Bachelor's thesis

Category theory and lambda calculus

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The first part of this thesis describes the untyped and the simply-typed **lambda calculus**. The **Brower-Heyting-Kolmogorov interpretation** of types leads to a full isomorphism between proofs in **Intuitionistic Propositional Logic** and simply-typed lambda calculus terms. We implement a complete lambda calculus interpreter in the Haskell programming language that can show the derivation tree in Gentzen's natural deduction style of any given term.

The second part describes categories, natural transformations, adjoints and Kan extensions. **Lawvere's algebraic theories** are discussed as a first example of categorical logic and **topoi** are studied as an intuitionistic generalization of the Lawvere's **Elementary Theory of the Category of Sets**. This categorical framework is then used to extend the previous isomorphism between simply-typed lambda calculus and propositional logic to **cartesian closed categories**. Robert Seely's 1984 paper refines this idea to a correspondence between locally closed cartesian categories and **Martin-Löf type theories**.

The thesis finishes with a discussion of the **Agda** programming language and how dependent type theories with the **Voevodsky's Univalence** axiom can be used to develop an intuitionistic foundation of mathematics.