**A Chemical Engineer Goes to the Horse Races**

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**Abstract**

This paper presents a mathematical model for the optimal wagers one should make on a horse race which uses the *pari-mutuel[[1]](#footnote-1)* wagering system. It is an extension of Rufus Isaacs’ model which he developed in 1953 while working for the RAND Corporation (Isaacs R., 1953). Isaacs’ solution, using the calculus of the Newton-Rhapson method, produced wagers that violated the model’s key assumption that the size of his wagers would not significantly affect the odds. It also produced “optimal wagers” that lost money even though the selected horse won the race. The purpose of this paper is to demonstrate that the mathematical method, *Monte Carlo Marching (MCM),* can be used to improve the solution of the Isaacs and other nonlinear models (Lashover, 2012). *MCM* was previously developed for chemical engineering calculations to guarantee convergence and to avoid taking the complicated calculus derivatives required for Newton-Rhapson solutions. While Isaacs showed his considerable prowess with calculus in solving his nonlinear model, it is doubtful that even he could have expressed and used calculus to solve the more rigorous model easily developed and solved with *MCM*. Using linear programming type constraints, *MCM* permitted improving the rigor of Isaacs’ model by eliminating “winning” wagers that lost money on multiple bets, and placing limits on the size of wagers to prevent them from significantly affecting the horse’s odds. A constraint was also added to prevent the bettor from “tapping out”—a situation which Isaacs did not address. While Isaacs’ model failed to prove profitable over a series of 540 races, the MCM model produced a profit of $177,314 over the same races with a 242% ROI (return on investment) and 27% winners. The longest losing streak was 9 with wealth drawdown of $6,171 on $100,000 of initial wealth. This demonstrated the efficacy of MCM for solving nonlinear equations other than those encountered in chemical engineering.

**Keywords:** *Pari-mutuel*, *Monte Carlo Marching,* optimal wagers, simulation, expected value, nonlinear models, convergence, linear programming, probability

**Introduction**

Rufus P. Isaacs, while a mathematician at the RAND Corporation in 1945-1955, worked with Richard Bellman (Dynamic Programming), John Nash[[2]](#footnote-2), and other prominent mathematicians to advance the field of mathematical optimization and control theory. RAND (Research ANd Development) is a “think tank” which was formed by the U. S. Army Air Force shortly after the end of World War II (as publicly stated) to focus on “global policy issues.” Located in Santa Monica, California, its first mission was to solve such problems as “The minimum time interception problem for fighter aircraft” … where …”due to the increased speed of aircraft, nonlinear terms no longer could be neglected” (Pesch, 2009). Many prominent mathematicians were brought on board, and much of their work involved the defense and avionics industries, and was classified. After leaving RAND in 1955, Isaacs became famous as a “game theorist” with his publication of *Differential Games* a decade later in 1965 (Isaacs, Differential Games, 1965). Earlier, in 1953, he had published *Optimal Horse Race Bets* where he used the Newton-Rhapson method to solve the nonlinear equation of his model (Isaacs, R. 1953).

The purpose of this paper is to demonstrate that the mathematical method, *Monte Carlo Marching (MCM),* previously developed for chemical engineering calculations to guarantee convergence and to avoid taking the complicated calculus derivatives required for Newton-Rhapson solutions, can be used to improve the solution of the Isaacs’ and other nonlinear models (Lashover, 2012). Use of MCM permitted modification of Isaacs’ calculus solution which violated his key assumption and failed to produce a profit. MCM’s use of linear programming type constraints eliminated “winning” wagers that lost money on multiple winning bets and prevented the size of optimal wagers from changing the horse’s odds and thus invalidating the model. Another constraint prevented the size of total wagers from leading to “Gambler’s Ruin” (running out of money) which Isaacs did not consider. Using MCM and identical probability and race data, the Isaacs’ model was converted from a non-profitable system to one which won 27% of the same races with a 242% ROI (return on investment). The longest losing streak was nine with wealth drawdown of $6,171 on $100,000 of initial wealth. Profit was $177, 314.

**Development of Isaacs’ model**

To mathematically describe the *pari-mutuel* model of Rufus Isaacs, we assign bettors B1, B2, ….. *Bn* handicapping a horse race involving *m* horses H1, H2, …..H*m*. We acquire an *n* x *m* matrix by assigning a subjective probability matrix {p*i j*}, where p*i j* designates the probability, in the opinion of B*i*, that H*j* will win the race. A sum b*i* > 0 is then wagered by B*i* in a manner that maximizes his mathematical expectation. That is, B*i* observes the *pari-mutuel* probabilities π1, π2, ….. π*m*, related to the subjective probabilities by the track take, and indicated by the track tote board as odds, that horses H1, H2, …..H*m* , respectively, might win the race. The real or objective probability of the horse winning the race, ρ*j*, can be determined by mathematical analysis of the key predictor variables. He then follows a strategy of distributing the amount b*i* among those horses H*j* for which the ratio ρi/π*j*, the expected value, E*i*, is a maximum.

Isaacs assumes that the sum b*i* is small with respect to the total amount wagered by the *n* bettors on the race and therefore does not significantly change the *pari-mutuel* probabilities or odds. He further assumes that each column of the matrix {p*i j*} contains at least one entry, otherwise, if the *j*th column consists of all zeroes, no bettor has selected horse H*j* and it can be eliminated.

