Machine Learning Assignment # 3
Universität Bern

Due date: 10/10/2018

Late submissions will incur a penalty. Submit your answers in ILIAS (as a pdf or as a picture of your written notes if the handwriting is very clear). Submission instructions will be provided via email. You are not allowed to work with others.

Probability theory review **[Total 100 points]**

Solve each of the following problems and show all the steps of your working.

1. Show that $var[X] = E[X^2] - E[X]^2$. **[10 points]**

2. Show that the variance of a sum is $var[X + Y] = var[X] + var[Y] + 2cov[X, Y]$, where $cov[X, Y]$ is the covariance between X and Y. **[10 points]**

3. Show that the covariance matrix is always symmetric and positive semidefinite. **[10 points]**

4. Show that the uniform distribution $f(x)$ integrates to 1 **[10 points]**

$$f(x) = \begin{cases} \frac{1}{b-a}, & \text{for } a \leq x \leq b \\ 0, & \text{otherwise.} \end{cases}$$

5. Show that the exponential distribution $f(x)$ integrates to 1 **[15 points]**

$$f(x) = \begin{cases} \lambda e^{-\lambda x}, & \text{for } 0 \leq x < +\infty \\ 0, & \text{otherwise.} \end{cases}$$

6. Let $X_1, X_2, ..., X_n$ be i.i.d. Poisson random variables, with $P(X = x) = \frac{\lambda^x e^{-\lambda}}{x!}$. **[15 points]**
Find the λ that maximizes the likelihood of $X_1, ..., X_n$.

7. $X \in R^n$ and $Y \in R^m$ are independent random variables. Their expectations and covariances are $E[X] = 0$, Cov$[X] = I$, $E[Y] = \mu$ and Cov$[Y] = \sigma I$, where I is the identity matrix of the appropriate size and σ is scalar. What are the expectation and covariance of the random variable $Z = AX + Y$, where $A \in R^{m \times n}$? **[15 points]**

8. Suppose X, Y are two points sampled independently and uniformly on the interval [0, 1]. **[15 points]**
What is the expectation of the left most point between X and Y?
For the leftmost point between X and Y use the definition of the minimum between two variables:

$$\min(x, y) = \frac{x + y - |x - y|}{2}.$$