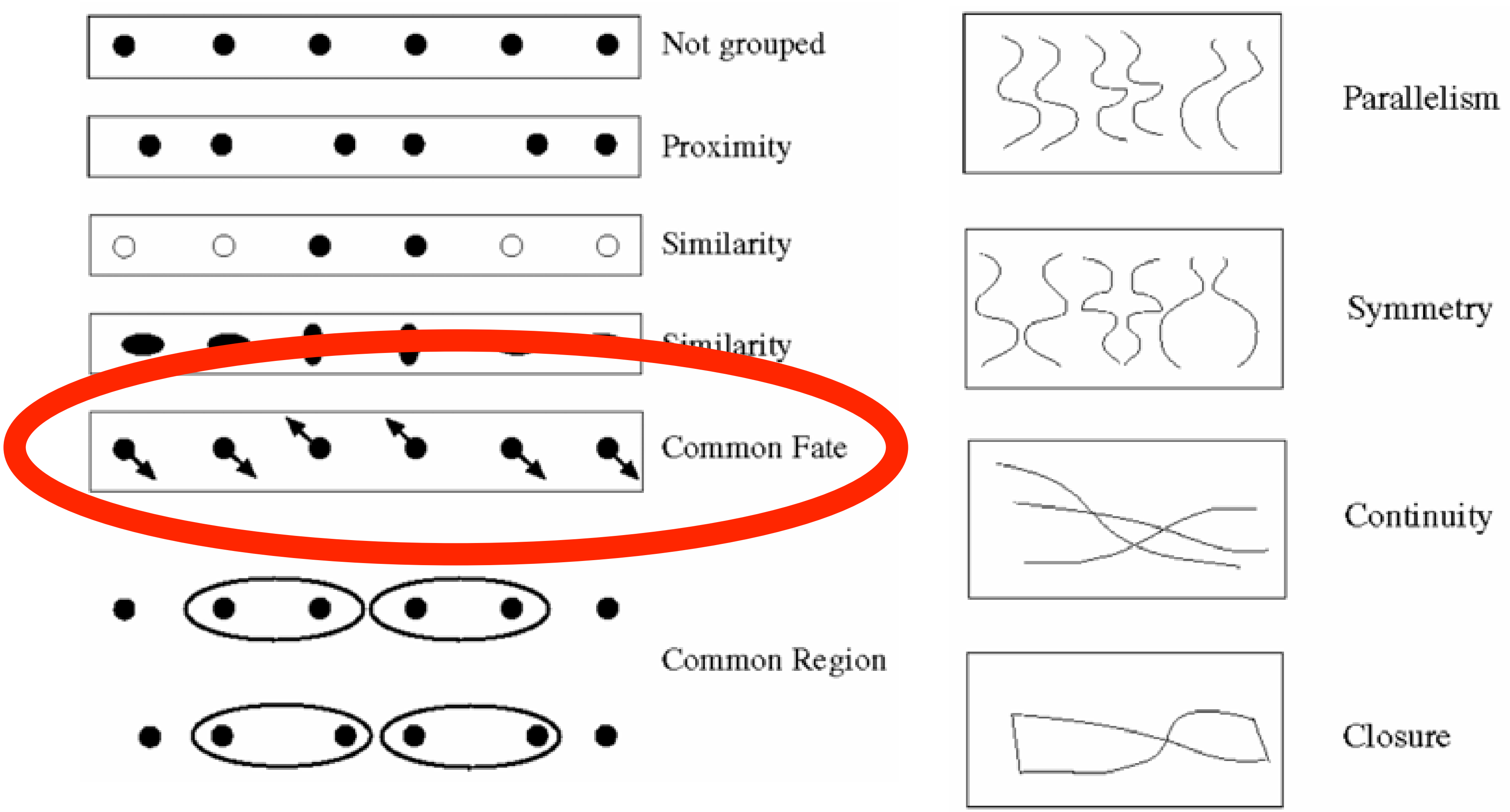
Visual motion

Many slides adapted from S. Seitz, R. Szeliski, M. Pollefeys

Motion and perceptual organization

- Sometimes, motion is the only cue



Motion and perceptual organization

- Even “impoverished” motion data can evoke a strong percept

G. Johansson, “Visual Perception of Biological Motion and a Model For Its Analysis", *Perception and Psychophysics 14, 201-211, 1973.*

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[YouTube video]

