102470 - Computer Vision Course
Institut für Informatik
Universität Bern

MOCK EXAM

15/12/2020

- **You can use one A4 sized hand-written sheet of paper.**
- **No books, notes, computers, calculators and cellular phones are allowed.**
- **The number of points of the exam is 50.**

Name: ____________________ **Student ID:** ____________________

Exercise	1	2	3	4
Total	10	10	11	19
Mark				

Multiple-Choice Questions [10 Points]

Correct answer: +1 Point, Wrong answer: -1 Point, No answer: 0 Points. Negative total points will be elevated to 0.

1. **True** **False** The depth of field is a function of the aperture.

2. **True** **False** If the rows of a 2D convolution filter are linearly independent, the filter is separable.

3. **True** **False** The gradient of the total variation TV(u) of a signal u is linear in u.

4. **True** **False** A Lambertian surface has the property that the reflected color does not change with the light direction.

5. **True** **False** Shape from shading uses a fixed camera and a still scene illuminated from a specific direction.

6. **True** **False** The only assumption in estimating optical flow is the brightness constancy.

7. **True** **False** A homography maps parallelograms to parallelograms.

8. **True** **False** Structure from motion can recover the absolute scale of the scene.

9. **True** **False** The SIFT feature extractor (not the descriptor) is invariant to the rotation of the image.

10. **True** **False** The epipolar geometry depends only on the intrinsic and extrinsic parameters of the cameras.

Photometry, Features, Filters & Photometric Stereo [11 points total]

[2 points]

1. Circle the factors below that affect the intrinsic parameters of a camera model.
   (a) Offset of the optical center
   (b) Camera orientation
   (c) Image resolution
   (d) Focal length

[5 points]

2. Suppose that the normal map $\mathbf{n}$ of a depth map d is given. Write the equation that relates the depth map to the normal map.

   Hint: Write the depth map as a 3D surface and relate it to its tangent vectors.

[3 points] 3. The median filter takes the middle element after sorting the elements in a patch. While this operation is quite robust to non Gaussian noise, it can lead to severe flattening of the texture when the size of the patch is large. How could we modify the median filter so that it retains the original texture while removing outliers?

Optical Flow [11 points total]

1. The *brightness constancy* constraint for optical flow states that for all x, y and $t > 0$ and some Δt

$$I(x, y, t) = I(x + \Delta x, y + \Delta y, t + \Delta t). \tag{7}$$

By assuming that Δt is small and by using the Taylor approximation, this can be reformulated as

$$\frac{\partial I(x, y, t)}{\partial x}\Delta x + \frac{\partial I(x, y, t)}{\partial y}\Delta y + \frac{\partial I(x, y, t)}{\partial t}\Delta t = 0. \tag{8}$$

[3 points] (a) Describe the meaning of the brightness constancy and all symbols involved in the approximation equation (8).

[2 points] (b) Transform eq. (8) into a form that contains the x- and y component (v_x, v_y) of the optical flow. *Hint: Recall that the optical flow relates to the velocity of moving points.*

[6 points] (c) Give three scenarios where optical flow fails. Justify your answer by referring to the formulas above.

Energy minimization & Bayesian estimation [19 points total]

1. Find the solution to the following energy minimization problem **[5 points]**

$$\arg\min_u |Au - f|^2 + \lambda |u - f|^2 \tag{10}$$

where $A \in \mathbf{R}^{n \times n}$ and $u, f \in \mathbf{R}^n$.

2. Suppose you are given a collection of images $Y_i \in \mathbb{R}^m, i = 1, \ldots n$, and you know Y_i are noisy measurements of an image $X \in \mathbb{R}^m$, such that

$$Y_i = X + \eta, \tag{15}$$

where the noise $\eta \sim \mathcal{N}(0, I)$ is assumed to be of zero mean and unit variance. Derive the maximum likelihood estimate of X.
Hint 1: $Y_i \sim \mathcal{N}(X, I)$.
Hint 2: The density of the multivariate normal distribution is

$$p(y; \mu, \Sigma) = \det(2\pi\Sigma)^{-\frac{1}{2}} e^{-\frac{1}{2}(y-\mu)^T \Sigma^{-1} (y-\mu)}. \tag{16}$$

[8 points]

3. Suppose we are given a task of fitting the parameters of a Gaussian Mixture Model (GMM) $p(x, z)$ to the data $\{x^{(1)}, \ldots, x^{(m)}\}$ consisting of m independent samples, where z denotes discrete latent variable. Each $z^{(i)}$ identifies the Gaussian from which the sample $x^{(i)}$ was generated.

   (a) Write the data log-likelihood under a Gaussian Mixture Model. [**3 points**]

   (b) Why do we need the EM algorithm to fit the parameters of GMM? Why do we not simply maximize the likelihood by setting $\nabla_\theta \ell(\theta)$ to 0? [**3 points**]