Shading and Photometric Stereo

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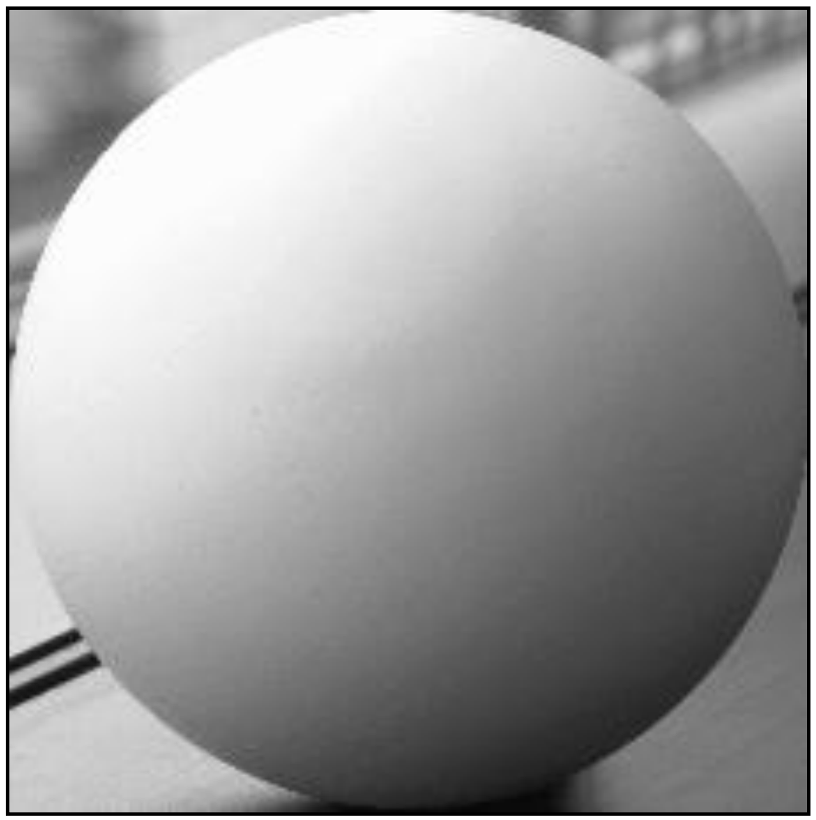
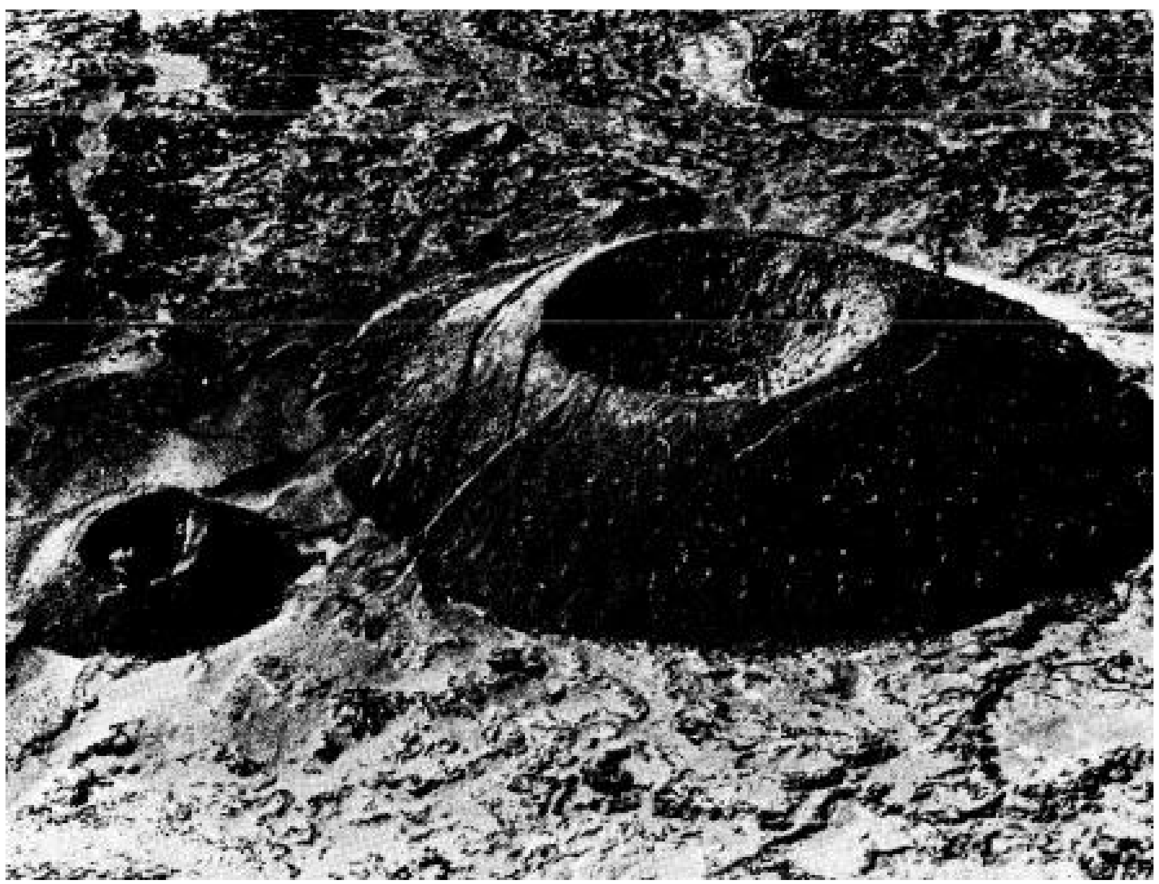
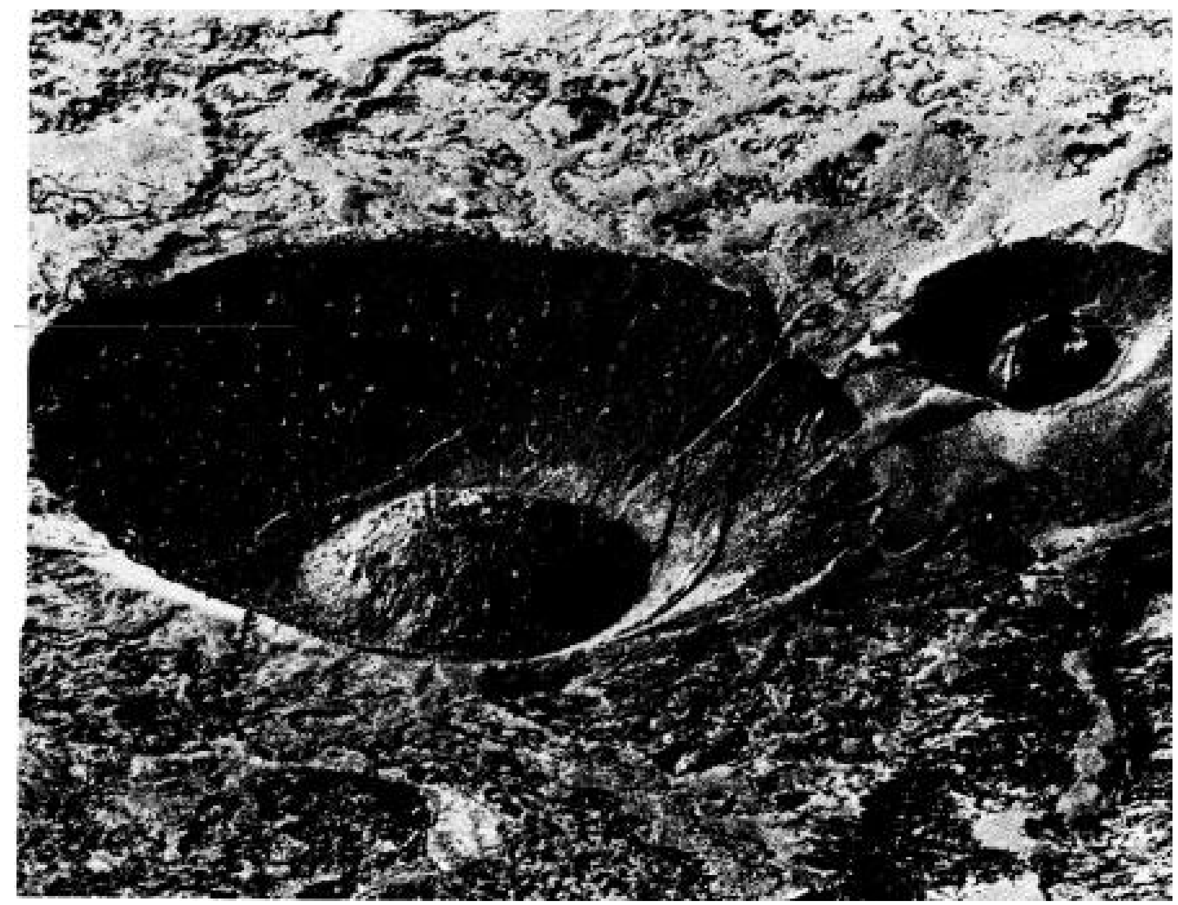
Image intensity and 3D geometry



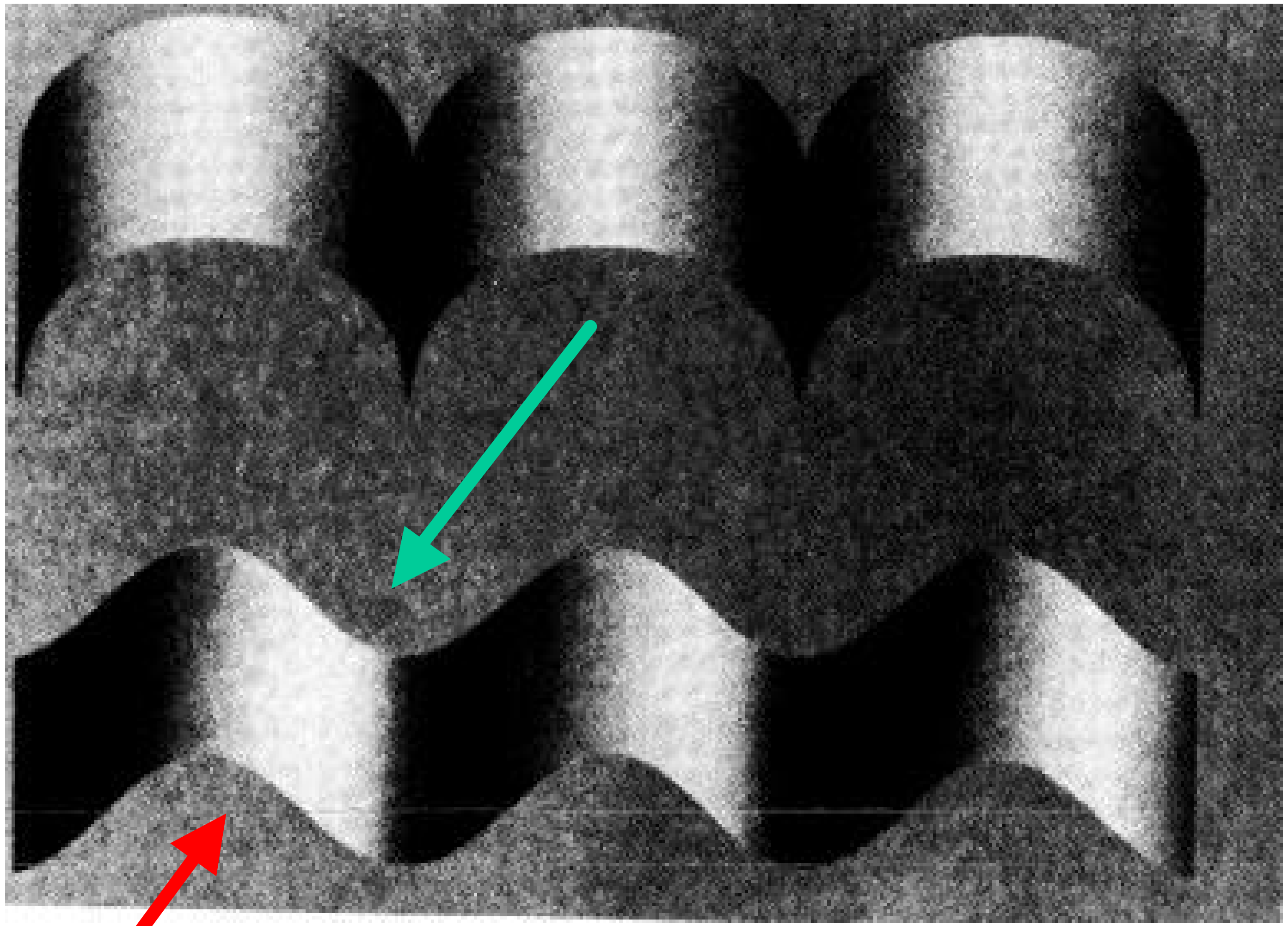
Shading as a cue for 3D shape reconstruction

