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Applied Optimization
Exercise 04

4.1 Lagrange Duality

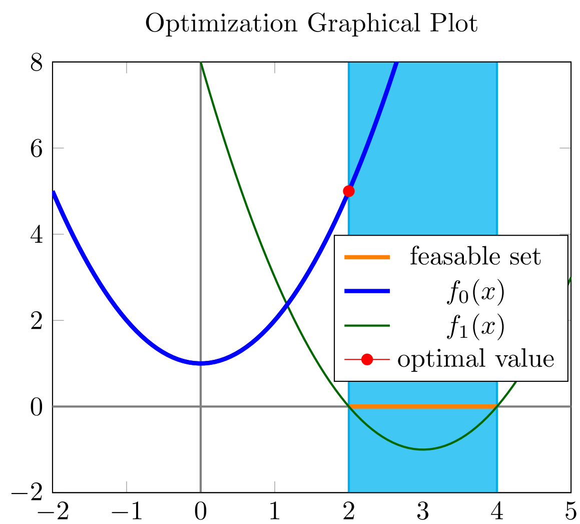
4.1.a

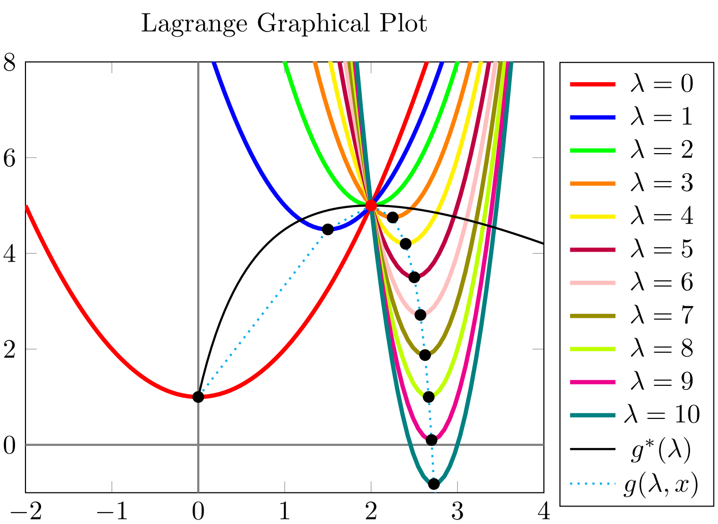
The feasable set is the following:

$$F = [x \mid x \in [2, 4]]$$

The optimal value is: $x = 2$, with $f_0(2) = 5$

4.1.b





4.1.c

Our start for the dual problem will look like the following function:

$$g(\lambda) = \inf_{x \in D} [x^2 + 1 + \lambda(x-2)(x-4)]$$

Now we compute the derivative of this function and the general solution for its minimum:

$$\begin{aligned} \triangledown g(\lambda, x) &= 2x + 2x\lambda - 6\lambda \\ \Rightarrow 2x + 2x\lambda - 6\lambda &= 0 \\ \Leftrightarrow x &= \frac{3\lambda}{1+\lambda} \end{aligned}$$

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This value is now put into the function $g(\lambda, x)$ (calling it $g^*(\lambda)$, it is also plotted in the second plot):

$$g^*(\lambda) = (\frac{3\lambda}{1+\lambda})^2 + 1 + \lambda(\frac{3\lambda}{1+\lambda} - 2)(\frac{3\lambda}{1+\lambda} - 4) = -\frac{(\lambda^2 - 9\lambda - 1)}{\lambda + 1}$$

It is obvious that this function is concave and we need to find its maximum, so we compute the derivative and the extremum of this function:

$$\begin{aligned} \triangledown g^*(\lambda) &= -\frac{\lambda^2 + 2\lambda - 8}{(\lambda+1)^2} \\ \Rightarrow -\frac{\lambda^2 + 2\lambda - 8}{(\lambda+1)^2} &= 0 \\ \Leftrightarrow \lambda^2 + 2\lambda - 8 &= 0 \\ \Leftrightarrow (\lambda + 4) \cdot (\lambda - 2) &= 0 \end{aligned}$$

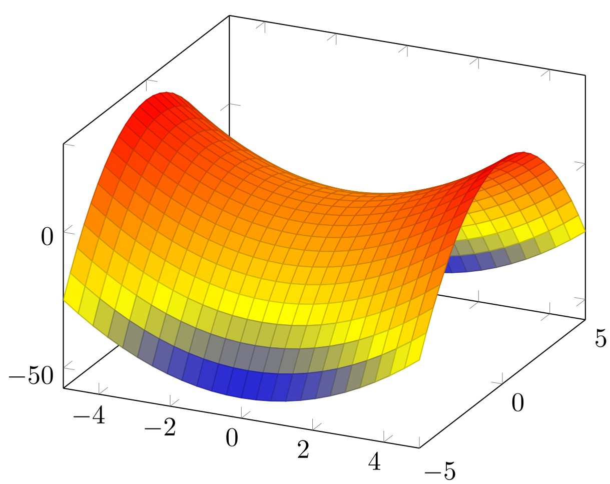
Because $\lambda > 0$, the result is 2. Because this is the exact optimal solution for x, the strong duality holds.

4.2 KKT Condition

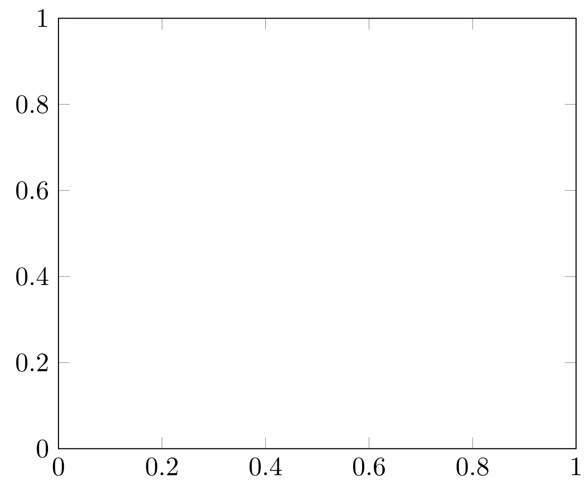
The feasable set is like the following:

$$F = [Point\ P(x_1, x_2) : x_1 + 4 = x_2,\ -10 \le x_1 \le -5\ \vee -2 \le x_1]$$

Objective Function Plot



Graphical Plot



Because we can see that the optimal solution must be on the function $f_2(x) = x_1 + 4$ and the further we go on it the smaller the value of the objective function becomes (because the contour lines are getting "bluish"), its values will get to ∞ and therefore its solution will be $-\infty$. This solution does not satisfy the KKT conditions.

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4.3 Programming