16-124-836
Marcel ZAUDER

2.1 Circuit for comparing two numbers

1.1.1 Algorithm for which number is bigger

The modified algorithm, for evaluating if a number x is bigger than a number y, with:

$$[x]_2 = x_{n-1}x_{n-2}...x_1x_0$$
$$[y]_2 = y_{n-1}y_{n-2}...y_1y_0$$

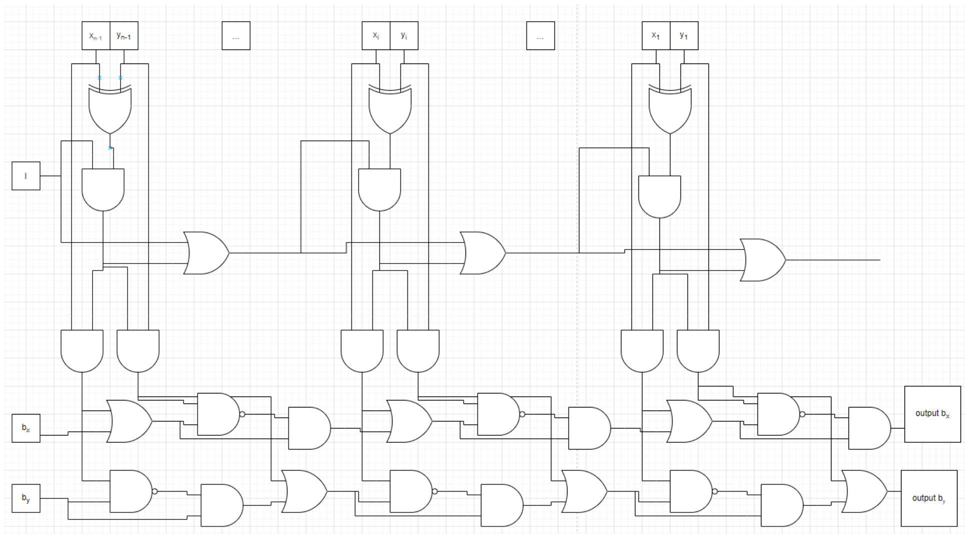
, can look like the following:

$$
\begin{array}{l}
i \leftarrow n \\
b_x \leftarrow 1 \\
b_y \leftarrow 1 \\
I \leftarrow 0 \\
\textit{while } i \ \geq \ 0 \textit{ do} \\
\quad i \leftarrow i - 1 \\
\quad \textit{if } (x_i \oplus y_i) \\
\quad\quad \textit{if } (x_i \wedge \neg I) \\
\quad\quad\quad b_x \leftarrow 1 \\
\quad\quad\quad b_y \leftarrow 0 \\
\quad\quad\quad I \leftarrow 1 \\
\quad\quad \textit{else if } (y_i \wedge \neg I) \\
\quad\quad\quad b_x \leftarrow 0 \\
\quad\quad\quad b_y \leftarrow 1 \\
\quad\quad\quad I \leftarrow 1 \\
\textit{return } (b_x, b_y)
\end{array}
$$

This algorithm searches for difference in the bitsequence of x and y. If there is a difference, the corresponding "indicate value" (b_x, b_y), will be set to 1 and the other to 0. Because we have a binary number the highest 1 bit is decisive to which value is bigger, therefore the "indication bit" I is needed so when b_x, b_y were overwritten it does not need to change them again.

1.1.2 Describe the corresponding circuit.

First we are checking if x_i and y_i differ. In that case the indication bit I is set to 1, therefore it blocks any new differences which will occur in a later, less significant bit pair. In the lower part of the diagram the signals only come through if the two bits were different and the I was 0. There the values would be overwritten according to which value was higher. In the end the values of b_x and b_y are returned; I is discarded.



1.2 Homomorphic Encryption

$$\textsc{Enc}(pk, m_1) \otimes \textsc{Enc}(pk, m_2) \;=\; \textsc{Enc}(pk, m_1 \oplus m_2)$$

1.2.1 ElGamal Encryption Scheme (Textbook)

The encoding process looks as follows:

$$Enc(pk, m) \;=\; (g^r, m \cdot Y^r)$$

For m_1 and m_2 we then get:

$$\begin{aligned}
Enc(pk, m_1) &= (g^{r_1}, m_1 \cdot Y^{r_1}) \\
Enc(pk, m_2) &= (g^{r_2}, m_2 \cdot Y^{r_2})
\end{aligned}$$

For the operations $\otimes$ and $\oplus$ we then get:

$$\begin{aligned}
Enc(pk, m_1) \;\otimes\; Enc(pk, m_2) &= (g^{r_1}, m_1 \cdot Y^{r_1}) \;\otimes\; (g^{r_2}, m_2 \cdot Y^{r_2}) \\
&= (g^{r_1} \;\otimes\; g^{r_2}, m_1 \cdot Y^{r_1} \;\otimes\; m_2 \cdot Y^{r_2}) \\
\textit{The } \otimes \textit{ can be replaced with a multiplication } (\cdot): & \\
&= (g^{r_1} \;\cdot\; g^{r_2}, m_1 \cdot Y^{r_1} \;\cdot\; m_2 \cdot Y^{r_2}) \\
&= (g^{r_1+r_2}, \underbrace{m_1 \;\cdot\; m_2}_{m_3} \cdot Y^{r_1+r_2}) \\
&= Enc(pk, m_3) \qquad\qquad\qquad with\ m_3 \;=\; m_1 \cdot m_2
\end{aligned}$$

1.2.2 RSA Encryption Scheme (Textbook)

The encoding process looks as follows:

$$Enc(pk, m) \ := \ m^{pk} \ \%N$$

For m_1 and m_2 we then get:

$$\begin{aligned} Enc(pk, m_1) \ &= \ m_1^{pk} \ \%N \\ Enc(pk, m_2) \ &= \ m_2^{pk} \ \%N \end{aligned}$$

For the operations $\otimes$ and $\oplus$ we then get:

$$\begin{aligned} Enc(pk, m_1) \ \otimes \ Enc(pk, m_2) \ &= \ m_1^{pk} \ \%N \ \otimes \ m_2^{pk} \ \%N \\ &= \ (m_1^{pk} \otimes m_2^{pk}) \ \%N \\ \textit{The} \ \otimes \ \textit{can be replaced with a multiplication} \ (\cdot): & \\ &= \ (m_1^{pk} \cdot m_2^{pk}) \ \%N \\ &= \ ((\underbrace{m_1 \cdot m_2}_{m_3})^{pk}) \ \%N \\ &= \ Enc(pk, m_3) \qquad\qquad \textit{with} \ m_3 \ = \ m_1 \cdot m_2 \end{aligned}$$