The *pari-mutuel* system is described by the following three conditions:

1) *j*=1∑*m*β*i j* = b*i* for all horses where β*i j* is the sum wagered by B*i* on H*j*. (1)

2) *i*=1∑*n* β*i j* = kπ*j* for all bettors where k is the proportionality constant relating the amount bet on each horse to its *pari-mutuel* probability. (2)

3) E*i*= ρi/π*j* > 1 meaning that each Bi bets only on horses with expected value > 1. (3)

*Pari-mutuel* probabilities for the Win are determined by the proportionality constant:

k = (1 – K) *i* = 1∑*n* b*i* (4)

where K is the “track take” so that

π*j* = *i* =1∑*n* β*i j* / k  (5)

The odds on each horse are calculated as (1 – π*j*) / π*j* to 1 (6)

He now has enough information to construct the model to be optimized:

First, he let the total wealth bet on the *j*th horse, kπ*j*, be partitioned into the sum, s*j* wagered by the subjective crowd and the amount t*j*be that contributed by the bettor. The bettor’s profit can be represented by the function F(t1, t2, …..,t*m*) from an investment spread over the *m* horses as follows:

F(t) = (1 – K)[*j*=1∑*m* (s*j*+ t*j*)] *j*=1∑*m* *ρj* t*j* / (s*j* + t*j* ) - *j*=1∑*m* tj (7)

Now it is desired to select values of t*j* >= 0 that will maximize F(t) where the maximal F(t) possesses a positive value. From this point on, Rufus Isaacs, a pure mathematician, used calculus to solve the problem. First, he showed that F has a positive maximum if[[3]](#footnote-3)

1 <= j <= m ρ*j*/ s*j* > 1 / (1 – K) ∑ s*j* (8)

Isaacs then used the Newton-Rhapson method and began by taking the derivative of F with respect to t and setting it equal to zero to find the maximum yields.[[4]](#footnote-4)

∂F/∂t = (1 – K) *j*=1∑*m* ρ*j* t*j*/ (s*j* + t*j*) + (1 – K) ρ*i* s*i* / (s*i* + t*i*)^2 j=1∑*m*(s*j*+ tavg) – 1 = 0 (9)

The quantity ρ*i* s*i* / (s*i* + t*i* avg)^2 exhibits the same value for all *i* such that t*i* avg > 0.

Defining this value by 1/λ^2 we have t*i*= λ(*ρi si*)^0.5- s*i* (10)

A maximal form of F never occurs for all t*i* avg > 0, i.e. the bettor never bets on all horses. Rather, there may exist some number of horses, r, whose “real” expectation is greater than the subjective expectation realized from the actual wealth invested on their behalf by the crowd.

Therefore we look for t*i*= λk (ρ*i* s*i*)^0.5- s*i* For *i* = k, k+1,…*m*, and … (11) λk^2 = (1 – K)  j=1∑k-1 s*j*[1 – (1 – K) j=k∑ m ρ*j* ]^-1. (12)

**Table 1. Rufus Isaacs’ Synthesized Horse Race**

**Horse Real Prob. $ Win Pool Crowd Prob.** **Calc. Odds Tote Odds Δ Prob.**

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| 1 | 0.4 | 35000 | 0.4666 | 1.1432 | 1 | -0.0666 |
| 2 | 0.18 | 10000 | 0.1334 | 6.496 | 6 | 0.0466 |
| 3 | 0.12 | 9000 | 0.1200 | 7.333 | 7 | 0 |
| 4 | 0.1 | 8000 | 0.1067 | 8.372 | 8 | -0.0067 |
| 5 | 0.1 | 7000 | 0.0933 | 9.718 | 10 | 0.0067 |
| 6 | 0.05 | 5000 | 0.0666 | 14.015 | 14 | -0.0166 |
| 7 | 0.05 | 1000 | 0.0133 | 74.188 | 74 | 0.0367 |
| **Total** |  | 75000 | 0.9999 |  |  |  |

Horses 2, 5, and 7 have real, positive expectations which are greater than their subjective expectations, i.e. their expected values, E*i*, are calculated as

E2 = 0.18/0.1334 = 1.35

E5 = 0.1/0.0933 = 1.07

E7 = 0.05/0.0133 = 3.76

And, λ^2 = {[0.85(35,000 + 9,000 + 8,000 + 5,000)] / [1 – 0.85(0.18 + 0.10 + 0.05]} ^ 0.5 = 259.5 when K = 0.15.

Thus the optimal wagers on each horse are calculated as:

• t2 =259.5 x [0.18 x 10,000]^0.5 - $10,000 = $1009.65

• t5 =259.5 x [0.10 x 7,000]^0.5 - $7,000 = < 0

• t7 =259.5 x [0.05 x $1,000]^0.5 - $1,000 = $834.94

One can easily see that the wager on Horse 7 will violate the assumption that the wagers do not affect the tote odds. The total of $834.94 wagered by Isaacs’ solution on Horse 7 will almost double the total of $1,000 by the crowd on Horse 7. The new total wager, s7 , of $1,834.94 will reduce the odds from 74 to 40 to one, thus reducing the pay-off should Horse 7 win, and invalidating the calculation of F. Further, no consideration is given to the bettor’s wealth, and the size of these wagers could easily lead to “tapping out” or “gambler’s ruin[[5]](#footnote-5)” over a series of races. Checking for wagers which would not produce an overall profit for the race was also neglected. Finally, a small point, but the win bet is an integer problem, as whole dollars, not dollars and cents, are the only acceptable win wagers.

**Development of the Monte Carlo Marching model**

The Monte Carlo Marching (MCM) method presented here provides robust convergence performance without derivatives and is insensitive to initial estimates of the variables. Application of MCM was demonstrated in an earlier paper on several non-linear models including Isaac Newton’s cubic polynomial, y^3 – 2y -5 = 0, (Cajori, 1911) which Newton used to first demonstrate his Newton-Raphson procedure. There have been no convergence failures with MCM. (Lashover, 2012)

To prove that *Monte Carlo Marching* can be applied to solution of general non-linear systems of equations, Isaacs’ synthetic case was solved, as presented, using MCM. The results were a $1,034 wager on #2 and an $839 wager on #7 as compared to $1,010 and $835, respectively by Isaacs. The maximum F calculated using MCM was 719.7185 whereas Isaacs’ maximum F was 719.6997. Isaacs’ model lost money over 540 races[[6]](#footnote-6). Appropriate constraints were then introduced into the MCM model and using an identical initial wealth of $100,000, a net profit of $177,314 was achieved over the same 540 races using the same probability values. The ROI was 242% with a win percentage of 27%. The longest losing streak was nine with wealth drawdown of $6,171.

