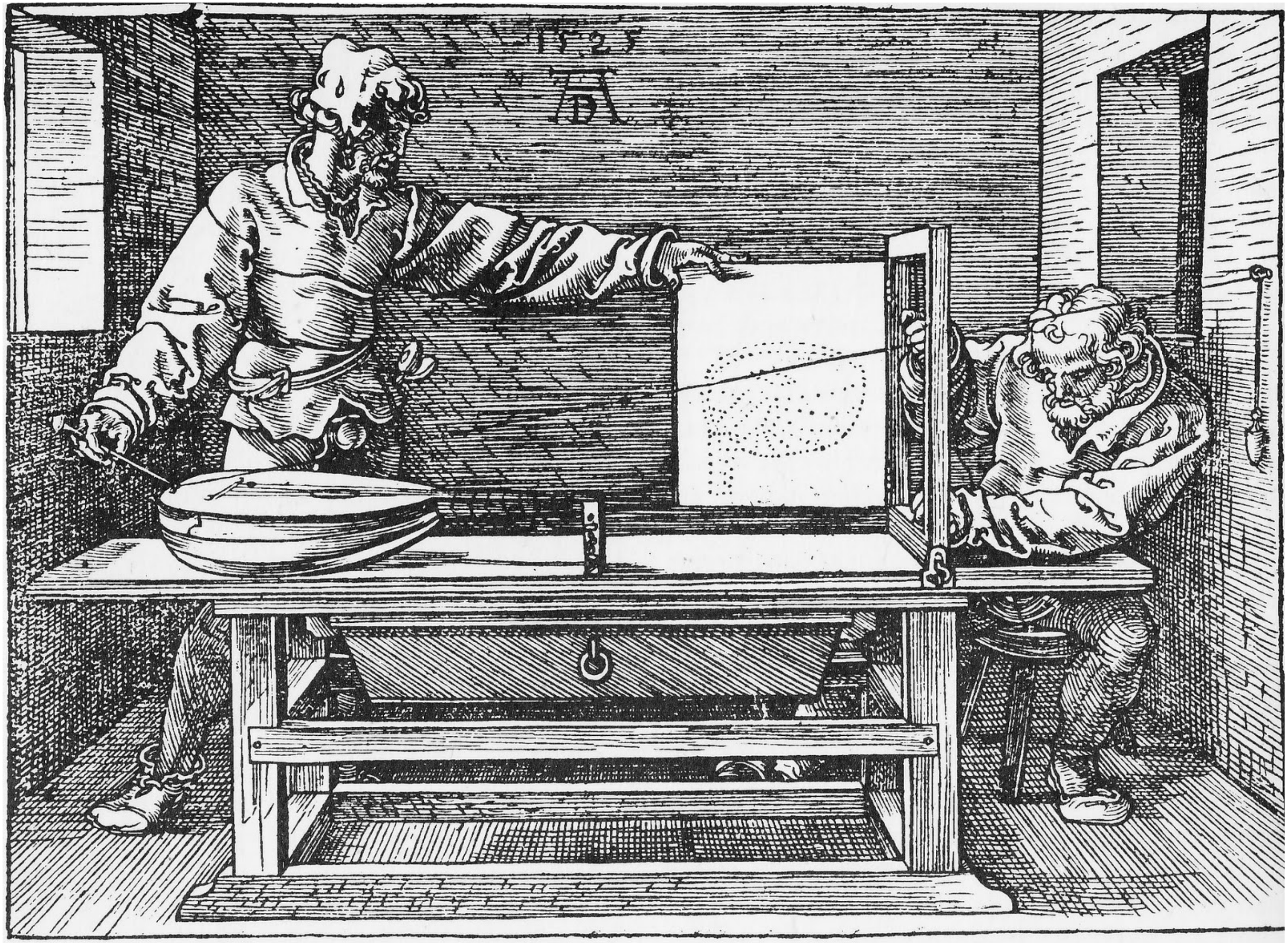
Perspective projection

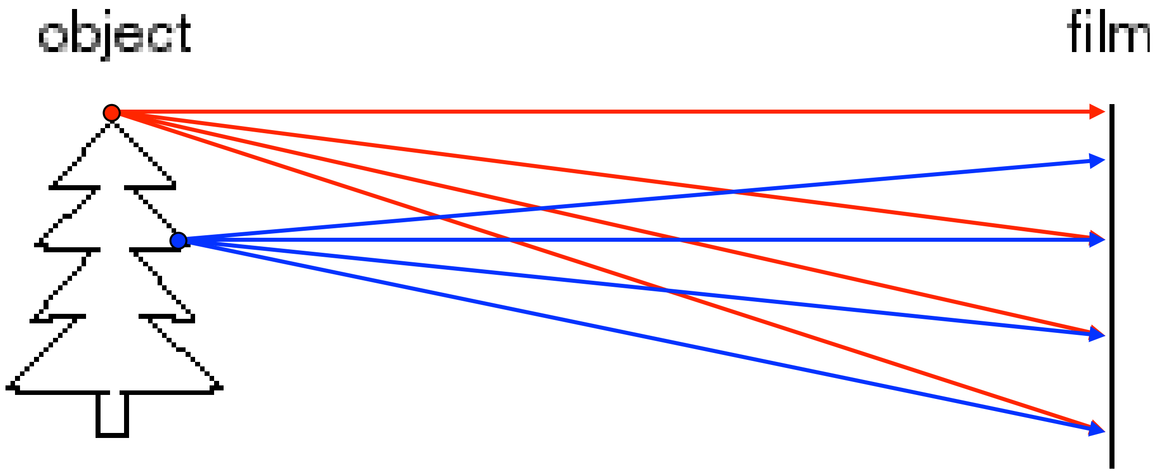


Mechanical creation of a perspective image, Albrecht Dürer, 1525

Overview of next two lectures

- The pinhole projection model
  - Qualitative properties
  - Perspective projection matrix
- Cameras with lenses
  - Depth of focus
  - Field of view
  - Lens aberrations

Let’s design a camera



Idea 1: put a piece of film in front of an object

Do we get a reasonable image?

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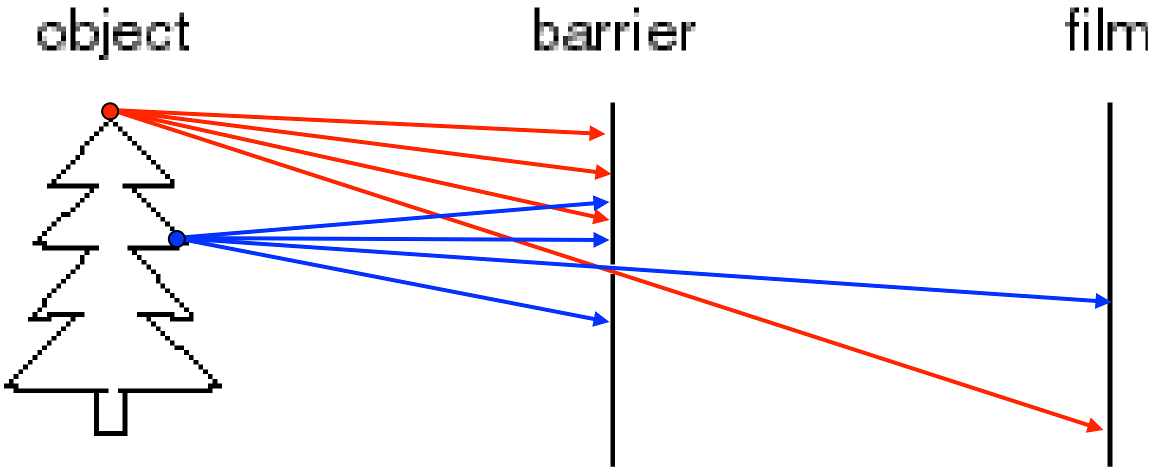
Pinhole camera