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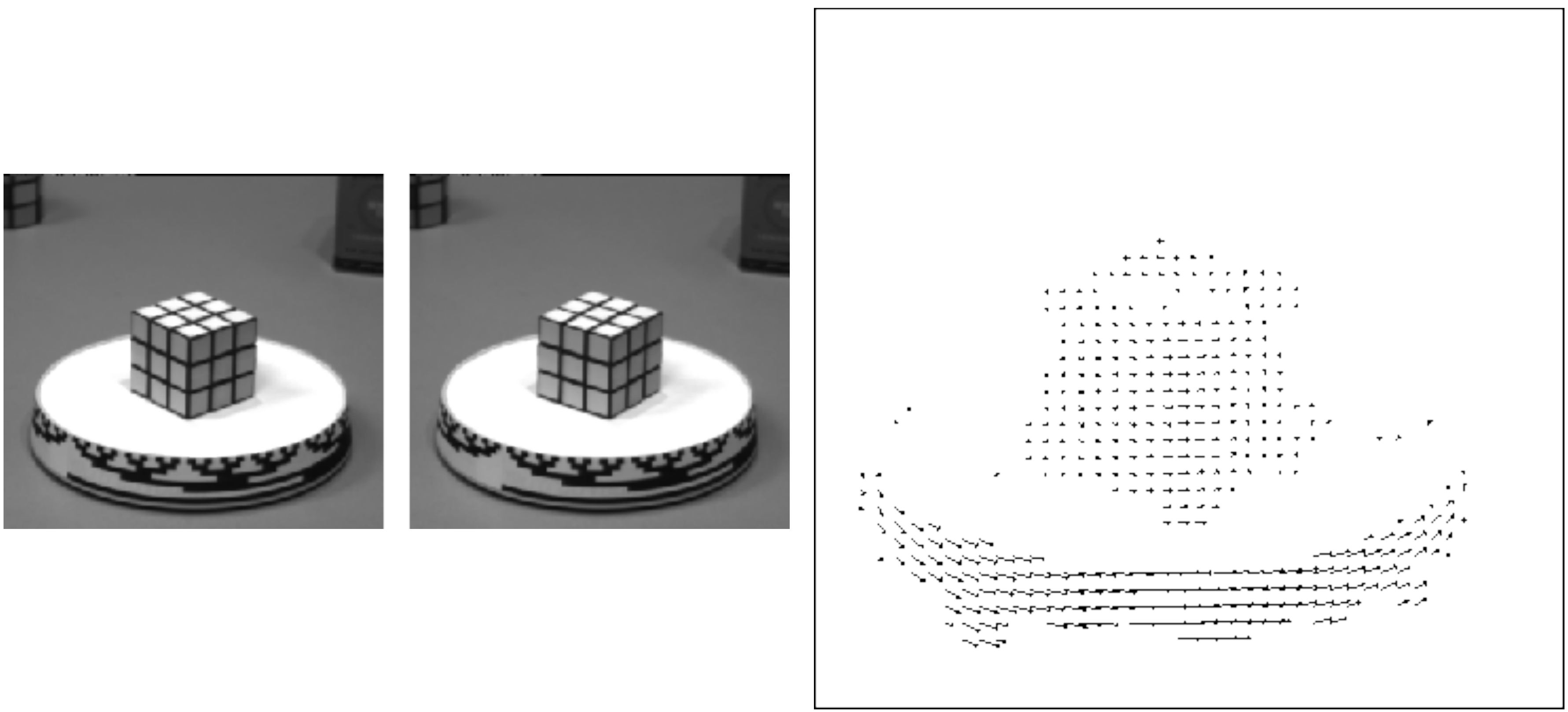
Uses of motion

- Estimating 3D structure
- Segmenting objects based on motion cues
- Learning and tracking dynamical models
- Recognizing events and activities

Motion field

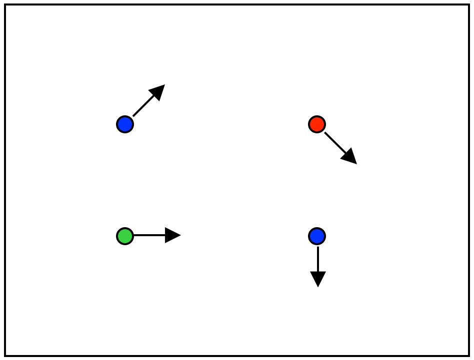
- The motion field is the projection of the 3D scene motion into the image



