

CLL:113-Tut-1(7.10.20)

Truncation Error:

1. Determine the number of terms necessary to approximate $\cos x$ to 8 significant figures using the Maclaurin series approximation:

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!}$$

Calculate the approximation using a value of $x = 0.3\pi$. Write a program to determine your result.

Round-off Error:

2. The infinite series

$$f(n) = \sum_{i=1}^n \frac{1}{i^4}$$

converges on a value of $f(n) = \pi^4/90$ as n approaches infinity. Write a program in single precision to calculate $f(n)$ for $n = 10,000$ by computing the sum from $i = 1$ to $10,000$. Then repeat the calculation but in reverse order—that is, from $i = 10,000$ to 1 using increments of -1 . In each case, compute the true percent relative error. Explain the results.

Total Error in Iterative Methods

The “divide and average” method, an old-time method for approximating the square root of any positive number a , can be formulated as

$$x = \frac{x + \frac{a}{x}}{2}$$

Write a computer program to evaluate the **true percent relative error** and the **approximate percent relative error** as a function of iteration steps, for $a=5$ and $x^0=0.005$ if $\epsilon_{\text{tol}}=10^{-6}$.