102470 - Computer Vision Course
Institut für Informatik
Universität Bern

EXAM

16.02.2021

- **You can use one A4 sized hand-written sheet of paper.**
- **You cannot use books/notes/computers/tablets/calculators/cellular phones to help you answer the questions.**
- **You can use the computer or a tablet to visualize the exam and the cellular phone to take pictures of the exam at the end.**
- **Make sure that the camera shows your desk at all times.**
- **The number of points of the exam is 100.**

Name: ______________________ **Student ID:** ______________________

Exercise	1	2	3	4	5
Total	10	14	27	30	19
Mark					

1. True or False Questions [10 Points Totoal]

Correct answer: +1 Point, Wrong answer: -1 Point, No answer: 0 Points. Negative total points will be elevated to 0.

1. **True** **False** Perspective projection is a linear transformation.
2. **True** **False** The rows/columns of a 2D convolution filter are always dependent.
3. **True** **False** The low pass filter eliminates high-frequency components from an image.
4. **True** **False** The solution of an ill-posed problem is not always unique.
5. **True** **False** Overfitting occurs when the test error is high and training error is low.
6. **True** **False** Straight lines remain straight lines under all 2D transformations (affine, perspective).
7. **True** **False** Harris and Hessian detectors are rotation-invariant.
8. **True** **False** Expectation Maximization is a gradient based optimization technique.
9. **True** **False** Mean Shift, in the context of segmentation, requires a predefined number of clusters k.
10. **True** **False** Structure from motion aims to find the 3D camera pose, given the 2D projections over time on the image plane of the camera of a rigidly moving 3D point cloud.

2. Photometry, Filters & Photometric Stereo [14 points total]

[2 points]

1. Circle **all factors** that affect the image intensity of a Lambertian surface:
   (a) The albedo of the surface
   (b) The viewing direction
   (c) The normal of the surface
   (d) The direction of the light source.

[2 points]

2. Which of the following sentences about the orthographic and the perspective projections **are correct**?
   (a) The projection lines in both projections are orthogonal to the projection plane.
   (b) The optical center is located at a finite point in the perspective projection and at an infinite point in the orthographic projection.
   (c) Vanishing points occur only in the perspective projection.
   (d) Both projections form a realistic picture of an object.

[2 points]

3. Shrinking the lens aperture of a camera can make the captured image sharper. Give a brief explanation of why we do not make the aperture as small as possible.

[2 points]

4. For an $n \times n$ image x and an $m \times m$ 2D filter k, compute the computational complexity (big O notation) of the convolution between x and k in both the case when the 2D filter k is separable and when it is not separable.

[2 points]

5. Explain briefly for which type of image noise one should use the Gaussian and the median filters?

[4 points]

6. Consider the 1×7 image **I** below, where each pixel intensity is denoted with a letter $a, \ldots, g$:

a	b	c	$\boldsymbol{d}$	e	f	g

(a) **[3 points]**
Apply a 1×3 averaging filter twice to the image I, that is, apply the averaging filter to I to obtain a smoothed image J and then apply the same filter again to J. Give the expression for the center pixel of the resulting image as a function of the original pixel values in I. Show all your calculations.

(b) **[1 point]**
Define a single 1D filter that, when applied only once to the image I, will produce the same result as applying the previous 1×3 averaging filter twice. Specify the dimension of the filter and its values.

3. Optical Flow, Tracking, Registration & Fitting [27 points total]

[5 points]

1. Write the definitions of the motion field and the optical flow. Then, give an example in which the motion field is zero, but the optical flow is non zero and an example in which the motion field is non zero, but the optical flow is zero.

[3 points]

2. What is the direction on the image plane along which the optical flow cannot be reliably estimated? Show your justification with mathematical formulas.

[6 points]

3. Obtain the explicit equations for the optical flow in the case of a global affine motion model. The affine motion model assumes that the flow $[u(x,y),\ v(x,y)]$ can be written as

$$u(x,y) = a_1 + a_2 x + a_3 y \tag{1}$$
$$v(x,y) = a_4 + a_5 x + a_6 y. \tag{2}$$

Recall the principles used to derive optical flow and then compute the first-order Taylor expansion of the intensity function $I(x,y,t)$ with respect to the affine motion parameters.
Moreover, compute how many linear constraints are needed to solve the equation in the unknown affine motion parameters $a_1,\ldots,a_6$.
Finally, cast the estimation of the motion parameters as a least squares minimization and show the closed form solution.
Note: You must show all the steps of your derivation.

[8 points]

4. When we have large displacement maps $[u,\ v]$, we can use **hierarchical optical flow**. This approach uses down-scaled images to obtain some initial u and v estimate. Explain in detail the problem it solves and how we can use this method recursively to estimate the final full-resolution displacement map.

[5 points]

5. Consider the equation of a parabola $y = ax^2 + x$. Compute the parameter a that best fits the points (1, 3), (-1, 1) and (2, 0.5) with the least squares method. Write the least squares objective and show all your calculations.

4. Epipolar Geometry, Stereo & Motion [30 points total]

[10 points] 1. (a) **[2 points]** What is the name of the coordinate system that is often used for convenience when working with 3D points and 2D projections? Describe with a mathematical expression one key property of these coordinates.

(b) **[5 points]** In Figure 1 we see two images of a scene taken from different viewpoints, annotated with red markers and colored epipolar lines. Pick a pair of corresponding epipolar lines and a pair of corresponding points and explain in words the relationship between them in both views. Name the lines and points with variables (draw directly on the pictures) and express their relationships with mathematical expressions.