Human perception



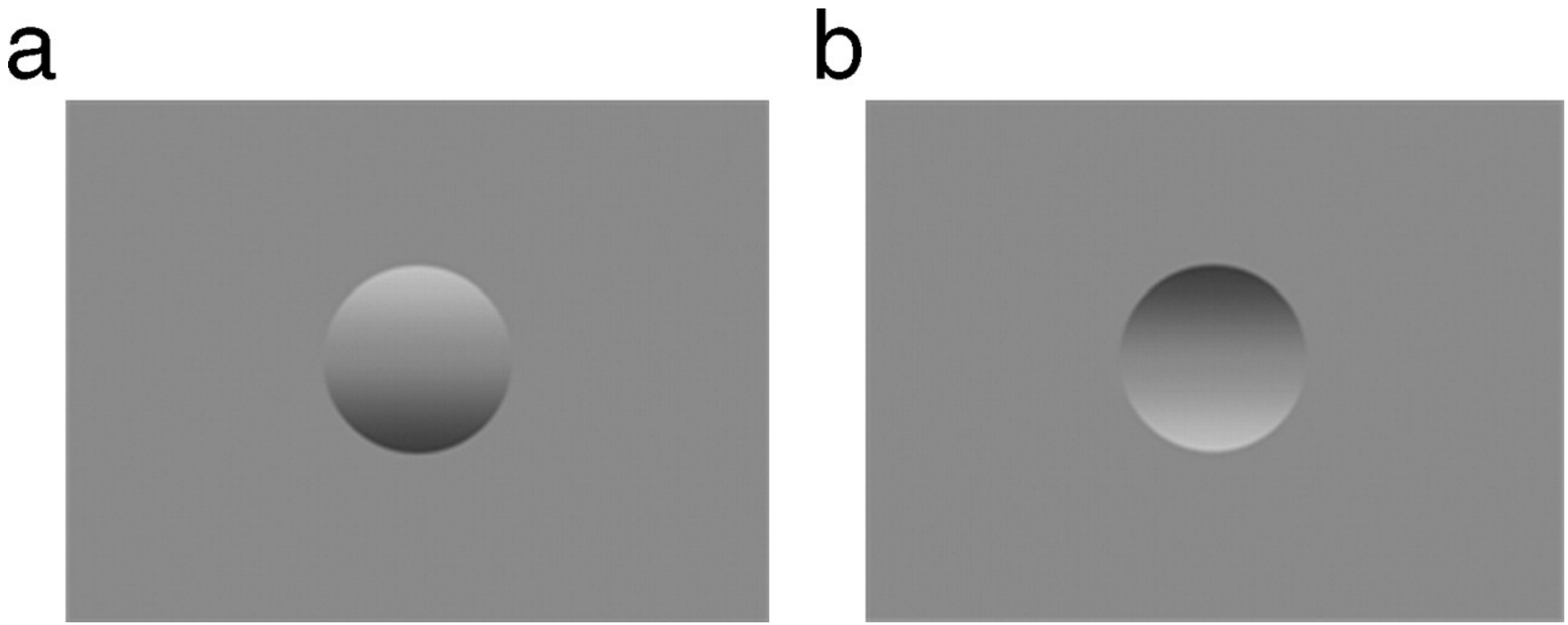


Human perception



2 possible illumination hypotheses

Shading perception



Thomas R et al. J Vis 2010;10:6

Human perception

- Our brain often perceives shape from shading
- Mostly, it makes many assumptions to do so (see V. Ramachandran http://psy.ucsd.edu/chip/ramabio.html)
- For example:
    - Light is coming from above (sun)
    - Biased by occluding contours

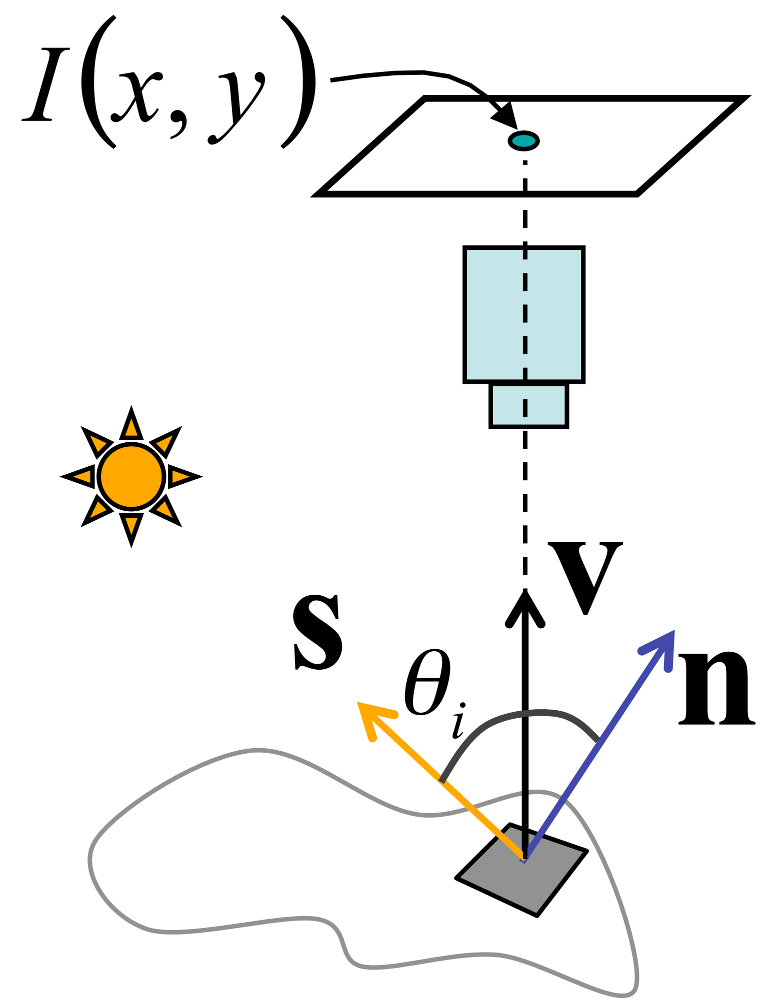
Modeling light interaction

- We assume surfaces are Lambertian (color does not change with viewing direction)

- L light source intensity
  ρ albedo
  $\mathbf{n}$ normal to the surface
  $\mathbf{s}$ light source direction
  $\mathbf{v}$ viewing direction

- Image intensity

$$I(x,y) = \rho(x,y) < \mathbf{n}(x,y), \mathbf{s} > L$$



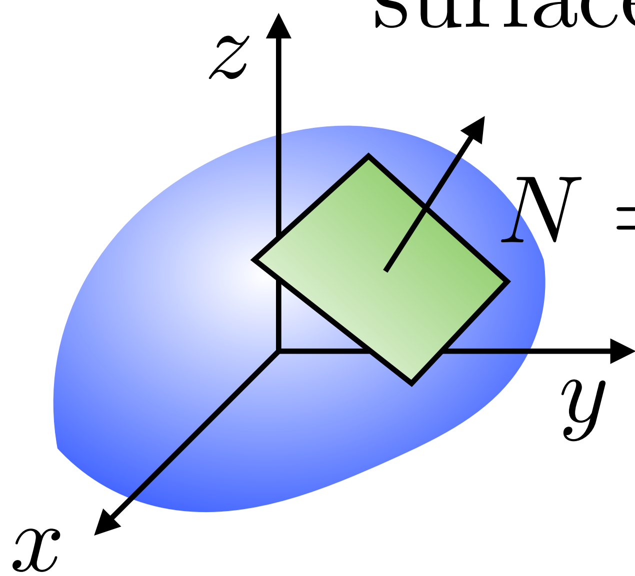
Modeling light interaction

- L light source intensity **fixed to 1**
  ρ albedo **fixed to 1 (constant color)**
  n normal to the surface
  s light source direction
  v viewing direction **(irrelevant — Lambertianity)**

- Image intensity

$$I(x,y) =< \mathbf{n}(x,y), \mathbf{s} >$$

Surface normal



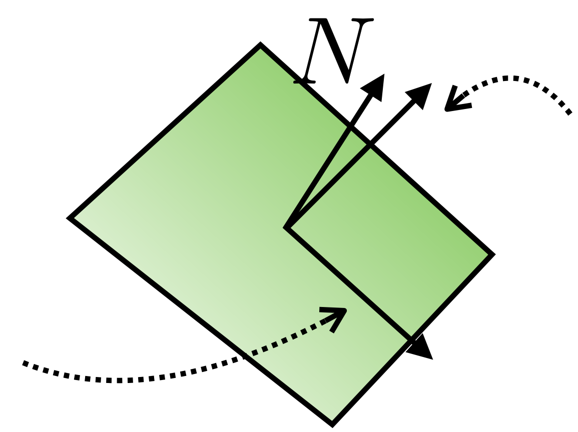
surface normal

$$N = \begin{bmatrix} p \\ q \\ 1 \end{bmatrix}$$

depth map $\begin{bmatrix} x \\ y \\ z(x,y) \end{bmatrix}$

tangent plane

$$\begin{bmatrix} 0 \\ 1 \\ \frac{\partial z(x,y)}{\partial y} \end{bmatrix}$$

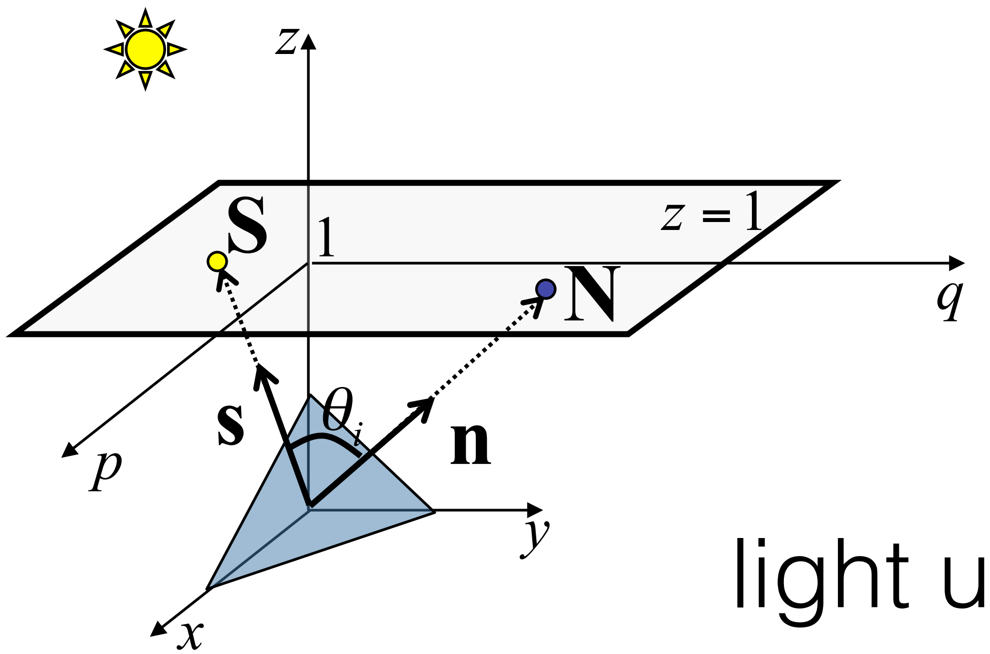


$$\begin{bmatrix} 1 \\ 0 \\ \frac{\partial z(x,y)}{\partial x} \end{bmatrix}$$

orthogonality $N^T \begin{bmatrix} 1 \\ 0 \\ \frac{\partial z(x,y)}{\partial x} \end{bmatrix} = 0 \qquad N^T \begin{bmatrix} 0 \\ 1 \\ \frac{\partial z(x,y)}{\partial y} \end{bmatrix} = 0$

$$p + \frac{\partial z(x,y)}{\partial x} = 0 \qquad q + \frac{\partial z(x,y)}{\partial y} = 0$$

