3 symmetric matrices. *Computer Methods in Applied Mechanics and Engineering*, 197(45-48):4007–4015, 2008.

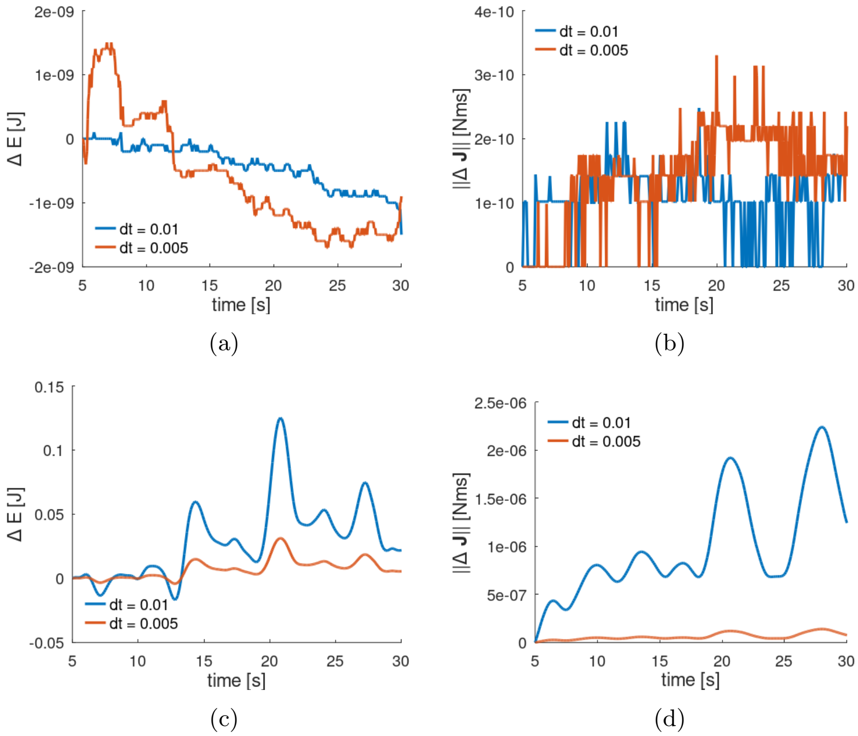
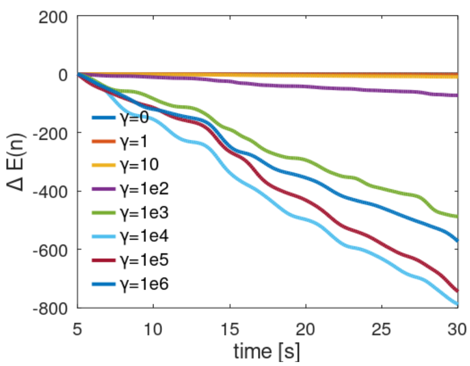
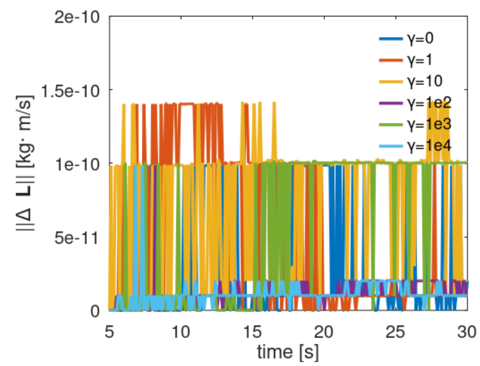


Figure 5: Absolute errors of total energy and total angular momentum. (a) and (b) are the cases of EM scheme. (c) and (d) are the cases of mid-point rule.

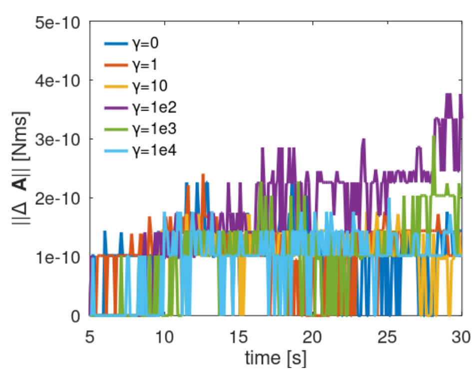
[4] J.C. Simo, N. Tarnow, and K.K. Wong. Exact energy-momentum conserving algorithms and symmetric schemes for nonlinear dynamics. *Computer Methods in Applied Mechanics and Engineering*, 100:63–116, 1992.



(a)



(b)



(c)

Figure 6: Evolution of total energy and total angular momentum within $[0 \sim 30]s$, $\Delta t = 0.01s$