UNIVERSITÄT BERN

Digital 3D Geometry Processing
Exercise 12 - Deformation

Handout date: 21.05.2019

Submission deadline: 28.05.2019, 13:00 h

Note

A .zip compressed file renamed to `Exercisen-GroupMemberNames.zip` where n is the number of the current exercise sheet. It should contain:

- Hand in **only** the files you changed (headers and source). It is up to you to make sure that all files that you have changed are in the zip.
- A `readme.txt` file containing a description on how you solved each exercise (use the same numbers and titles) and the encountered problems.
- Other files that are required by your `readme.txt` file. For example, if you mention some screenshot images in `readme.txt`, these images need to be submitted too.
- Submit your solutions to ILIAS before the submission deadline.

Goal

In this exercise you will implement deformation of the triangular mesh. Given a set of fixed vertices and a set of displaced vertices, the goal is to compute smooth displacements of the remaining vertices.

User Interface

You do not need to implement anything for this part. The framework provided with Exercise 12 offers an interface for selecting fixed vertices, displaced vertices and displacement vector.

- To select vertices, one can use the `Selections` in the toolbox. Click on the icon and then activate the `Select vertices`. The selection can be done one by one or using other selection method such as `Sphere Selection` to select multiple vertices at a time.

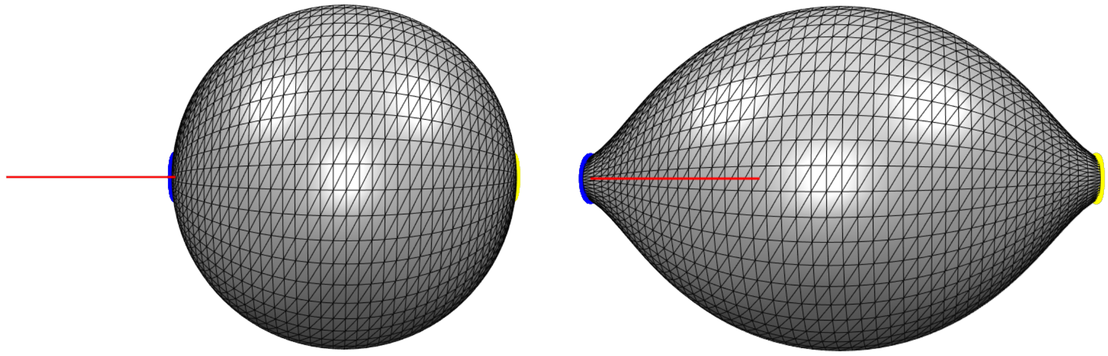

Figure 1: Visualization of deforming a sphere.

- To specify fixed vertices, choose `Fixed vertices` in the combo box. Press the `Select Vertices` button in the plugin tab. All the selected vertices will be marked as fixed vertices and colored in yellow.
- To specify displaced vertices, choose `Displaced vertices` in the combo box. Press the `Select Vertices` button in the plugin tab. All the selected vertices will be marked as displaced vertices and colored in blue.
- **Important Note:** Make sure that you do not select the same vertex both as fixed and displaced. These constraints are mutually contradicting.
- To specify the displacement vector, enter its X, Y and Z components in the line edit in order. The displacement vector can be visualized by clicking the `Visualize Displacement Vector` button. It will be shown as a red segment starting from the center of displaced vertices. To edit the displacement vector, enter the new values in the line edit and press the button again.

Figure 1 shows the selected vertices on a sphere mesh and the displacement vector. On the right is the mesh after deformation.

Solving Laplace Equations

To compute the smooth displacements of the unconstrained vertices, solve a linear system $\mathbf{L}^2\mathbf{x} = \mathbf{b}$, where $\mathbf{L}^2$ is squared Discrete Laplace-Beltrami matrix, $\mathbf{x}$ are the unknown values of the displacements, and $\mathbf{b}$ is equal to 0 for all vertices except for the ones for which the values of displacements $\mathbf{x}_i$ are constrained.

In order to obtain a smooth mesh deformation, one needs to constrain the displacements of the two sets of vertices. The displacements of the selected fixed vertices should be set to zero and the displacements for the selected displaced vertices should be equal to the specified displacement vector. To constrain the displacement $\mathbf{x}_i$, replace the i-th row of $\mathbf{L}^2$ by $[0, ...0, 1, 0...0]$ with 1 at position i.

Hint: Once you set up the **sparse** squared Discrete Laplace-Beltrami matrix, Eigen library only allows to traverse it column-wise. Since the matrix $\mathbf{L}^2$ is symmetric, edit the columns $\mathbf{A}$ to set the constraints and transpose the result.