Gradient space



$$n = \frac{N}{|N|} = \frac{[p \ q \ 1]^T}{\sqrt{p^2 + q^2 + 1}}$$

light unit vector $$s = \frac{S}{|S|} = \frac{[p_s \ q_s \ 1]^T}{\sqrt{p_s^2 + q_s^2 + 1}}$$

image brightness $$\cos\theta_i = n \cdot s = \frac{pp_s + qq_s + 1}{\sqrt{p^2 + q^2 + 1}\sqrt{p_s^2 + q_s^2 + 1}}$$

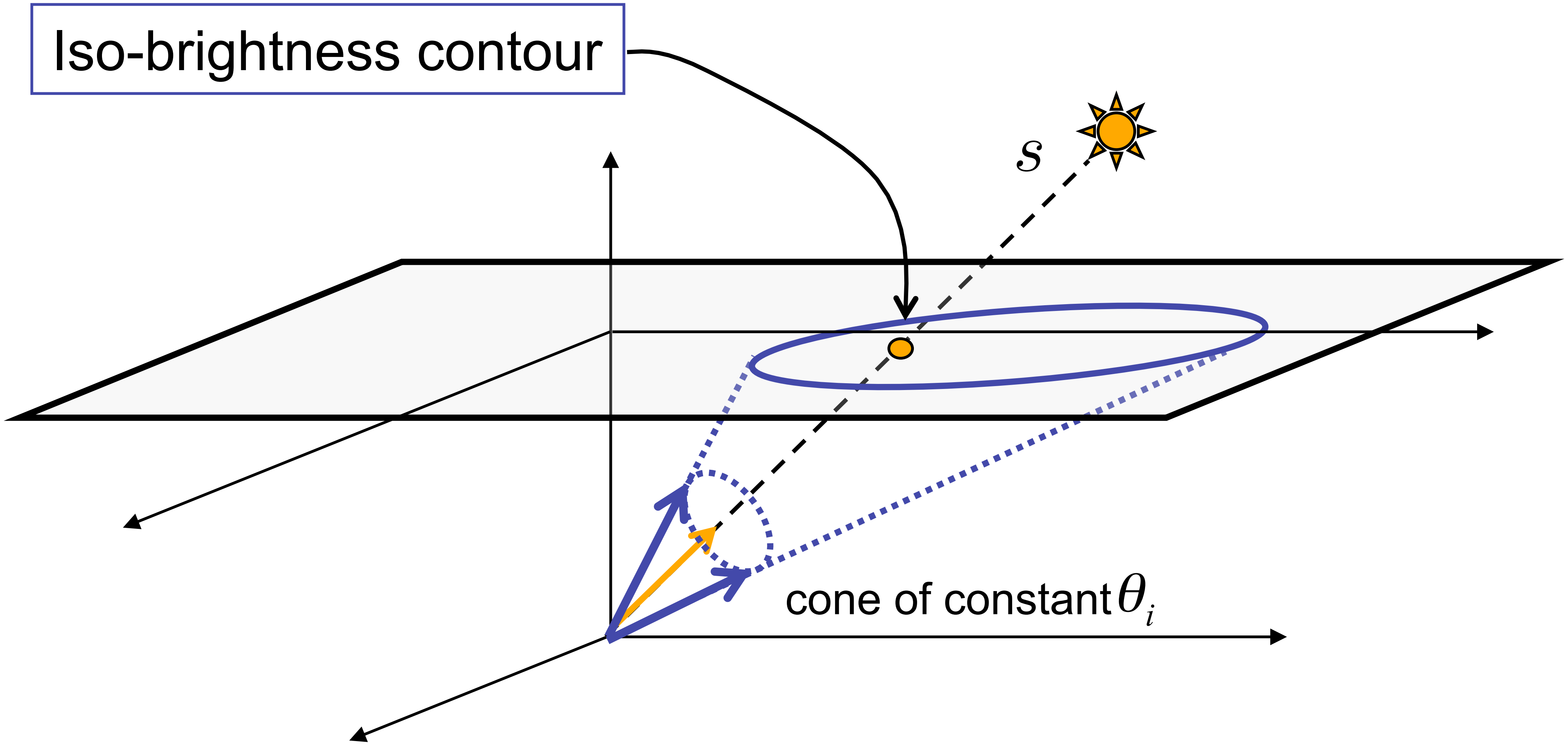
$z = 1$ is called the **gradient space** (p-q plane)

every point on it corresponds to a surface orientation

Reflectance map

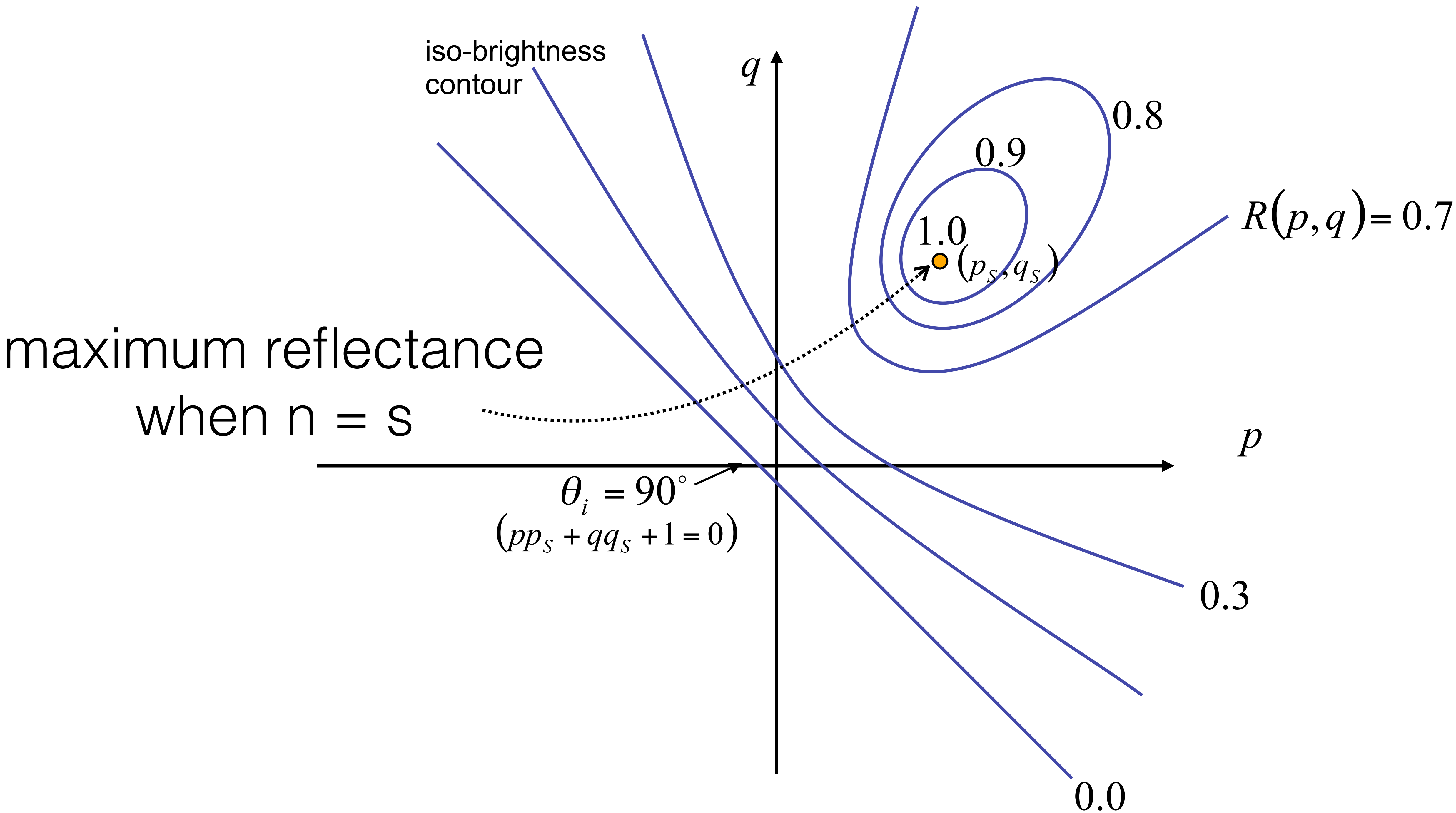
Lambertian assumption

$$R(p,q) = \cos\theta_i = n \cdot s = \frac{pp_s + qq_s + 1}{\sqrt{p^2+q^2+1}\sqrt{p_s^2+q_s^2+1}}$$



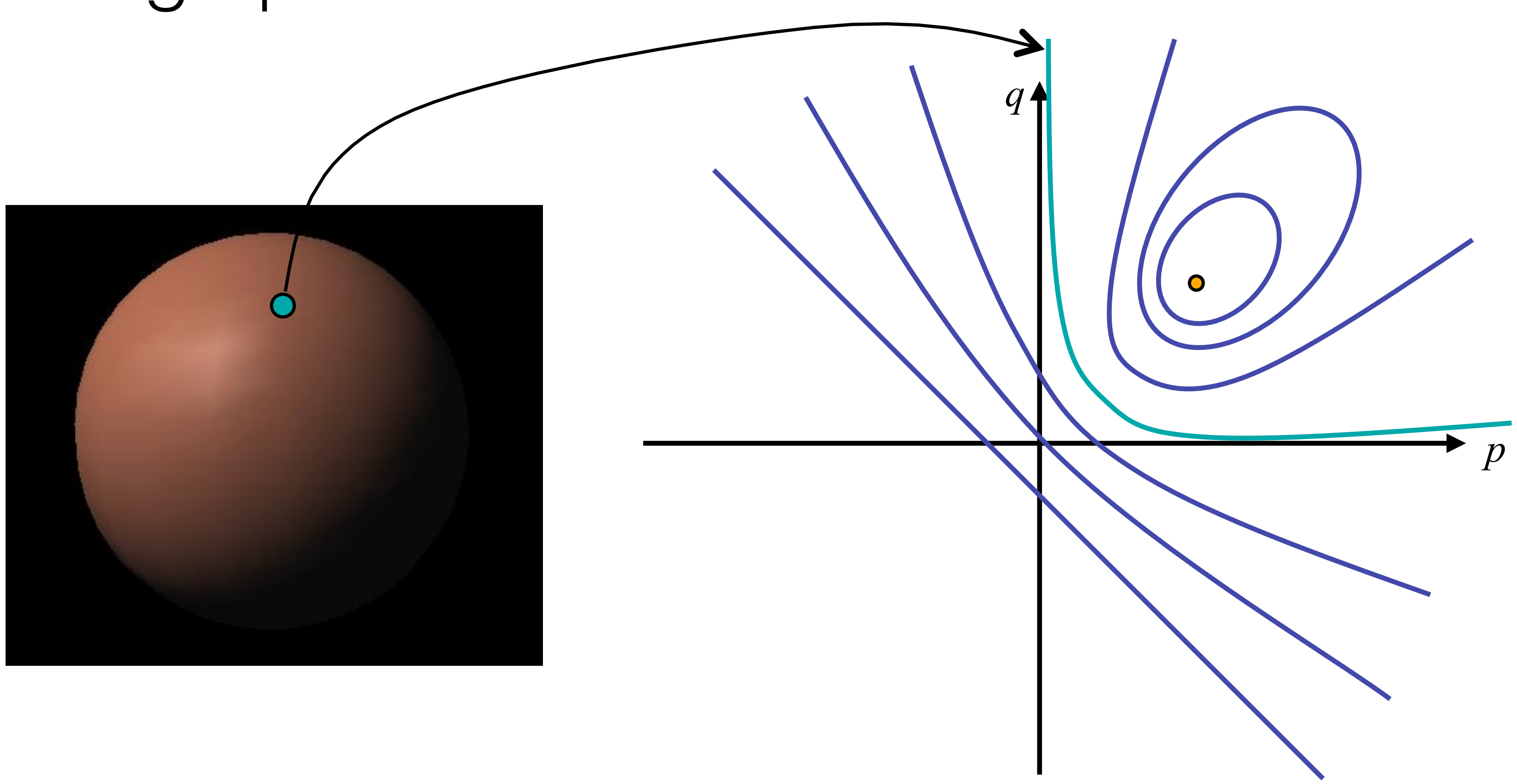
Reflectance map

top view of the gradient space



Shape from single image?