**Examples**

“Marching”, “Binary-chop”, “Interval-halving” or “Bi-section” is a simple convergence procedure which guarantees a solution to “bounded” variables. Two examples are discussed below.

**Example No. 1**

Consider the solution of Equation 6 above for the probability, πj, when the tote board odds are known. These odds on each horse are calculated as (1 – π*j*) / π*j* to 1. This non-linear equation is not solvable using algebra. The following simple QB64[[7]](#footnote-7) program using MCM easily solves for πj. The probability, P, varies from 0 to 1. When solving for O, odds, using “Marching”, the lower limit, PL is set to zero and the upper limit, PU, is set equal to 1.0. The initial value of P is calculated as P = (PL + PU)/ 2 = 0.50. If the first iteration produces a negative difference, DEL, between the actual value of O, odds, and the calculated odds, OO, PL is set equal to P. PU is left at 1.0. If the difference, DEL, is positive, PU is set equal to P, and PL is left at 0. The next value of P to be tried is again calculated as P = (PL + PU)/2 and the process is repeated. So, if P is too low, the lower limit, PL, is raised to P as no solution exists below P. Similarly, if P is too high, values above PU can be eliminated. When the “correct” probability is calculated there will be no difference (within tolerance) between the actual and calculated odds. DEL eventually becomes very small, and below the acceptable tolerance, TOL. This method always converges and eliminates one-half of the range of feasible solutions after each iteration. This can be represented algebraically by the function f(x) = R / (X*n*) where R = the range of solutions, X = 2, and *n* = 1, 2, 3….*n*, the number of the iteration. The limit of this function as *n* approaches infinity is 0 where the entire feasible range of solutions has been examined. It does not produce an exact solution like algebra or calculus can, however it will be shown that this convergence procedure permits use of constraints which are not easily included when using algebra or calculus.

The QB64 statements demonstrating this method follow:

REM SOLVE NONLINEAR ODDS/PROBABILITY EQUATION

INPUT " TOTE BOARD ODDS= ", O

LP = 0.0: UP = 1.0: TOL = 0.003

**2** P = (LP + UP) / 2.0

OO = (1 - P) / P

DEL = O - OO

IF ABS(DEL) <= TOL THEN PRINT: PRINT " P= "; P: GOTO **9** 'SOLVED

IF DEL < 0 THEN LP = P: GOTO **2**

IF DEL > 0 THEN UP = P: GOTO **2**

**9** END

Computer output demonstrating the calculation of probability for tote odds of “2” follows:

**Table 2. Convert Tote Odds to Probability**

DATE= 06-20-2014 TIME= 17:43:04

ODDS FOR WHICH PROBABILITY IS NEEDED= 2

**LP UP P OO O DEL LP UP CYCLE**

0 1 0.5 1 2 +1 0 0.5 1

0 0.5 0.25 3 2 -1 0.25 0.5 2

0.25 0.5 0.375 1.667 2 +0.333 0.25 0.375 3

**LP UP P OO O DEL LP UP CYCLE**

0.25 0.3750 0.3125 2.2 2 -0.200 0.3125 0.375 4

0.3125 0.3750 0.3438 1.909 2 +0.091 0.3125 0.3438 5

0.3125 0.3438 0.3281 2.048 2 -0.048 0.3281 0.3438 6

0.3281 0.3438 0.336 1.976 2 +0.024 0.3281 0.3360 7

0.3281 0.3360 0.332 2.012 2 -0.012 0.3320 0.3360 8

0.3320 0.3360 0.334 1.994 2 +0.006 0.3320 0.3340 9

0.3320 0.3340 0.333 2.000 2 0.000 10

P= .333

**Example No. 2** (See Lashover, J. H. (2012). *Monte Carlo Marching.* Baton Rouge, Louisiana: "Academia.edu" for a more detailed explanation of this example.)

Consider the chemical engineering calculation of the fraction of vapor flashed, V, from a liquid stream which varies from 0 to 100 % or from 0 to 1.0 as a fraction. The concentration of the liquid stream (feed) is known as are its temperature and pressure, however the concentrations of both the vapor and liquid streams formed are unknown. To complicate matters further, the composition of the vapor stream formed is related to the concentration of the liquid stream formed by a complex, non-linear relationship, i.e. the composition of the vapor stream is dependent on the yet unknown liquid stream. The mass of the feed must also equal the total mass of the resulting liquid and vapor streams (conservation of mass). There have been many solutions presented, including the author's using MCM, but others fail frequently, especially when V is close to 0 or 1.

When solving for V using “Marching”, the lower limit, VL is set to zero and the upper limit, VU, is set equal to 1.0. The liquid and vapor concentrations formed are also bounded between 0 and 1 when expressed as mole fractions and the total molar concentration of each stream must also sum to unity. An overall mass balance must be satisfied and calculation of the total pressure must equal the pressure specified. When one mole of feed is flashed and the total pressure is equal to one atmosphere, all variables can have a lower limit of 0 and an upper limit of 1. Since the vapor composition is dependent on the liquid composition, the liquid compositions become the unknown independent variables. With three components, there are then four unknown independent variables when V is included. Actually, there are only two unknown compositions as the third equals 1.0 minus the sum of the other two, however, all three are considered unknown to avoid losing the constraint requiring the sum of the compositions to be unity.