Add a barrier to block off most of the rays

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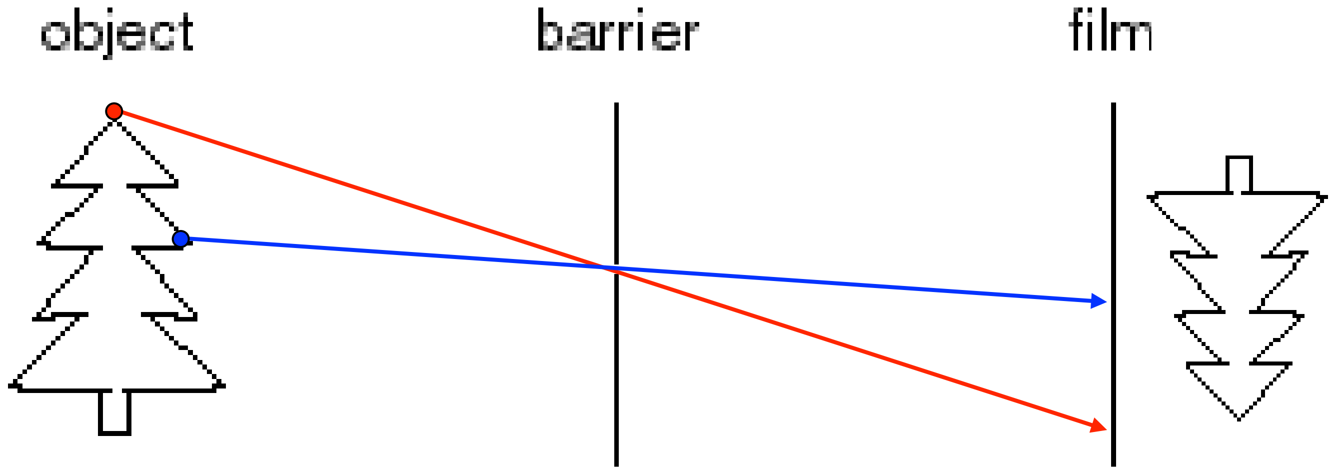
Pinhole camera



- Captures **pencil of rays** – all rays through a single point: **aperture, center of projection, focal point, camera center**
- The image is formed on the **image plane**

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Pinhole cameras everywhere

Tree shadow during a solar eclipse

photo credit: Nils van der Burg

http://www.physicstogo.org/index.cfm

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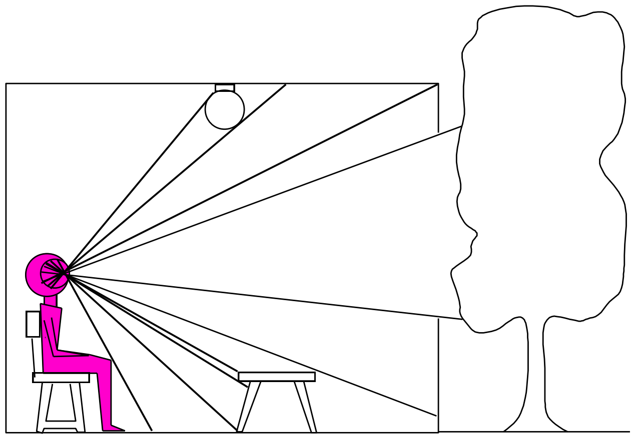
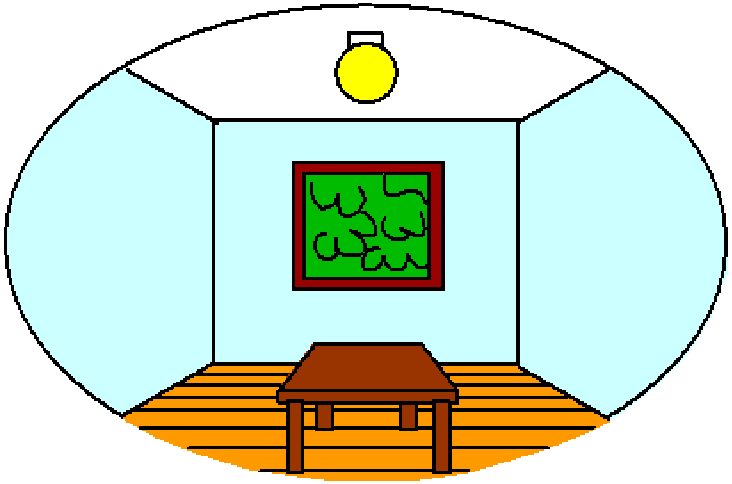
Dimensionality reduction: from 3D to 2D

3D world



Point of observation

2D image



What is preserved?

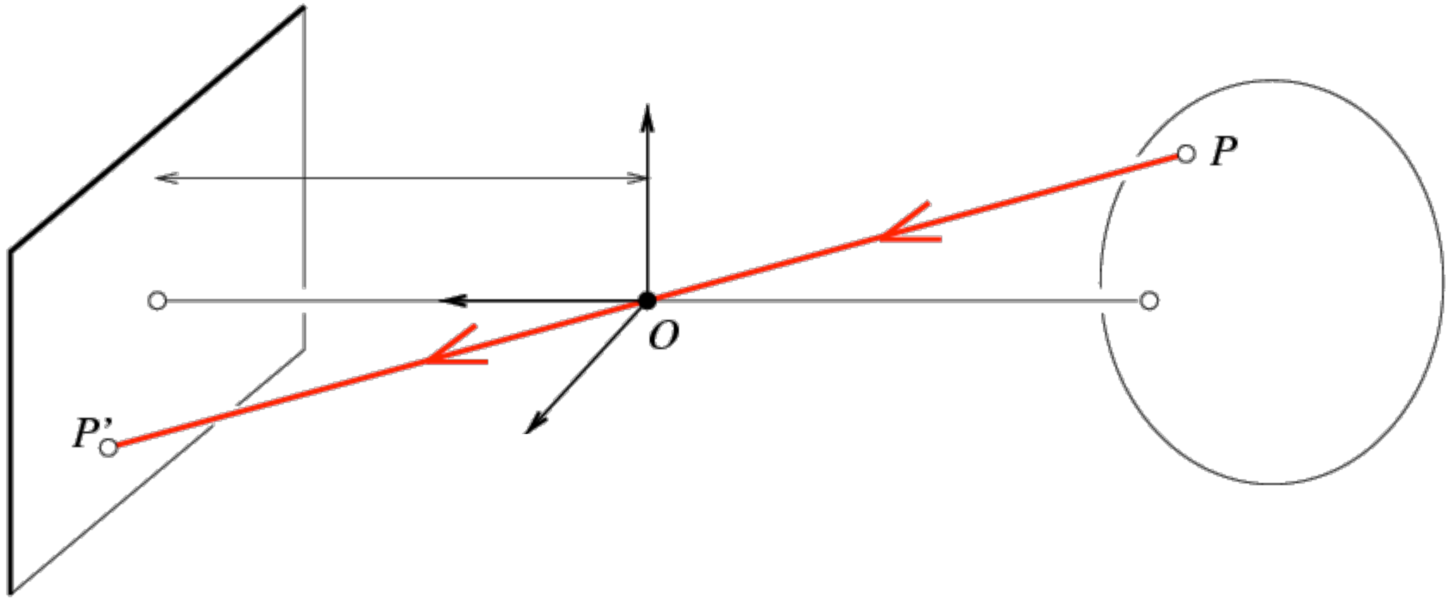
- Straight lines, incidence

What is not preserved?