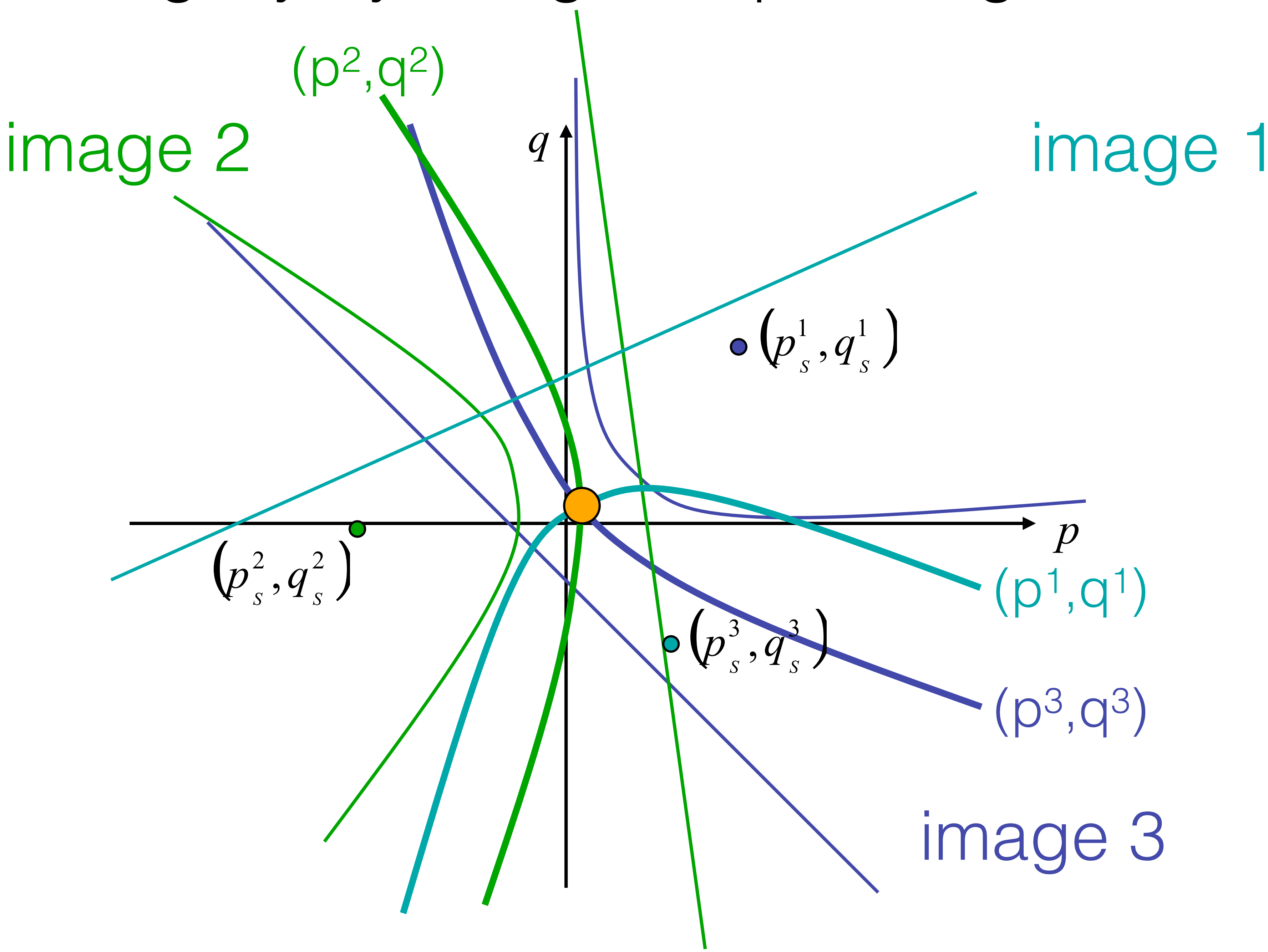
- Given a single image of an object with known surface reflectance taken under a known light source, can we recover the shape of the object?
- Given R(p,q) can we determine (p,q) uniquely for each image point?



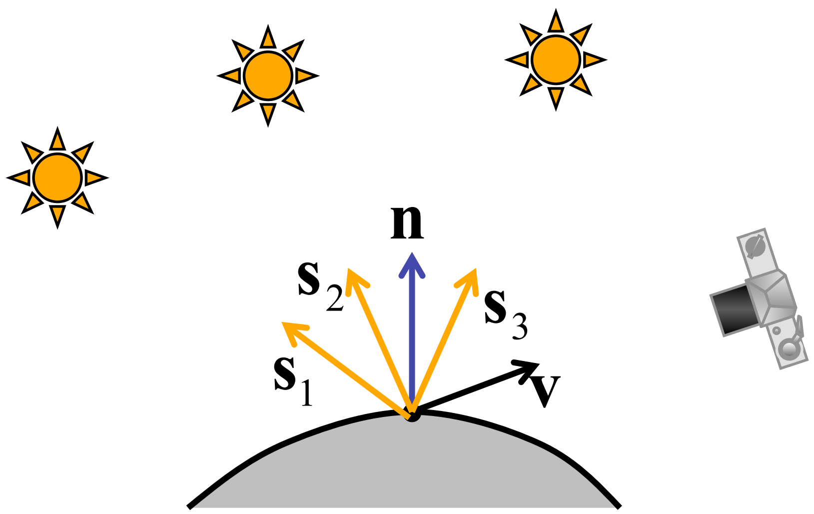
no
need additional constraints

Photometric stereo

- Fix ambiguity by using multiple images



Photometric stereo



light intensity

$$I_1 = \rho n \cdot s_1 L_1$$

$$I_2 = \rho n \cdot s_2 L_2$$

$$I_3 = \rho n \cdot s_3 L_3$$

in matrix form per pixel

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} L_1 s_1^T \\ L_2 s_2^T \\ L_3 s_3^T \end{bmatrix} n\rho$$

3x1 3x3 3x1

Estimating normals

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} L_1 s_1^T \\ L_2 s_2^T \\ L_3 s_3^T \end{bmatrix} n\rho \quad \Rightarrow \quad I = S\hat{N}$$

non-unit normal $\hat{N} = S^{-1} I$

surface color $\rho = |\hat{N}|$

unit normal $n = \dfrac{\hat{N}}{\rho} = \dfrac{\hat{N}}{|\hat{N}|}$

More than 3 images

With m lights we have m equations

$$\begin{bmatrix} I_1 \\ \dots \\ I_m \end{bmatrix} = \begin{bmatrix} L_1 s_1^T \\ \dots \\ L_m s_m^T \end{bmatrix} n\rho$$

Least squares solution for $\underset{\text{mx1}}{I} = \underset{\text{mx3}}{S} \underset{\text{3x1}}{\hat{N}}$

Left-multiply $S^T I = S^T S \hat{N}$

Invert $\hat{N} = \left(S^T S\right)^{-1} S^T I$

$$\rho = |\hat{N}| \qquad n = \frac{\hat{N}}{\rho} = \frac{\hat{N}}{|\hat{N}|}$$

Moore-Penrose pseudo inverse

$$\left(S^T S\right)^{-1} S^T$$

Color images

- With color images we have 3 sets of equations, one per color channel

$$I_R = Sn\rho_R$$
$$I_G = Sn\rho_G$$
$$I_B = Sn\rho_B$$

- A simple solution: use only one set as before and then use the estimated normal *n* to compute the color channels $\rho_R \quad \rho_G \quad \rho_B$

Handling shadows

- Because shadows are dark, their intensities are approximately 0
- Equations corresponding to shadows should be discarded
- Equations of the type 0=0 do not have an effect
- Left-multiply each equation by the same image intensity. If the pixel is dark (shadows) the equation becomes 0=0

Handling shadows

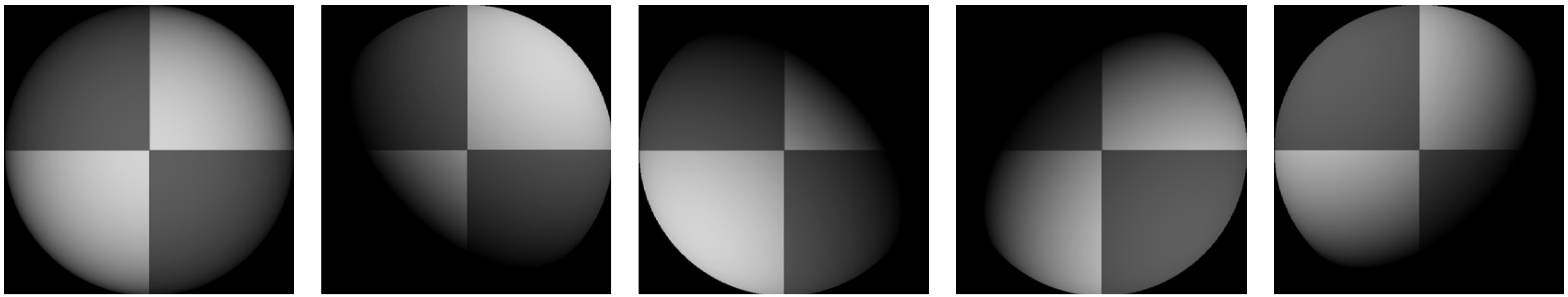
- Left-multiply each equation by the same image intensity. If the pixel is dark (shadows) the equation becomes 0=0

$$I[I] = I[S\hat{N}]$$

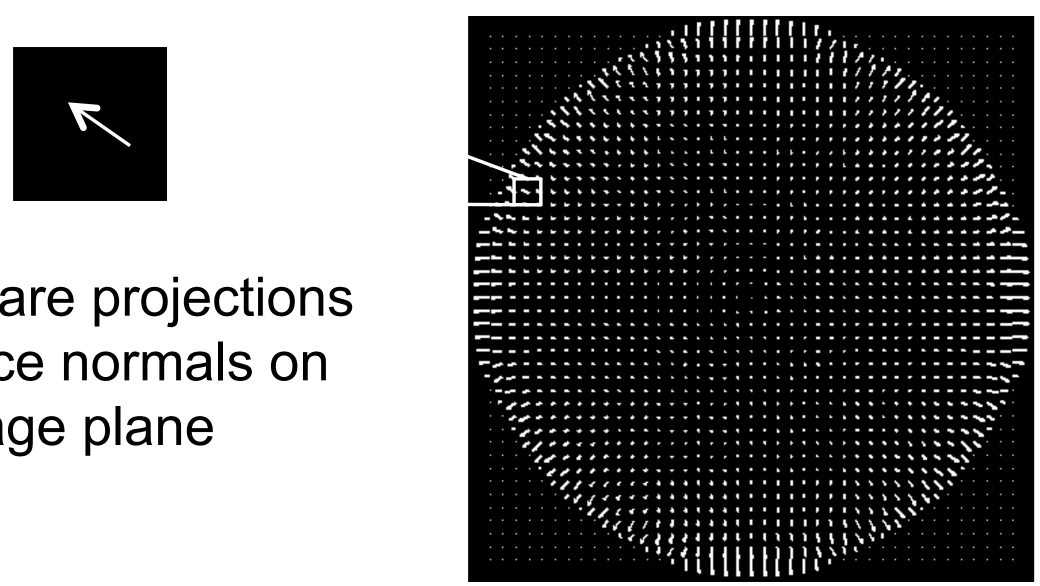
$$\begin{bmatrix} I_1^2 \\ \dots \\ I_m^2 \end{bmatrix} = \begin{bmatrix} I_1 L_1 s_1^T \\ \dots \\ I_m L_m s_m^T \end{bmatrix} \hat{N}$$