In MCM, the simple marching technique shown previously is applied to the multiple variables. These unknown variables are essentially improved from their lower boundary values toward their higher boundary values by analyzing linear regions which are inverse multiples of powers of two. This procedure emulates the guaranteed conversion procedure of “Marching” and satisfies the mathematical theories of hyper-rectangles. (Faloutsos, 1903) Remember, after the first iteration of marching, the region left to study has decreased by ½ or 1/ (2^1). After the second iteration, the region left to study has decreased to ¼ or 1/ (2^2), and so forth. The feasible region of solutions for V, after being reduced to ½ by searching either 0.5 to 1.0 or 0 to 0.5 is next reduced to ¼ by searching either 0.75 to 1.0 or 0 to 0.25. By normalizing all independent variables to 0 to 1 or 0 to 100, no one variable can overly influence the objective function.

Finally, the objective function is the sum of the convex constraint equations which are crafted as the absolute differences of the constraint values from the desired constraint values at each iteration, i.e., for the V calculation, G1 = ABS(Σ X – 1.0) would be one constraint, G2 = ABS(ΣY – 1.0) would be the second, G3 = ABS(∑P – 1.0) would be the third, and G4 = ABS(∑Z – V\*∑Y – (1 – V)\*∑X). Thus G, profit in most “Simplex” calculations, would equal to G1 + G2 + G3 + G4[[8]](#footnote-8) and comprise the convex objective function which would be minimized in the calculation of V. The justification for the addition of these constraints is similar to that used in “Maximum Likelihood” methods. (Banbura, "Maximum Likelihood Estimation of Factor Models on Data Sets with Arbitrary …) The objective function becomes convex by being built of convex functions and can thus be optimized. The feasible solutions are bounded so as to insure that they are realistic parts of the model “set”. Further proof of the linearity (ability to “add” constraints) of the simultaneous marching schema used in this work can be found in Analysis of the n-dimensional quadtree decomposition for arbitrary hyper-rectangles by Christos Faloutsos et.al. at <http://drum.lib.umd.edu/bitstream/1903/678/2/CS-TR-3381.pdf> (Faloutsos, 1903).

In the horse race model, F replaces G and is maximized. When maximizing, bets which produce an F below the present best are rejected. Also, in the Isaacs' model, Eq. 7 is the objective function, and the constraints on bet size, etc. are tested separately.

**Solving the Isaacs’ Model using MCM**

The same nomenclature used for the Isaacs’ model is used here and begins with the solution of Eq. 7. Before the solution is started, races with less than 10 entries, W, and with total win pool, TWP, less than $8,000, at three minutes to “Post” (all horses are in the starting gate) are screened out. See **Appendix A: Background for the horse racing *MCM* model** for the origin of these and other constraints. TWP and W are parameters whose values were evaluated by simulation.

>BETA is the maximum percentage of the bettor’s wealth which can be risked on any one race. This is to avoid “tapping out” or “gambler’s ruin”. The author has used a conservative 5% as opposed to Quirin’s 8% where his simulations required a winning percent of 30 along with an ROI of 25% to avoid “tapping out”. Quirin’s simulations also showed that betting 8+ to 10% of one’s bankroll per race led to a 50-50 chance of making a profit or tapping out (William L. Quirin, 1979).

>BTEFF is the maximum percentage of the win pool dollars that can be wagered on any horse to avoid significantly changing the tote board odds. The author has used 5% here. At 5% the bet on #7 in Isaacs’ sample race would be limited to $50 (0.05 x $1,000) as opposed to his $838 wager which cut the odds from 74 to 40 (46 %).[[9]](#footnote-9) The $50 wager cut the odds from 74 to 70 (5.4 %). The wager on #2 was reduced to $500 (0.05 x $10,000) and the odds were reduced from 6.4 to 6.1 (4.7%) as compared to 6.4 to 5.9 (9.3%) for the Isaacs’ solution. These calculations show only the effects of the optimal bets and do not consider any dollars wagered by the crowd.[[10]](#footnote-10)

In each iteration, a number of feasible solutions are calculated for the marching region and the highest F with its t’s (bets on each horse) is stored as a local maximum, M. The number of solutions examined can easily be increased to insure location of a global stationary point with little penalty in computing time. The values of the variables used for each solution are obtained by random (Monte Carlo) selection of values in the iteration ranges. These random values are first tested to insure that they are within the current boundaries of the region to be tested. If not, they are rejected and a new set of values are determined.

It should be noted that the solution to the Isaacs’ model was quickly determined with a relatively coarse grid of solutions. First, even though there were 7 horses in the race, the final solution only had to consider those horses with expected values above 1.0. This reduced the number of possible wagers from 7 to 3. Taking a helicopter view of the model, it appears optimal to bet the maximum that one can on the horses with the highest expected values. However, as mentioned previously, this can lead to wagers on some horses that are not profitable if they win. Further, the length of losing streaks encountered will be smallest when betting on the horses with the highest probability of winning. As one can intuit, the ideal case will be that of the race favorite having the highest expected value. In fact, this often occurs during the last race at the track when gamblers who are losing for the day bet long shots to try and recover their losses letting favorites go off at profitable odds. Favorites win about 1/3 of their races, but usually show a negative profit due to being over bet, i.e., they are underlays. The exception here being favorites with odds less than or equal to 1 to 1 as predicted by the favorite-long shot bias. These favorites are frequently overlays.

Long losing streaks provide a serious psychological dilemma to gamblers. With each successive loss, the bettor sees his wealth becoming smaller and begins to doubt his probability model and whether he is properly executing his betting system. The modified Isaacs’ model had a maximum losing streak of 9 during which the initial wealth was reduced by about 6%. Higher percentages would provide serious stress to most bettors.

Another issue which was learned from stock market technical analysts is that no more than 25% of the profit from any investment system should come from any one investment or wager. Such a large profit from just one investment might be the only reason that the system was successful and may not occur again in the future. The technical analysts also caution that only ¾ of the database should be used for developing parameters, while saving the last ¼ of the database to test the system. Using the total database for developing parameters and then testing the system on the total database may mean that the method will only work on “that database.” This fallacious procedure produces what analysts call a “back-optimized” system.