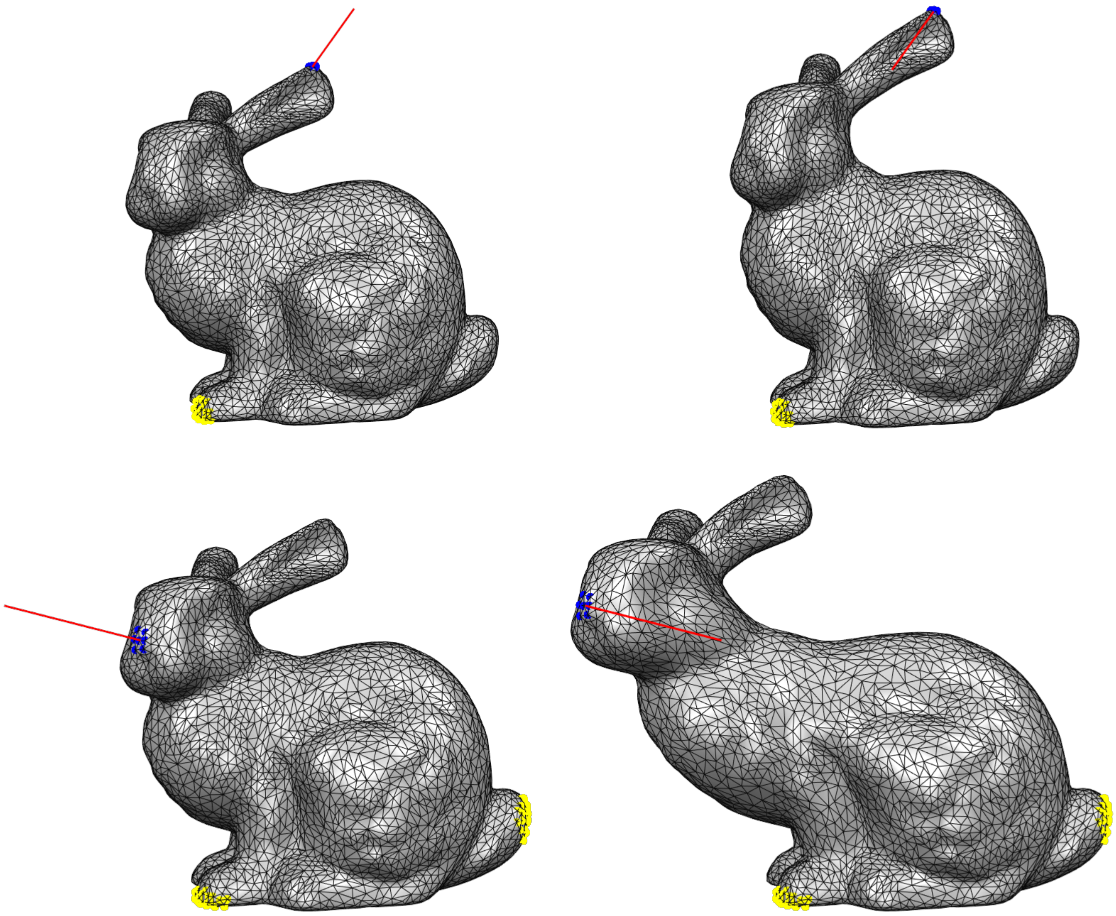

Figure 2: Deforming a bunny.

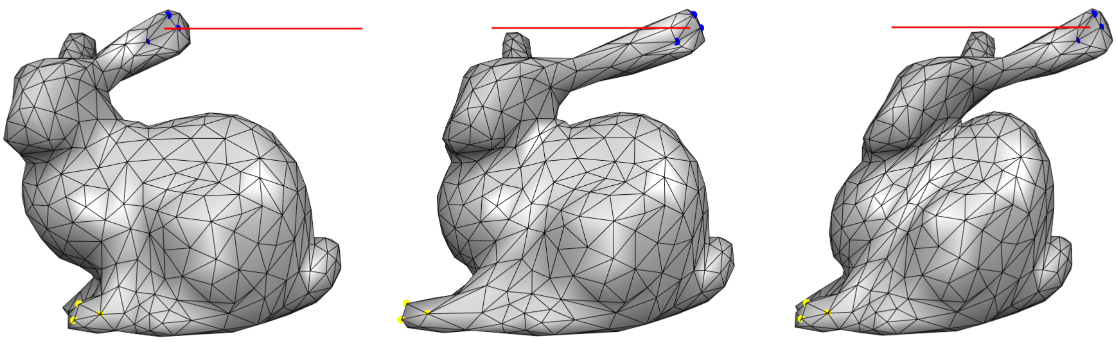

Figure 3: $\mathbf{Lx} = \mathbf{b}$ (middle) versus $\mathbf{L}^2\mathbf{x} = \mathbf{b}$ (right).

Implement the linear solve for the deformation in the function `deform(...)`.

Comparing $\mathbf{Lx} = \mathbf{b}$ and $\mathbf{L}^2\mathbf{x} = \mathbf{b}$

Replace squared Discrete Laplace-Beltrami matrix $\mathbf{L}^2$ by Discrete Laplace-Beltrami matrix $\mathbf{L}$ (Figure 3). Comment on the difference between the results.

Comparing Different Laplacian Weights

Replace the cotangent weight used in Discrete Laplace-Beltrami matrixLby the uniformweight. Comment on the difference between the results.

Comparing with Physical Deformation

Comment on how the computed deformations differ from a real physical material, e.g.an elastic rubber membrane or a thin metal sheet. Create your own example shapes and deformations where those differences would be easily explainable. **Please upload your(.obj/.stl) model as well as images of its deformations in the submission.**