5.1 Which occurences are bound and which are free?

5.1.1 $(\lambda x.x)\ y\ (\lambda y.yx)\ x$

The first x (This does not count the λx so for me the first one is always without the λ) is bound by the lambda calculus, the first y is free, the second y is bound by the lambda calculus, and both last x are free.

5.1.2 $((\lambda x.\lambda y.\lambda z.xyz)\ (\lambda x.yx)\ y\ (\lambda x.zx)))$

The first x, y, z are bounded by the lambda calculus, the second x is bounded, the second and third y are free, the third x is bounded and the second z is free.

5.1.3 $\lambda y.(\lambda x.\ z\ (x\ (\lambda x.\ y\ (z))))\ (\lambda z.\ y(x\ (z)))$

All x except the last one and all y are bounded, whereas the last x and the first two z are free, and the last z is bounded.

5.2 Reducing lambda-expressions to their normal form

5.2.1 $(\lambda x.(\lambda z.\ zy)\ x)\ (\lambda x.\ x)$

$$
\begin{aligned}
(\lambda x.(\lambda z.\ zy)\ x)\ (\lambda x.\ x) &= (\lambda z.\ zy)\ (\lambda x.\ x) && \textit{β-reduction}\\
&= (\lambda x.\ xy) && \textit{β-reduction}\\
&= xy
\end{aligned}
$$

5.2.2 $(\lambda x.\ xxy)\ (\lambda x.\ xxy)$

$$
\begin{aligned}
(\lambda x.\ xxy)\ (\lambda x.\ xxy) &= (\lambda x.\ xxy)\ (\lambda x.\ xxy)y && \textit{β-reduction}\\
&= ((\lambda x.\ xxy)\ (\lambda x.\ xxy)y)y && \textit{β-reduction}\\
&= ((\lambda x.\ xxy)\ (\lambda x.\ xxy)y)y)y && \textit{β-reduction}\\
&= \textit{ad infinum} \Rightarrow\ \textit{no normal form available}
\end{aligned}
$$

5.2.3 $P \equiv (\lambda x.\ x\ (xy))I$ where $I \equiv \lambda u.\ u$

$$
\begin{aligned}
(\lambda x.\ x\ (xy))\ (\lambda u.\ u) &= (\lambda u.\ u)(\lambda u.\ uy) && \textit{β-reduction}\\
&= (\lambda u.\ uy) && \textit{β-reduction}\\
&= uy && \textit{β-reduction}
\end{aligned}
$$