$$\rho = |\hat{N}| \qquad n = \frac{\hat{N}}{\rho} = \frac{\hat{N}}{|\hat{N}|}$$

Experiments

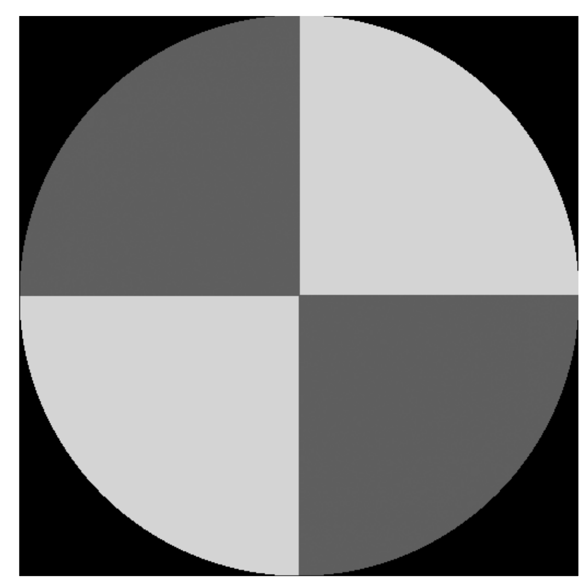


Input Images



Needles are projections of surface normals on image plane

Estimated Surface Normals

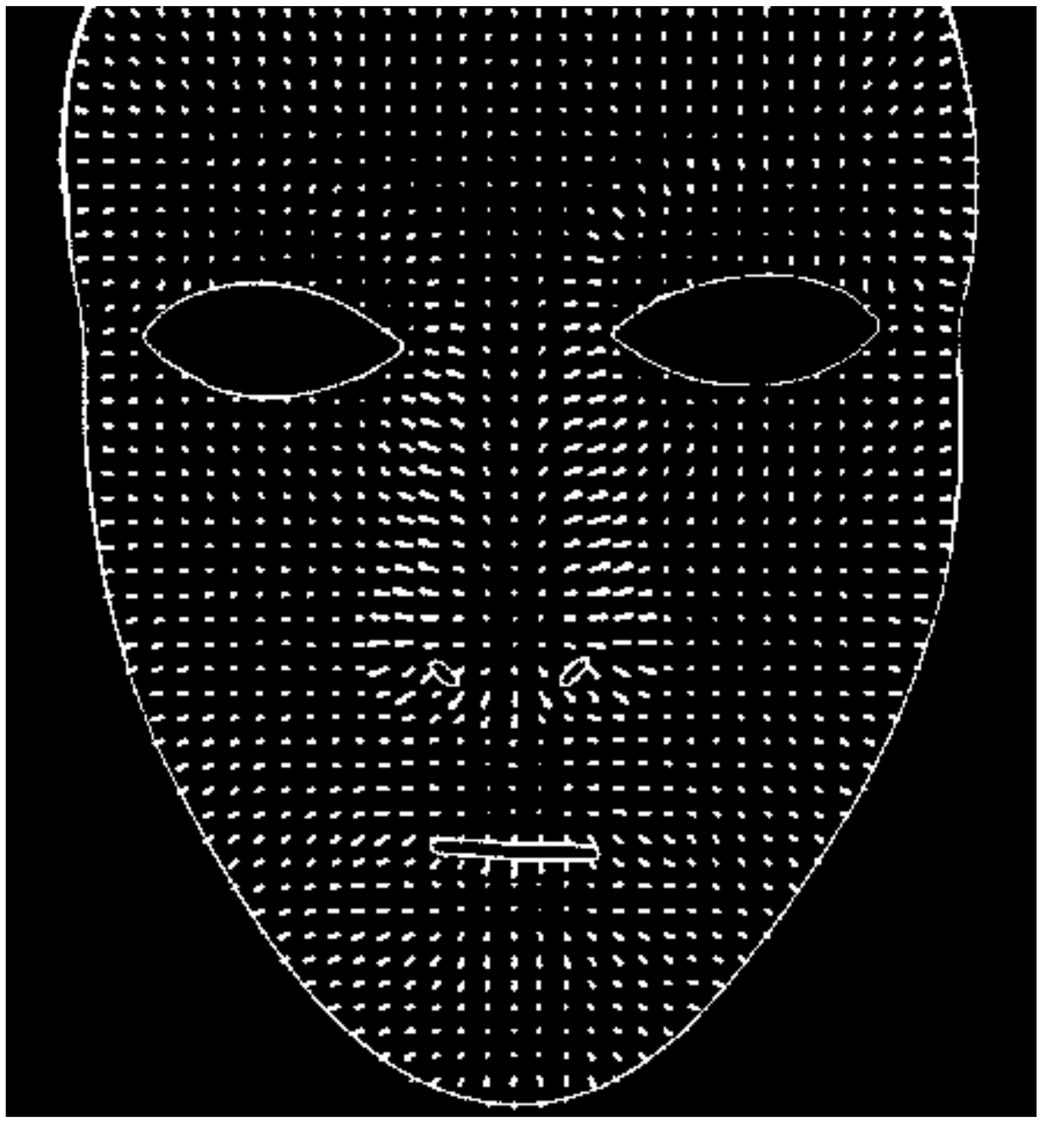


Estimated Albedo

Experiments

Estimated albedo and normals

albedo



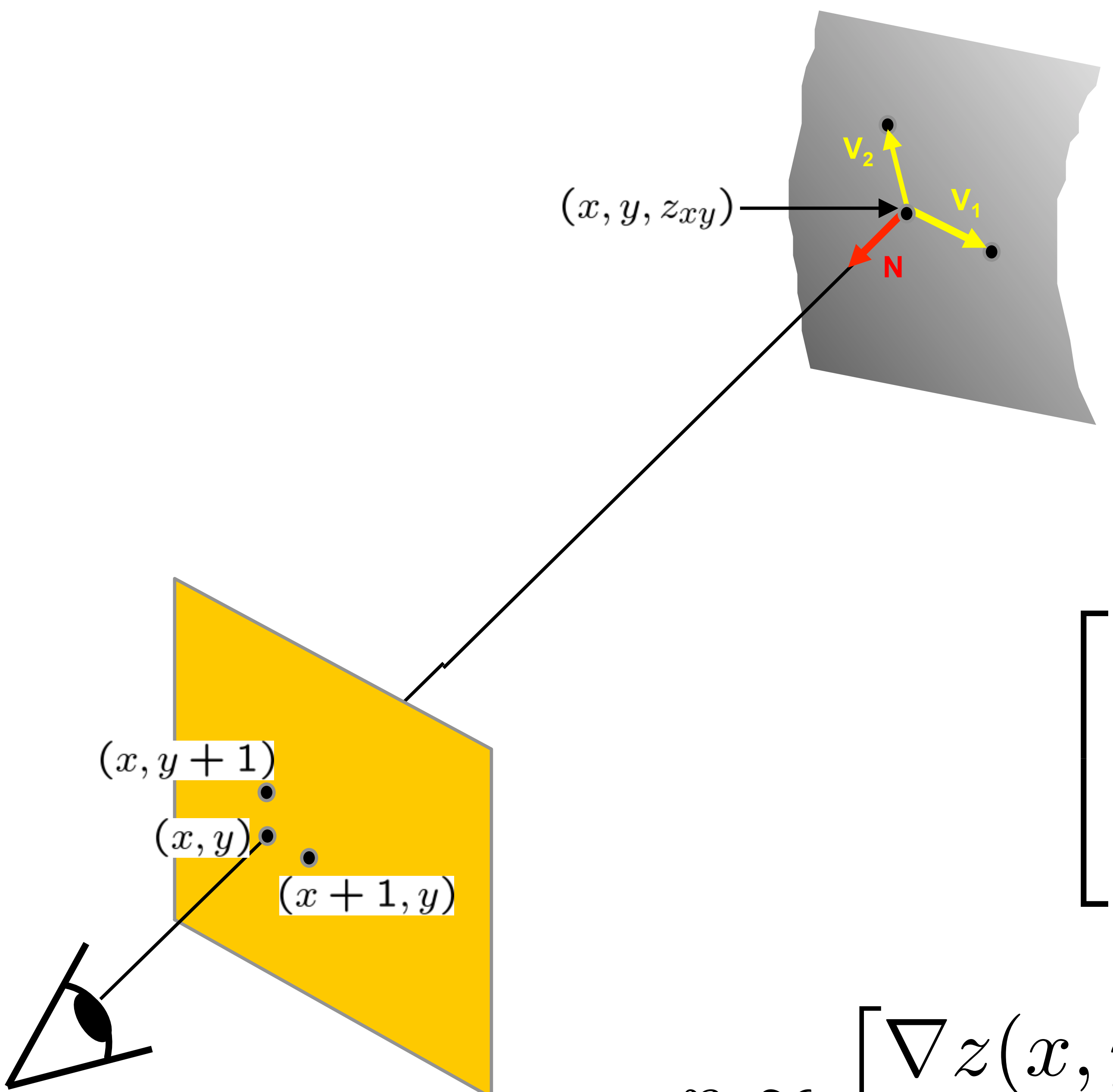
normals

Experiments (real data)

