Teoría de Categorías

Introducción y aplicación a la programación

Mario Román

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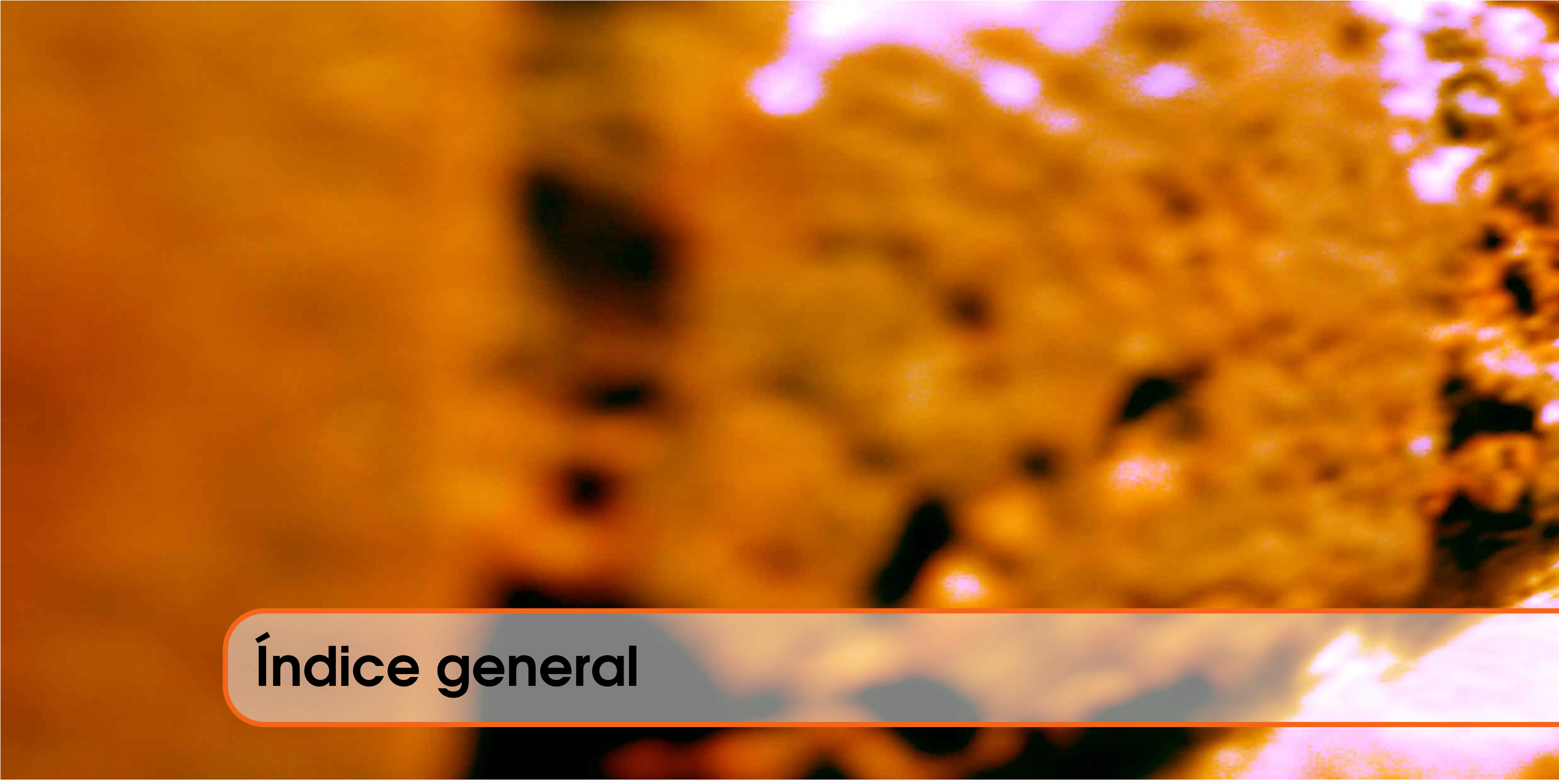


Índice general

I Teoría de Categorías

Teoría de Categorías

1. Categorías

1.1 Motivación

Varias estructuras matemáticas (grupos, espacios vectoriales, espacios topológicos ...) cuentan con morfismos que preservan las estructura subyacentes entre ellas. Como ejemplos:

Conjunto	Morfismos
Grupos	Homomorfismos de grupos
Espacios topológicos	Funciones continuas
Espacios métricos	Funciones cortas
Conjuntos	Funciones
Espacios vectoriales sobre $\mathbb{K}$	Funciones lineales sobre $\mathbb{K}$

Si estudiamos axiomáticamente las propiedades abstractas de estas estructuras y sus morfismos, obtendremos teoremas particularizables a todos estos casos, útiles por sí mismos. Una categoría la formarán una clase de estos espacios con estructura y los morfismos entre estos espacios; y los teoremas que deduzcamos para todas las categorías podrán aplicarse a cada uno de los espacios.