- Angles, lengths

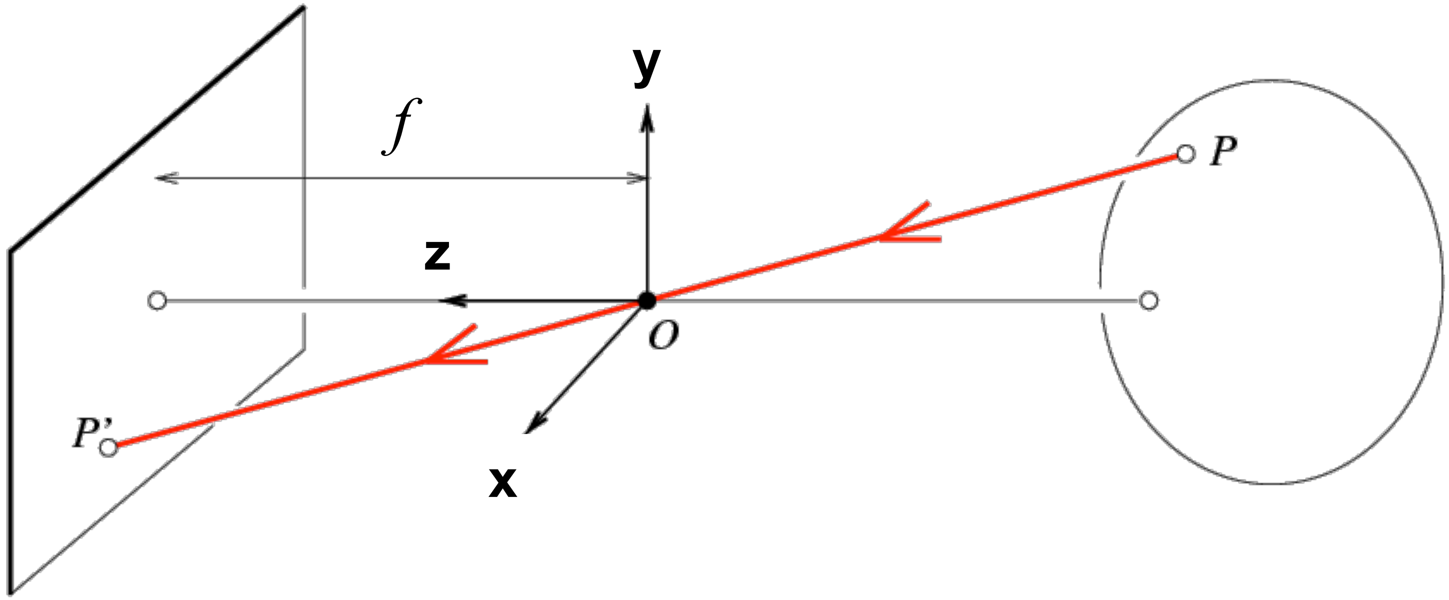
Slide by A. Efros

Modeling projection



- To compute the projection *P'* of a scene point *P*, form the **visual ray** connecting *P* to the camera center *O* and find where it intersects the image plane
  - All scene points that lie on this visual ray have the same projection in the image
  - Are there scene points for which this projection is undefined?

Modeling projection



The coordinate system

- The optical center (***O***) is at the origin
- The image plane is parallel to xy-plane (perpendicular to z axis)

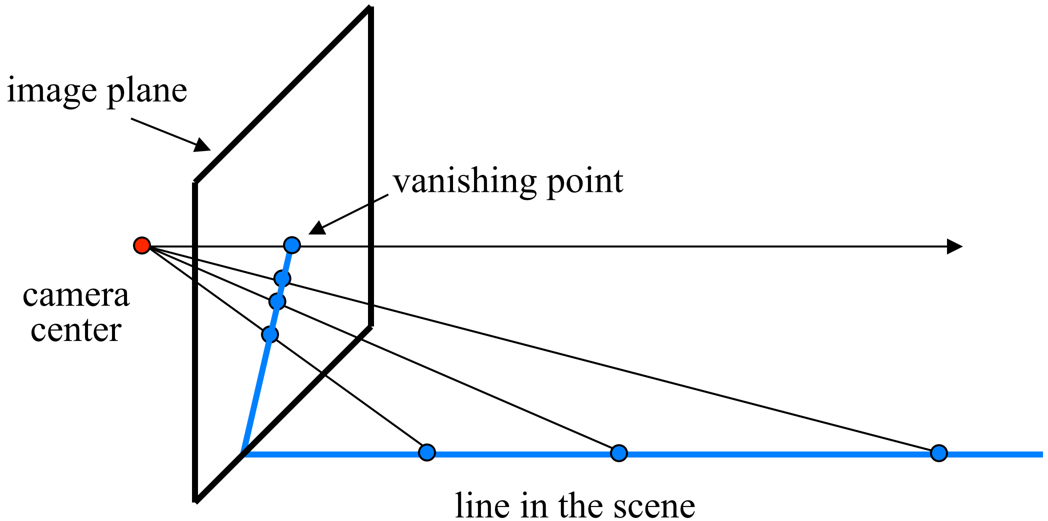
Projection equations

- Derived using similar triangles:

$$(x, y, z) \rightarrow (f\frac{x}{z}, f\frac{y}{z})$$

Source: J. Ponce, S. Seitz

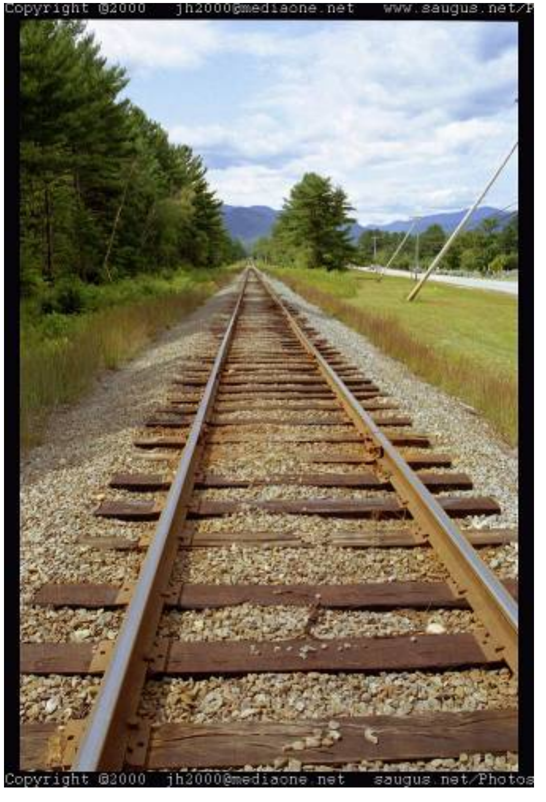
Projection of a line



- What if we have another line in the scene parallel to the first one?

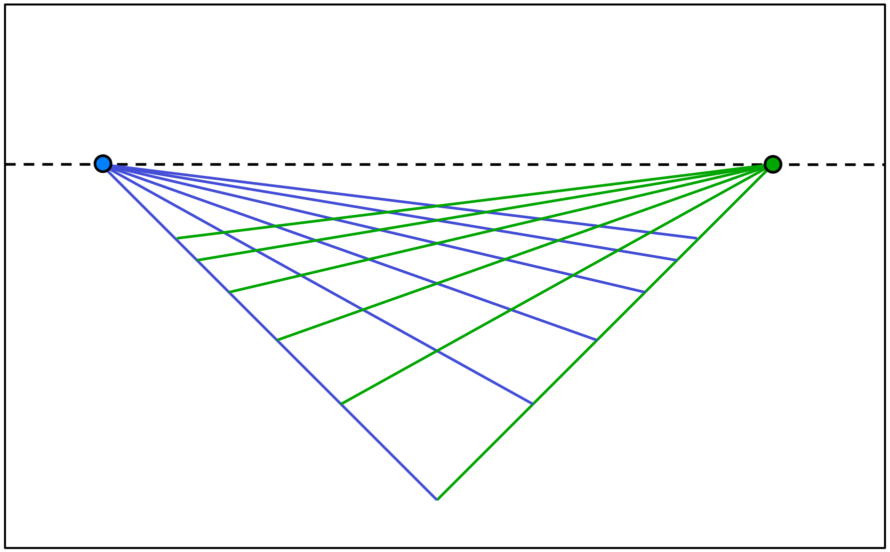
Vanishing points

- Each direction in space has its own vanishing point
  - All lines going in that direction converge at that point
  - Exception: directions parallel to the image plane



Vanishing points

- Each direction in space has its own vanishing point
  - All lines going in that direction converge at that point
  - Exception: directions parallel to the image plane
- What about the vanishing line of a plane?



