Lecture 3

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STATS 67 - UCI

Permutations and Combinatorics

Equally likely outcomes.

- Say all events in the sample space are equally likely.
- Let X denote the number of ways event A can occur.
- Let n denote the number of possible outcomes (number of elements in the sample space).
- Then $P(A) = \frac{X}{n}$.

Example: Say you flip a coin 3 times,

- What is the sample space?
- Let the event A be the event that at least 2 heads show up in the 3 flips. What is the event space for A?
- What is the probability of having at least 3 heads show up in 3 coin flips?

Permutations and Combinatorics

- *Permutation*: A permutation is an arrangement of objects in a definite order.

- *Combination*: A combination is a selection of objects without regard to order,

- Assume we have 4 people: Amy, Bruce, Chad, and Dina, and we are going to select two of them to go on a trip.
  - *Permutation*: In a permutation, the order matters. So the sets {Amy, Bruce} is not the same as {Bruce, Amy}.
    - You can think of {Amy, Bruce} as picking Amy first and then Bruce, while {Bruce, Amy} is picking Bruce first and then Amy.
  - *Combination*: In a combination, the order does not matter, The set {Chad, Dina} is the same as {Dina, Chad}.
  - We will have more ways to create a permutation than a combination (due to order mattering).

Permutations and Combinatorics

Permutation.

- In how many ways can we select r many objects from a total of n many to choose from?

- The formula for this is $\mathbb{P}_{r,n} = \dfrac{n!}{(n-r)!} = n*(n-1)*...*(n-r+1)$

  - The notation n! (read as n-factorial) is computed as follows: $n! = n*(n-1)*(n-2)*...*2*1$.

  - Also $(n-r)! = (n-r)*(n-(r-1))*...*2*1$.

  - And so $\dfrac{n!}{(n-r)!} = n*(n-1)*...*(n-r+1)$.

- We can think of a permutation as positioning r many objects, selected from n many in total, into slots.

  - The first slot will have n many options to pick from, the second slot will have $n-1$ objects to choose from,..., and the r-th slot will have $n-(r-1) = n-r+1$ many objects to choose from.

Permutations and Combinatorics

- Now think of the case where order does not matter.
- In the previous example, this would mean that the duo {Amy, Bruce} is the same as {Bruce, Amy}.
- This is a combination.
- If we select r many objects from a total of n possible objects, where order does not matter.
- The formula is $\mathbb{C}_{n,r} = \binom{n}{r} = \dfrac{n!}{r!(n-r)!}$
- Similar to the permutation formula, this accounts for the notion that order does not matter among the r many selected (hence the division by $r!$).

Permutations and Combinatorics

Example: Say we have 5 people: Audry, Bruce, Colin, Daniel, and Emily.

- How many ways can we select 2 people if the order matters?

- How many ways can we select 3 people if the order matters?

- How many ways can we select 2 people if the order does not matter?

- How many ways can we select 3 people if the order does not matter?

Counting

Permutations and Combinatorics are a type of selection process where the objects selected are not replaced. Once selected, the object is removed from the remaining possible objects to be selected.

- Assume we have n many total objects, and want to create a grouping of r many of them, where order matters and objects are selected with replacements (i.e. in each group, an objects can occupy several places).

  - An example is creating a password using only lower case letters.

  - Each password spot (which character, going from left to right) can be one of 26 objects (a,b,c,...,x,y, or z).

  - it is possible use a single letter numerous times. For example abcda or aacde or aaaaaa.

  - Note: The ordering of the objects matters. For example abcdef and fedcba are different passwords.

Counting

Say we have n many total objects, and want to create a grouping of r many of them, where order matters and objects are selected with replacements (i.e. in each group, an objects can occupy several places).

- The formula for this is $\prod_{i=1}^{r} n = n^r$

In general, each spot can be occupied by an object with n_i many possible objects. Example is a password where certain spots can be letters only and some numbers only.

- The formula for this is $\prod_{i=1}^{r} n_i$
- Example: Lets say you want to create a password that is 5 characters long, using only lower case letters.
  - What does n equal?
  - What does r equal?
  - How many possible passwords are there that are 5 lowercase letters long?

Counting

- Example: Say we are asked to create a password with 5 lower case letters followed by two numbers.
  - How many ways can we create a password with 5 lowercase letters?
  - How many ways can we create a password that has two numbers?
  - What is the total number of ways we can create a password with 5 lowercase letters followed by two numbers?
  - Notationally, we say that $\prod_{i=1}^{r} n_i =$