# 8) Steganography

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(originally presented at CMS 2005)

# *Steganography ≠ Cryptography*

- Cryptography hides the content of communication
  - Adversary knows that communication exists
  - Conceals data
- Steganography hides the existence of communication
  - Adversary should not discover existence of communication
  - Conceals also metadata

# *Information-hiding concepts*

Embed information in a "cover" (carrier signal)

Steganography

- Hide presence of hidden information
- To avoid censorship and surveillance
- Presence not known; can easily be removed

Watermarking

- Hide information in a robust way
- To authenticate (multimedia) data, control ownership
- Presence is known; should be difficult to remove

Fingerprinting

- Like watermarking, but hides user-specific information for tracing and litigation

# ***Prisoner's problem***


- Alice and Bob want to coordinate their escape
- Communication through passive observer (warden, censor ...)
  - innocent communication is allowed
  - talking about escape plans is forbidden
- Formulated by Simmons (1983)

# *Steganography = hidden communication*

How to be convinced that it is "really" hidden?

Many attacks

- Correlation, histogram, transforms ...

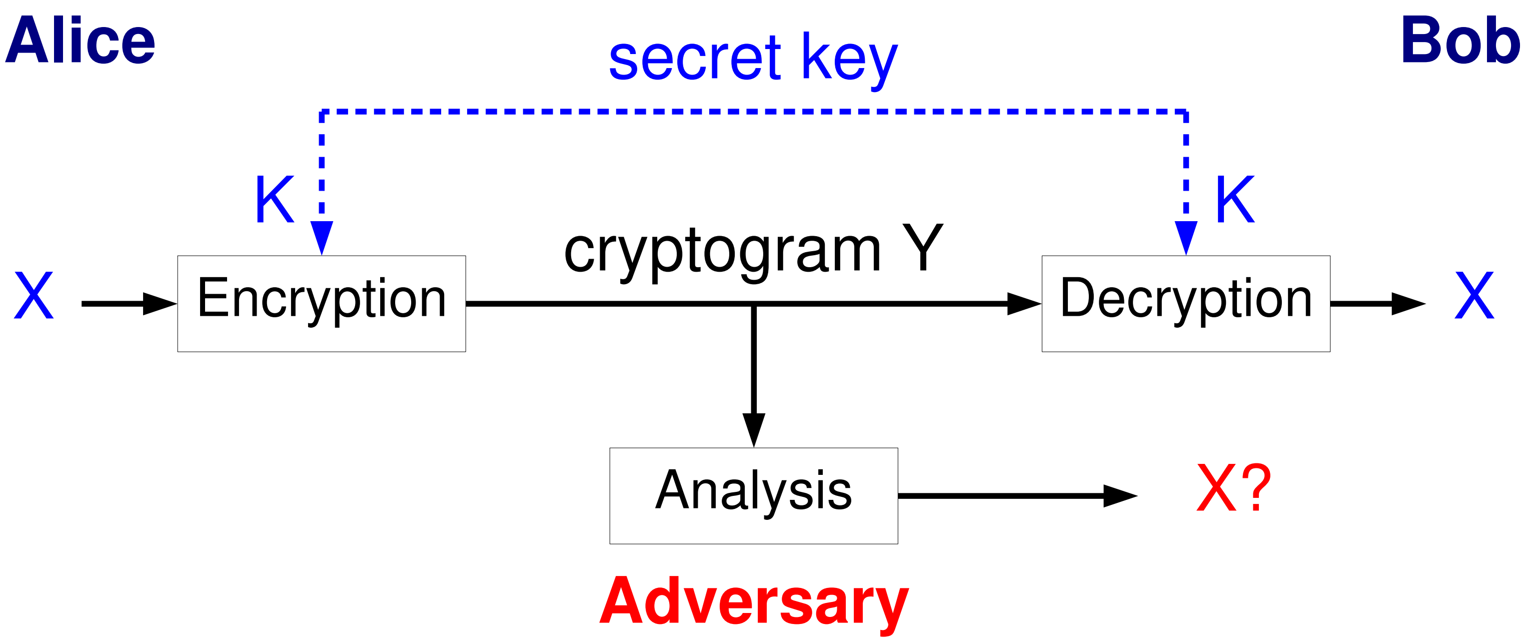
Analogy to cryptology

- Ages of broken cryptosystems until ~1980
- Theory of cryptology with provably secure cryptosystems since ~1990