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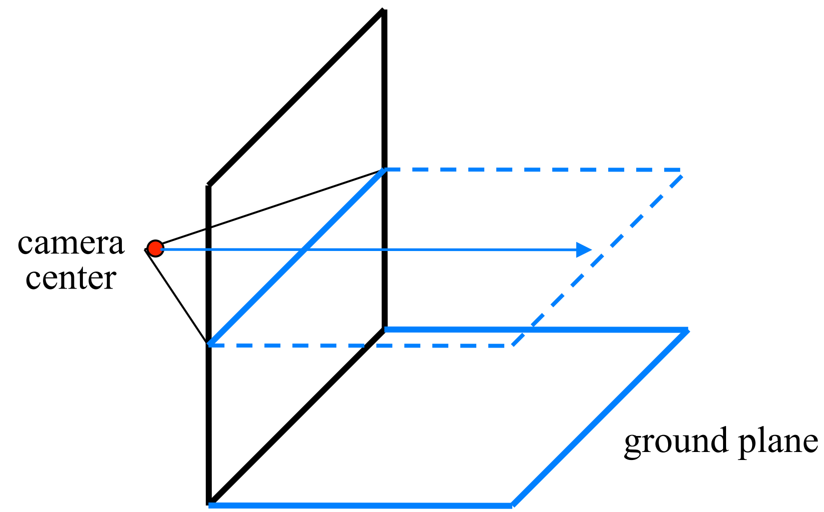
The horizon



- Vanishing line of the ground plane
  - All points at the same height as the camera project to the horizon
  - Points higher than the camera project above the horizon
  - Provides way of comparing height of objects

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The horizon

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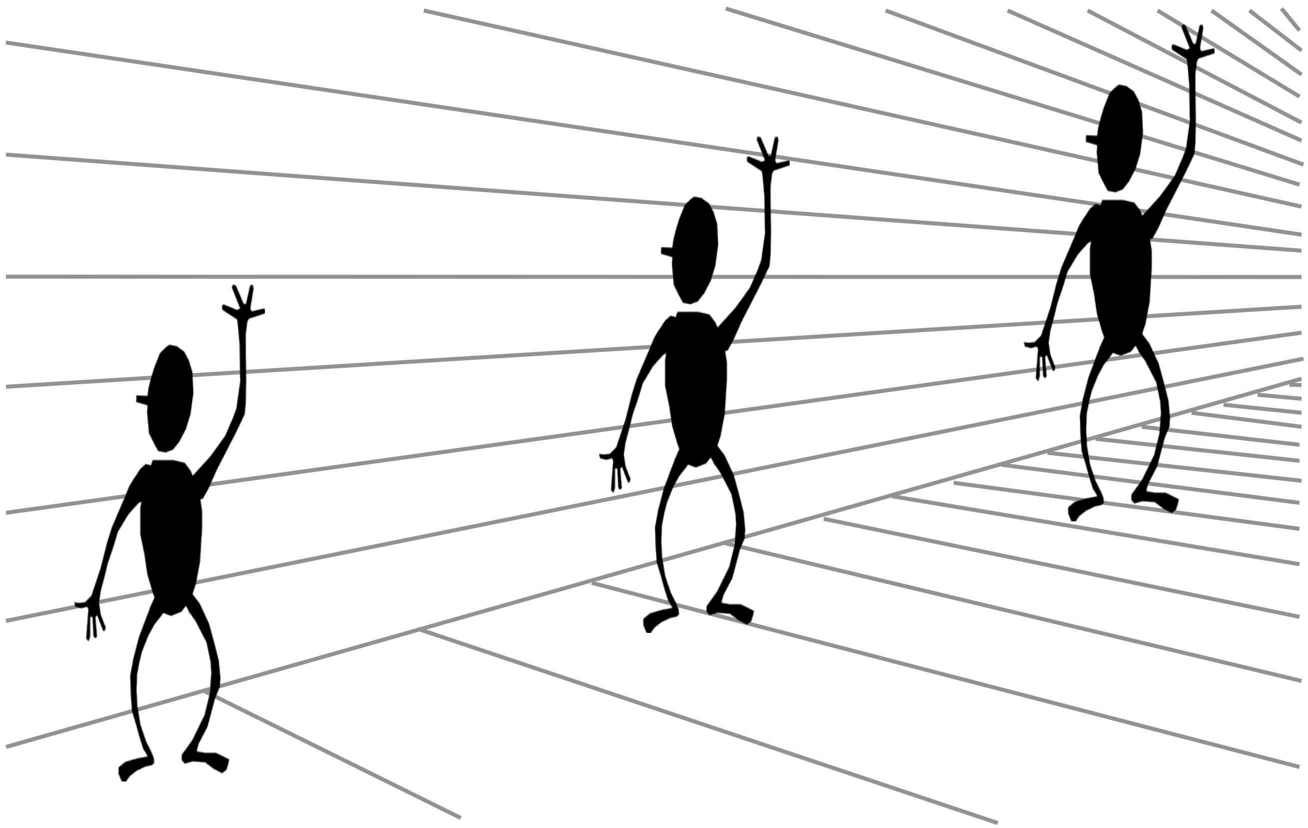
Perspective cues



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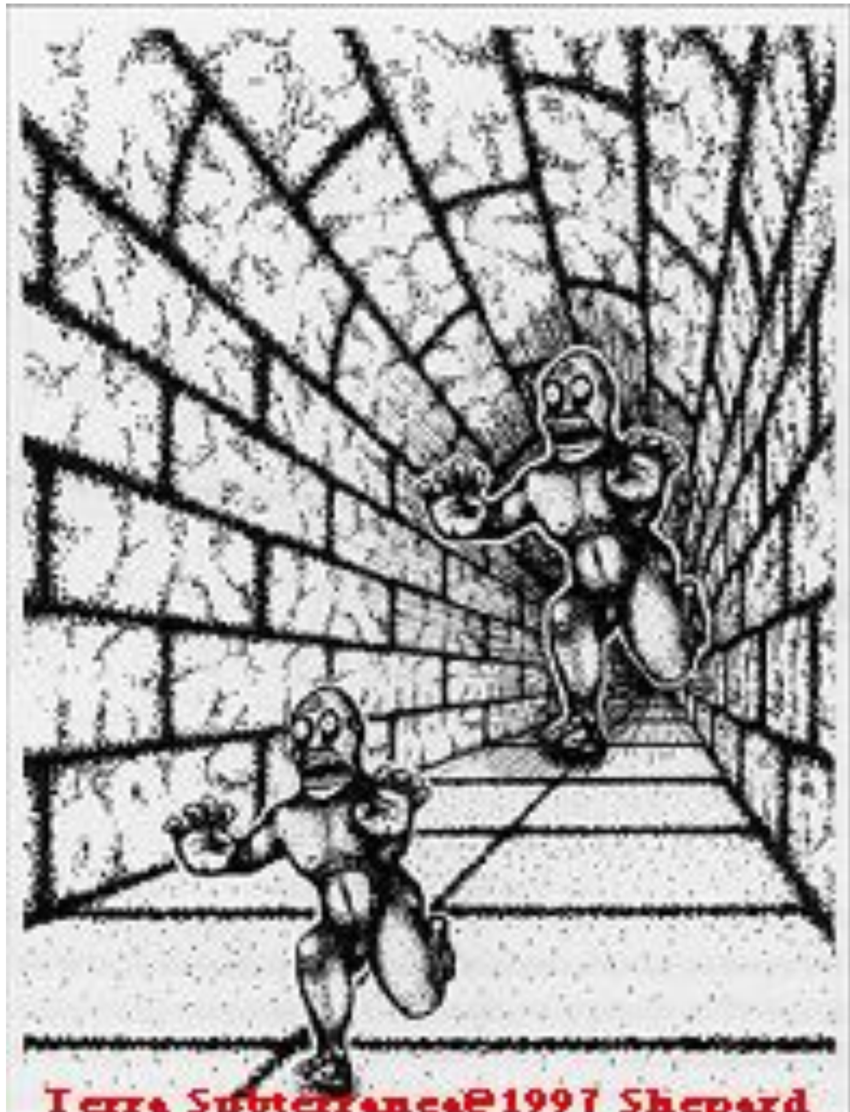
Perspective cues



