102470 - Computer Vision Course
Institut für Informatik
Universität Bern

EXAM

13/02/2018

- **You can use one A4 sized hand-written sheet of paper.**
- **No books, notes, computers, calculators and cellular phones are allowed.**
- **The number of points of the exam is** 100. **The questions are divided into** 4 **groups of** 25 **points each.**

Multiple-Choice Questions (10 Points)

Correct answer: +1 Point, Wrong answer: -1 Point, No answer: 0 Points.
Negative total points will be elevated to 0.

1. **True** **False** The values of the smoothing filter can be negative.

2. **True** **False** The values of the derivative filters sum to 1.

3. **True** **False** Gaussian filter is high-pass filter.

4. **True** **False** Median filter is more effective to remove ”pepper and salt” noise (impulsive noise) than Gaussian filter.

5. **True** **False** RANSAC does not need to fit the model to all samples to find the global optimum.

6. **True** **False** RANSAC can only be used when the number of outliers is less than 50%.

7. **True** **False** The rank of the fundamental matrix is 3.

8. **True** **False** Structure from motion can recover the absolute scale of the scene.

9. **True** **False** The mean-shift algorithm is suitable for multiple segmentations.

10. **True** **False** The SIFT feature descriptor is robust to any shift over sub-patches in the image because it doesn’t preserve the spacial information.

Photometry, Features & Filters [21 points total]

1. Shrinking the lens aperture of a camera can make the captured image sharper. Why do we not make the aperture as small as possible? [**2 points**]

2. Why are 2D separable kernels (e.g., the Gaussian filter) useful? [**2 points**]

3. Let us consider an image x. Let also $p, q \in \mathbf{R}^2$ be two pixels (represented as two 2D vectors) in x. We define *self-similarity* as the property that

$$x[q] = x[\alpha(q-p)+p] \qquad \forall q : |q-p| \leq \rho \tag{1}$$

where $\rho > 0$ is the radius of a ball around p. See Fig. 1. In other words, rays originating at p should have constant image intensity. Notice that, in particular, the self-similarity property is satisfied at corners and at edges. If x satisfies eq. (1) at a ball around p, what orientation will the gradient of x have at q? [**6 points**]

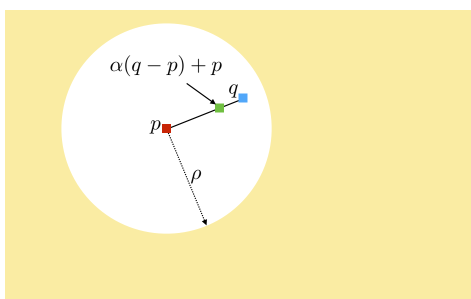

Figure 1: Gradients and self-similarity.

4. The relationship between a 3D point at world coordinates (X, Y, Z) and its corresponding 2D pixel at image coordinates (u, v) can be defined as a projective transformation, i.e. a 3×4 camera projection matrix P. How many degrees of freedom does the projection matrix P have in the most general case? Briefly justify your answer. [**4 points**]

5. Suppose that the normal map $\mathbf{n}$ of a depth map d is given. Write the equation that relates the depth map to the normal map. [**7 points**]

Optical Flow, Tracking, Registration, Fitting & Recognition [26 points total]

1. Briefly describe a general way to track over many frames in a video. [**4 points**]

2. Does the K-Means Algorithm always converge to the same solution when run multiple times on the same data? Justify your answer. [**4 points**]

3. Briefly describe a way to align two images when the correspondences of the two images are not given. [**4 points**]

4. The task of optical flow is to find the motion field u and v by minimizing the functional

$$E[u,v] = |I_x^{t-1}u + I_y^{t-1}v + I^{t-1} - I^t|^2 + \lambda(|\nabla u| + |\nabla v|), \tag{10}$$

where I is the grayscale image, $|\nabla u|$ and $|\nabla v|$ are the total variation on u and v, and t is the index of the video frame. This is derived from the Taylor series expansion (up to the first order) of the brightness constancy equation

$$I(x - u(x,y), y - v(x,y), t-1) = I(x,y,t). \tag{11}$$

Give three scenarios (based on motion and brightness) where optical flow fails. Justify your answer by using the formulas above. [**6 points**]

5. Consider the equation of a parabola $y = ax^2 + x$. Compute the parameter a that best fits the points (1, 3), (-1, 1) and (2, 0.5) with the least squares method. Write the least squares objective and show all your calculations. [**8 points**]

Epipolar Geometry, Multiple Views & Motion [16 points total]

1. Why is image rectification useful in stereo matching? [**4 points**]

2. Epipolar geometry is the intrinsic projective geometry between two views I and I'. It depends only on the camera intrinsic parameters and their relative pose (rotation and translation between the camera centers). The Fundamental Matrix F is a 3×3 matrix.

   (a) How are the fundamental matrices F, going from I to I', and F' going from I' to I, related? [**2 points**]

   (b) What is the geometric meaning of the epipoles **e** and **e**$'$? How are they (algebraically) related to the fundamental matrix? [**4 points**]

   (c) What is the effect of applying the fundamental matrix F to a point x? [**2 points**]

3. Briefly describe one way to improve window-based stereo matching. [**4 points**]

Energy minimization & Bayesian estimation [27 points total]

1. Find the solution to the following energy minimization problem [**6 points**]

$$\arg\min_u |Au - f|^2 + \lambda |u - f|^2 \tag{13}$$

where $A \in \mathbf{R}^{n \times n}$ and $u, f \in \mathbf{R}^n$.

2. Infer the probability that a coin shows up heads, given a series of observed coin tosses. Suppose $X_i \sim$ Ber(θ), where X_i = 1 represents "heads", X_i = 0 represents "tails", and $\theta = p(X_i \equiv$ "head") is the probability of heads. Note: Assume the data samples are iid (independent and identically distributed). [**10 points**]

3. Suppose we are given a task of fitting the parameters of a Gaussian Mixture Model (GMM) $p(x, z)$ to the data $\{x^{(1)}, \ldots, x^{(m)}\}$ consisting of m independent samples, where z denotes discrete latent variable. Each $z^{(i)}$ identifies the Gaussian from which the sample $x^{(i)}$ was generated.

   (a) Write the data log-likelihood under a Gaussian Mixture Model. [**3 points**]

   (b) Why do we need the EM algorithm to fit the parameters of GMM? Why do we not simply maximize the likelihood by setting $\nabla_\theta \ell(\theta)$ to 0? [**4 points**]

   (c) Describe the two main steps of the EM algorithm. [**4 points**]