UNIVERSITÄT BERN

Applied Optimization
Exercise 4 - Duality

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Submission deadline: 30.10.2019, 23:59 h

Hand-in instructions:

Please hand-in **only one** compressed file named after the following convention: `Exercise`n`-GroupMemberNames.zip`, where n is the number of the current exercise sheet. This file should contain:

- **Only** the files you changed (headers and source). It is up to you to make sure that all files that you have changed are in the zip.
- A `readme.txt` file containing a description on how you solved each exercise (use the same numbers and titles) and the encountered problems.
- Other files that are required by your `readme.txt` file. For example, if you mention some screenshot images in `readme.txt`, these images need to be submitted too.
- Submit your solutions to ILIAS before the submission deadline.

Lagrange Duality (3 pts)

Consider the optimization problem

$$\begin{array}{ll} \text{minimize} & x^2+1 \\ \text{subject to} & (x-2)(x-4) \leq 0 \end{array}$$

with variable $x \in \mathbb{R}$.

(a) Analysis of primal problem. Give the feasible set, the optimal value, and the optimal solution.

(b) Lagrangian and dual function. Plot the objective x^2+1 versus x. On the same plot, show the feasible set, optimal point and value, and plot the Lagrangian $L(x,\lambda)$versus x for a few positive values of λ. Verify the lower bound property ($p^* \geq inf_x L(x,\lambda)$ for $\lambda \geq 0$) for. Derive and sketch the Lagrange dual function g.

(c) Lagrange dual problem. State the dual problem, and verify that it is a concave maximization problem. Find the dual optimal value and dual optimal solution λ. Does strong duality hold?

KKT Condition (2 pts)

Sketch the feasible set and objective function of the following optimization problem. Solve the problem graphically and see if the solution satisfies the KKT conditions.

$$\begin{aligned} \text{minimize} \quad & x_1^2 - 2x_2^2 \\ \text{subject to} \quad & (x_1+4)^2 - 2 \le x_2 \\ & x_1 - x_2 + 4 = 0 \\ & x_1 \ge -10 \end{aligned}$$

Programming (5 pts)

For the following optimization problem in the standard form:

$$\begin{aligned} \text{minimize} \quad & f_0(x) \\ \text{subject to} \quad & f_i(x) \le 0, i = 1, ..., m \\ & h_i(x) = 0, i = 1, ..., p \end{aligned}$$

with variable $x \in \mathbb{R}^n$. Using the KKT conditions, implement an optimality checker to see if a query point x is a local optimum w.r.t. a user specified tolerance ϵ. Write your code in the function `is_local_optimum (...)` in the `OptimalityChecker.cc` file. The input to the function is the problem, query point and the corresponding λ and ν.

As a special case, you could test the optimality checker with the quadratically constrained quadratic program:

$$\begin{aligned} \text{minimize} \quad & 1/2x^T A_0 x + b_0^T x + c_0 \\ \text{subject to} \quad & 1/2x^T A_i x + b_i^T x + c_i \le 0, i = 1, ..., m \\ & Cx = d \end{aligned}$$

with variable $x \in \mathbb{R}^n$, $C, A_i \in \mathbb{R}^{n \times n}$, $b_i \in \mathbb{R}^n$ and $c_i, d \in \mathbb{R}$. The class `FunctionQuadraticND` can be reused in this case. To setup the optimization problem, please go to the main function, where you can setup the n dimension quadratic function by manually giving the A and b. Store the equality constraints and the inequality constraints, as well as λ and ν, in the corresponding vectors that are provided. About more details, please refer to the code. To verify the correctness of the program, simply try the `KKT Condition` example.