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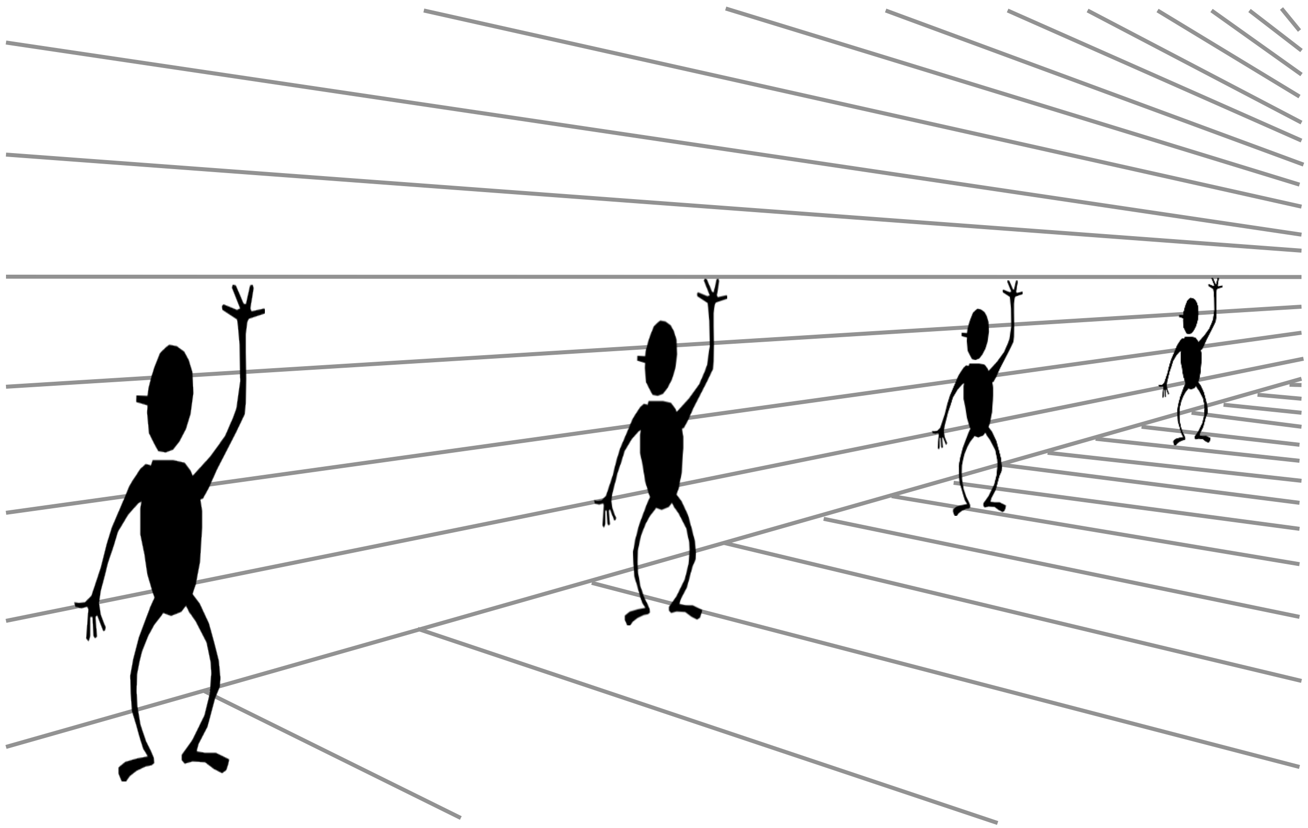
Perspective cues



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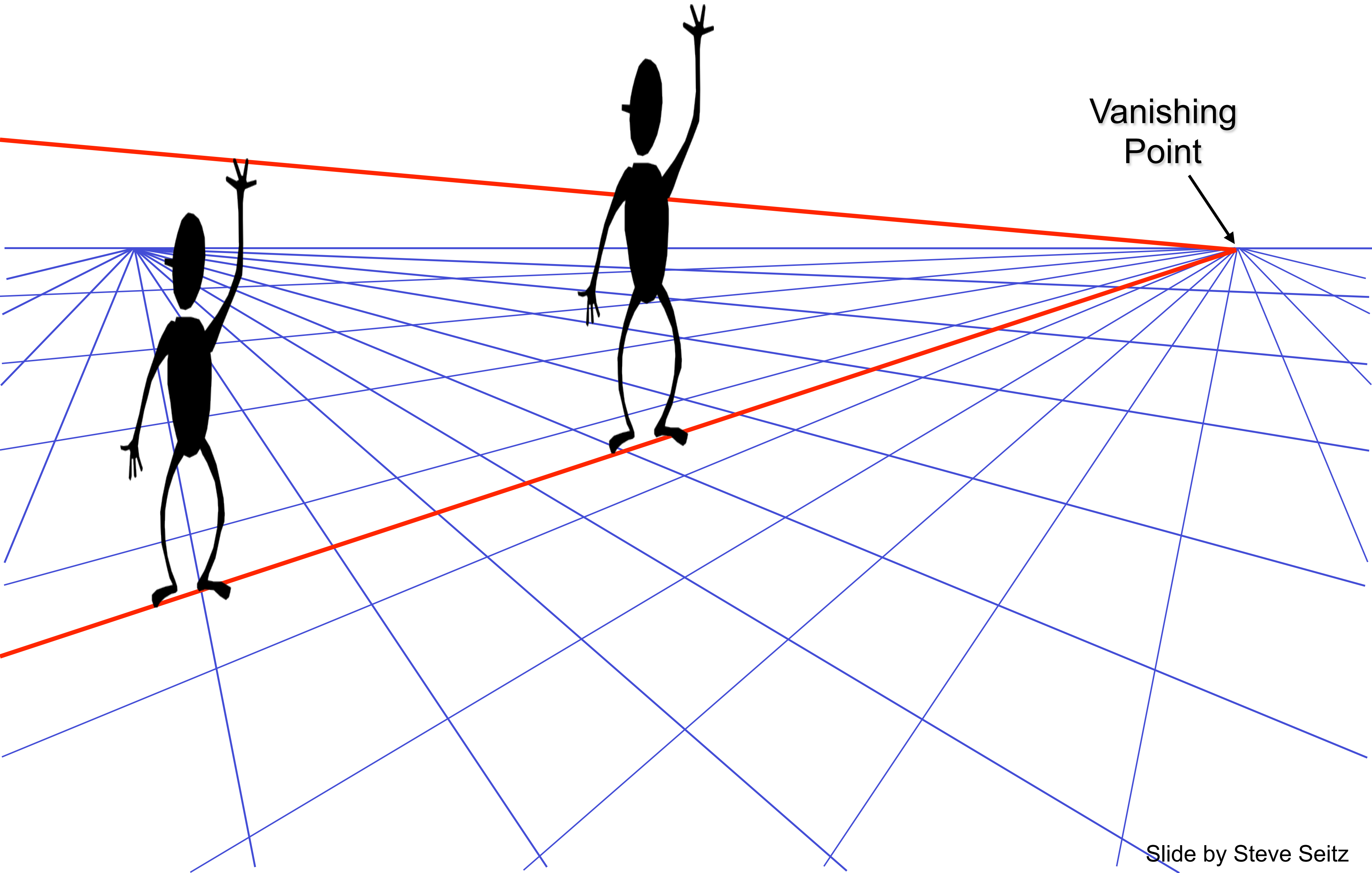
Comparing heights



