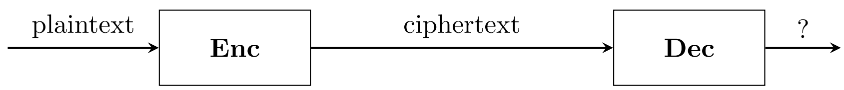
# Cryptography

# 7 Security against chosen-plaintext attacks




increasing adversary power $\downarrow$ 

known-plaintext
chosen-plaintext $\Big\}$ attack
chosen ciphertext

One-time security

- not very useful
- chooses a fresh key per encryption cell
  $\Rightarrow$ relax this!

## Definition

A encryption scheme $\Sigma$ is secure against chosen-plaintext attacks if $L^{\Sigma}_{cpa-L} \approx L^{\Sigma}_{cpa-R}$, where:

| $L^{\Sigma}_{cpa-L}$ | $L^{\Sigma}_{cpa-R}$ |
|---|---|
| $k \leftarrow \{0,1\}^{\lambda}$ | $k \leftarrow \{0,1\}^{\lambda}$ |
| $\textsc{Eavesdrop}(m_L, m_R)$ | $\textsc{Eavesdrop}(m_L, m_R)$ |
| if $\mid m_L \mid \neq \mid m_R \mid$ | if $\mid m_L \mid \neq \mid m_R \mid$ |
| then return $\textsc{Error}$ | then return $\textsc{Error}$ |
| $c := \Sigma.Enc(k, m_L)$ | $c := \Sigma.Enc(k, m_R)$ |
| return c | return c |

NOTE 1: Lengths must be equal. That allows $\Sigma$ to be used for plaintext of different lengths:
$\Sigma.M = \{0,1\}^{\star}, \quad \mid m \mid = \textit{length in bits}$

- Traffic analysis reveals information about plaintext sizes
- Steganography hides the existence of a hidden message

NOTE 2: Often called IND-CPA security (indistinguishable CPA)

NOTE 3: Almost same notion for public key crypto.

## Lemma

CPA-secure encryption schemes cannot be deterministic.

## Proof

*Suppose it is:*
Then:

$$\begin{aligned} &c_x := EAVESDROP(x, x); \\ &c_y := EAVESDROP(x, y); \\ &\textbf{return } c_x \stackrel{?}{=} c_y \end{aligned}$$

distinguishes between $L^{\Sigma}_{cpa-L}$ and $L^{\Sigma}_{cpa-R}$.
***Need probabilistic encryption!***

### How to make encryption non-deterministic?

1. Stateful encryption
   - Keep state (counter) inside Σ.Enc()
   - Complex to implement:
     requires synchronisation between Enc() and Dec()
2. Randomization in encryption algorithm
   - Σ.Enc uses randomness $r$
   - $r$ becomes part of ciphertext:
     ⇒ increases length
   - most popular
3. Nonce-based encryption
   - add a nonce to Σ.Enc()
   - nonce: number used once
   - caller must ensure that Enc() is never called with the same nonce twice

## Pseudorandom ciphertext

1. Second notion for CPA-secure symmetric encryption
2. Often more useful than CPA

## Definition

An encryption scheme $\Sigma$ has pseudorandom ciphertexts against chosen-plaintext attacks if $L^{\Sigma}_{cpa\$-real} \approx L^{\Sigma}_{cpa\$-rand}$

$$\boxed{\begin{array}{l} \underline{L^{\Sigma}_{cpa\$-real}} \\ k \leftarrow \{0,1\}^{\lambda} \\ \\ \underline{\text{CTXT}(m)} \\ \quad c := \Sigma.Enc(k,m) \\ \quad \text{return c} \end{array}} \qquad \boxed{\begin{array}{l} \underline{L^{\Sigma}_{cpa\$-rand}} \\ \\ \underline{\text{CTXT}(m)} \\ \quad r \leftarrow \Sigma.C\,|_{length(r)=length(\Sigma.Enc(k,m))} \\ \quad \text{return r} \end{array}}$$

## Lemma

CPA\$-security $\Rightarrow$ CPA-security ($\not\Leftarrow$)

## Proof

Exercise

## CPA-secure encryption from a PRF

<u>Idea:</u> Use $F(k,r)$ with a different $r$ for each encryption call.
<u>How to make $r$ distinct?</u>

- statful encryption: $r$: counter/state, complex
- Randomized: $r \leftarrow \{0,1\}^{\lambda}$
- Delegate it: use nonce, $r \;:=\; nonce$

## Construction:

CPA-secure $\Sigma$ from PRF $F$

$$F : \{0,1\}^{\lambda} \times \{0,1\}^{\lambda} \rightarrow \{0,1\}^{len}$$

$$\begin{array}{lcl} \Sigma.M &=& \{0,1\}^{\lambda} \\ \Sigma.C &=& \{0,1\}^{\lambda} \times \{0,1\}^{len} \\ \Sigma.K &=& \{0,1\}^{\lambda} \end{array} \qquad \begin{array}{l} \underline{\text{KeyGen():}} \\ k \leftarrow \{0,1\}^{\lambda} \\ \textbf{return } k \end{array}$$

$$\begin{array}{l} \underline{\Sigma.\text{Enc(k,m):}} \\ r \leftarrow \{0,1\}^{\lambda} \\ x := F(k,r) \oplus m \\ \textbf{return } \text{F(r,x)} \end{array} \qquad \begin{array}{l} \underline{\Sigma.\text{Dec(r,x):}} \\ \textbf{return } F(k,r) \oplus x \end{array}$$

## Lemma

If F use a secure PRF then construction has CPA\$-security.

## Proof idea

$$\begin{array}{|l|}\hline \underline{L^{\Sigma}_{cpa\$-real}} \\ k \leftarrow \{0,1\}^{\lambda} \\ \\ \underline{\textsc{CTxt}(m)} \\ \quad r \leftarrow \{0,1\}^{\lambda} \\ \quad x := F(k,r) \oplus m \\ \quad \textbf{return } (r,x) \\ \hline \end{array} \qquad \begin{array}{|l|}\hline \underline{L^{\Sigma}_{cpa\$-rand}} \\ r \leftarrow \{0,1\}^{\lambda} \\ x \leftarrow \{0,1\}^{len} \\ \textbf{return } (r,x) \\ \hline \end{array}$$

# 8 Block ciphers in practice

- Blockcipher as a PRP
- How to encrypt long messages?
- **Mode of operation**
  Needed to implement CPA-secure encryption of long messages.
- **Padding scheme**