albedo 

 normals

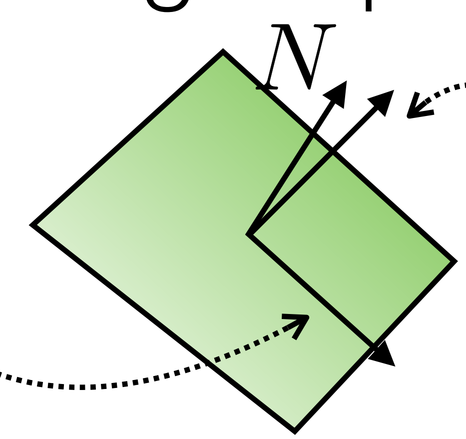
Depth from normals



normal at a pixel $n = \begin{bmatrix} n_1 \\ n_2 \\ n_3 \end{bmatrix}$

recall: normal orthogonal to tangent plane

$$\begin{bmatrix} 0 \\ 1 \\ \frac{\partial z(x,y)}{\partial y} \end{bmatrix}$$

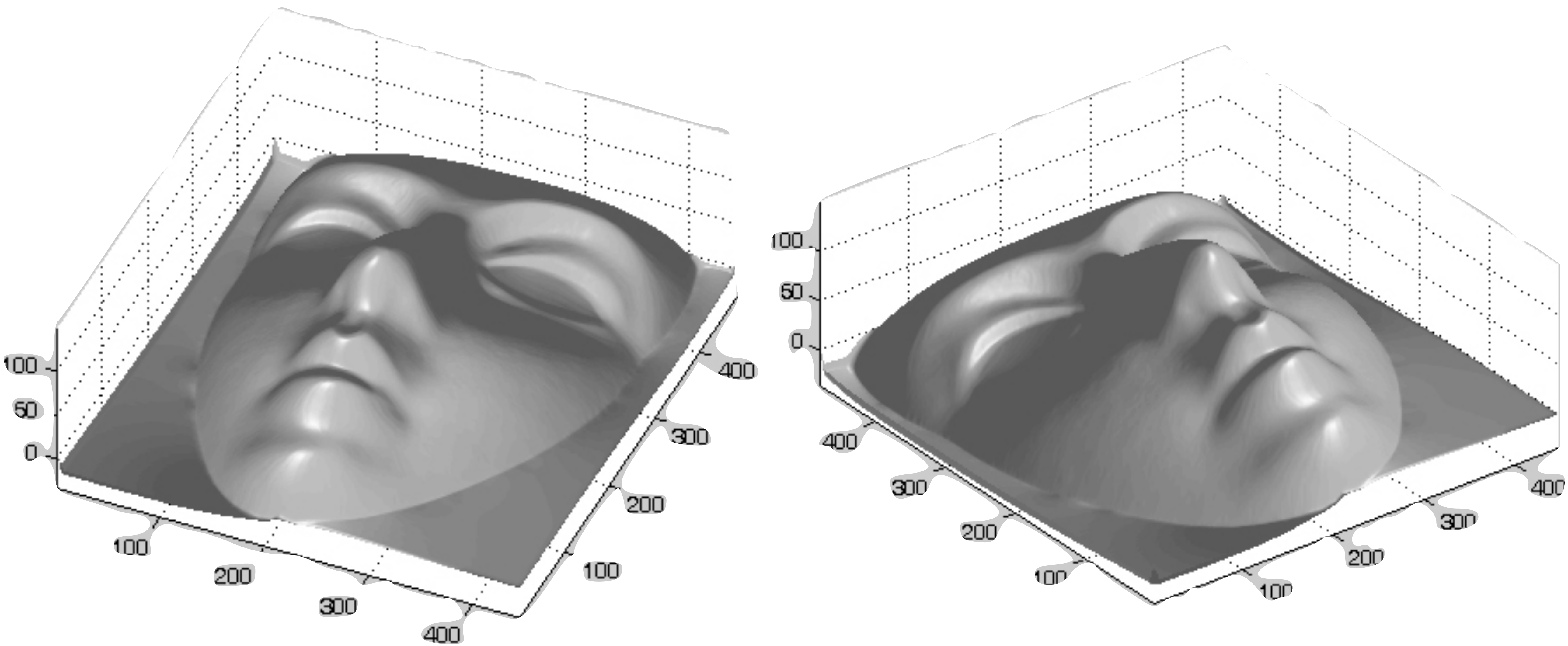


$$\begin{bmatrix} 1 \\ 0 \\ \frac{\partial z(x,y)}{\partial x} \end{bmatrix}$$

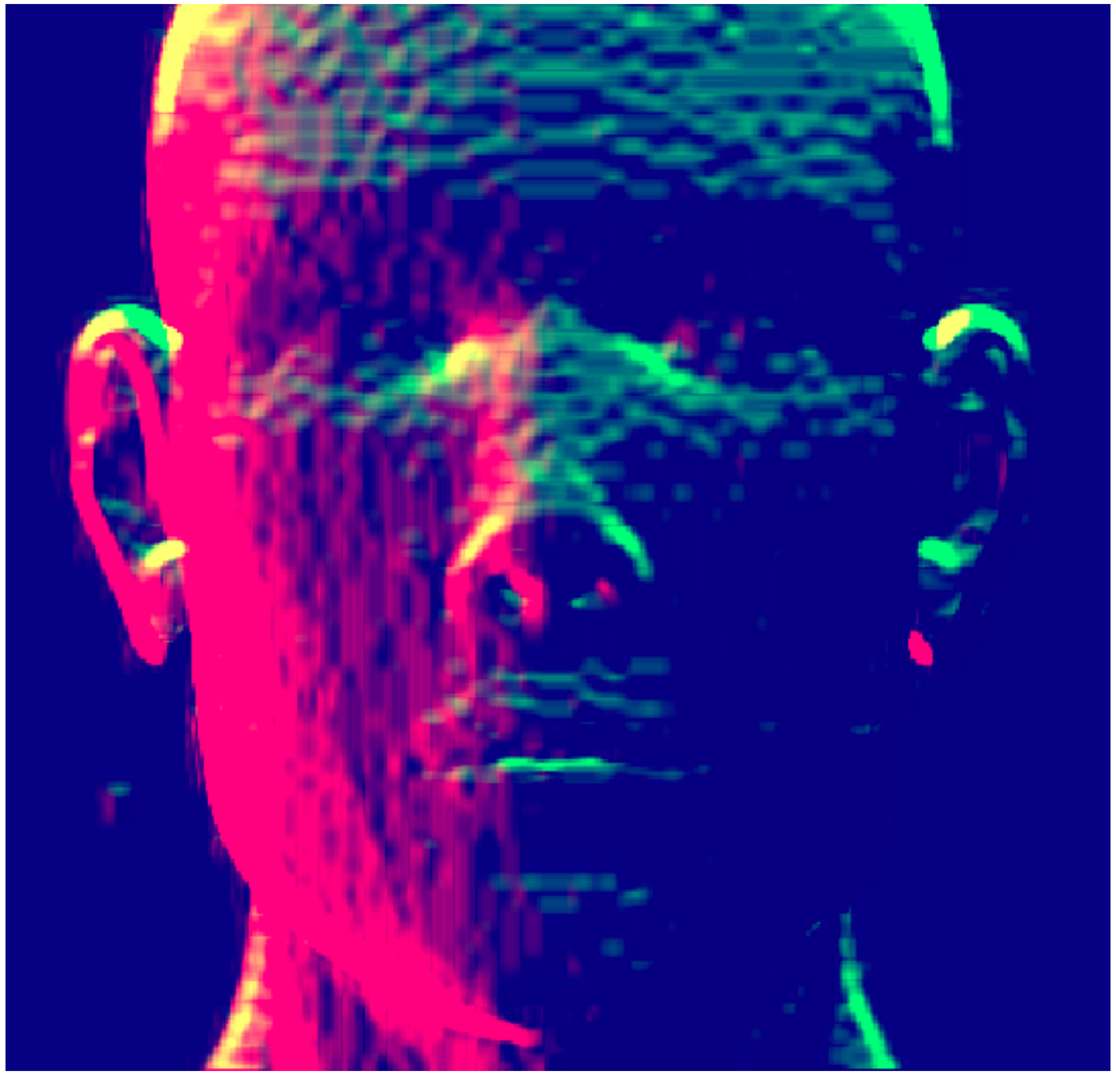
$$n \propto \begin{bmatrix} \nabla z(x,y) \\ -1 \end{bmatrix} \Rightarrow \nabla z(x,y) = -\begin{bmatrix} \frac{n_1}{n_3} \\ \frac{n_2}{n_3} \end{bmatrix}$$

see Tutorial 3 Problem 4.1

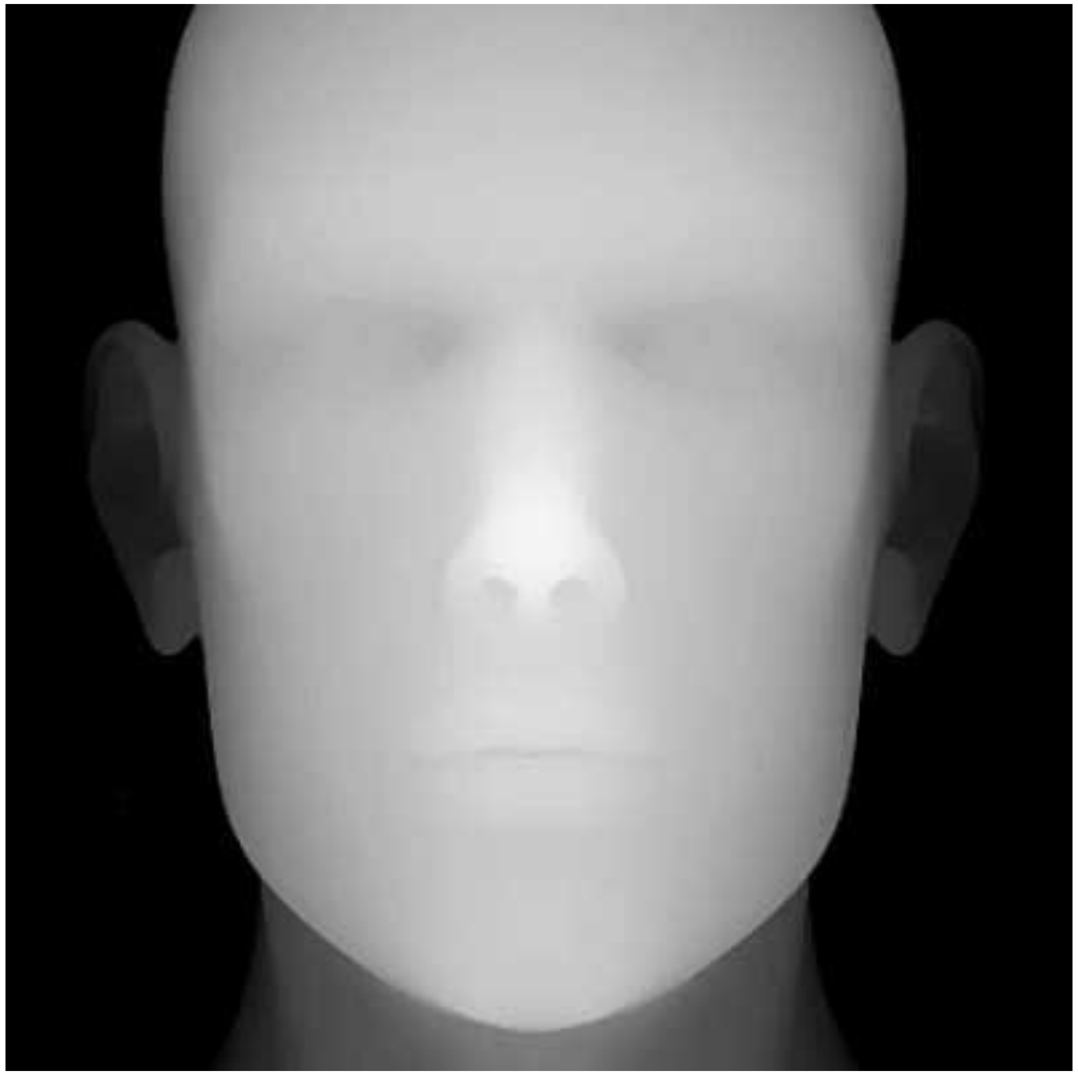
Reconstructed depth map



Reconstructed depth map

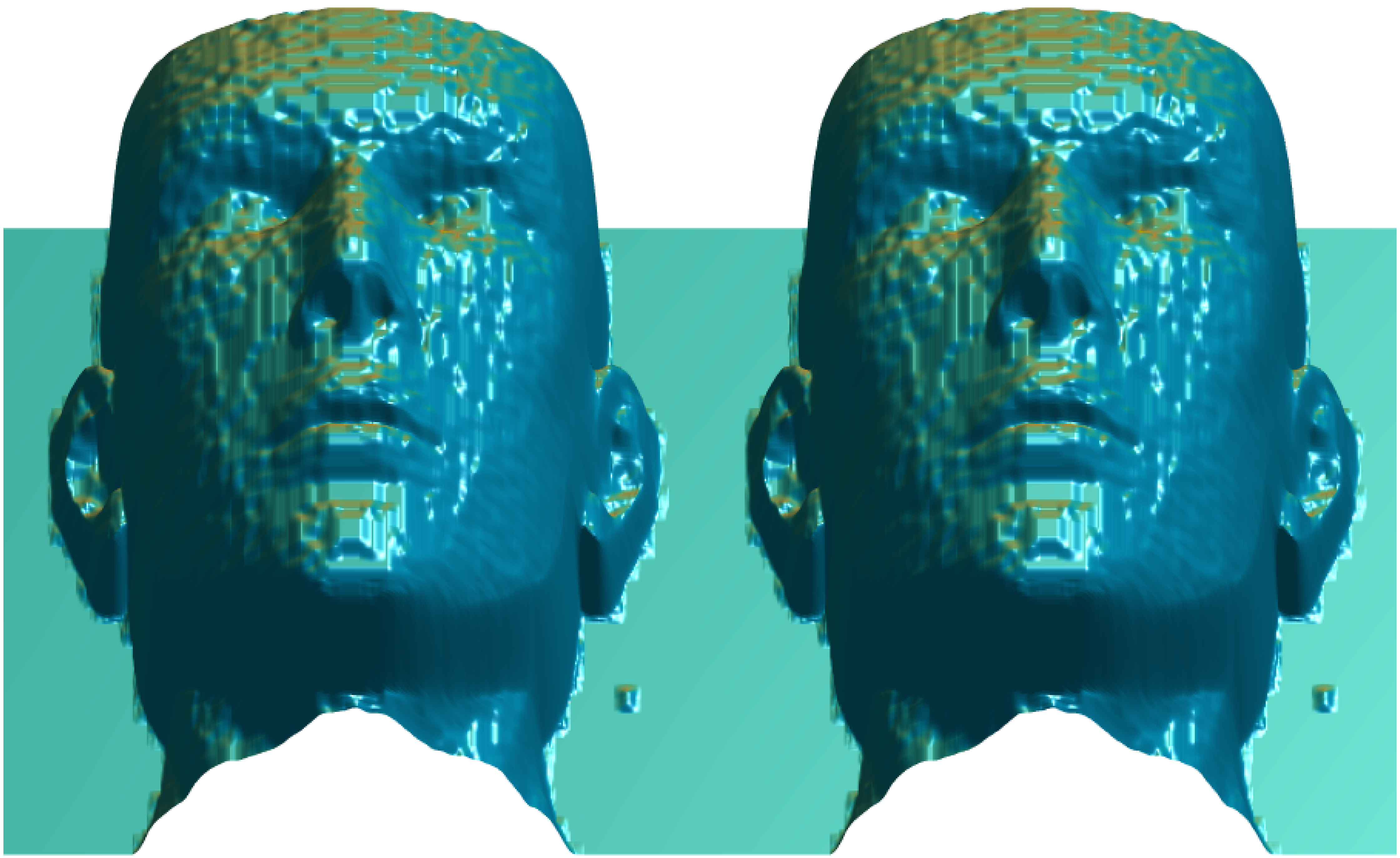


normal map



depth map

Normal map integration



reconstructed original

Reconstruction results
(real data)