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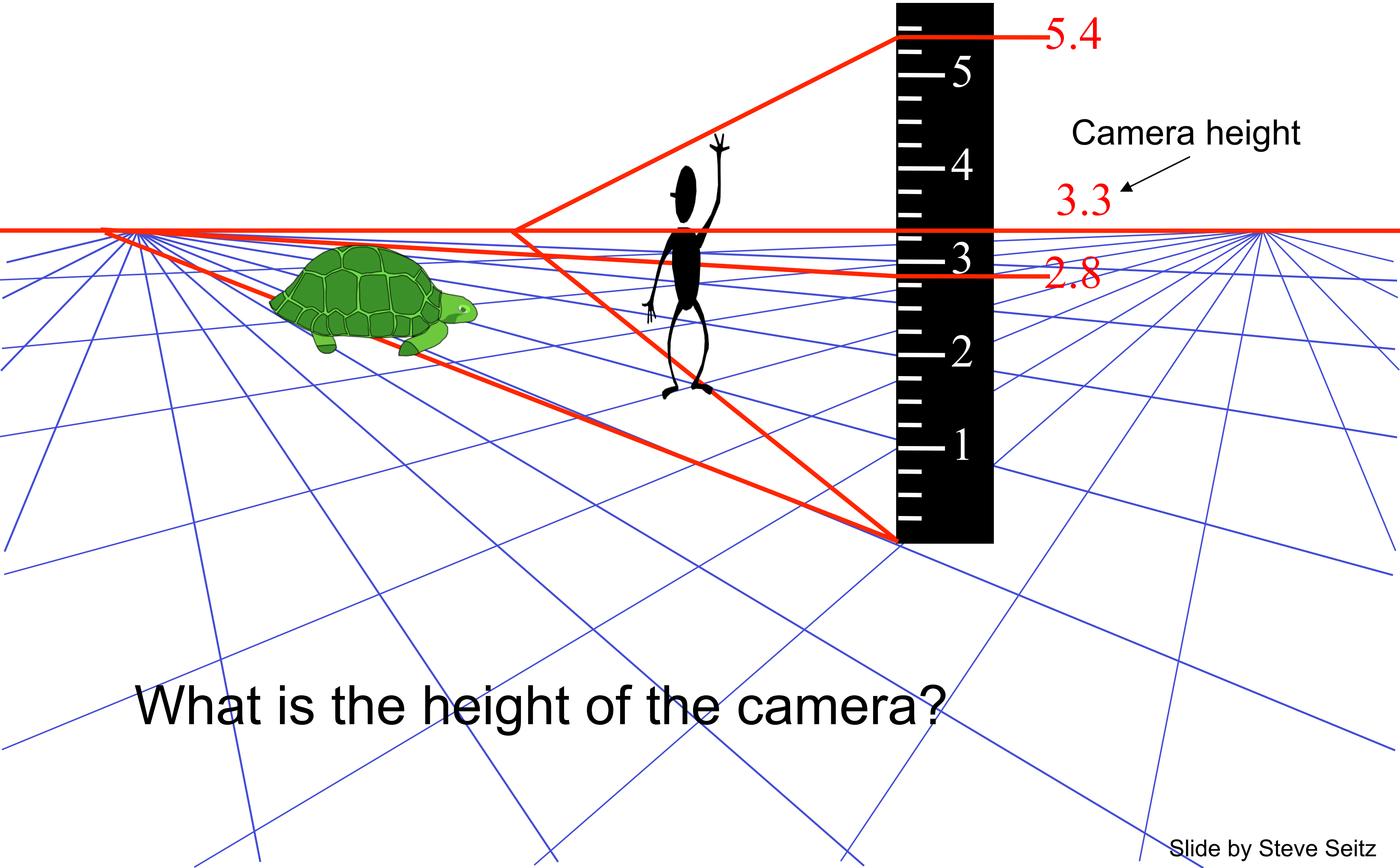
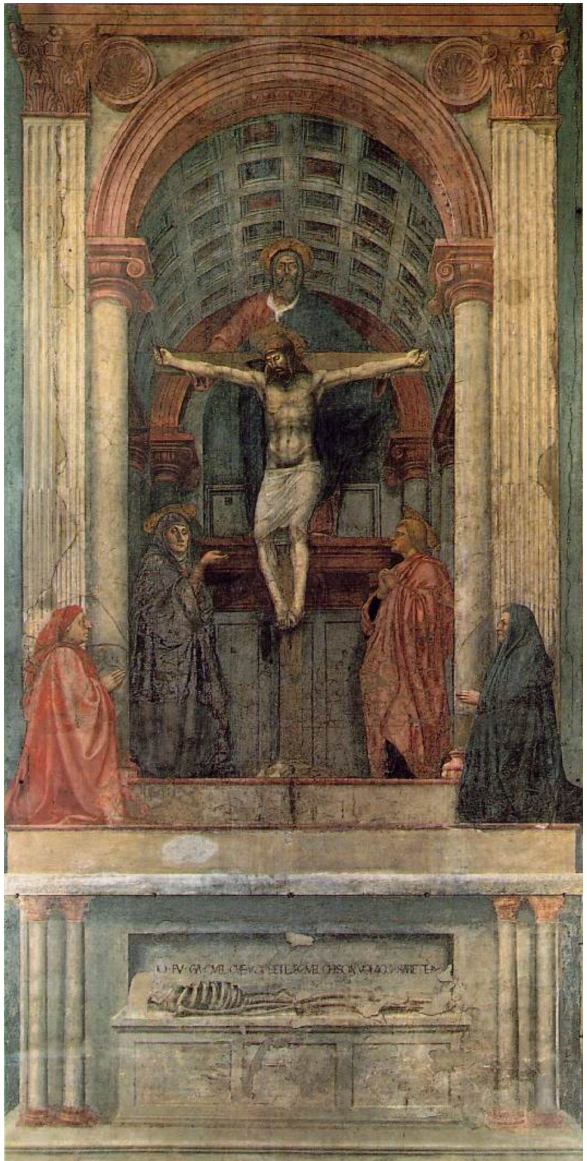
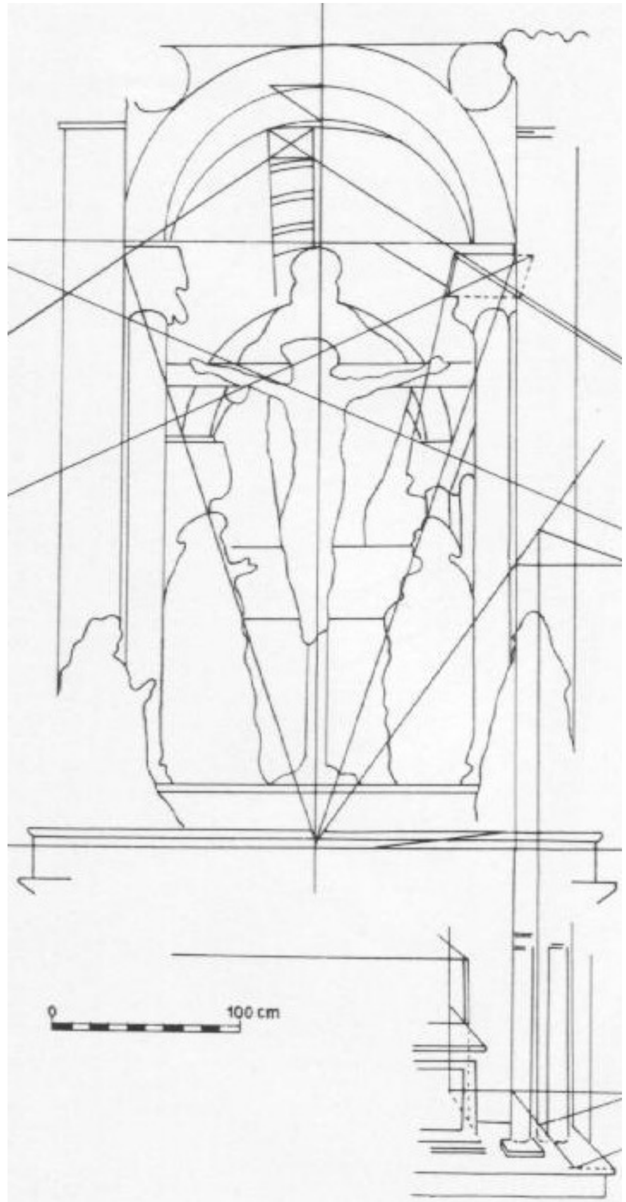
Measuring height



What is the height of the camera?

Perspective in art





Masaccio, *Trinity*, Santa Maria Novella, Florence, 1425-28

One of the first consistent uses of perspective in Western art

Perspective distortion

- What does a sphere project to?