1.2 Definición formal

Definition 1.2.1 Una **categoría** $\mathscr{C}$ está definida por:

- Una clase de objetos de la categoría, $Obj(\mathscr{C})$.
- Un conjunto de morfismos $Hom_{\mathscr{C}}(A,B)$, poblado o no, entre cada par de objetos $A,B \in Obj(\mathscr{C})$.

Cumpliendo sus morfismos las siguientes propiedades:

- Para dos morfismos $f \in Hom(A,B)$, $g \in Hom(B,C)$, existe su morfismo composición $f \circ g$.
- La composición es asociativa: $f \circ (g \circ h) = (f \circ g) \circ h$
- Todos los objetos tienen un morfismo identidad, $1_A \in Hom(A,A)$, neutro para la composición: $\forall f \in Hom(A,B) : f \circ 1_A = 1_B \circ f = f$

Exercise 1.1 Demostrar que la identidad es el único elemento neutro para la composición. ■

$$
\begin{tikzcd}
A \arrow[r, "f"] \arrow[dr, "g\circ f"'] & B \arrow[d, "g"] \\
& C
\end{tikzcd}
\qquad
\begin{tikzcd}
A \arrow[r, "f"] \arrow[dr, "g\circ f"'] & B \arrow[d, "g"] \arrow[dr, "h\circ g"] & \\
& C \arrow[r, "h"] & D
\end{tikzcd}
\qquad
\begin{tikzcd}
A \arrow[loop right, "1_A"]
\end{tikzcd}
$$

Diagramas conmutativos de las propiedades básicas.

1.3 Texto en castellano con acentuación

Este frase contiene signos acentuación, eñes, ¿y signos de interrogación?

1.4 Citation

This statement requires citation [**book_key**]; this one is more specific [**article_key**].

1.5 Lists

Lists are useful to present information in a concise and/or ordered way[1].