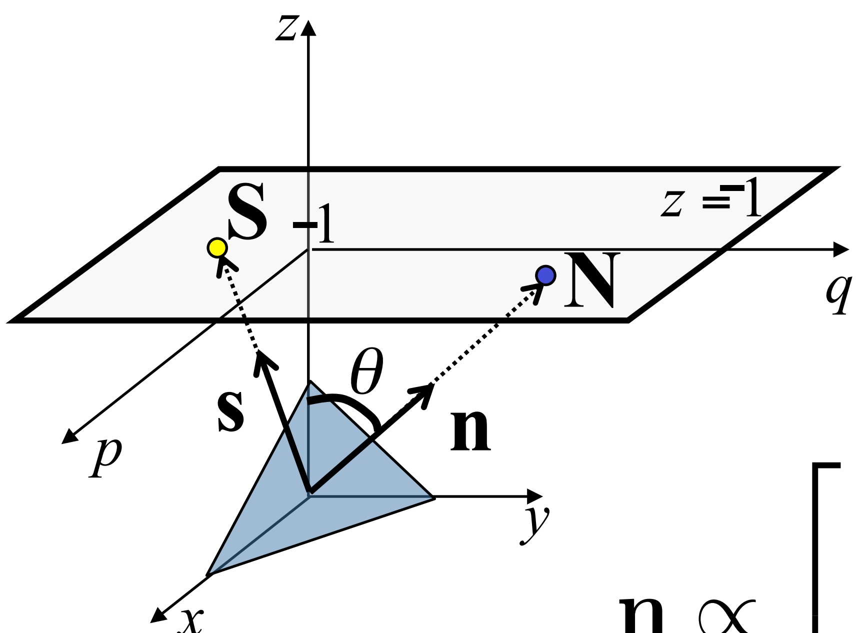
1 input image | estimated normals | estimated albedo | estimated depth map | simulated image

Shape from shading

- Use only 1 image
- Fix ambiguity by introducing additional regularization

Normal representation

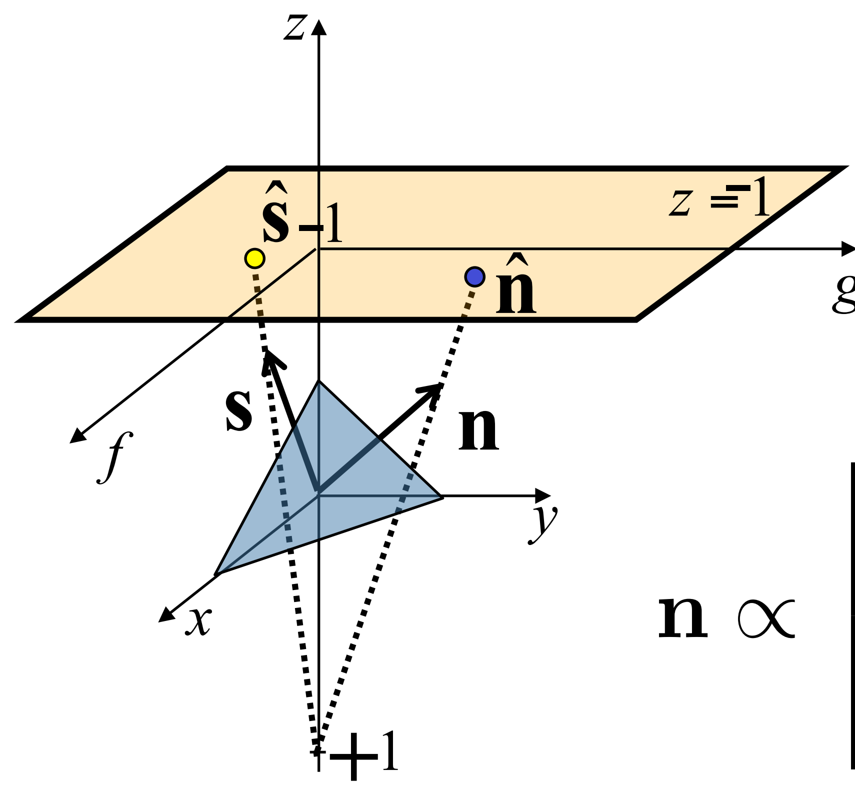
(p,q) space



$$\mathbf{n} \propto \begin{bmatrix} p \\ q \\ -1 \end{bmatrix}$$

(p,q)=∞ when $\theta = \dfrac{\pi}{2}$

(f,g) space



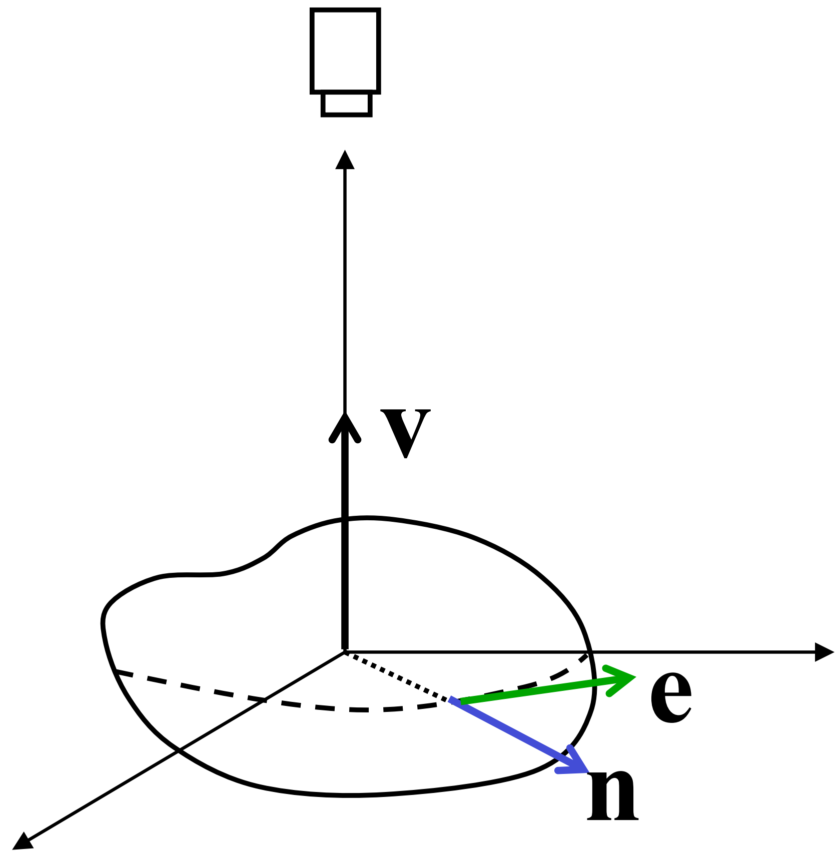
$$\mathbf{n} \propto \begin{bmatrix} 4f \\ 4g \\ f^2 + g^2 - 4 \end{bmatrix}$$

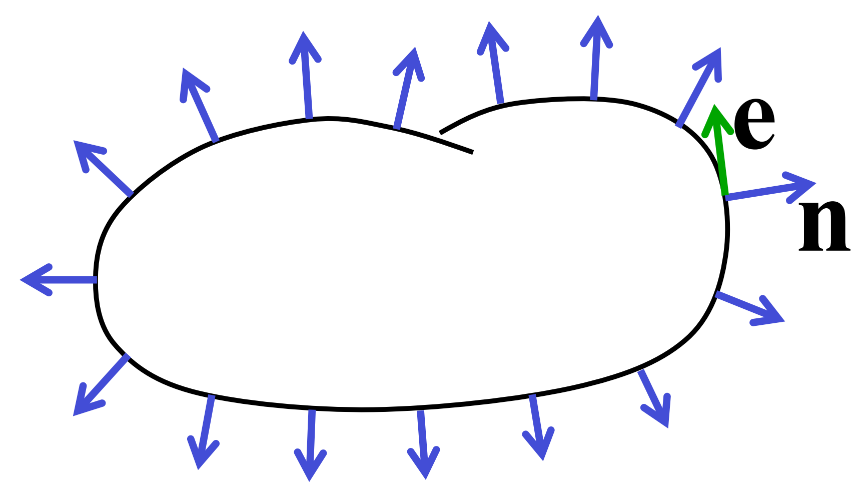
$$f = \frac{2p}{1 + \sqrt{1 + p^2 + q^2}} \qquad g = \frac{2q}{1 + \sqrt{1 + p^2 + q^2}}$$

$$p = \frac{4f}{4 - f^2 - g^2} \qquad q = \frac{4g}{4 - f^2 - g^2}$$

Further constraints

$$\mathbf{n} \perp \mathbf{v}$$
$$\mathbf{n} \perp \mathbf{e}$$
$$\mathbf{n} = \mathbf{e} \times \mathbf{v}$$





- The **n** values on the occluding boundary can be used as boundary conditions

Energy formulation

- *I(x,y)* are the measurements
- *(f,g)* are the unknowns (the normal map representation)

$$E[f,g] = \iint \left(I(x,y) - \frac{4s_1 f(x,y) + 4s_2 g(x,y) + s_3(f^2(x,y) + g^2(x,y) - 4)}{4 + f(x,y)^2 + g(x,y)^2} \right)^2 dxdy$$
$$+ \lambda \iint |\nabla f(x,y)|^2 + |\nabla g(x,y)|^2 dxdy$$

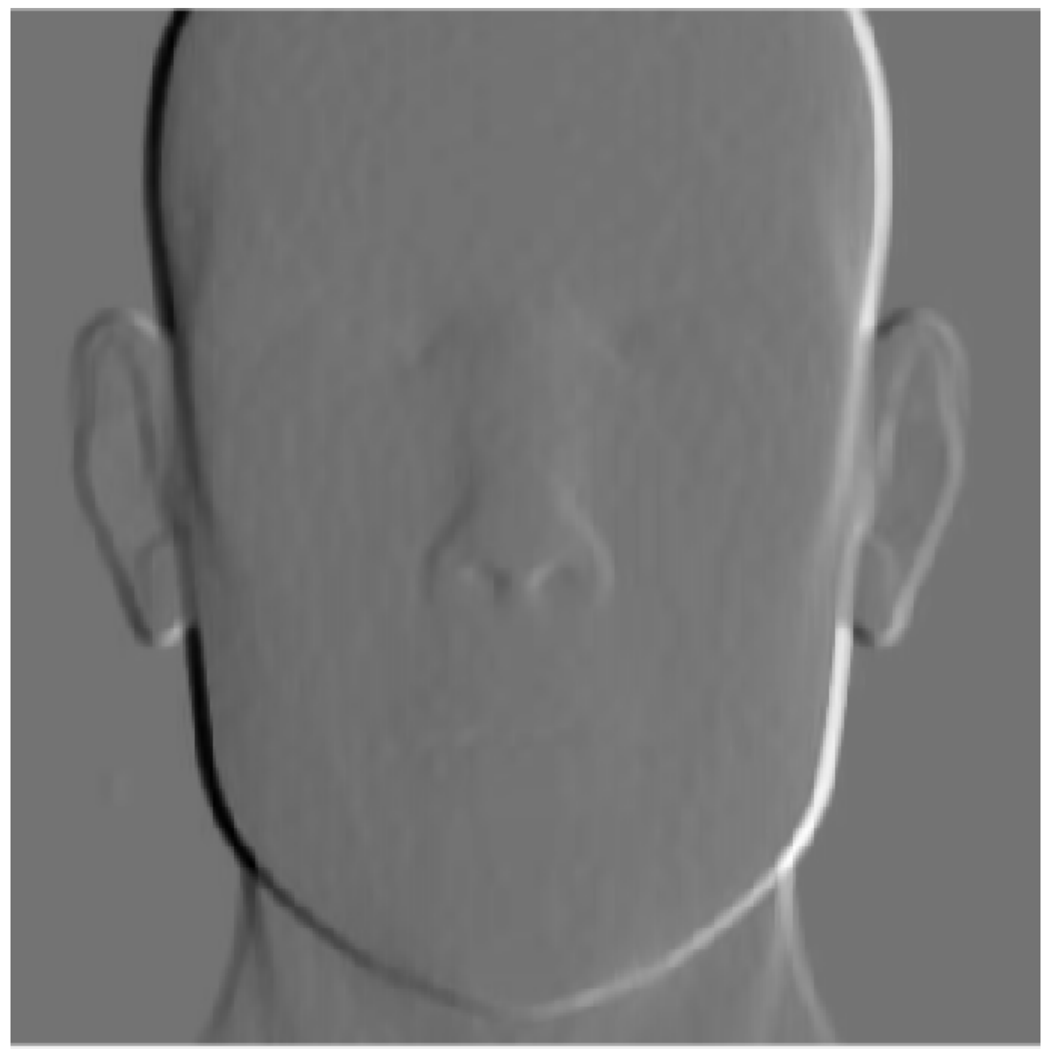
- The first term is the **data fidelity** (model-data matching) and the second term the **regularization** (smoothness)