As has been shown above, it takes significant expertise to modify an academic model to succeed in the real world. Much of the optimization work in the literature features very simplistic examples which are not verified using actual data from real situations. As an unknown statistician once lamented, “Numbers are like people, if you torture them long enough eventually they will tell you anything that you want to hear.” MCM can solve simultaneous nonlinear equations, and its facility for constraints has shown it to be adaptable for work on real world problems. (Lashover, 2012).

**QB64 Computer code for the modified Isaacs’ model**

The MCM strategy is to create a large W-dimensional[[11]](#footnote-11) hyper-rectangle inside the limits of feasible solutions, always storing the best answers so far, and re-centering the rectangle about those best answers; then reducing the W-dimensional rectangle in width in each dimension and repeating the process. This is done until the W-dimensional rectangle moves around and surrounds the solution, and then further narrows and finds it when the maximum value no longer changes.

REM PARIMUTUEL BETTING BY JACK LASHOVER 032914

CLS

DIM A%(20), B%(20), N%(20), LL%(20), UL%(20), S(20), T%(20), P(20)

DIM O(20), E(20), PIE(20) ‘ “%” indicates integer variable

DEFSNG A-Z

DEFINT I-K

RACEFIL$ = " SYNTHETIC RACE"

REM INPUT DATA FOR PARIMUTUEL CALCS. USING ISAACS’ SYNTHETIC DATA"

PRINT: PRINT RACEFIL$

W = 7 ‘NUMBER OF HORSES

TT = 0.15 'TRACK TAKE

REM DURING ACTUAL WAGERING OR SIMULATIONS, THERE IS A SUBROUTINE HERE THAT SCRAPES THE WAGER AMOUNTS FROM AN INTERNET SITE AND ANOTHER WHICH CALCULATES THE (HOPEFULLY) REAL PROBABILITY OF EACH HORSE WINNING. THE FOLLOWING NUMBERS ARE FROM ISAACS’ EXAMPLE PROBLEM:

P(1) = 0.40 'TRUE PROBABILITIES OF ENTRY WINNING FROM ISAACS

P(2) = 0.18

P(3) = 0.12

P(4) = 0.10

P(5) = 0.10

P(6) = 0.05

P(7) = 0.05

S(1) = 35000 'AMOUNT BET BY CROWD ON EACH ENTRY FROM ISAACS

S(2) = 10000

S(3) = 9000

S(4) = 8000

S(5) = 7000

S(6) = 5000

S(7) = 1000

TWP = $75,000 REM TOTAL WIN WAGERS BY CROWD

REM CALCULATE PROBABILITIES & ODDS FROM AMOUNTS WAGERED

FOR I = 1 TO W

PIE(I) = S(I) / ((1 - TT) \* TWP)

O(I) = (1 - PIE(I)) / PIE(I)

NEXT I

REM CALCULATE EXPECTED VALUES

FOR I = 1 TO W

E(I) = P(I) / PIE(I)

NEXT I

REM THE FOLLOWING CODE IS THEN USED TO MANIPULATE THE HYPER-RECTANGLES TOWARDS CONVERGENCE TO A SOLUTION:

REM CALCULATE FIRST GUESSES AT BETS

FOR I = 1 TO W

B%(I) = 2 ‘MINIMUM BET ACCEPTED BY TRACK

N%(I) = 5000 ‘MAXIMUM EXPECTED BET = 5% of $100,000 wealth

A%(I) = (B%(I) + N%(I)) / 2 ‘FIRST GUESS BY MARCHING

NEXT I

RANDOMIZE (TIMER)' USE OF A CONSTANT IN PLACE OF 'TIMER' PERMITS OBTAINING SAME RANDOM NUMBERS EACH RUN

M = 0

BTEFF = 0.05 'BETS CANNOT EXCEED 0.05 \* S(K) TO AVOID CHANGING ODDS

BETA = 0.05 'TOTAL BETS CANNOT EXCEED 0.05 \* TOTAL WEALTH TO AVOID TAPPING OUT

WINWEALTH = 100000

REM ASSUME 5000 (5 x 1000) SOLUTIONS IS ENOUGH TO FIND GLOBAL MAXIMUM

**1** J = 1 TO 5 ‘Exponent of 2 which reduces the size of the hyper-rectangle

I = 1 TO 1000 ’Size of grid which determines how many solutions are checked

K = 1 to W ‘Number of horses

IF E(K) < 1.0 THEN A%(K) =0 : T%(K) = 0: GOTO **70** 'SKIP THIS HORSE

IF A%(K) – N%(K) / 2 ^ J < 0 THEN GOTO **10**

GOTO **20**

**10** LL%(K) = B%(K) ‘$2 MIN BET

GOTO **30**

**20** LL%(K) = A%(K) – N%(K) / 2 ^ J

**30** IF A%(K) + N%(K) / 2 ^ J > N%(K) THEN GOTO **40**

GOTO **50**

**40** UL%(K) = N%(K) – LL%(K)

GOTO **60**

**50** UL%(K) = A%(K) + N%(K) / 2 ^ J – LL%(K)

**60** T%(K) = LL%(K) + INT(RND \* UL%(K) + 0.5)

IF T%(K) > (BTEFF \* S(K)) THEN T%(K) = INT(BTEFF \* S(K) + 0.5) 'NOT USED BY ISAACS METHOD

**70** NEXT K

These T%(K)’s are used to calculate F (Profit) in Isaacs Eq. 7. If F is larger than the previous best F then the T%(K)’s are converted to A%(I)’s which are now the best bets calculated so far, and this F becomes the new maximum. These calculations are repeated with as fine a grid as necessary until the F’s or maximum ceases to change. By observing how many new F’s are obtained per iteration of J, the size of the hyper-rectangle, one can use a finer grid by increasing J and I.

