

5.1 Which occurrences are bound and which are free?

5.1.1 $(\lambda x.x) y (\lambda y.yx) x$

The first x (This does not count the λx so for me the first one is always without the λ) is bound by the lambda calculus, the first y is free, the second y is bound by the lambda calculus, and both last x are free.

5.1.2 $((\lambda x.\lambda y.\lambda z.xyz) (\lambda x.yx) y (\lambda x.zx))$

The first x, y, z are bounded by the lambda calculus, the second x is bounded, the second and third y are free, the third x is bounded and the second z is free.

5.1.3 $\lambda y.(\lambda x. z (x (\lambda x. y (z)))) (\lambda z. y(x (z)))$

All x except the last one and all y are bounded, whereas the last x and the first two z are free, and the last z is bounded.

5.2 Reducing lambda-expressions to their normal form

5.2.1 $(\lambda x.(\lambda z. zy) x) (\lambda x. x)$

$$\begin{aligned} (\lambda x.(\lambda z. zy) x) (\lambda x. x) &= (\lambda z. zy) (\lambda x. x) && \beta\text{-reduction} \\ &= (\lambda x. x)y && \beta\text{-reduction} \\ &= (\lambda y. y) && \beta\text{-reduction} \\ &= y \end{aligned}$$

5.2.2 $(\lambda x. xxy) (\lambda x. xxy)$

$$\begin{aligned} (\lambda x. xxy) (\lambda x. xxy) &= (\lambda x. xxy) (\lambda x. xxy)y && \beta\text{-reduction} \\ &= ((\lambda x. xxy) (\lambda x. xxy)y)y && \beta\text{-reduction} \\ &= ((\lambda x. xxy) (\lambda x. xxy)y)y)y && \beta\text{-reduction} \\ &= \text{ad infinitum} \Rightarrow \text{no normal form available} \end{aligned}$$

5.2.3 $P \equiv (\lambda x. x (xy))I$ where $I \equiv \lambda u. u$

$$\begin{aligned} (\lambda x. x (xy)) (\lambda u. u) &= (\lambda u. u)((\lambda u. u)y) && \beta\text{-reduction} \\ &= (\lambda u. u)y && \beta\text{-reduction} \\ &= (\lambda y. y) && \beta\text{-reduction} \\ &= y \end{aligned}$$