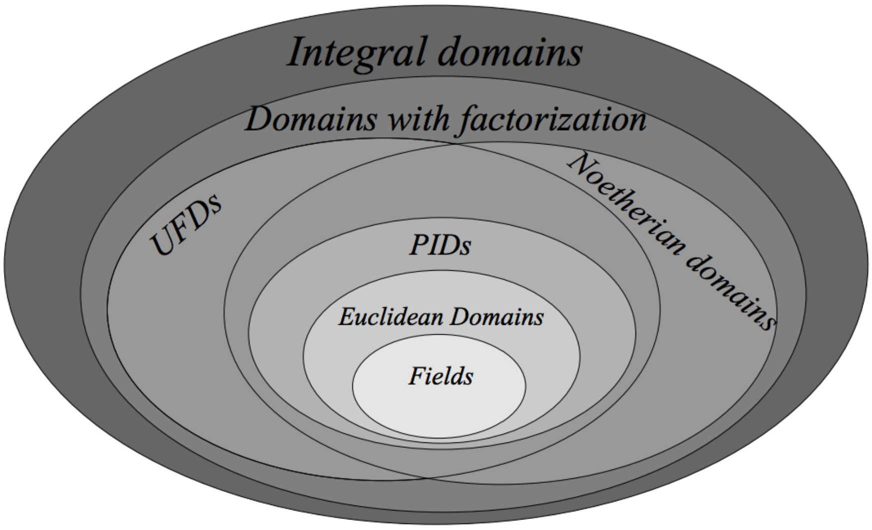

Definitions

Integral domain

The product of any nonzero elements is nonzero.

Domain with factorization

Every nonzero non-unit element can be written as a finite product of irreducible elements.

UFD

Every nonzero element can be written as a product of prime elements and a unit. Equivalently, every nonzero element can be written as an unique product (up to permutations and associates) of irreducible elements.

Noetherian ring

Verifies the ascending chain condition on ideals. Any chain:

$$I_1 \subset I_2 \subset I_3 \subset \cdots \subset I_n \subset \dots$$

stabilizes after a finite number of steps.

PID

Every ideal is principal (can be generated by a single element).

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Euclidean domain

Ring endowed with an euclidean function, a function $\Phi : R \to \mathbb{N}$ satisfiying:

$$\forall a, b \in R \setminus \{0\} : \exists q, r \in R : \quad a = bq + r, \text{ and } \Phi(r) < \Phi(b)$$

Field

Every element has an inverse.

Examples

Integral domain

$$\mathbb{K}\left[x, x^{\frac{1}{2}}, x^{\frac{1}{4}}, \dots\right]$$

Domain with factorization:

$$\mathbb{Z}\left[\sqrt{-5}\right]$$

UFD:

$$\mathbb{K}\left[x_1, x_2, x_3 \dots\right]$$

Noetherian ring:

$$\mathbb{Z}[x]$$

PID:

$$\mathbb{Z}\left[\tfrac{1+\sqrt{19}}{2}\right]$$

Euclidean domain:

$$\mathbb{Z}$$

Field:

$$\mathbb{Q}$$

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