→ Formal model for steganography

# *Models for cryptosystems*

# *Model of a cryptosystem*




**Shannon (1949)**

Adversary is *passive*

# *Security of cryptosystems*

Perfect security (Shannon 1949)

$$I(X;Y) = H(X) - H(X|Y) = 0$$

- Information-theoretic
- Unbounded adversary obtains no information
- Implies also that $H(K) \geq H(X)$ ☹
- Ex. one-time pad

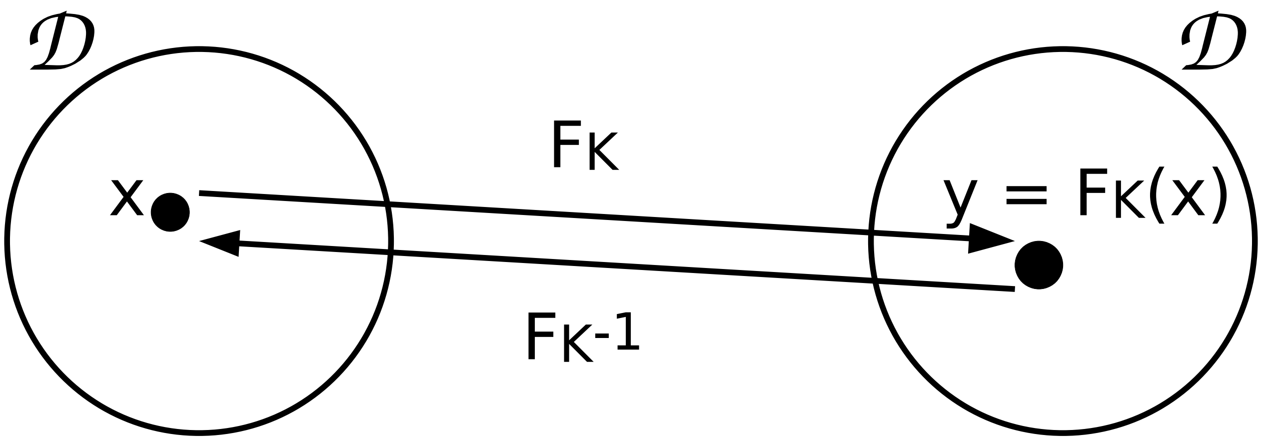
But ... practical cryptography uses block- and stream-ciphers with short keys.

