



# Teoría de Categorías

Introducción y aplicación a la programación

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DOBLE GRADO EN MATEMÁTICAS E INFORMÁTICA DE LA UNIVERSIDAD DE GRANADA

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# Teoría de Categorías

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# 1. Categorías

## 1.1 Motivación

Varias estructuras matemáticas (grupos, espacios vectoriales, espacios topológicos ...) cuentan con morfismos que preservan las estructura subyacentes entre ellas. Como ejemplos:

Conjunto	Morfismos
Grupos	Homomorfismos de grupos
Espacios topológicos	Funciones continuas
Espacios métricos	Funciones cortas
Conjuntos	Funciones
Espacios vectoriales sobre $\mathbb{K}$	Funciones lineales sobre $\mathbb{K}$

Si estudiamos axiomáticamente las propiedades abstractas de estas estructuras y sus morfismos, obtendremos teoremas particularizables a todos estos casos, útiles por sí mismos. Una categoría la formarán una clase de estos espacios con estructura y los morfismos entre estos espacios; y los teoremas que deduzcamos para todas las categorías podrán aplicarse a cada uno de los espacios.

## 1.2 Definición formal

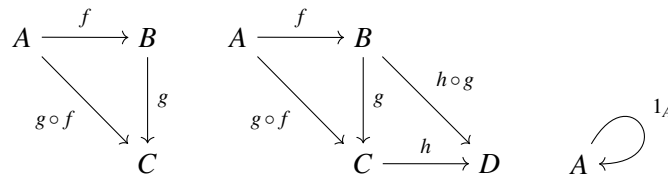
**Definition 1.2.1** Una **categoría**  $\mathcal{C}$  está definida por:

- Una clase de objetos de la categoría,  $Obj(\mathcal{C})$ .
- Un conjunto de morfismos  $Hom_{\mathcal{C}}(A, B)$ , poblado o no, entre cada par de objetos  $A, B \in Obj(\mathcal{C})$ .

Cumpliendo sus morfismos las siguientes propiedades:

- Para dos morfismos  $f \in Hom(A, B)$ ,  $g \in Hom(B, C)$ , existe su morfismo composición  $f \circ g$ .
- La composición es asociativa:  $f \circ (g \circ h) = (f \circ g) \circ h$
- Todos los objetos tienen un morfismo identidad,  $1_A \in Hom(A, A)$ , neutro para la composición:  
 $\forall f \in Hom(A, B) : f \circ 1_A = 1_B \circ f = f$

**Exercise 1.1** Demostrar que la identidad es el único elemento neutro para la composición. ■



*Diagramas conmutativos de las propiedades básicas.*

### 1.3 Texto en castellano con acentuación

Este frase contiene signos acentuación, eñes, ¿y signos de interrogación?

### 1.4 Citation

This statement requires citation [**book\_key** ]; this one is more specific [**article\_key** ].

### 1.5 Lists

Lists are useful to present information in a concise and/or ordered way<sup>1</sup>.

#### 1.5.1 Numbered List

1. The first item
2. The second item
3. The third item

#### 1.5.2 Bullet Points

- The first item
- The second item
- The third item

#### 1.5.3 Descriptions and Definitions

**Name** Description

**Word** Definition

**Comment** Elaboration

---

<sup>1</sup>Footnote example...



## 2. In-text Elements

### 2.1 Theorems

This is an example of theorems.

#### 2.1.1 Several equations

This is a theorem consisting of several equations.

**Theorem 2.1.1 — Name of the theorem.** In  $E = \mathbb{R}^n$  all norms are equivalent. It has the properties:

$$||\mathbf{x}|| - ||\mathbf{y}|| \leq ||\mathbf{x} - \mathbf{y}|| \quad (2.1)$$

$$||\sum_{i=1}^n \mathbf{x}_i|| \leq \sum_{i=1}^n ||\mathbf{x}_i|| \quad \text{where } n \text{ is a finite integer} \quad (2.2)$$

#### 2.1.2 Single Line

This is a theorem consisting of just one line.