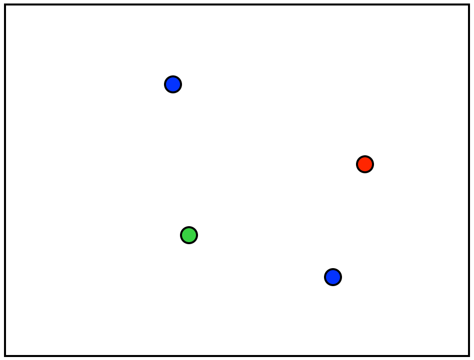
Optical flow

- Definition: optical flow is the *apparent* motion of brightness patterns in the image
- Ideally, optical flow would be the same as the motion field
- Have to be careful: apparent motion can be caused by lighting changes without any actual motion
  - Think of a uniform rotating sphere under fixed lighting vs. a stationary sphere under moving illumination

Estimating optical flow



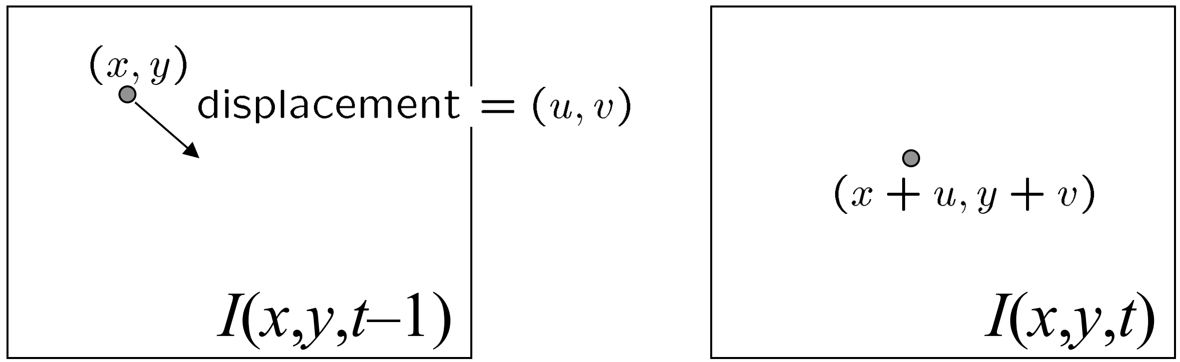
$I(x,y,t-1)$



$I(x,y,t)$

- Given two subsequent frames, estimate the apparent motion field $u(x,y)$ and $v(x,y)$ between them
- Key assumptions
  - **Brightness constancy:** projection of the same point looks the same in every frame
  - **Small motion:** points do not move very far
  - **Spatial coherence:** points move like their neighbors

The brightness constancy constraint



Brightness Constancy Equation:

$$I(x, y, t-1) = I(x + u(x, y), y + v(x, y), t)$$

Linearizing the right side using Taylor expansion:

$$I(x, y, t-1) \approx I(x, y, t) + I_x \cdot u(x, y) + I_y \cdot v(x, y)$$

Hence, $I_x \cdot u + I_y \cdot v + I_t \approx 0$

The brightness constancy constraint

$$I_x \cdot u + I_y \cdot v + I_t = 0$$

- How many equations and unknowns per pixel?
  - One equation, two unknowns
- What does this constraint mean?

$$\nabla I \cdot (u, v) + I_t = 0$$

- The component of the flow perpendicular to the gradient (i.e., parallel to the edge) is unknown

