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Applied Optimization
Exercise 04

4.1 Lagrange Duality

4.1.a

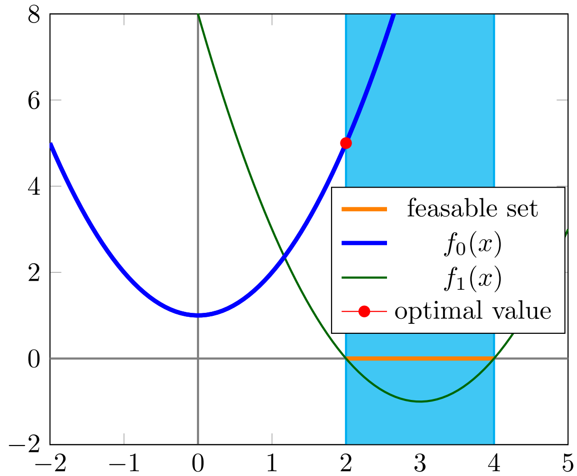
The feasable set is the following:

$$F \;=\; [x \mid x \in [2,4]]$$

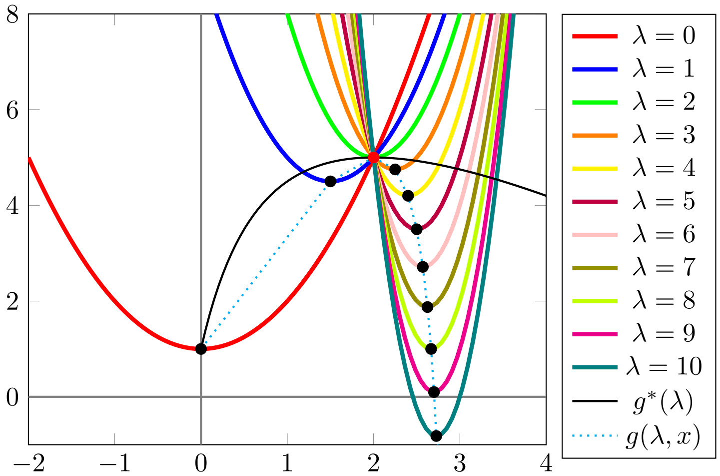
The optimal value is: $x = 2$, with $f_0(2) = 5$

4.1.b

Optimization Graphical Plot



Lagrange Graphical Plot



4.1.c

Our start for the dual problem will look like the following function:

$$g(\lambda) \;=\; \inf_{x \in D} [x^2 + 1 + \lambda(x-2)(x-4)]$$

Now we compute the derivative of this function and the general solution for its minimum:

$$\begin{aligned} \bigtriangledown g(\lambda, x) \;&=\; 2x + 2x\lambda - 6\lambda \\ \Rightarrow 2x + 2x\lambda - 6\lambda \;&=\; 0 \\ \Leftrightarrow x \;&=\; \frac{3\lambda}{1+\lambda} \end{aligned}$$

This value is now put into the function $g(\lambda, x)$ (calling it $g^*(\lambda)$, it is also plotted in the second plot):

$$g^*(\lambda) \;=\; (\frac{3\lambda}{1+\lambda})^2 + 1 + \lambda(\frac{3\lambda}{1+\lambda} - 2)(\frac{3\lambda}{1+\lambda} - 4) \;=\; -\frac{(\lambda^2 - 9\lambda - 1)}{\lambda + 1}$$

It is obvious that this function is concave and we need to find its maximum, so we compute the derivative and the extremum of this function:

$$\begin{aligned} \bigtriangledown g^*(\lambda) &= -\frac{\lambda^2 + 2\lambda - 8}{(\lambda+1)^2} \\ \Rightarrow -\frac{\lambda^2 + 2\lambda - 8}{(\lambda+1)^2} &= 0 \\ \Leftrightarrow \lambda^2 + 2\lambda - 8 &= 0 \\ \Leftrightarrow (\lambda + 4) \cdot (\lambda - 2) &= 0 \end{aligned}$$

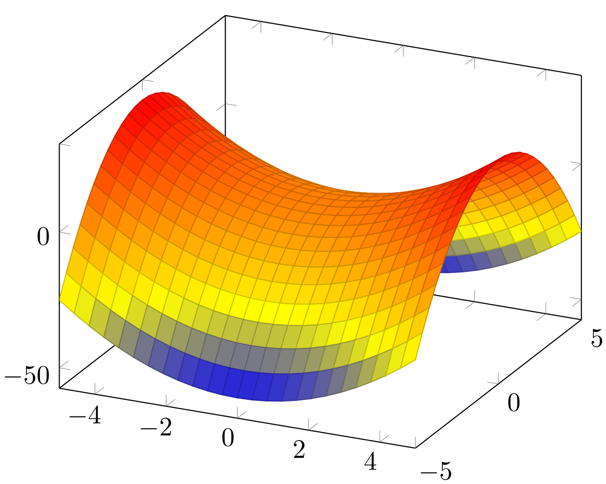
Because $\lambda > 0$, the result is 2. Because this is the exact optimal solution for x, the strong duality holds.

4.2 KKT Condition

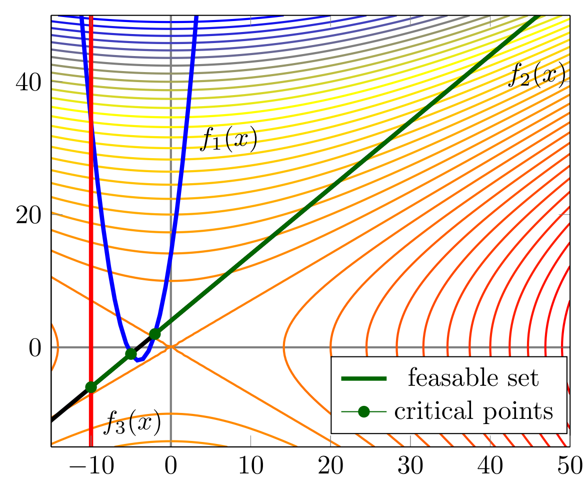
The feasable set is like the following:

$$F \;=\; [Point\ P(x_1, x_2) : x_1 + 4 = x_2,\ -10 \leq x_1 \leq -5\ \vee -2 \leq x_1]$$

Objective Function Plot



Graphical Plot



Because we can see that the optimal solution must be on the function $f_2(x) = x_1 + 4$ and the further we go on it the smaller the value of the objective function becomes (because the contour lines are getting "bluish"), its values for x_1 and x_2 will get to ∞ and therefore its solution will be $-\infty$. This solution does not satisfy the KKT conditions.

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4.3 Programming

The OptimumChecker function should be alright now.
We tried to add the KKT Example in the main function but we were unsure how to add the constraints to the vectors.