# *Practical cryptosystems*

Family of one-way permutations

$\mathcal{F} = \{F_K\}, F_K : \mathcal{D} \to \mathcal{D}$

indexed by key K

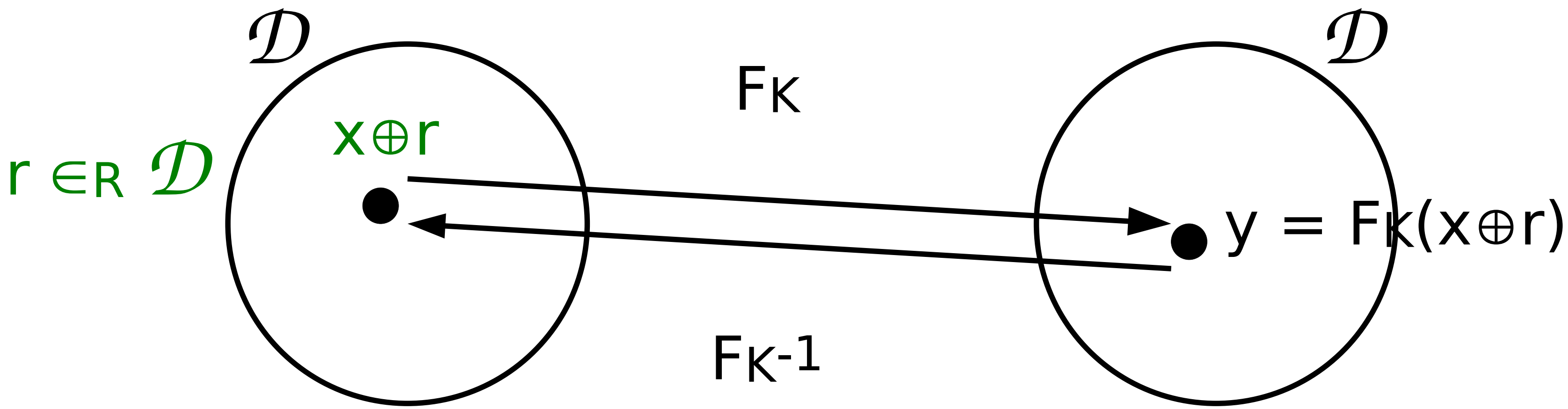



Encryption and decryption are deterministic

leaks information

# *Probabilistic encryption*

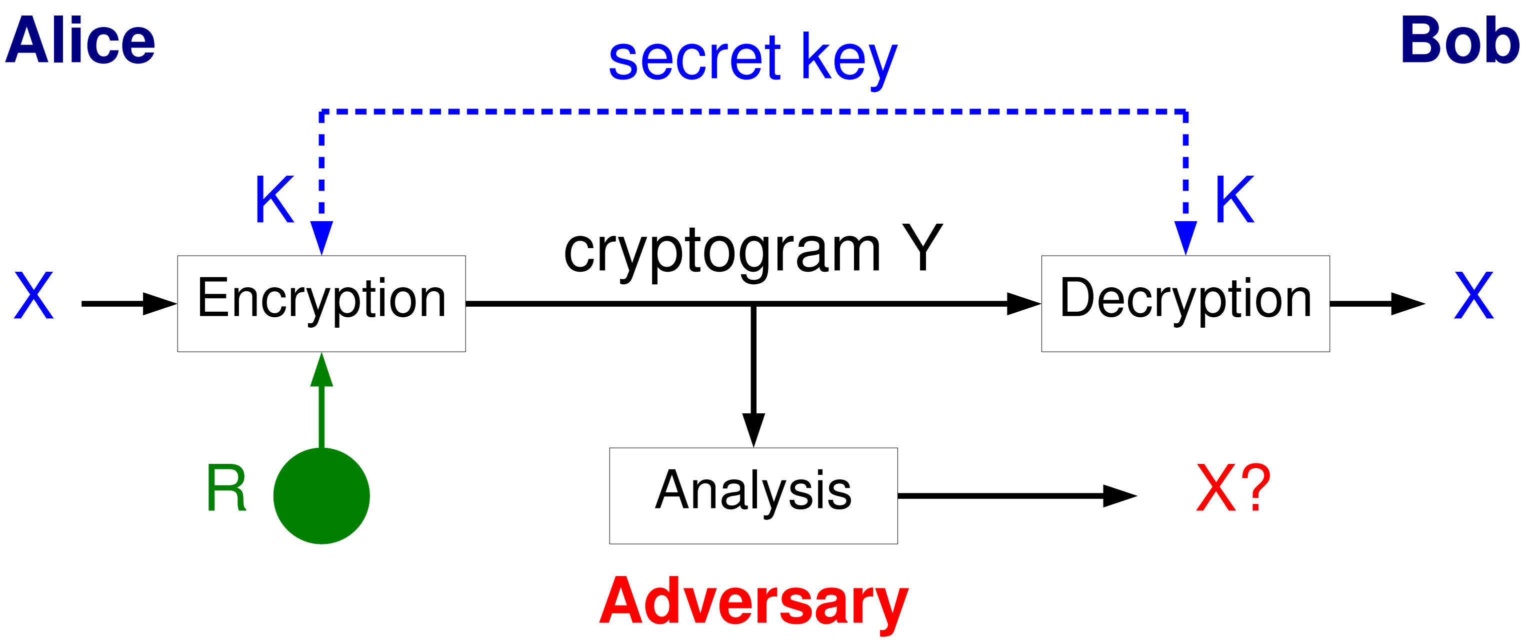
Randomize input to encryption function




Ciphertext is (r, y)

- does not repeat for same plaintext x
- better protection

# Probabilistic cryptosystems




Private random source R

# *Computational security for cryptosystems*

Formal security model (Goldwasser-Micali, 1985)

- Chosen-plaintext attacks (passive adversary)
- Semantic security ⇔ indistinguishability of ciphertexts

Defined by experiment with adversary A, a probabilistic polynomial-time (PPT) algorithm:

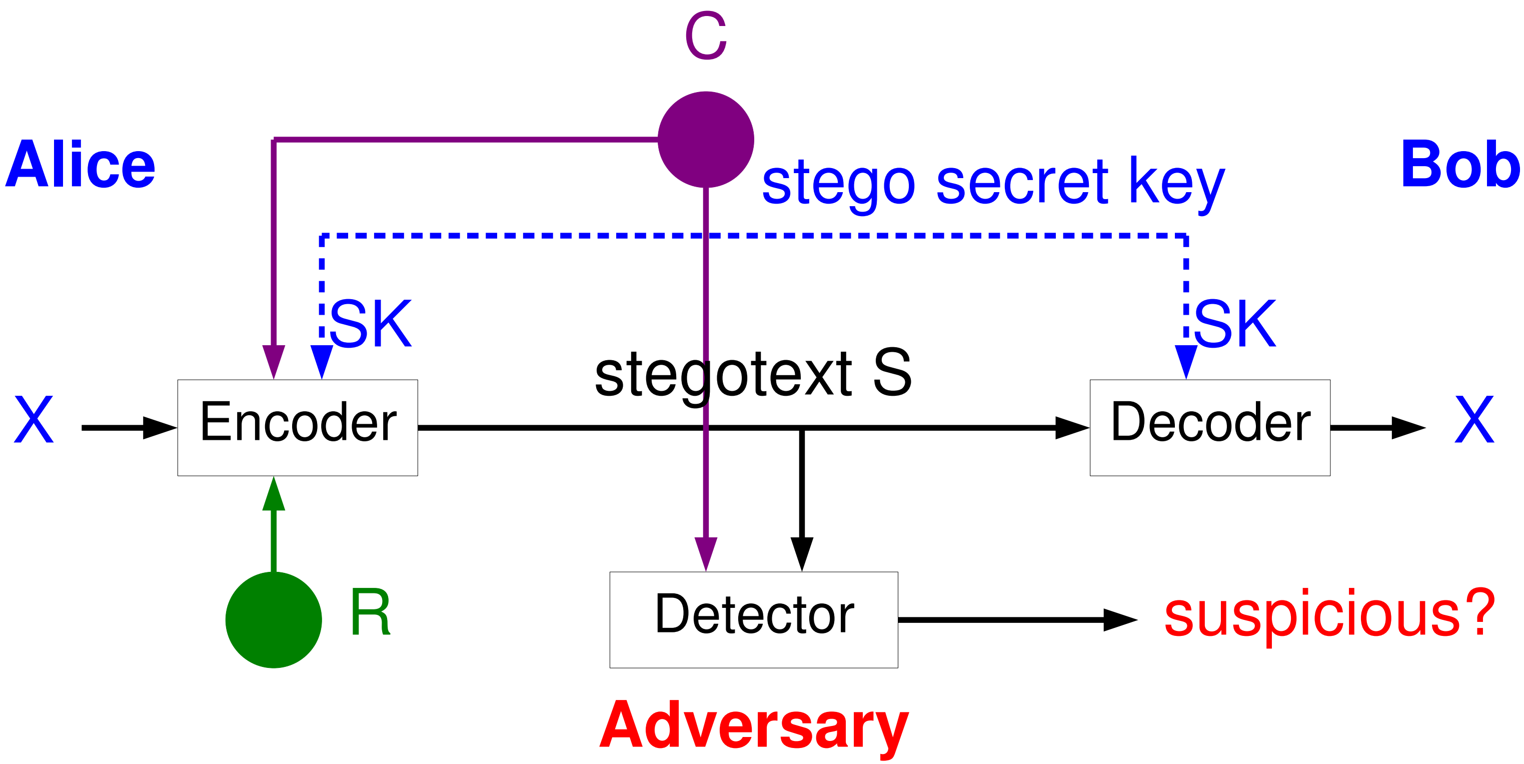
$$
\begin{aligned}
K &\leftarrow KG(1^k) \\
(x_0,x_1) &\leftarrow A_1(K) \\
b &\in_R \{0,1\} \\
c^* &\leftarrow E(K,x_b) \\
b^* &\leftarrow A_2(x_0,x_1,c^*)
\end{aligned}
$$

Then

$\forall$ PPT $A=(A_1,A_2)$ : $\Pr[\, b = b^* \,] \le \frac{1}{2}$ + negl.

# *Models for stegosystems*

# *Model of a stegosystem*




Covertext C and stegotext S over domain Y
Adversary is *passive*

# *How to define security for stegosystems?*

Attempt 1

Based on information theory and mutual entropy:

$$I(X;S|C) = 0$$

Fails to differentiate stegosystem from cryptosystem.

Attempt 2

Based on distortion measure $d(C,S) = \|P_C - P_S\|_2$ :

$$d(c,s) \to 0$$

Fails for some examples, e.g., when $C$ is a random $n$-bit string, but $S$ is a random $n$-bit string with even parity.