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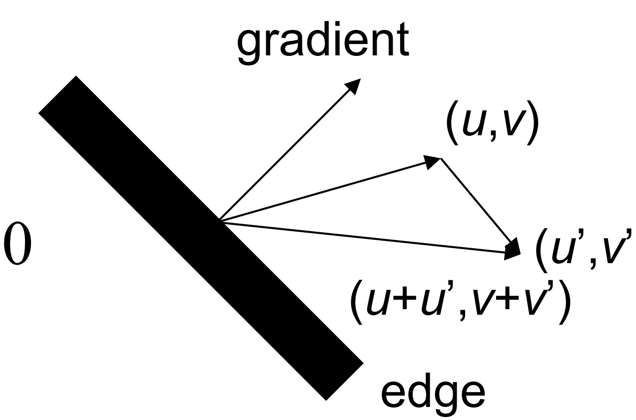
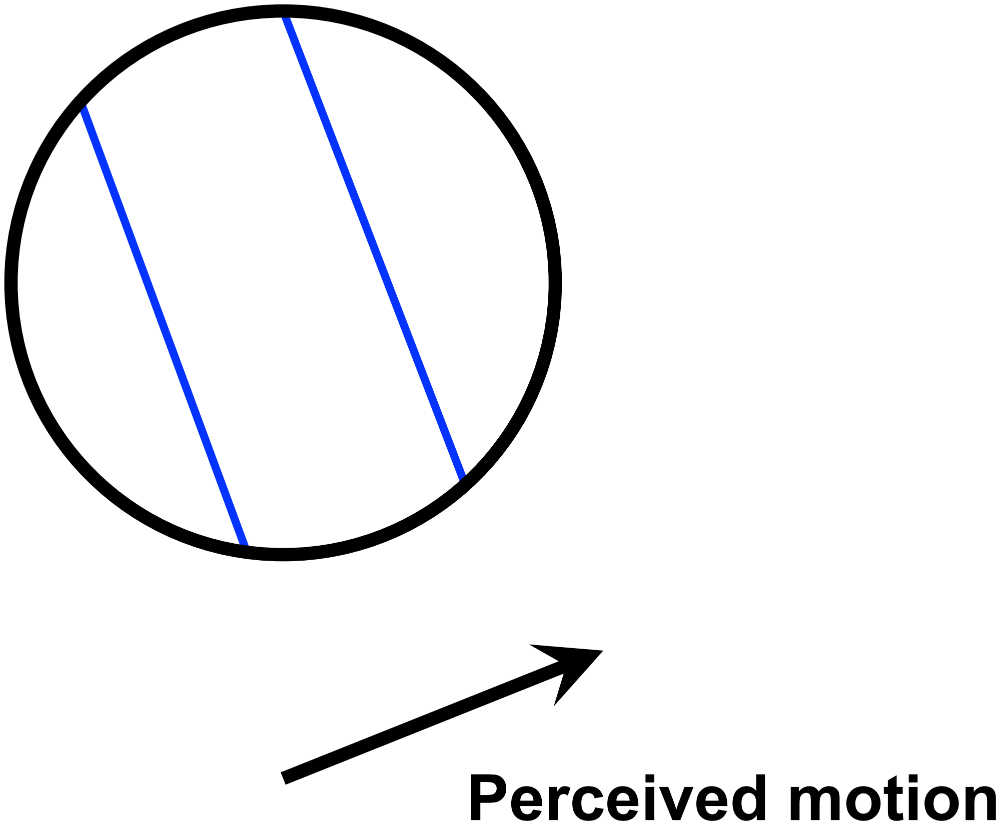
If (u, v) satisfies the equation,
so does (u+u', v+v') if $\nabla I \cdot (u', v') = 0$