1.5.1 Numbered List

1. The first item
2. The second item
3. The third item

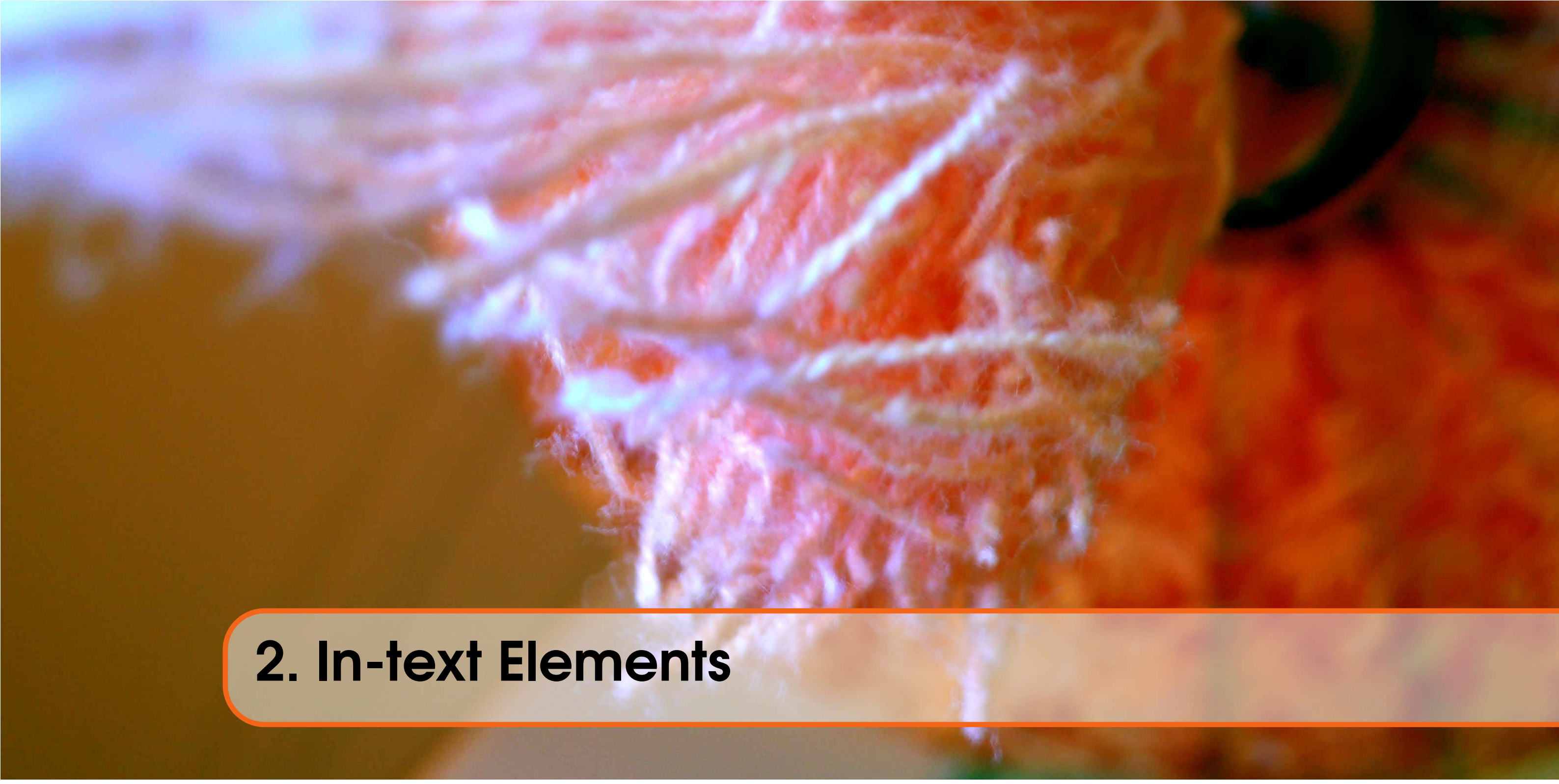
1.5.2 Bullet Points

- The first item
- The second item
- The third item

1.5.3 Descriptions and Definitions

Name Description
Word Definition
Comment Elaboration

[1] Footnote example...



2. In-text Elements

2.1 Theorems

This is an example of theorems.

2.1.1 Several equations

This is a theorem consisting of several equations.

Theorem 2.1.1 — Name of the theorem. In $E=\mathbb{R}^n$ all norms are equivalent. It has the properties:

$$\big|||\mathbf{x}||-||\mathbf{y}||\big| \leq ||\mathbf{x}-\mathbf{y}|| \tag{2.1}$$

$$||\sum_{i=1}^{n}\mathbf{x}_i|| \leq \sum_{i=1}^{n}||\mathbf{x}_i|| \quad \text{where } n \text{ is a finite integer} \tag{2.2}$$

2.1.2 Single Line

This is a theorem consisting of just one line.

Theorem 2.1.2 A set $\mathscr{D}(G)$ in dense in $L^2(G)$, $|\cdot|_0$.

2.2 Definitions

This is an example of a definition. A definition could be mathematical or it could define a concept.

Definition 2.2.1 — Definition name. Given a vector space E, a norm on E is an application, denoted $||\cdot||$, E in $\mathbb{R}^+ = [0, +\infty[$ such that:

$$||\mathbf{x}|| = 0 \Rightarrow \mathbf{x} = \mathbf{0} \tag{2.3}$$

$$||\lambda\mathbf{x}|| = |\lambda| \cdot ||\mathbf{x}|| \tag{2.4}$$

$$||\mathbf{x}+\mathbf{y}|| \leq ||\mathbf{x}|| + ||\mathbf{y}|| \tag{2.5}$$

2.3 Notations

Notation 2.1. *Given an open subset G of $\mathbb{R}^n$, the set of functions φ are:*

1. *Bounded support G;*
2. *Infinitely differentiable;*

a vector space is denoted by $\mathscr{D}(G)$.

2.4 Remarks

This is an example of a remark.

R The concepts presented here are now in conventional employment in mathematics. Vector spaces are taken over the field $\mathbb{K} = \mathbb{R}$, however, established properties are easily extended to $\mathbb{K} = \mathbb{C}$.

2.5 Corollaries

This is an example of a corollary.

Corollary 2.5.1 — Corollary name. The concepts presented here are now in conventional employment in mathematics. Vector spaces are taken over the field $\mathbb{K} = \mathbb{R}$, however, established properties are easily extended to $\mathbb{K} = \mathbb{C}$.

2.6 Propositions

This is an example of propositions.

2.6.1 Several equations

Proposition 2.6.1 — Proposition name. It has the properties:

$$\big| ||\mathbf{x}|| - ||\mathbf{y}|| \big| \leq ||\mathbf{x}-\mathbf{y}|| \tag{2.6}$$

$$||\sum_{i=1}^{n} \mathbf{x}_i|| \leq \sum_{i=1}^{n} ||\mathbf{x}_i|| \quad \text{where } n \text{ is a finite integer} \tag{2.7}$$

2.6.2 Single Line

Proposition 2.6.2 Let $f, g \in L^2(G)$; if $\forall \varphi \in \mathscr{D}(G)$, $(f, \varphi)_0 = (g, \varphi)_0$ then $f = g$.

