102470 - Computer Vision Course
Institut für Informatik
Universität Bern

# MOCK EXAM

18/12/2018

- **You can use one A4 sized hand-written sheet of paper.**
- **No books, notes, computers, calculators and cellular phones are allowed.**

# Optical Flow, Tracking, Registration, Fitting & Recognition [14 points total]

1. The task of optical flow is to find the motion field $u$ and $v$ by minimizing the functional

$$E[u, v] = |I_x^{t-1}u + I_y^{t-1}v + I^{t-1} - I^t|^2 + \lambda(|\nabla u| + |\nabla v|), \tag{1}$$

where $I$ is the grayscale image, $|\nabla u|$ and $|\nabla v|$ are the total variation on $u$ and $v$, and $t$ is the index of the video frame. This is derived from the Taylor series expansion (up to the first order) of the brightness constancy equation

$$I(x - u(x, y), y - v(x, y), t - 1) = I(x, y, t). \tag{2}$$

Give three scenarios (based on motion and brightness) where optical flow fails. Justify your answer by using the formulas above. **[6 points]**

2. Consider the equation of a parabola $y = ax^2 + x$. Compute the parameter $a$ that best fits the points (1, 3), (-1, 1) and (2, 0.5) with the least squares method. Write the least squares objective and show all your calculations. **[8 points]**

# Epipolar Geometry, Multiple Views & Motion [12 points total]

1. Why is image rectification useful in stereo matching? [**4 points**]

2. Epipolar geometry is the intrinsic projective geometry between two views $I$ and $I'$. It depends only on the camera intrinsic parameters and their relative pose (rotation and translation between the camera centers). The Fundamental Matrix $F$ is a $3 \times 3$ matrix.

   (a) How are the fundamental matrices $F$, going from $I$ to $I'$, and $F'$ going from $I'$ to $I$, related? [**2 points**]

   (b) What is the geometric meaning of the epipoles $\mathbf{e}$ and $\mathbf{e}'$? How are they (algebraically) related to the fundamental matrix? [**4 points**]

   (c) What is the effect of applying the fundamental matrix $F$ to a point $x$? [**2 points**]

# Energy minimization & Bayesian estimation [23 points total]

1. Find the solution to the following energy minimization problem **[6 points]**

$$\arg\min_u |Au - f|^2 + \lambda |u - f|^2 \tag{4}$$

where $A \in \mathbf{R}^{n \times n}$ and $u, f \in \mathbf{R}^n$.

2. Suppose you are given a collection of images $Y_i \in \mathbb{R}^n, i = 1, \ldots n$, and you know $Y_i$ are noisy measurements of an image $X \in \mathbb{R}^n$, such that

$$Y_i = X + \eta, \tag{9}$$

where the noise $\eta \sim \mathcal{N}(0, I)$ is assumed to be of zero mean and unit variance. Derive the maximum likelihood estimate of X.
**Hint 1:** $Y_i \sim \mathcal{N}(X, I)$.
**Hint 2:** The density of the multivariate normal distribution is

$$p(y; \mu, \Sigma) = \det(2\pi\Sigma)^{-\frac{1}{2}} e^{-\frac{1}{2}(y-\mu)^T \Sigma^{-1}(y-\mu)}. \tag{10}$$

**[10 points]**

3. Suppose we are given a task of fitting the parameters of a Gaussian Mixture Model (GMM) $p(x, z)$ to the data $\{x^{(1)}, \ldots, x^{(m)}\}$ consisting of $m$ independent samples, where $z$ denotes discrete latent variable. Each $z^{(i)}$ identifies the Gaussian from which the sample $x^{(i)}$ was generated.

   (a) Write the data log-likelihood under a Gaussian Mixture Model. **[3 points]**

   (b) Why do we need the EM algorithm to fit the parameters of GMM? Why do we not simply maximize the likelihood by setting $\nabla_\theta \ell(\theta)$ to 0? **[4 points]**