The aperture problem



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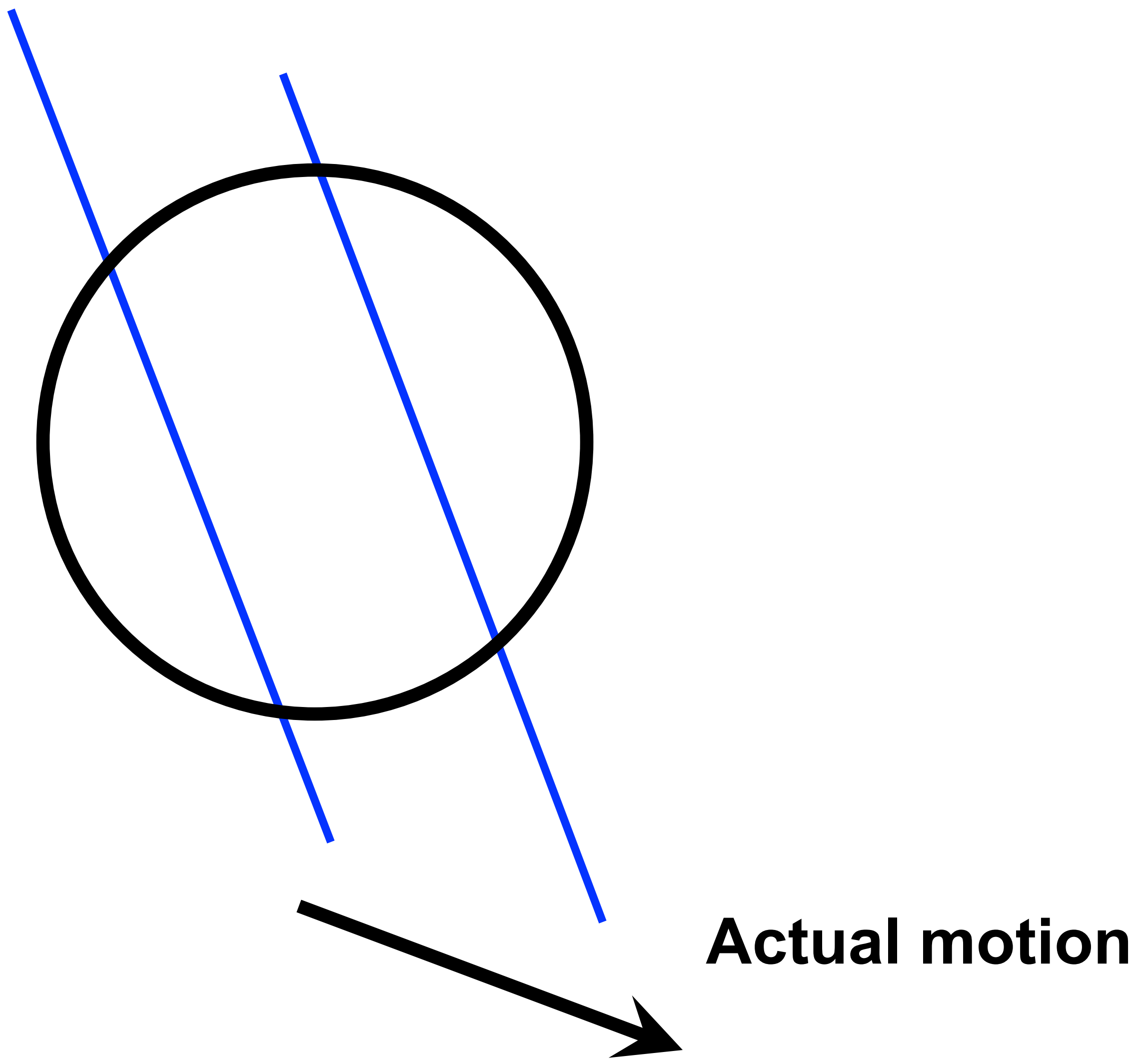

The barber pole illusion

http://en.wikipedia.org/wiki/Barberpole_illusion

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Solving the aperture problem

- How to get more equations for a pixel?
- **Spatial coherence constraint:** pretend the pixel's neighbors have the same (u,v)
  - E.g., if we use a 5x5 window, that gives us 25 equations per pixel

$$\nabla I(\mathbf{x}_i)\cdot[u,v]+I_t(\mathbf{x}_i)=0$$

$$\begin{bmatrix} I_x(\mathbf{x}_1) & I_y(\mathbf{x}_1) \\ I_x(\mathbf{x}_2) & I_y(\mathbf{x}_2) \\ \vdots & \vdots \\ I_x(\mathbf{x}_n) & I_y(\mathbf{x}_n) \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = - \begin{bmatrix} I_t(\mathbf{x}_1) \\ I_t(\mathbf{x}_2) \\ \vdots \\ I_t(\mathbf{x}_n) \end{bmatrix}$$

B. Lucas and T. Kanade. An iterative image registration technique with an application to stereo vision. In *Proceedings of the International Joint Conference on Artificial Intelligence*, pp. 674–679, 1981.

Solving the aperture problem

- Least squares problem:

$$\begin{bmatrix} I_x(\mathbf{x}_1) & I_y(\mathbf{x}_1) \\ I_x(\mathbf{x}_2) & I_y(\mathbf{x}_2) \\ \vdots & \vdots \\ I_x(\mathbf{x}_n) & I_y(\mathbf{x}_n) \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = - \begin{bmatrix} I_t(\mathbf{x}_1) \\ I_t(\mathbf{x}_2) \\ \vdots \\ I_t(\mathbf{x}_n) \end{bmatrix}$$

- When is this system solvable?
  - What if the window contains just a single straight edge?

B. Lucas and T. Kanade. An iterative image registration technique with an application to stereo vision. In *Proceedings of the International Joint Conference on Artificial Intelligence*, pp. 674–679, 1981.