# *Information-theoretic security*

Steganography as hypothesis testing (C. 1998)

- Adversary distinguishes stegotext from covertext

Quantified using relative entropy

- Perfect security

$$D(P_C \| P_S) = 0$$

- Statistical security

$$D(P_C \| P_S) < \varepsilon$$

Discrimination or relative entropy

$$D(P_C \| P_S) = \sum_{y \in Y} P_C(y) \log \left(P_C(y)/P_S(y)\right) \geq 0$$

# *Bounds on detection*

Adversary distinguishes stegotext S ($H_1$) from covertext C ($H_0$)

- Deciding S with signal C is type-I error, prob. $\alpha$
- Deciding C with signal S is type-II error, prob. $\beta$

Statistical security $D(P_C \| P_S) < \varepsilon$,

$$d(\alpha,\beta) \leq \varepsilon$$

- binary discrimination $d(\bullet,\bullet)$

For $\alpha=0$, this implies

$$\beta > 2^{-\varepsilon}$$

# *Modeling the covertext*

Probabilistic source or channel

$\mathbf{C} = C_0, C_1, C_2 \ldots$

Distribution known

**C** is a stochastic process

Random variable (i.i.d. → ergodic → ...)

Unrealistic in practice

Real-world communication does not come with specification of distribution

# *Universal stegosystems*

What if distribution of **C** is not known?

→ Universal stegosystems

→ No knowledge of cover distribution needed

**C** is an algorithm, given as oracle

Can be queried on arbitrary history

$\mathbf{C}(h,n) = C_{|h|+1}, C_{|h|+2}, \ldots, C_{|h|+n}$

Synthetic cover signal **C** from machine learning

# *Example stegosystem*

Known distribution $P_C$ over domain Y

Message $x \in \{0,1\}$

Let

- $Y_0 = \min_{Y' \subseteq Y} |\Pr[C \in Y'] - \Pr[C \notin Y']|$ (prob. =: $\varepsilon$)
- $Y_1 = Y \setminus Y_0$
- Key $sk \in_R \{0,1\}$
- RV $C_0$ is $C$ restricted to $Y_0$
- RV $C_1$ is $C$ restricted to $Y_1$

Stego-encoder: $SE(sk,x) = C_{x \oplus sk}$

Stego-decoder: $SD(sk,y) = 0$ iff $y \in Y_{sk}$

**Thm.**: This is an $\varepsilon^2/\ln 2$-statistically secure stegosystem (in terms of relative entropy).

# *Computationally secure stegosystems*

Analogous to computational security model for cryptosystems

- Universal stegosystem
  - covertext given as oracle **C**
- Chosen-plaintext attacks (CPA)
  - passive adversary
- Indistinguishability of covertext and stegotext

Secret-key stegosystem: SKG, SE, SD

- $\mathrm{SKG}(1^k) \rightarrow sk$
- $\mathrm{SE}(sk,x) \rightarrow c$
- $\mathrm{SD}(sk,c) \rightarrow x$ or $\perp$

# *Computational stegosystems (2)*

Robustness

$$SD(sk, SE(sk,x)) = x$$

SS-CPA security defined by experiment with SA

$K \leftarrow SKG(1^k)$

$(x^*,s) \leftarrow SA_1(K)$

$b \in_R \{0,1\}$

if $b=0$ then $c^* \leftarrow SE(K,x^*)$ else $c^* \leftarrow_R \mathbf{C}$ fi

$b^* \leftarrow SA_2(x^*,c^*,s)$

Then

$$\forall \text{ PPT } SA=(SA_1,SA_2) : \Pr[\, b = b^* \,] \leq \tfrac{1}{2} + \text{negl.}$$

Stegotext and covertext are indistinguishable

# Construction – Assumptions

Given

- Sym. cryptosystem (KG,E,D) with pseudo-random ciphertexts & universal hash function G

SKG($1^k$)

- Run KG($1^k$) → sk and pick g $\in_R$ G → (sk,g)