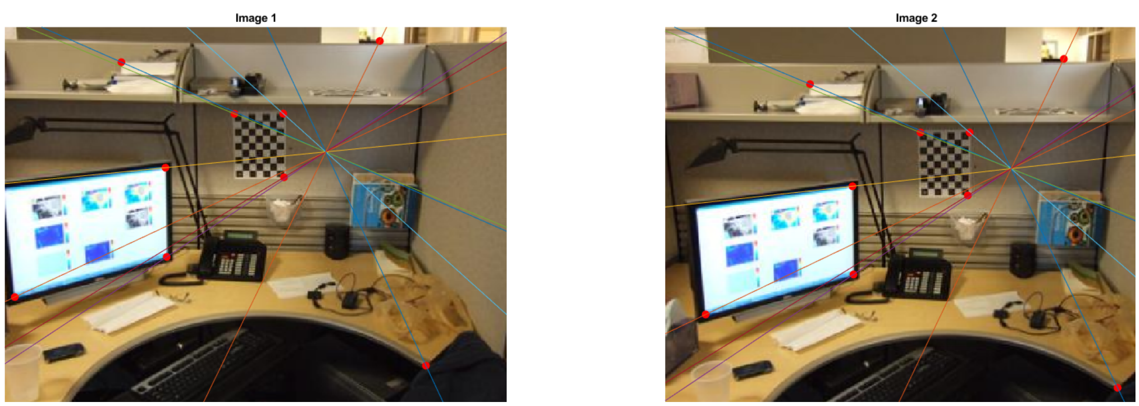

Figure 1: The Epipolar Geometry in two views.

(c) **[3 points]** Explain the meaning of the point where all epipolar lines intersect. What does the configuration of epipolar lines in Figure 1 tell us about the relative position of the cameras?

[14 points] 2. We are given two cameras with calibration matrices $K_l = \text{diag}(2, 2, 1)$ and $K_r = \text{diag}(0.5, 0.5, 1)$. The essential matrix E is also known,

$$E = [t]_\times R = \begin{bmatrix} 1 & 2 & 1 \\ -2 & -3 & 1 \\ 3 & 5 & 0 \end{bmatrix}, \tag{3}$$

and is determined by the relative pose (R, t) of the two cameras.

(a) **[2 points]** Verify that E has rank 2.

(b) **[5 points]** Compute the fundamental matrix F.

(c) **[7 points]** Given the point $x_l = \begin{bmatrix} 2 & -3 & 5 \end{bmatrix}^T$ in homogeneous coordinates on the left image plane, determine the parameters a and b of the epipolar line $y = ax + b$ in the right image. If you were not able to solve the question 2. (b) above, assume $F = E$.

[6 points]

3. Two identical pinhole cameras are placed side by side. The focal length of the cameras is f and the distance between the camera centers is b. The point $P = [X, Y, Z]^T \in \mathbb{R}^3$ is located in front of the cameras and its disparity $d(x, y)$ is the distance between the corresponding image points $p = [x, y] \in \mathbb{R}^2$ and $p' = [x', y'] \in \mathbb{R}^2$, i.e., $p' = p + d(p)$.

   (a) **[3 points]** Derive a formula for the Z-coordinate of P (the depth) that depends only on the parameters b and f and the disparity d. Show all the steps of your derivation.

   (b) **[3 points]** At a given depth Z, how does a measurement error in the disparity affect the value of Z? What can be done to counteract the depth error in stereo?
   Hint: Compute the derivative of the formula derived in question 3 (a) with respect to the disparity and then write the disparity as a function of the depth and the camera parameters.

5. Bayesian Methods & Energy Minimization [19 points total]

[7 points] 1. We observe m iid samples $x_1, ..., x_m$ from a distribution p parameterized by σ

$$p(x|\sigma) = \frac{1}{2\sigma} \exp\left(-\frac{|x|}{\sigma}\right). \tag{4}$$

Assume that σ is an independent random variable with Gaussian distribution and an extremely large variance, so that we can claim $p(\sigma) \simeq$ const. Write the Maximum a Posteriori objective to find the estimate of σ. Explain all the steps of the derivation, find the expression for the estimate of σ and explain the relation to the Maximum Likelihood estimate of σ.

[9 points]

2. You are given a noisy discrete signal $f : \mathbb{R}^N \to \mathbb{R}$. You are asked to denoise f by solving

$$\min_u \frac{1}{2} \sum_i (u[i] - f[i])^2 + \frac{\lambda}{2} \sum_i |\nabla u[i]|^2 \tag{5}$$

with regularization parameter $\lambda \geq 0$.

(a) **[2 points]** What would be the result of the optimization if we set $\lambda = 0$? What would be the result if we set it to an extremely large value?

(b) **[1 point]** Choose a finite difference discretization for the gradient $\nabla u[i]$.

$\nabla u[i] =$

(c) **[4 points]** Compute the gradient of the above energy with respect to u. You can ignore the special cases at the boundaries. Show all the steps in your calculations.

(d) **[2 points]** By assuming that the gradient formula is available, write the gradient descent algorithm that could be used to optimize the objective.

[3 points]

3. Recall the Majorization-Minimization (MM) Algorithm. State the two conditions that need to be satisfied for a function $g(\theta|\theta^t)$ to majorize another function $f(\theta)$ at a point θ^t. Illustrate the relationship between f and g with a sketch.