

## 5.1 Reduction Sequence

We get the following reduction sequence:

$$\begin{aligned} ((Y\ t)1)k &\rightarrow \text{if } (\text{isZero } 1) \ 0 \ (\text{add } k \ ((Y\ t) \ (\text{pred } 1)k)) \\ &\rightarrow \text{if } (\text{False}) \ 0 \ (\text{add } k \ ((Y\ t) \ 0\ k)) \\ &\rightarrow \text{add } k \ (\text{if } (\text{isZero } 0) \ 0 \ (\text{add } k \ ((Y\ t) \ (\text{pred } 0)k)) \\ &\rightarrow \text{add } k \ (\text{if } (\text{True}) \ 0 \ (...)) \\ &\rightarrow \text{add } k \ 0 \\ &\rightarrow k \end{aligned}$$

## 5.2 List Operators

### 5.2.1 Non-Recursive From

$$\text{append} = \lambda f. \lambda l_1. \lambda l_2. \text{if } (\text{null } l_1) \ l_2 \ (\text{cons } (\text{head } l_1)) \ (f \ (\text{tail } l_1) \ l_2)$$

### 5.2.2 Test your Result

We get the following sequence with  $L_1 = \text{cons } 1 \ (\text{cons } 2 \ \text{nil})$  and  $L_2 = \text{cons } 3 \ \text{nil}$ :

$$\begin{aligned} \text{append } L_1 \ L_2 &\rightarrow \text{if } (\text{null } L_1) \ L_2 \ (\text{cons } (\text{head } L_1) \ (f \ (\text{tail } L_1) \ L_2)) \\ &\rightarrow \text{if } (\text{False}) \ L_2 \ (\text{cons } 1 \ (f \ (\text{tail } (\text{cons } 2 \ \text{nil})) \ L_2)) \\ &\rightarrow \text{cons } 1 \ (\text{if } (\text{null } (\text{cons } 2 \ \text{nil})) \ L_2 \ (\text{cons } (\text{head } (\text{cons } 2 \ \text{nil})) \ (f \ (\text{tail } (\text{cons } 2 \ \text{nil})) \ L_2)) \\ &\rightarrow \text{cons } 1 \ (\text{if } (\text{False}) \ L_2 \ (\text{cons } 2 \ (f \ (\text{nil}) \ L_2)) \\ &\rightarrow \text{cons } 1 \ (\text{cons } 2 \ (f \ (\text{nil}) \ L_2)) \\ &\rightarrow \text{cons } 1 \ (\text{cons } 2 \ (\text{if } (\text{null } \text{nil}) \ L_2 \ (\text{cons } (\text{head } \text{nil}) \ (f \ (\text{tail } \text{nil}) \ L_2))) \\ &\rightarrow \text{cons } 1 \ (\text{cons } 2 \ (\text{if } (\text{True}) \ \text{cons } 3 \ \text{nil} \ (...)) \\ &\rightarrow \text{cons } 1 \ (\text{cons } 2 \ (\text{cons } 3 \ \text{nil})) \end{aligned}$$