# *Construction – Rejection sampler*

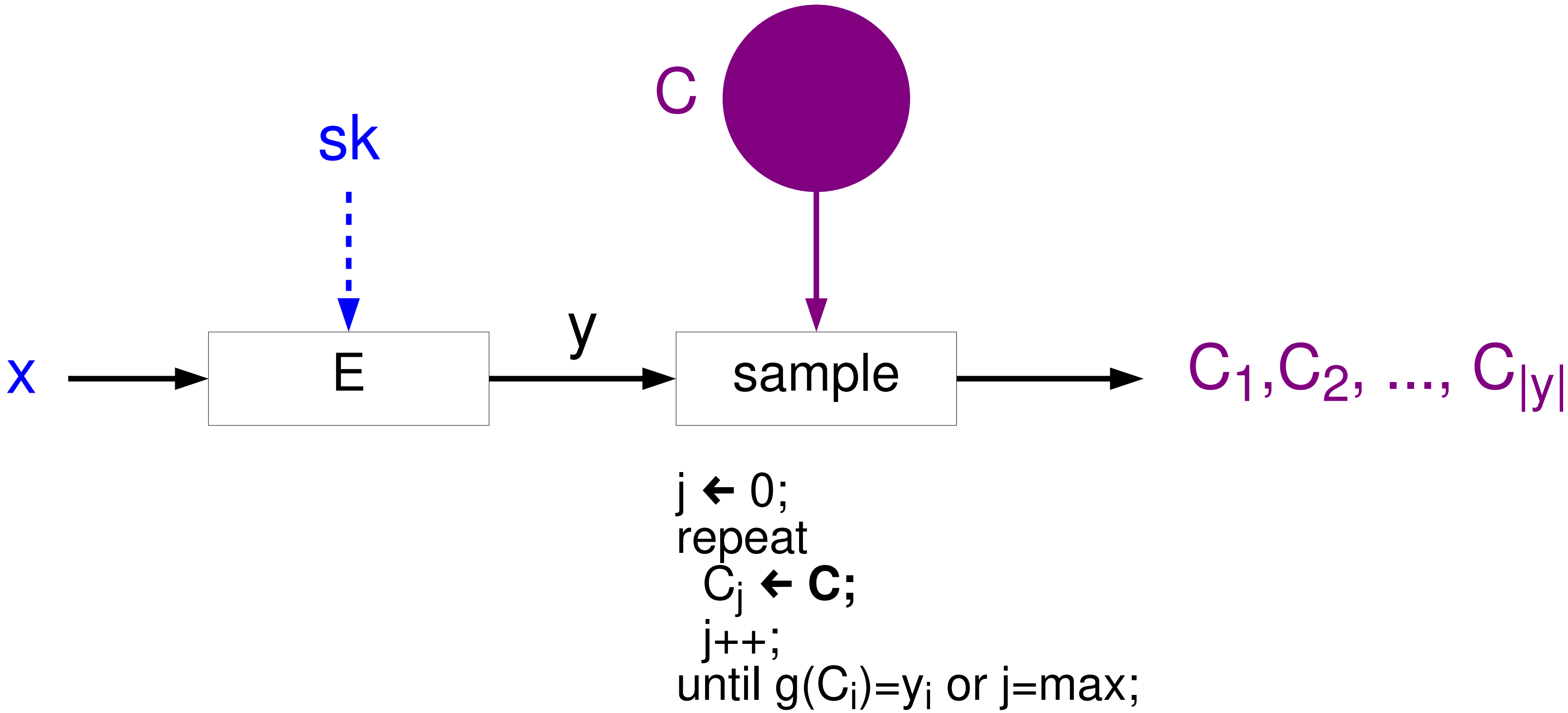
SE(sk,m)

  $z \leftarrow E(sk,x)$
  **for** $i = 1 \ldots |z|$ **do**
    **repeat**
      sample $C_i \leftarrow_R \mathbf{C}$
    **until** $g(C_i) = z_i$
  output $C_1 \ldots C_{|z|}$

SD(sk, $C_1 \ldots C_{|z|}$)