Minimization

- Discretize integrals and gradients
- Compute EL equations
- Ikeuchi and Horn 89: Use a time-explicit approximation only on the derivatives of the regularization term

$$
\begin{aligned}
-\Delta f_{i,j} &\simeq 2(f_{i,j}^{t+1} - \bar{f}_{i,j}^{t}) \\
\bar{f}_{i,j}^{t} &\doteq \frac{1}{4}(f_{i-1,j}^{t} + f_{i+1,j}^{t} + f_{i,j-1}^{t} + f_{i,j+1}^{t})
\end{aligned}
$$

Results (synthetic data)



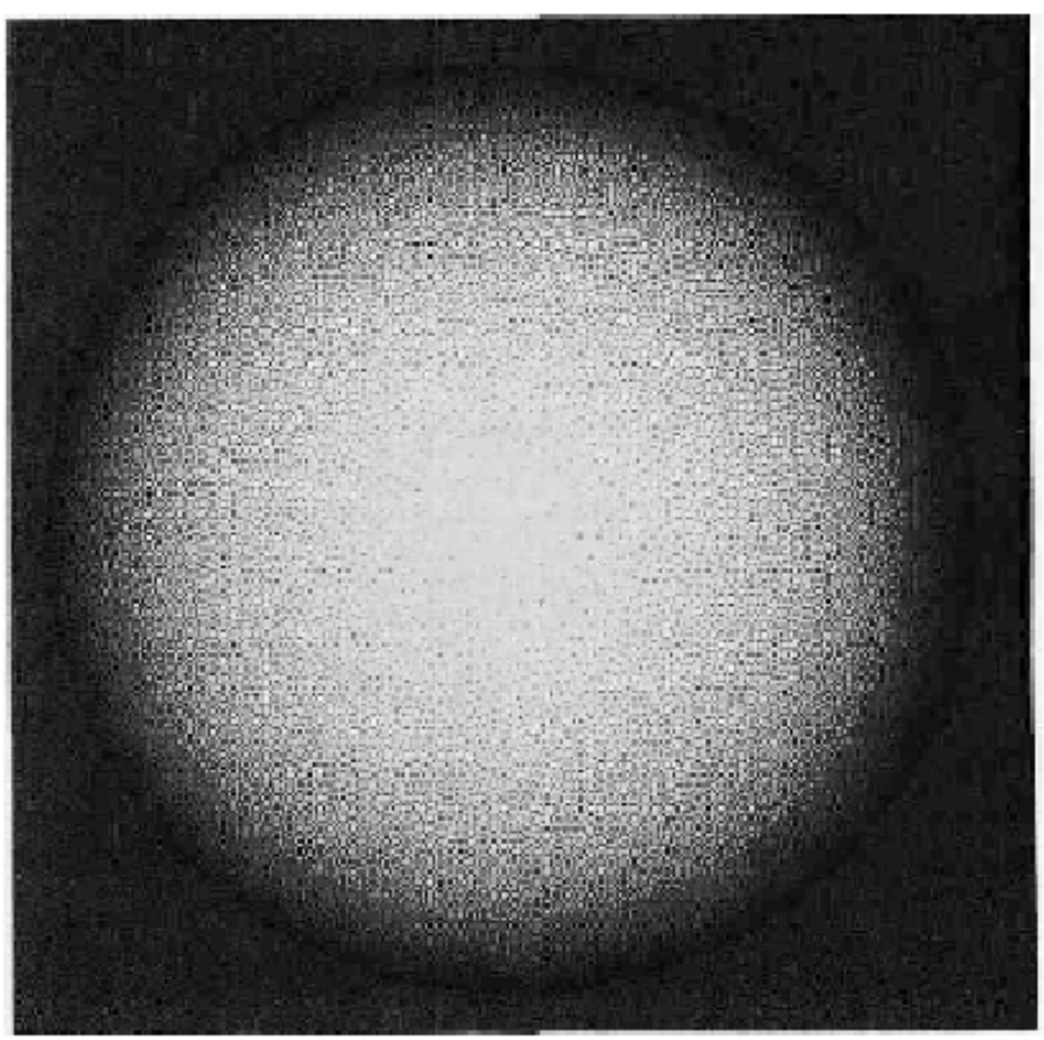
input image
(grayscale)

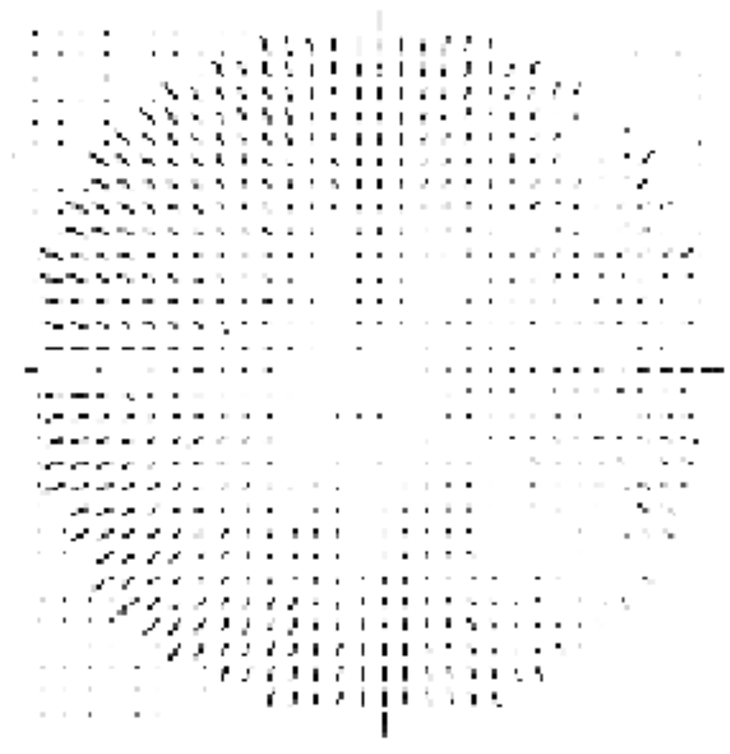
Results (synthetic data)

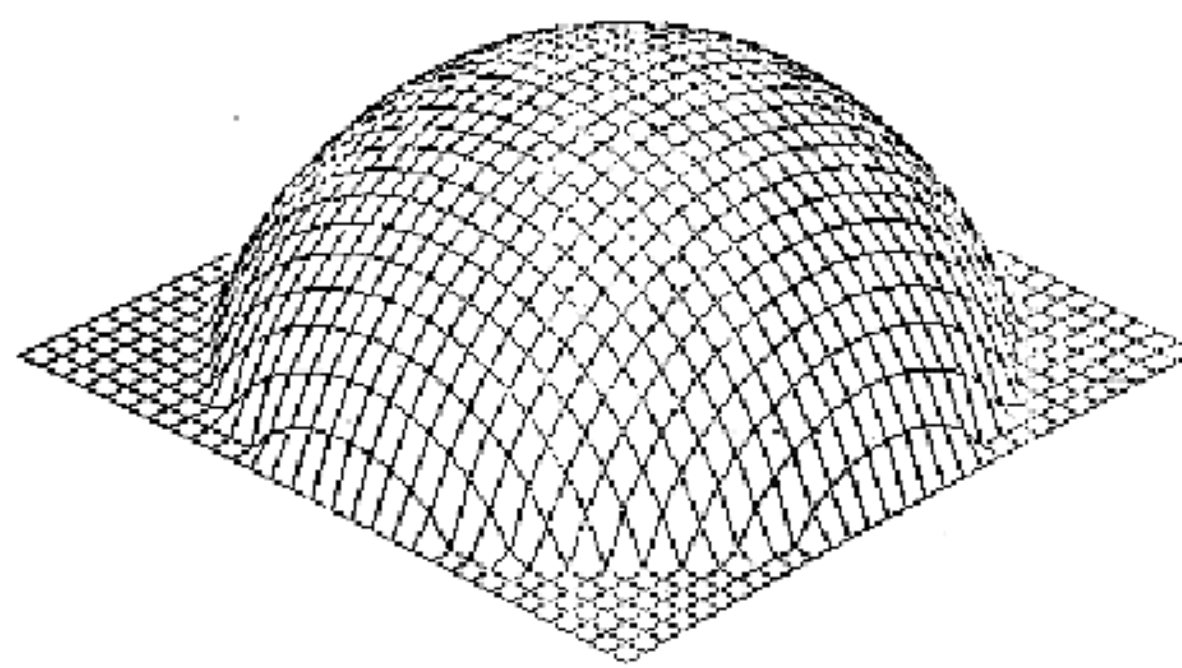
reconstructed

original

Results

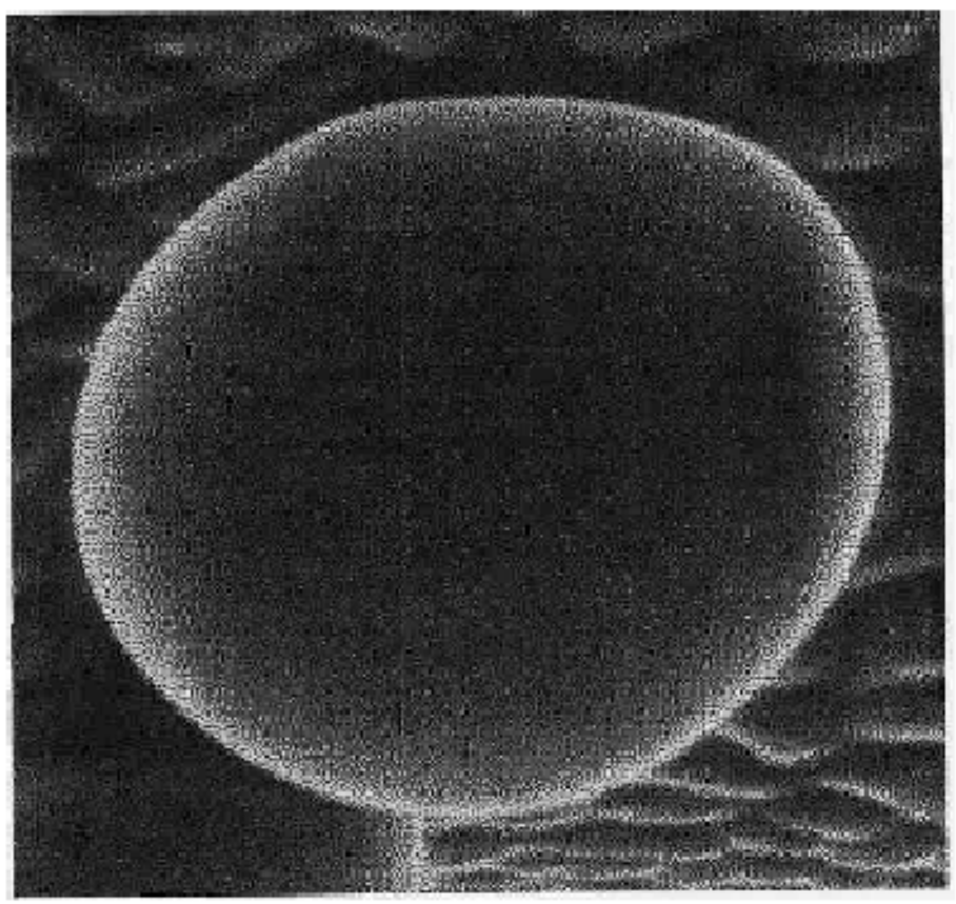






by Ikeuchi and Horn

Results



Scanning Electron Microscope image (inverse intensity)

by Ikeuchi and Horn