REM CHECK TOTAL AMOUNT OF ALL BETS & PROFIT

TOTALBETS = 0

FOR II = 1 TO W

TOTALBETS = TOTALBETS + T%(II)

NEXT II

IF TOTALBETS > (BETA \* WINWEALTH) THEN PRINT: PRINT " MAXIMUM TOTAL BETS EXCEED GAMBLER'S RUIN.": GOTO **1** ‘NOT USED BY RUFUS ISAACS METHOD

REM CHECK PROFITABILITY OF BETS

FOR II = 1 TO W

IF T%(II) <= 0 THEN GOTO **2**

IF T%(II) \* (O(II) + 1) < TOTALBETS THEN GOTO **1**

**2** NEXT II

REM CALCULATE FUNCTION (EQ. 7)TO BE MAXIMIZED

REM BREAK UP FUNCTION INTO PARTS FOR EASIER CALCULATION

PART1 = 0: PART2 = 0: PART3 = 0

FOR JJ = 1 TO W

IF E(JJ) <= 1 THEN T%(JJ) = 0

PART1 = PART1 + (1 - TT) \* (S(JJ) + T%(JJ))

PART2 = PART2 + (P(JJ) \* T%(JJ)) / (S(JJ) + T%(JJ))

PART3 = PART3 + T%(JJ)

F = (PART1 \* PART2) - PART3

NEXT JJ

PRINT: PRINT " F= "; F; PART1; PART2; PART3

IF F > M THEN GOTO **21**’ NEW MAXIMUM FOUND

GOTO **22**’NEW MAXIMUM NOT FOUND

**21** FOR II = 1 TO W

A%(II) = T%(II)

NEXT II

M = F

**22** NEXT I

NEXT J

CLS: PRINT: PRINT " FINAL RESULTS "

FOR I = 1 TO W

IF T%(I) = 0 THEN GOTO **25**

PRINT: PRINT “ BET ON HORSE= “; I; “ = “; T%(I)

**25** NEXT I

PRINT: PRINT " TOTAL BETS= "; TOTALBETS

PRINT " MAXIMUM F= "; M

END

**Table 3. Race Odds Data from Internet**

Santa Anita Time: 6:20 11/02/2013

Race 9 2 mins. To Post

No. M/L[[12]](#footnote-12) Odds Win Place Show

1 15 24 43581 18397 13225

2 30 83 13361 5309 5529

3 20 32 33370 14910 13578

4 6 5 176178 64529 39410

5 30 67 16567 6030 5717

6 30 75 14782 5133 5707

7 3 8/5 431320 92080 66800

8 4 4 216684 75385 53215

9 9/2 8 121328 47828 29521

10 6 8 124802 56633 37321

11 8 12 84867 25352 17906

12 8 17 59848 24161 16607

Pool Totals: 1336688 435747 304536

**Table 4. SIMULATION RESULTS**

WELCOME 05-26-2014

BEGINNING RACE= 13,000 ENDING RACE= 13,540

BASE BET= 2 BETA= .05 TRP= .79 TRACK TAKE= .21[[13]](#footnote-13)

BTEFF= .05 TWP MIN= $8,000.

ALL INITIAL WEALTHS=$100,000.

RACE FILES 13,000-13,540

AFTER RACENO= 13,540 TOT WIN BET= $73,383 TOT WIN PAY= $250,697 ROI= 242%

WIN BETCOUNT= 66 WINCOUNT= 18 WPCT= 27

WIN PROFIT= $177,314 WIN WEALTH= $277,314

CURRENT LOSING STREAK= 0 DRAWDOWN= 0

LONGEST LOSING STREAK= 9 LARGEST DRAWDOWN= $6,171

AVERAGE WAGER = $1,112 WITH MIN = $42 AND MAX = $5,000

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**APPENDIX A: Background for the horse racing MCM model**

***Pari-Mutuel Wagering***

*Pari-Mutuel* wagering, now used at all U. S. horserace tracks, is based on the average of probabilities, hence called subjective, assigned to a field of horses by the crowd of bettors. The total amount bet on each horse by all bettors is shown on a tote board next to the horse’s odds to win which are based on the percentage of the wagers on each horse to the total wagers on all horses (win pool). The method was invented in France by Pierre Oller in 1865 and insures a profit to the track operator by claiming a fixed percentage (track take) of the win pool before pay-offs of the remainder are distributed among the bettors holding tickets on the winning horses. An additional 1-2% loss is sustained by each winner due to the “breakage” rule where pay-offs are rounded down to the next lower 5, 10 or 20 cent portion of the decimal portion of the pay-off. With 10% breakage, a pay-off of $6.37 for a $2 bet would be rounded down to $6.30. Track take is usually 15 to 17%. Some off-shore betting sites offer lower track takes, but offer more risk as to the safety of accounts and the reliability of pay-offs. At this time, online wagering on horse racing is legal in the U.S. and includes races from Australia, England, Japan, and other countries.

Place and Show wagers are not addressed in this work, nor are exotic bets like Quinellas, Trifectas, Superfectas, and Daily Doubles. Exactas, which require picking the winner and placer in perfect order, can use the same probabilities as win bets, and, like the futures market are related to common stocks, exacta bets are related to win bets. Like some stock market traders look for stock trends in the future’s markets, some gamblers look for hidden trends in the exacta pools that might lead to a horse which is under bet to win (Hope, 2014).

***A Chemical Engineering approach to wagering at horse races***

How would chemical engineers wager at the track? Some handicappers have listed as many as 70 different variables to be considered. The statistics involved are called “non-stationary” as opposed to the fixed sizes of vessels and pipes (stationary) upon which chemical engineering calculations are usually based. There is no doubt that any approach would involve use of a computer and the building of a data base.

What follows is the chemical engineering author’s method for wagering on horse races. The following strategy and related methodology were used to extend Isaacs’ work to an actual wagering system. It is organized into three major tasks as follows:

1) Finding a financially safe wagering site which pays track odds with minimum transaction costs--primarily “track take.” Real time odds must be available in a digital format that can be converted to computer input. A high speed computer, with programs that “scrape” the latest odds from an internet tote board such as “PHONEBET”[[14]](#footnote-14) and quickly calculate the optimal wagers, is needed since about 40% of all wagers are placed in the last minute before a race starts (Ziemba, 1981). Getting the most accurate odds therefore requires betting as late as possible.