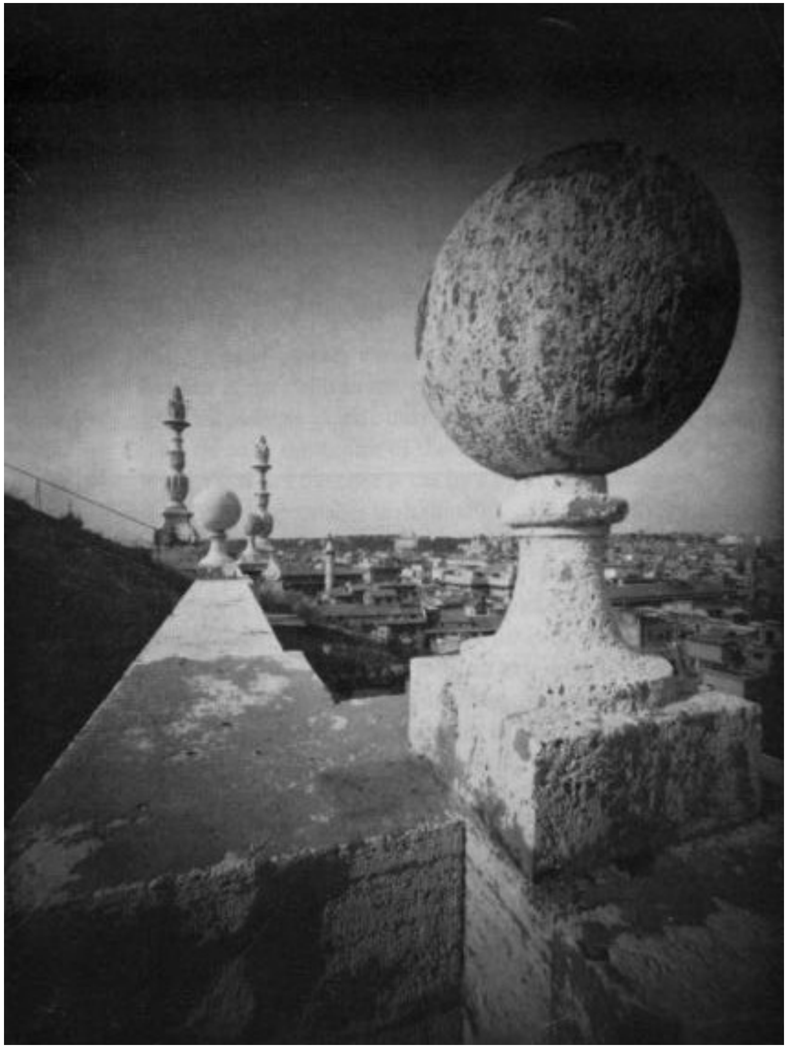
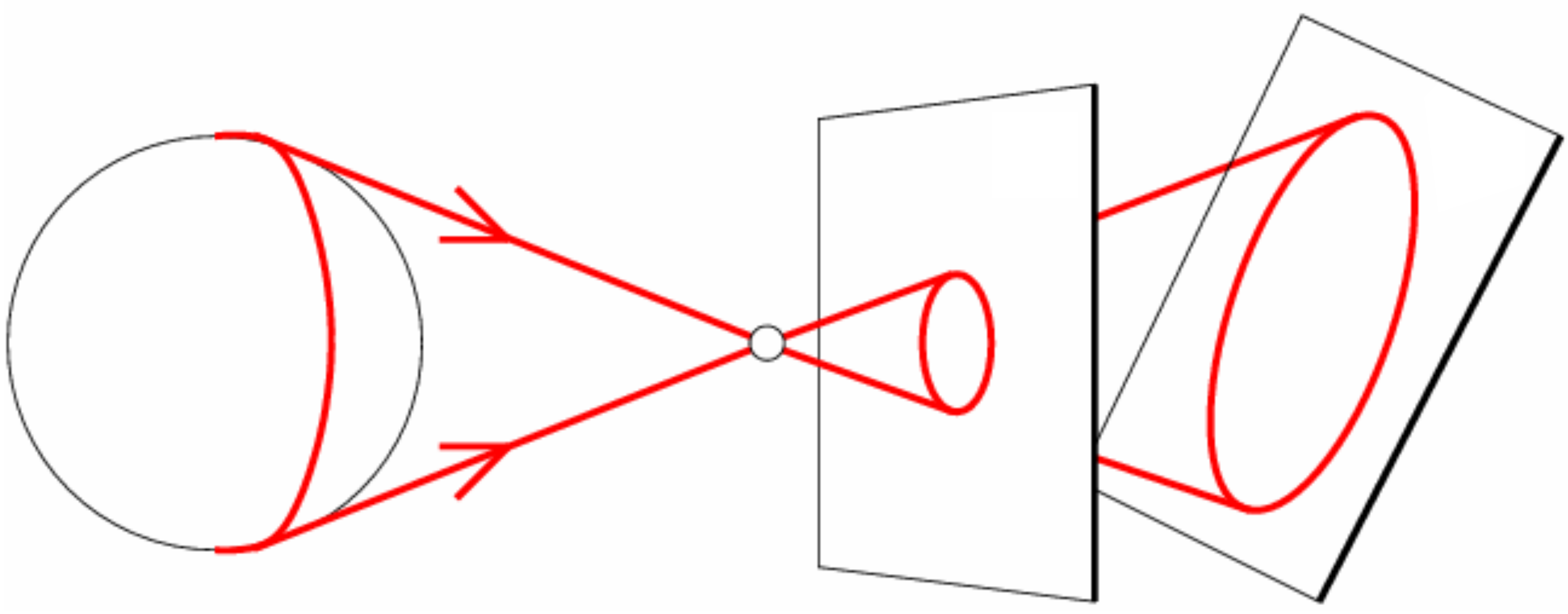


Image source: F. Durand

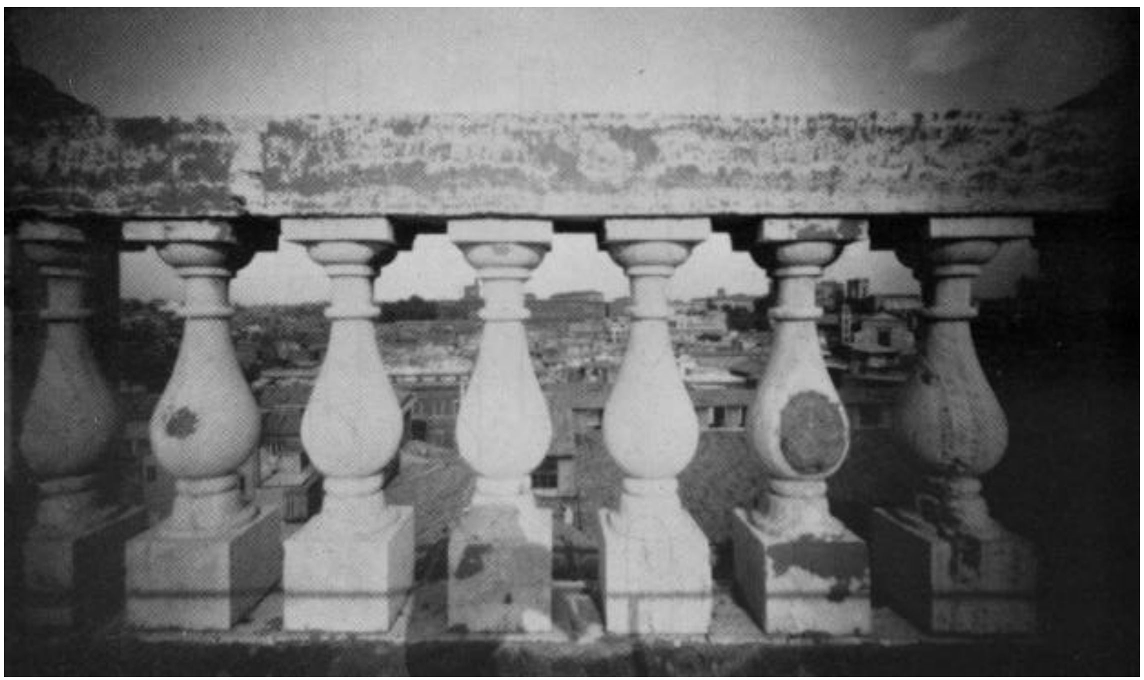
Perspective distortion

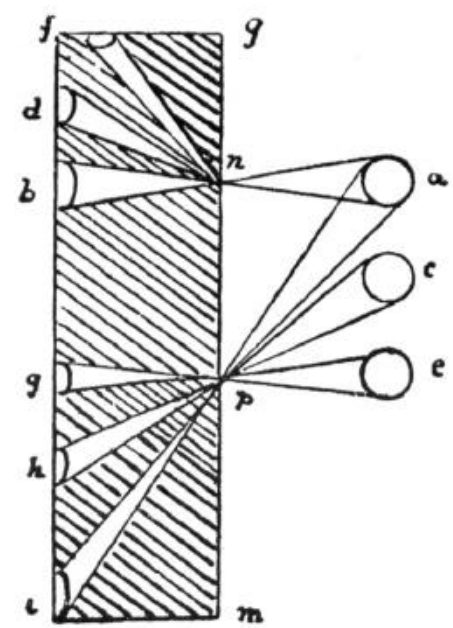
- What does a sphere project to?