2.7 Examples

This is an example of examples.

2.7.1 Equation and Text

■ **Example 2.1** Let $G = \{x \in \mathbb{R}^2 : |x| < 3\}$ and denoted by: $x^0 = (1,1)$; consider the function:

$$f(x) = \begin{cases} e^{|x|} & \text{si } |x - x^0| \leq 1/2 \\ 0 & \text{si } |x - x^0| > 1/2 \end{cases} \tag{2.8}$$

The function f has bounded support, we can take $A = \{x \in \mathbb{R}^2 : |x - x^0| \leq 1/2 + \varepsilon\}$ for all $\varepsilon \in]0;5/2 - \sqrt{2}[$. ■

2.7.2 Paragraph of Text

■ **Example 2.2 — Example name.** Lorem ipsum. ■

2.8 Exercises

This is an example of an exercise.

Exercise 2.1 This is a good place to ask a question to test learning progress or further cement ideas into students' minds. ■

2.9 Problems

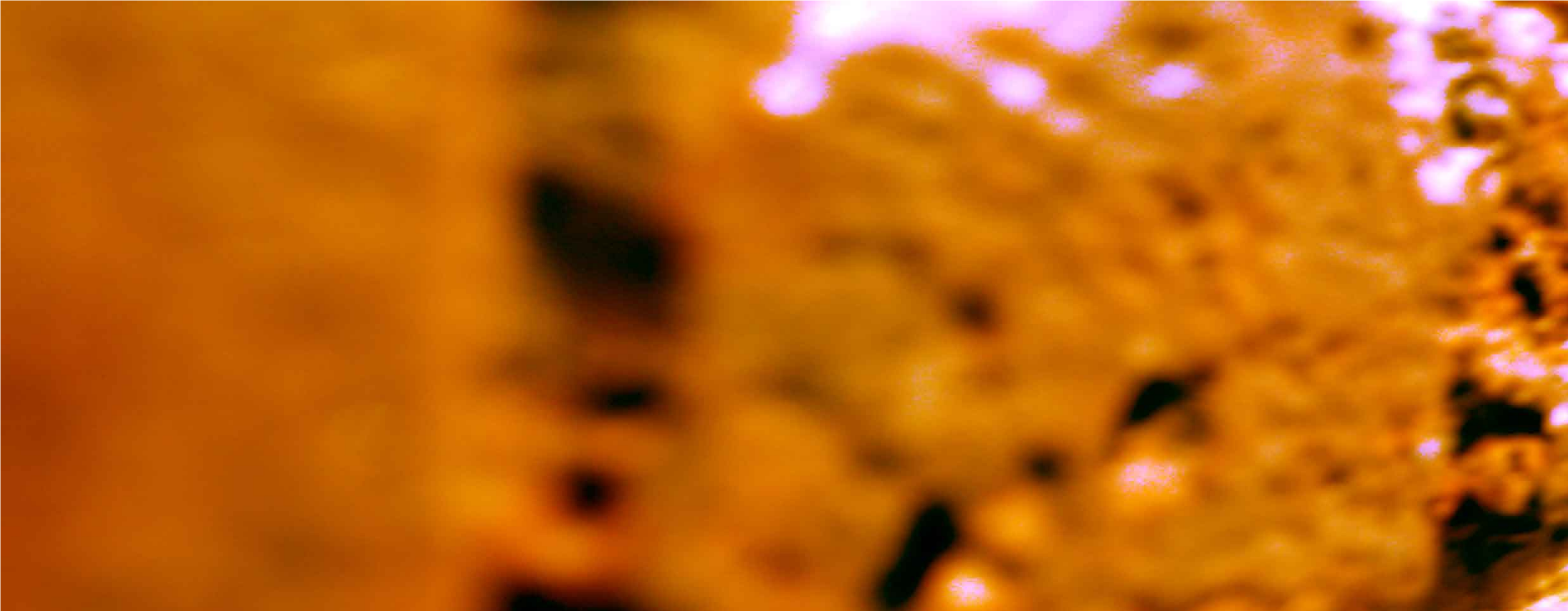
Problem 2.1 What is the average airspeed velocity of an unladen swallow?

2.10 Vocabulary

Define a word to improve a students' vocabulary.

Vocabulary 2.1 — Word. Definition of word.