  **for** $i = 1 \ldots |z|$ **do**
    $z_i \leftarrow g(C_i)$
  $x \leftarrow D(sk,z)$

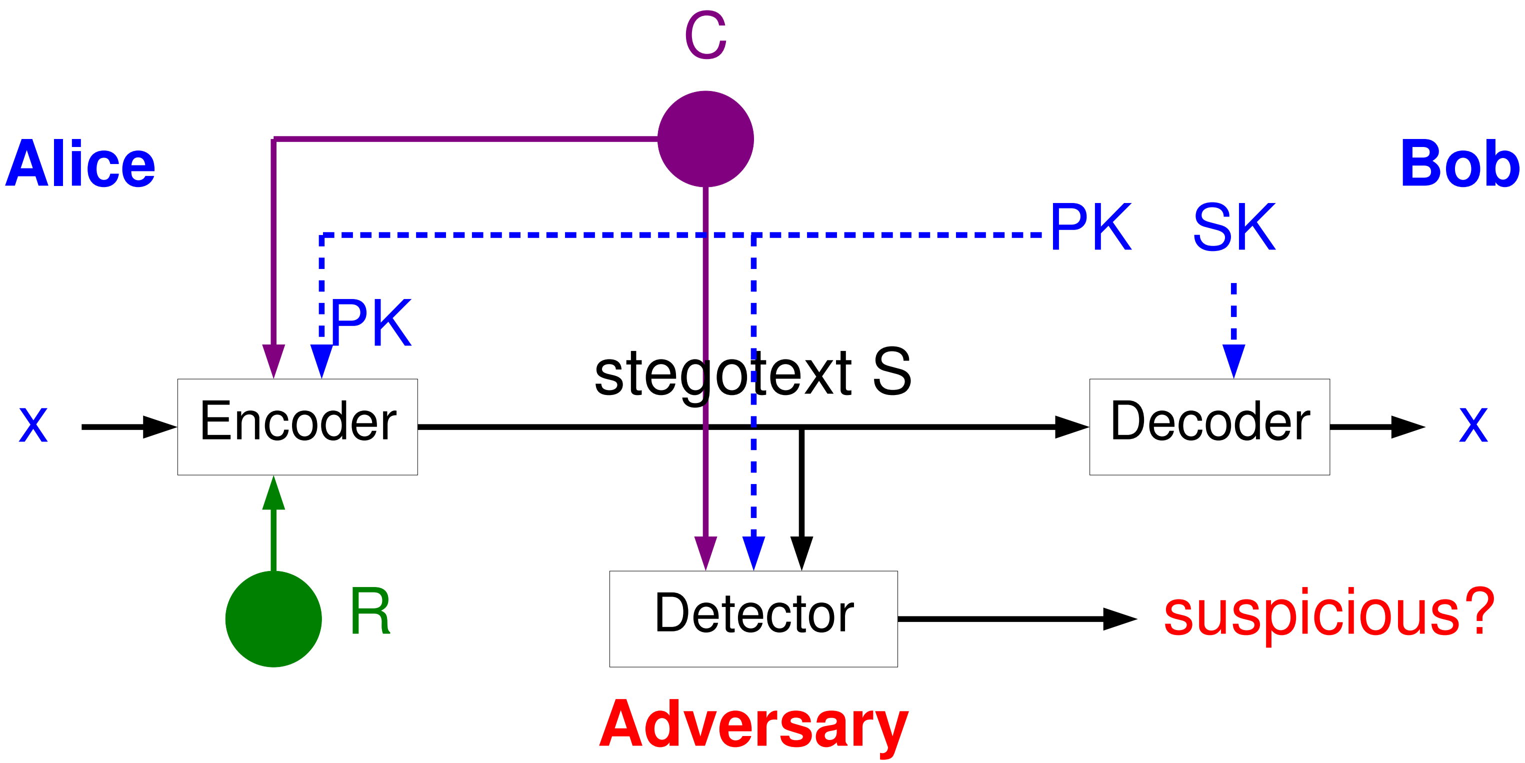
# *Rejection sampler*




If **C** has large enough min-entropy, then

$$\| \langle g, C_1, C_2 \ldots C_{|y|} \rangle - \langle g, \mathbf{C}^{|y|} \rangle \| \leq \text{negl.}$$