Perspective distortion

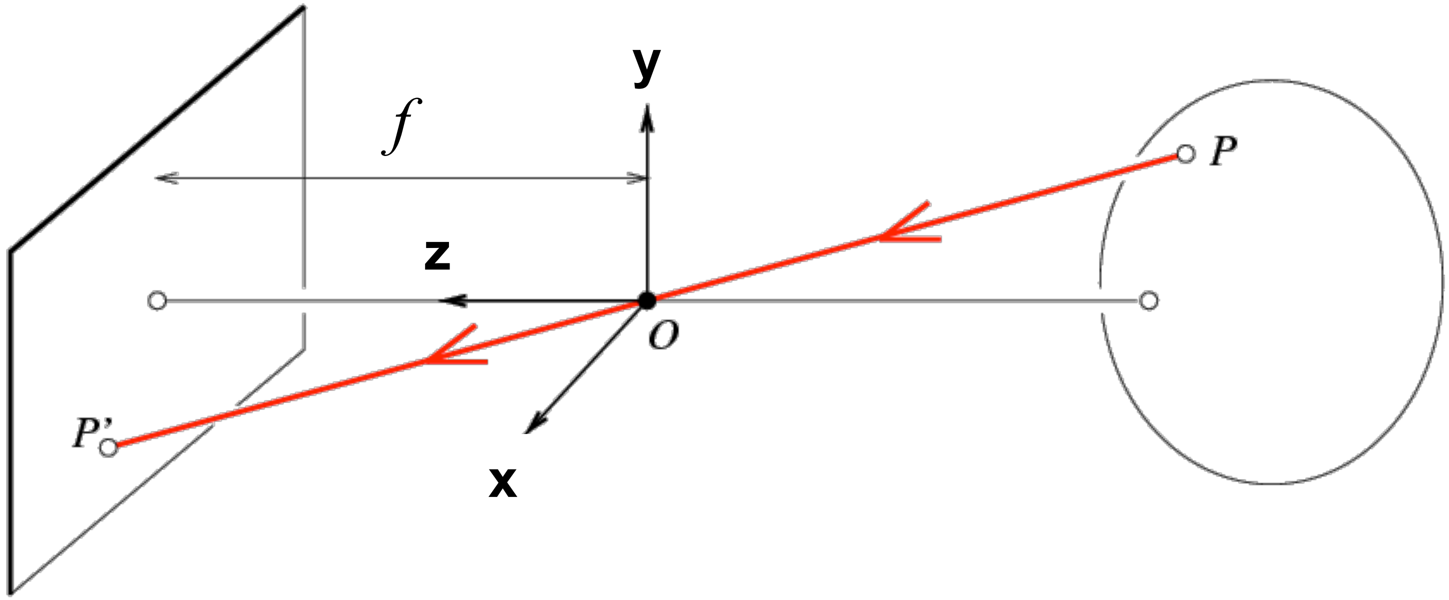
- The exterior columns appear bigger
- The distortion is not due to lens flaws
- Problem pointed out by Da Vinci





Perspective distortion: People

Modeling projection



Projection equation: $(x, y, z) \rightarrow (f\,\frac{x}{z}, f\,\frac{y}{z})$

Source: J. Ponce, S. Seitz

Homogeneous coordinates

$$(x, y, z) \rightarrow (f\frac{x}{z}, f\frac{y}{z})$$

Is this a linear transformation?

- no—division by z is nonlinear

Trick: add one more coordinate:

$$(x, y) \Rightarrow \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

homogeneous image coordinates

$$(x, y, z) \Rightarrow \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

homogeneous scene coordinates

Converting *from* homogeneous coordinates

$$\begin{bmatrix} x \\ y \\ w \end{bmatrix} \Rightarrow (x/w, y/w)$$

$$\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} \Rightarrow (x/w, y/w, z/w)$$

Slide by Steve Seitz

Perspective Projection Matrix

Projection is a matrix multiplication using homogeneous coordinates

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1/f & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z/f \end{bmatrix} \Rightarrow \left(f\frac{x}{z}, f\frac{y}{z}\right)$$

divide by the third coordinate

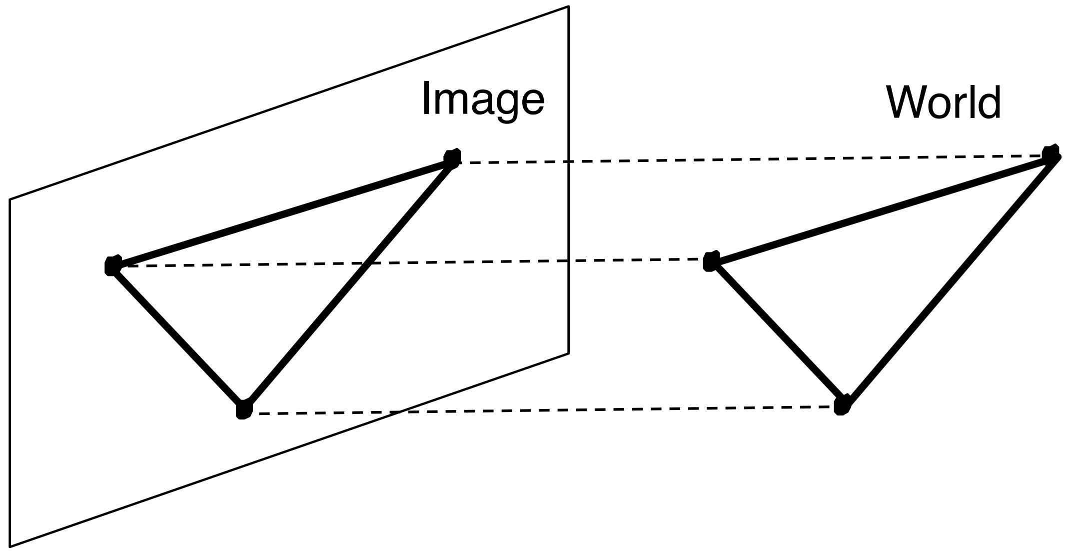
In practice: lots of coordinate transformations…

$$\left(\begin{array}{c}\text{2D} \\ \text{point} \\ \text{(3x1)}\end{array}\right) = \left(\begin{array}{c}\text{Camera to} \\ \text{pixel coord.} \\ \text{trans. matrix} \\ \text{(3x3)}\end{array}\right) \left(\begin{array}{c}\text{Perspective} \\ \text{projection matrix} \\ \text{(3x4)}\end{array}\right) \left(\begin{array}{c}\text{World to} \\ \text{camera coord.} \\ \text{trans. matrix} \\ \text{(4x4)}\end{array}\right) \left(\begin{array}{c}\text{3D} \\ \text{point} \\ \text{(4x1)}\end{array}\right)$$

Orthographic Projection

Special case of perspective projection

- Distance from center of projection to image plane is infinite



- Also called “parallel projection”
- What’s the projection matrix?

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \Rightarrow (x, y)$$

Slide by Steve Seitz