Part Two

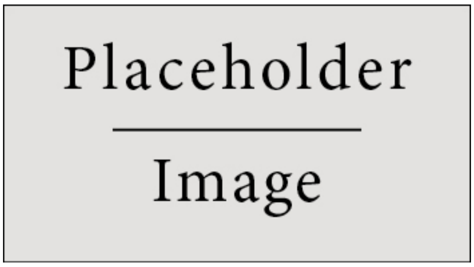


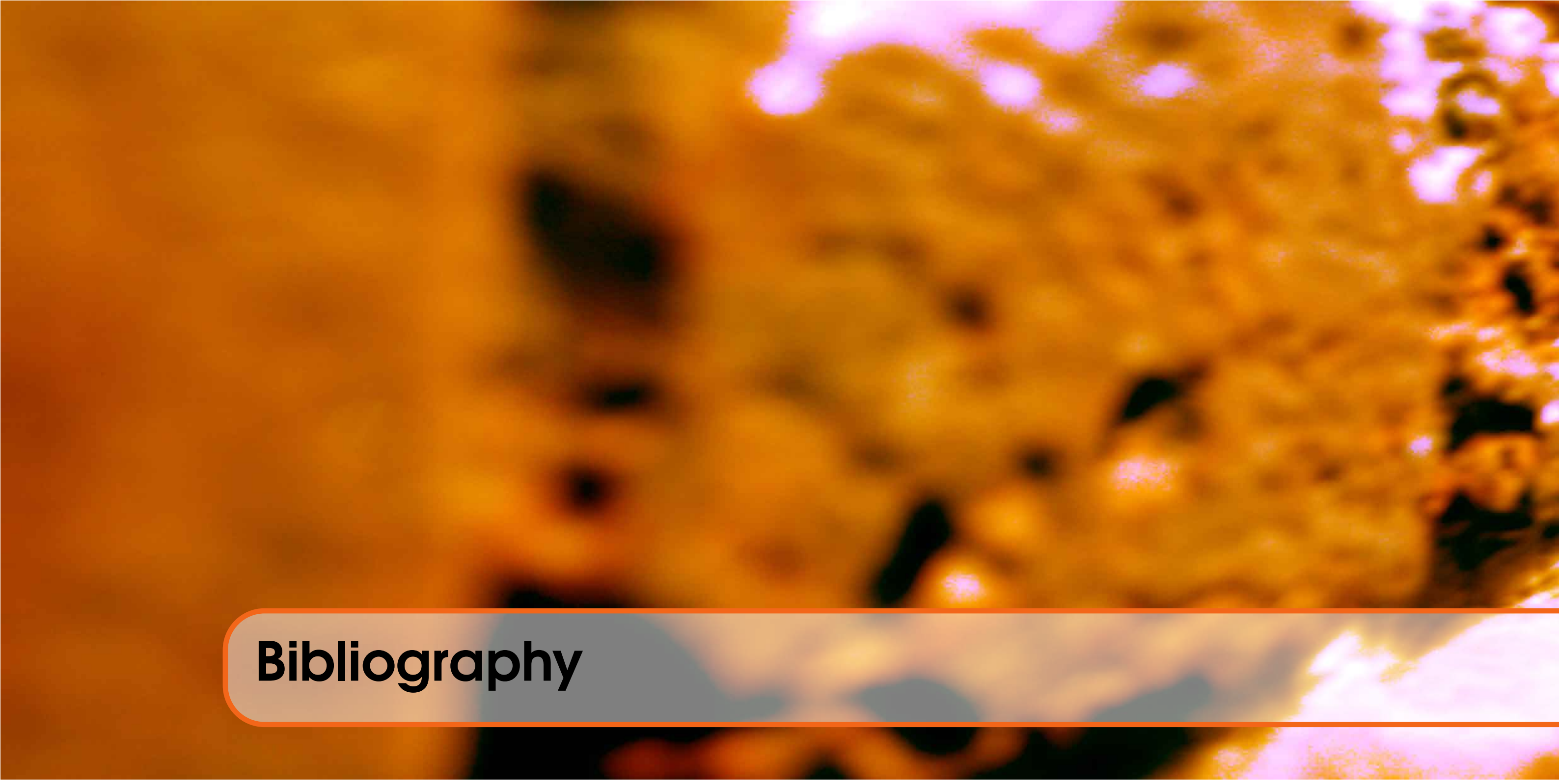
3. Presenting Information

3.1 Table

Treatments	Response 1	Response 2
Treatment 1	0.0003262	0.562
Treatment 2	0.0015681	0.910
Treatment 3	0.0009271	0.296

3.2 Figure





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