Lucas-Kanade flow

Linear least squares problem

$$\begin{bmatrix} I_x(\mathbf{x}_1) & I_y(\mathbf{x}_1) \\ I_x(\mathbf{x}_2) & I_y(\mathbf{x}_2) \\ \vdots & \vdots \\ I_x(\mathbf{x}_n) & I_y(\mathbf{x}_n) \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = - \begin{bmatrix} I_t(\mathbf{x}_1) \\ I_t(\mathbf{x}_2) \\ \vdots \\ I_t(\mathbf{x}_n) \end{bmatrix}$$

$$\underset{n\times 2}{A}\ \underset{2\times 1}{d} = \underset{n\times 1}{b}$$

Solution given by $(A^T A)\ d = A^T b$

$$\underbrace{\begin{bmatrix} \sum I_x I_x & \sum I_x I_y \\ \sum I_x I_y & \sum I_y I_y \end{bmatrix}}_{A^T A} \begin{bmatrix} u \\ v \end{bmatrix} = - \underbrace{\begin{bmatrix} \sum I_x I_t \\ \sum I_y I_t \end{bmatrix}}_{A^T b}$$

The summations are over all pixels in the window

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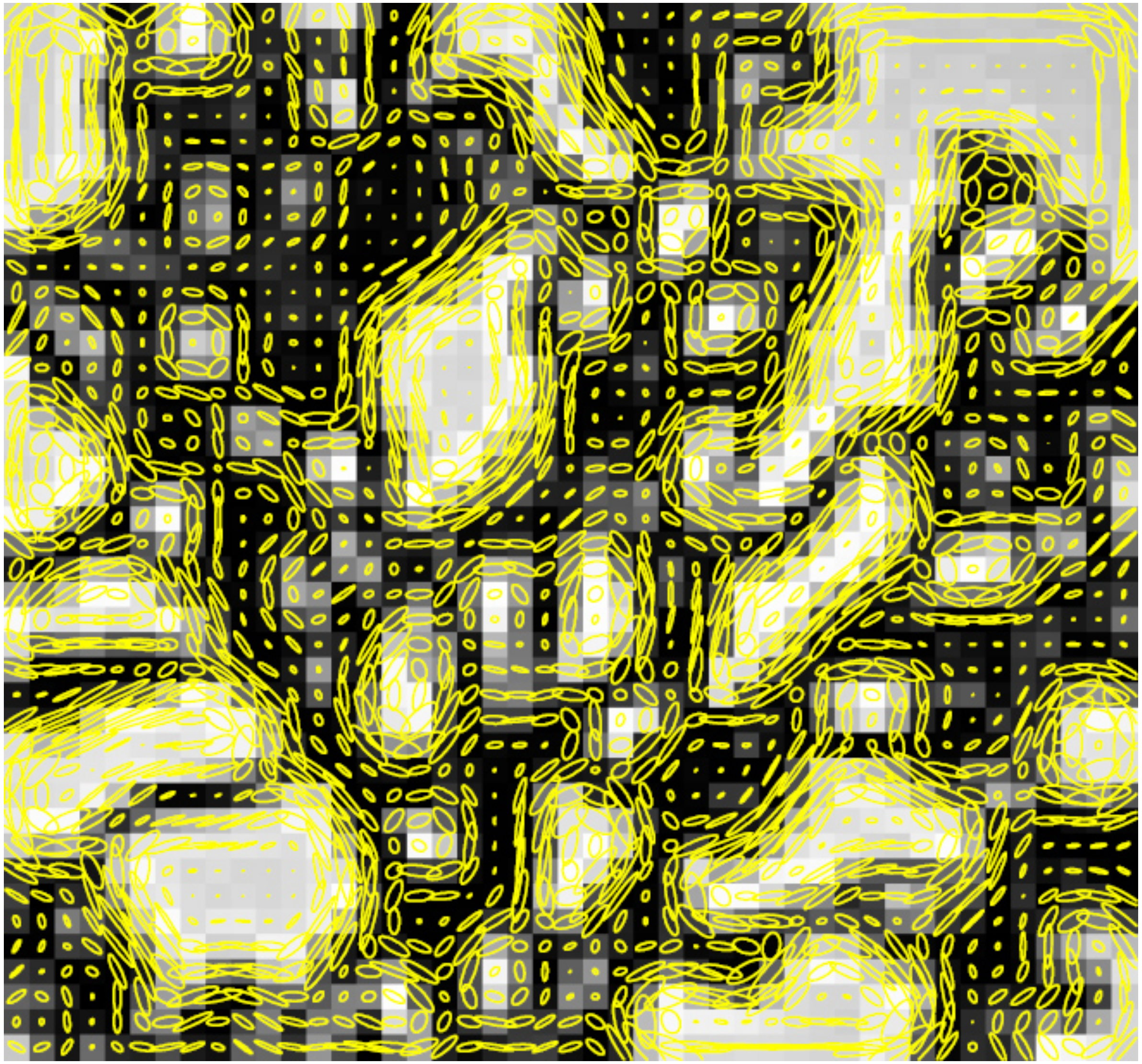
Lucas-Kanade flow