2) Technical analysis and handicapping methods must provide superior win probabilities for the entries. This requires large data bases and data mining computer programs[[15]](#footnote-15). William Quirin, a mathematician and university professor, offers one of the best and most comprehensive studies of the effects of the major variables which he weighted to scientifically determine the probabilities of horses winning races (William L. Quirin, 1979). He used the statistical technique of multiple regression analysis to correlate win probabilities with the predictor variables. He also discussed money management techniques[[16]](#footnote-16) and concluded through simulations that when winning 25% of the time with a 30% ROI, no more than 8% of one’s wealth should be invested in any one race to avoid tapping out. Both flat betting (same amount for each wager) and progressive betting (percentage of one’s present wealth) were considered. The MCM model presented here used progressive betting. Quirin’s system produced an ROI of 8% over 1,000 races when all races were bet.

There are speed differences between tracks depending on the type of surfaces used. West coast tracks have previously had faster times on average than eastern tracks in the U.S., but new synthetic surfaces like AWT, all weather turf, are now being installed which make handicapping more interesting. There are also turf (on the grass) races and those on earthen soil. Whether the track is dry (fast), muddy (slow), or drying is significant.

3) Mathematical models which calculate the optimal wagers, if any, for each race use methods similar to those used for allocating resources for stock market investments. Technical analysis of market parameters provide excellent formats for determining the performance of various methods (Hartle, 1998) when simulating the models on thousands of races.

Hundreds of different methods for calculating optimal wagers have been published (Press, 2011). One of the most scientific methods was developed by William Ziemba, an economist who calls himself “Dr. Z”. Dr. Z noted in his prolific publications that his work showed that the racetrack market was efficient for all bets except place and show (Ziemba, 1981). He defined a market as being “…efficient if current security prices fully reflect all available relevant information. If this is the case, experts should not be able to achieve higher than average returns with regularity.” He and his colleagues developed elaborate formulas to determine the amounts that should be wagered on favorable place and show bets which he found to be the only “inefficient” (bettable) wagers.

Dr. Z’s optimal betting schemes feature adjustment of probabilities for “the favorite-longshot bias” coupled with calculation of the expected value of a wager. As calculated by Isaacs in Equation 3 above, expected value, E*i*, is the real (objective) win probability divided by the crowd (subjective) probability. The expected value of a wager on a horse can also be estimated as the probability of success (objective) times the odds + 1. If this value, E*i*[[17]](#footnote-17), is above 1.0, and the probability is correct, then the wager will produce a profit. This wager is called an “overlay”. When the E*i* is below 1.0, the wager is called an “underlay”. Dr. Z recommends not wagering unless the E*i* is above 1.15 to provide for margin of error, especially if the win pool is small and the odds change significantly in the last minutes of wagering. Another key feature is Dr. Z’s use of the Kelly criterion (J. L. Kelly, 1956) to finally decide his optimal wagers. Kelly worked for Bell Labs and was trying to minimize the amount of cable required for telephone calls. His model has been found to parallel methods useful for calculating optimal horse race wagers. The author has successfully used the Kelly criterion for exactas.

**Investing at the racetrack**

As noted above, the racetrack shares many of the characteristics of the securities markets and the tools used by market technical analysts are also useful for horse race investing or wagering. This author agrees that all races are not ‘inefficient” or suitable for profitable wagering. Being a third generation handicapper and in market parlance, a technical analyst, a list of constraints was determined from “What if?” analyses in thousands of simulations:

•The race must have at least 10, preferably 12 entrants. More entries give more advantage to the high speed computer over manual handicapping.[[18]](#footnote-18) Remember, with the *pari-mutuel* system, you are competing against all of the other bettors.

•There must be at least a total of $8,000 in the win pool with three minutes to post time (all horses in the starting gate). This TWP, total win pool, of $8,000 will permit betting on typical weekday races. Here again, more is better as there will be more stable odds upon which to base calculations. As noted previously , larger wagers are possible in higher graded races with correspondingly larger purses. Screening out races with win pools less than $100,000 provided fewer bettable races but led to higher wagers and profits. Not surprisingly, the higher quality horses in these races ran more consistently and also led to higher win percentages and ROI's.

•Another assumption is that all of the odds must not change significantly before the bets are placed. We are looking to place our bets last. The “triple crown” races, including the Kentucky Derby, and the Breeder’s Cup races have millions of dollars in the pools hours before post time so calculations can begin earlier.[[19]](#footnote-19) As opposed to the “fast traders” of the stock market who seek to make their investments millionths of a second before other traders, we are seeking to make our investments at the last possible moment when the odds are less likely to change significantly (Hope, 2014).

•Similar to Isaacs, to be wagered upon, the expected value of the horse must be greater than 1.0. Our probabilities must be more accurate than the crowd’s for this to hold. This value can easily be changed to 1.15 or some other more conservative number.

•Regardless of the outcome, all race wagers have limited liability, i.e. in about two minutes one will know whether he has won or lost and one can only lose the amount of the wager. This unequivocal outcome and an associated rate of return within a finite time frame provide an objective benchmark to measure the quality of an investment. In England, the winning pay-offs are called “dividends”.

•If a horse or entry is so heavily bet that his odds go below 10 cents on a dollar, this is called a “minus pool” and the track must pay a minimum of 10 cents for each dollar wagered on this heavy favorite if he wins. Income tax is withheld using IRS form W2-G for individual race winnings over a certain amount at the track or OTB (off-track betting parlor). Internet accounts also provide the IRS with information on your profits. This is another factor to consider in calculating your ROI.

