Problem Set 7

Computer Vision 2020
University of Bern

1 Epipolar Geometry

1. In this exercise we will derive the essential and fundamental matrices for a pair of cameras related by a rigid transformation $[\mathbf{R}|\mathbf{t}]$, by looking at the common 3D point $\mathcal{P}$.

   Let $\mathbf{X}_1 = [X_1\ Y_1\ Z_1]^T \in R^3$ be the coordinates of $\mathcal{P}$ defined in the first camera coordinate system with origin in the first camera center C_1. Let $\mathbf{R}$ be the rotation matrix and $\mathbf{t}$ be the translation vector that transform the first camera coordinate system into the second camera coordinate system with origin C_2. Remember that $\mathbf{R}$ is a 3×3 orthogonal matrix with $\mathbf{R}\mathbf{R}^T = \mathbf{I}$ and $\mathbf{t} \in R^3$. Finally, let the coordinates of the 3D point $\mathcal{P}$ in the second camera coordinate system be $\mathbf{X}_2 = [X_2\ Y_2\ Z_2]^T \in R^3$.

   (a) How do you compute the projection $\mathbf{m}_1 = [x_1\ y_1\ 1]^T$ of $\mathcal{P}$ in the first coordinate system using the coordinate of $\mathbf{X}_1$ and how do you compute the projection $\mathbf{m}_2 = [x_2\ y_2\ 1]^T$ of $\mathcal{P}$ in the second coordinate system using the coordinate of $\mathbf{X}_2$?

   (b) Give the relationship between $\mathbf{X}_1$ and $\mathbf{X}_2$ using $\mathbf{R}$ and $\mathbf{t}$.

   (c) How are the projections $\mathbf{m}_1$ and $\mathbf{m}_2$ related (always taking the first image plane as coordinate system)?

   (d) Suppose that $\mathcal{P}$ lies on a plane $\mathcal{P} \in \pi$, where π has a normal vector $\mathbf{n}_1$ (expressed in the first camera coordinate system) and its distance to C_1 is d_1. Assuming that $\mathbf{n}_1$ verifies $\|\mathbf{n}_1\| = \sqrt{\mathbf{n}_1^T \mathbf{n}_1} = 1$, give the equation of the plane π.

(e) Use the results above, including $\mathcal{P} \in \pi$, to find a transformation of the form $\mathbf{m}_2 = T\mathbf{m}_1$.

(f) Given the normalized image coordinates $\mathbf{m}_1$ and $\mathbf{m}_2$ express the transformation of two points $\mathbf{p}_1$ and $\mathbf{p}_2$ in pixel coordinates with the help of the intrinsic matrix $\mathbf{K}$.

(g) Try to find a relationship between $\mathbf{m}_1$ and $\mathbf{m}_2$ without knowing the normal of the plane passing from $\mathcal{P}$.

(h) Give the relation between the points $\mathbf{p}_1$ and $\mathbf{p}_2$ in pixel coordinates.

2. The epipolar geometry is the intrinsic projective geometry between two views I and I'. It depends only on the cameras' intrinsic parameters and their relative pose (rotation and translation between the camera centers). The **Fundamental Matrix F** is a 3×3 matrix with $\text{rank}(\mathbf{F}) = 2$.

   (a) How is the fundamental matrix $\mathbf{F}$ related to pairs of corresponding points x, x' in the two images?

   (b) How are the fundamental matrices $\mathbf{F}$, going from I and I', and **F'**, going from I' and I, related?

   (c) What is the geometric meaning of the epipoles $\mathbf{e}$ and **e'**? How are they related to the fundamental matrix (algebraically)?

   (d) Are the epipoles always visible in the two views?

   (e) How can one determine the epipolar line **l'** passing trough a given point **x'**?

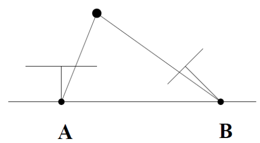
   (f) What is the effect of applying the fundamental matrix $\mathbf{F}$ to a point $\mathbf{x}$?

2 Calibrated Reconstruction

For a given essential matrix E with singular value decomposition $E = U\text{diag}(1, 1, 0)V^T$ and first camera matrix $P = [I \mid 0]$, there are four possible choices for the second camera matrix P', namely

$$P' = [UWV^T \mid u_3] \quad \text{or} \quad [UWV^T \mid -u_3] \quad \text{or} \quad [UW^TV^T \mid u_3] \quad \text{or} \quad [UW^TV^T \mid -u_3],$$

where W is a specific orthogonal matrix and u_3 is the third column of U.
The Figure below illustrates the geometric interpretation of the first solution. Draw the three missing configurations.



(a) (b)

(c) (d)