

Exercise 6

6.1 Soundness error (2pt)

The *soundness error* is the probability that a verifier V does not detect a false proof by a cheating prover P .

Determine the soundness errors of these zero-knowledge proofs we have discussed so far and discuss the influence of this parameter on the protocol:

- ZKP for Graph Isomorphism;
- ZKPK of knowledge of a discrete logarithm (Schnorr proof);
- ZKPK of knowledge of an RSA-inverse (Ex. 5.2).

6.2 Proof-of-knowledge protocol of a representation (REP) (3pt)

We have introduced a ZKPK for knowledge of a representation of a value y with respect to multiple bases g_1, \dots, g_n , abbreviated

$$\text{PK}\left\{(\alpha_1, \dots, \alpha_n) : y = g_1^{\alpha_1} \cdot \dots \cdot g_n^{\alpha_n}\right\}.$$

Prove soundness and zero-knowledge for the case $n = 2$.

Hint: The soundness property for this ZKPK means that given two accepting transcripts with the same commitment but different challenges, you can extract $\alpha_1, \dots, \alpha_n$ such that the above relation holds (but not necessarily that you can compute any discrete logarithm).

6.3 Encrypting a vote (5pt)

Consider the additively homomorphic ElGamal cryptosystem.

- For a given public key y , describe a protocol and a corresponding ZKPK that allows a party P to encrypt a value $i \in \mathbb{Z}_q$ and prove to V that it knows the encrypted value.
- Now P participates in an e-voting protocol and encrypts its vote under y . A vote must be $v = 0$ or $v = 1$. Develop a protocol for P to encrypt v and to prove the correctness of the encrypted vote to V .