$$\underbrace{\begin{bmatrix} \sum I_x I_x & \sum I_x I_y \\ \sum I_x I_y & \sum I_y I_y \end{bmatrix}}_{A^T A} \begin{bmatrix} u \\ v \end{bmatrix} = - \underbrace{\begin{bmatrix} \sum I_x I_t \\ \sum I_y I_t \end{bmatrix}}_{A^T b}$$

- Recall the Harris corner detector: $M = A^T A$ is the *second moment matrix*
- We can figure out whether the system is solvable by looking at the eigenvalues of the second moment matrix
  - The eigenvectors and eigenvalues of *M* relate to edge direction and magnitude
  - The eigenvector associated with the larger eigenvalue points in the direction of fastest intensity change, and the other eigenvector is orthogonal to it

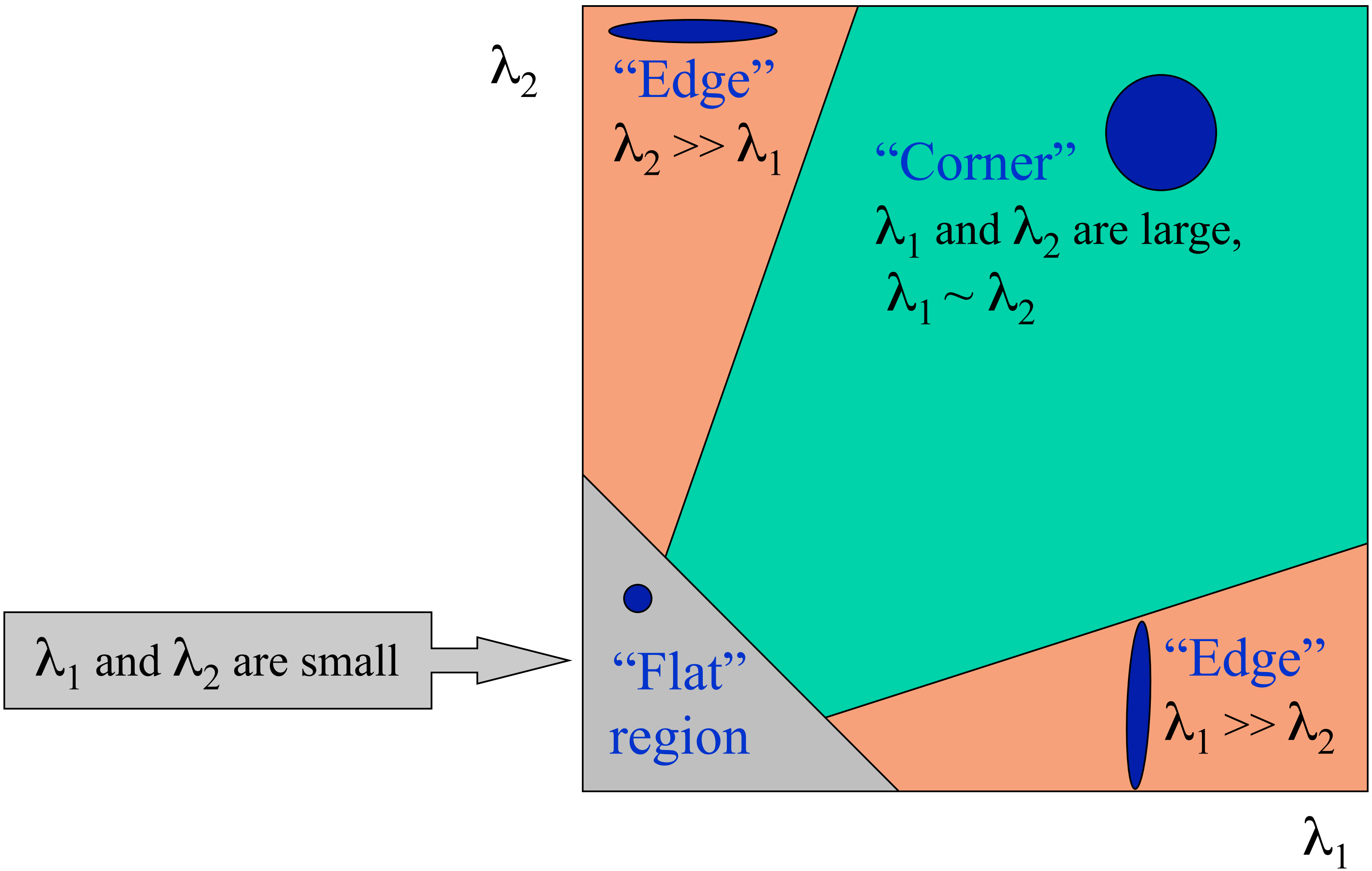
Visualization of second moment matrices

Visualization of second moment matrices



Interpreting the eigenvalues

Classification of image points using eigenvalues of the second moment matrix:



Uniform region

– gradients have small magnitude
– small λ_1, small λ_2
– system is ill-conditioned

Edge

– gradients have one dominant direction
– large λ_1, small λ_2
– system is ill-conditioned

High-texture or corner region

– gradients have different directions, large magnitudes
– large λ_1, large λ_2
– system is well-conditioned

Errors in Lucas-Kanade

- The motion is large (larger than a pixel)
  - Iterative refinement
  - Coarse-to-fine estimation
  - Exhaustive neighborhood search (feature matching)
- A point does not move like its neighbors
  - Motion segmentation
- Brightness constancy does not hold
  - Exhaustive neighborhood search with normalized correlation

Energy formulation

The optical flow can be formulated as an optimization problem

$$\min_{u,v} \sum_{i} \left| \nabla I[i,j,t-1] \cdot \begin{bmatrix} u[i,j] \\ v[i,j] \end{bmatrix} + I[i,j,t] - I[i,j,t-1] \right|_2^2 + \lambda \left(|\nabla u[i,j]|_2 + |\nabla v[i,j]|_2 \right)$$

The data term matches images to the optical flow model

$$\nabla I[i,j,t-1] \cdot \begin{bmatrix} u[i,j] \\ v[i,j] \end{bmatrix} + I[i,j,t] - I[i,j,t-1] = 0$$