**Theorem 2.1.2** A set  $\mathcal{D}(G)$  is dense in  $L^2(G)$ ,  $|\cdot|_0$ .

### 2.2 Definitions

This is an example of a definition. A definition could be mathematical or it could define a concept.

**Definition 2.2.1 — Definition name.** Given a vector space  $E$ , a norm on  $E$  is an application,

denoted  $\|\cdot\|$ ,  $E$  in  $\mathbb{R}^+ = [0, +\infty[$  such that:

$$\|\mathbf{x}\| = 0 \Rightarrow \mathbf{x} = \mathbf{0} \quad (2.3)$$

$$\|\lambda \mathbf{x}\| = |\lambda| \cdot \|\mathbf{x}\| \quad (2.4)$$

$$\|\mathbf{x} + \mathbf{y}\| \leq \|\mathbf{x}\| + \|\mathbf{y}\| \quad (2.5)$$

## 2.3 Notations

**Notation 2.1.** Given an open subset  $G$  of  $\mathbb{R}^n$ , the set of functions  $\varphi$  are:

1. Bounded support  $G$ ;
2. Infinitely differentiable;

a vector space is denoted by  $\mathcal{D}(G)$ .

## 2.4 Remarks

This is an example of a remark.

**R** The concepts presented here are now in conventional employment in mathematics. Vector spaces are taken over the field  $\mathbb{K} = \mathbb{R}$ , however, established properties are easily extended to  $\mathbb{K} = \mathbb{C}$ .

## 2.5 Corollaries

This is an example of a corollary.

**Corollary 2.5.1 — Corollary name.** The concepts presented here are now in conventional employment in mathematics. Vector spaces are taken over the field  $\mathbb{K} = \mathbb{R}$ , however, established properties are easily extended to  $\mathbb{K} = \mathbb{C}$ .

## 2.6 Propositions

This is an example of propositions.

### 2.6.1 Several equations

**Proposition 2.6.1 — Proposition name.** It has the properties:

$$\left| \|\mathbf{x}\| - \|\mathbf{y}\| \right| \leq \|\mathbf{x} - \mathbf{y}\| \quad (2.6)$$

$$\left\| \sum_{i=1}^n \mathbf{x}_i \right\| \leq \sum_{i=1}^n \|\mathbf{x}_i\| \quad \text{where } n \text{ is a finite integer} \quad (2.7)$$

### 2.6.2 Single Line

**Proposition 2.6.2** Let  $f, g \in L^2(G)$ ; if  $\forall \varphi \in \mathcal{D}(G)$ ,  $(f, \varphi)_0 = (g, \varphi)_0$  then  $f = g$ .

## 2.7 Examples

This is an example of examples.

### 2.7.1 Equation and Text

■ **Example 2.1** Let  $G = \{x \in \mathbb{R}^2 : |x| < 3\}$  and denoted by:  $x^0 = (1, 1)$ ; consider the function:

$$f(x) = \begin{cases} e^{|x|} & \text{si } |x - x^0| \leq 1/2 \\ 0 & \text{si } |x - x^0| > 1/2 \end{cases} \quad (2.8)$$

The function  $f$  has bounded support, we can take  $A = \{x \in \mathbb{R}^2 : |x - x^0| \leq 1/2 + \varepsilon\}$  for all  $\varepsilon \in ]0; 5/2 - \sqrt{2}[$ . ■

### 2.7.2 Paragraph of Text

■ **Example 2.2 — Example name.** Lorem ipsum. ■

## 2.8 Exercises

This is an example of an exercise.

**Exercise 2.1** This is a good place to ask a question to test learning progress or further cement ideas into students' minds. ■

## 2.9 Problems

**Problem 2.1** What is the average airspeed velocity of an unladen swallow?

## 2.10 Vocabulary

Define a word to improve a students' vocabulary.

**Vocabulary 2.1 — Word.** Definition of word.





# Part Two

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## 3. Presenting Information

### 3.1 Table

Treatments	Response 1	Response 2
Treatment 1	0.0003262	0.562
Treatment 2	0.0015681	0.910
Treatment 3	0.0009271	0.296

### 3.2 Figure









## Bibliography

Books

Articles