•Due to what is called “the favorite-longshot bias” the expected returns decline as risk increases, i.e. longshots lose proportionately more than favorites do according to odds probabilities. In the stock market, one expects additional reward for more risk.

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1. See Appendix A, "Background for the MCM horse racing model" for explanation of *pari-mutuel.* [↑](#footnote-ref-1)
2. Played by Russell Crowe in the movie, “A Beautiful Mind.” [↑](#footnote-ref-2)
3. Isaacs did not take into account that a positive maximum could be achieved with wagers on some winning horses not producing an overall profit for the race. Colloquially, this is called “betting against yourself” and happens frequently when bettors make numerous bets on a race without adequate analysis. [↑](#footnote-ref-3)
4. Taking the derivative of the model function(s) is not necessary when using *Monte Carlo Marching.* [↑](#footnote-ref-4)
5. It can be shown that for a series of 200 races, with a probability of losing and winning of 0.5, that the longest run of wins or losses is expected to be 7. The 95% confidence limits with p = 0.5 are ± 3 so one would expect the longest runs of wins or losses to be 7 ± 3 or between 4 and 10. A bettor with $100 to wager over 10 races must therefore not wager more than $10 per race when betting on odds-on favorites. When the probability of losing is higher, the losing streaks can be expected to be longer, i.e. with p(losing) = 0.6, streaks of 9 are expected ± confidence limits. (Schilling, 2012) [↑](#footnote-ref-5)
6. The races were the most recent in the author’s data bank of over 13,000 races with the last race at Santa Anita in November, 2013. Objective probabilities were obtained from the author’s probability model which is not shown here. [↑](#footnote-ref-6)
7. QB64 is a modern version of Microsoft QUICKBASIC© which Microsoft no longer supports. It is an open source programming language that will run on 32 or 64 bit machines with Windows XP, VISTA, Windows 7, Linux and MAC OSX. It is compatible with VBA and has many features such as stereo sound, graphics loading and transformations, TCP/IP internet capabilities, devices (joysticks), screen capture, TTF fonts, UNICODE and IME input; and clipboard access. [↑](#footnote-ref-7)
8. V is the fraction of vapor flashed; Z, X, and Y are the feed, liquid, and vapor mole fractions, respectively; and P is the total system pressure in kPa. The sum of the Z's, X's, and Y's must equal 1. G4 is the overall mass balance. [↑](#footnote-ref-8)
9. This precipitous drop in horse #7’s odds invalidates the model as the horse will pay significantly less than the odds used in the calculations. [↑](#footnote-ref-9)
10. Many bettors or “punters” as they are called in England “handicap the handicappers”\*, i.e. they watch for significant wagers on a horse as seen by the drop in his odds on the tote board. They assume that these bettors have “inside information” and therefore also wager heavily on the same horse further driving down the odds. [↑](#footnote-ref-10)
11. W is the number of horses. [↑](#footnote-ref-11)
12. M/L stands for Morning Line which are the odds proposed by the track handicapper before the race. [↑](#footnote-ref-12)
13. Note that Isaacs used 0.15 which should lead to more favorable profits, i.e. more of the win pool is available to bettors. [↑](#footnote-ref-13)
14. The web site is [www.parxracing.com](http://www.parxracing.com/) and is run by Philadelphia Park race track. EXCEL© or parsing of the data can be used to import the odds to a spreadsheet where a macro can perform the necessary calculations. [↑](#footnote-ref-14)
15. The author has a data base of over 13,000 races run at major North American tracks. For past odds information, the Daily Racing Form ([www.drf.com](http://www.drf.com/)) provides PDF results charts of races at most North American race tracks. Data on major predictor variables comes from “The Daily Racing Form”, programs printed prior to each racing day. The Equibase Company provides detailed historical information back to the year 2000 and includes video race replays. The web site is [www.equibase.com](http://www.equibase.com/) which also offers the hobbyist a chance to own a “virtual” stable. The “TrackMaster Pocket Handicapper” was designed for iPhones and Android Smartphones for “handicapping on the go.” As advertised, “…Let’s you go deep inside the vast amounts of exclusive ratings and statistics of each horse race.” The TVG internet wagering site at [www.tvg.com](http://www.tvg.com) has an “iPhone App” that permits wagering from an iPhone at 150 major racetracks. [↑](#footnote-ref-15)
16. Called “asset allocation” by stock brokers. [↑](#footnote-ref-16)
17. For “odds-on” favorites which pay even money or $1 for every dollar bet, if the “objective” probability found from thousands of races is 0.53, the expected value would be calculated as 0.53 x (odds + 1) or 0.53 x 2 = 1.06, an overlay. For a $2 wager, the bettor would receive $2 for odds of 1 to 1 plus his original bet for a total of $4.00. [↑](#footnote-ref-17)
18. The Jockey Club ([www.jockey.com](http://www.jockey.com/)), which was incorporated in New York in 1894, is an organization dedicated to “the improvement of Thoroughbred breeding and racing”. Their primary responsibility is the maintenance of the “American Stud Book”, which tracks horse lineage and approves names for all American horses. They publish an annual statistics book called “The Jockey Club Fact Book” which shows data through 2013 in the 2014 version. It shows that the 2013 "average" field for the 48,580 races in North America was 7.85 horses. The highest ever "average" field was 9.07 in 1950. This shows that there will sometimes be long waits for fields of 10 or higher. [↑](#footnote-ref-18)
19. Jockey Club data show that the gross purses for North America in 2013 were $1,235,300,000 or an average of $25,428 for 48,580 total races. The 2013 total pari-mutuel handle was $1,087,600,000 including worldwide commingling of wagers on North American tracks. There was $35,080,700 in the win pool at Churchill Downs for the $2,000,000 purse Kentucky Derby won by California Chrome on May 2, 2014. Interestingly, only about 10.9% of 2013 bets were placed at the tracks, with the balance coming from off-track wagering. Note that this only considers legal wagering. [↑](#footnote-ref-19)