The regularization imposes smoothness in the flow field at every pixel

Feature tracking

- So far, we have only considered optical flow estimation in a pair of images
- If we have more than two images, we can compute the optical flow from each frame to the next
- Given a point in the first image, we can in principle reconstruct its path by simply "following the arrows"

Tracking challenges

- Ambiguity of optical flow
  - Need to find good features to track
- Large motions, changes in appearance, occlusions, disocclusions
  - Need mechanism for deleting, adding new features
- Drift – errors may accumulate over time
  - Need to know when to terminate a track

Tracking over many frames

- Select features in first frame
- For each frame:
  - Update positions of tracked features
    - Discrete search or Lucas-Kanade (or a combination of the two)
  - Terminate inconsistent tracks
    - Compute similarity with corresponding feature in the previous frame or in the first frame where it's visible
  - Find more features to track

Shi-Tomasi feature tracker

- Find good features using eigenvalues of second-moment matrix
  - Key idea: “good” features to track are the ones whose motion can be estimated reliably
- From frame to frame, track with Lucas-Kanade
  - This amounts to assuming a translation model for frame-to-frame feature movement
- Check consistency of tracks by *affine* registration to the first observed instance of the feature
  - Affine model is more accurate for larger displacements
  - Comparing to the first frame helps to minimize drift

J. Shi and C. Tomasi. Good Features to Track. CVPR 1994.

Tracking example

Figure 1: Three frame details from Woody Allen's *Manhattan*. The details are from the 1st, 11th, and 21st frames of a subsequence from the movie.

Figure 2: The traffic sign windows from frames 1,6,11,16,21 as tracked (top), and warped by the computed deformation matrices (bottom).

J. Shi and C. Tomasi. [Good Features to Track](). CVPR 1994.