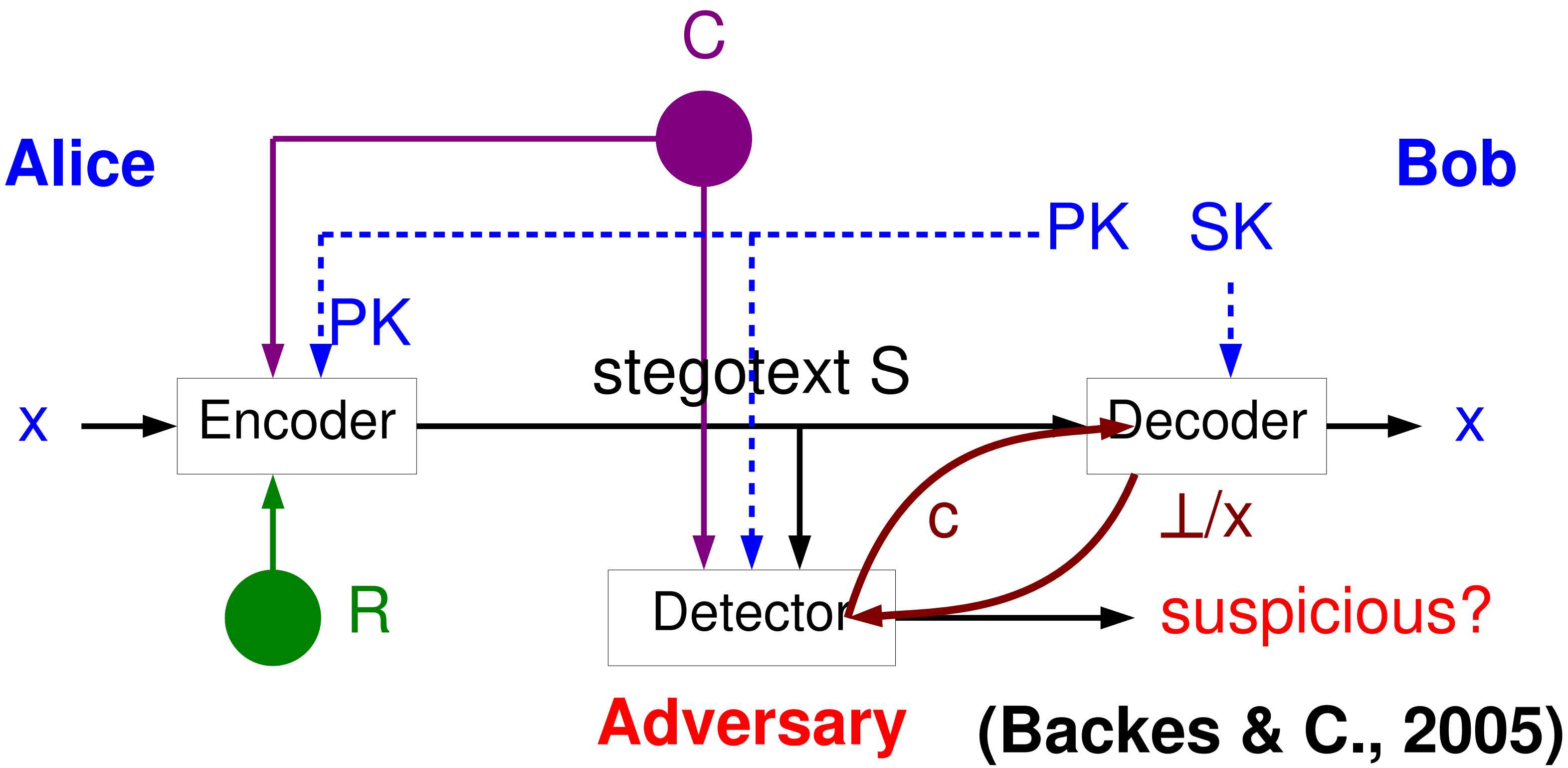
# *Public-key stegosystems*




**(v. Ahn & Hopper, 2004)**

Bob has public-key/secret-key pair (PK,SK)
Adversary is passive (CPA) or active (CCA)

# *Modeling active attacks*




Adversary may ask Bob if he considers c to be stegotext

Bob answers 0 (cover) or 1 (stego)

- Analogous to adaptive chosen-ciphertext attacks for public-key cryptosystems

# *Practical steganography*

Many tools for image and audio steganography

- Ca. 1990-2005
- Starting from making imperceptible modifications to the least significant bit (LSB, in pixel or audio data)

Not relevant in practice today

- Steganography is often very easy to detect
- with forensic image analysis tools

Recently – New ideas from machine learning

# *Stegosystems using ML*

Meteor (2021)

- https://meteorfrom.space
- https://eprint.iacr.org/2021/686

Sampler uses ML to generate realistic covers

- Natural-language text
- Uses OpenAI's GPT-2 (Generative Pre-trained Transformer) that produces human-like texts

# *Meteor*

- Examples 1-3
- Long explanation: https://meteorfrom.space

# *Summary*

Definition of stegosystems

Security for (secret-key) stegosystems

- Perfect → statistical → computational
- Always relative to covertext distribution

Public-key stegosystems

- Computational security

Realistic stegosystems using machine learning

- Automatically generate covertext
- Permitted covertext distribution is critical

# *Executive summary*

Stegosystems are cryptosystems with prescribed distribution of ciphertext