UNIVERSITÄT BERN

Applied Optimization
Exercise 3 - Convex Functions And Convex Problems

Heng Liu Nicolas Gallego

Handout date: 10.10.2019

Submission deadline: 16.10.2019, 23:59 h

Hand-in instructions:

Please hand-in **only one** compressed file named after the following convention: `Exercisen-GroupMemberNames.zip`, where n is the number of the current exercise sheet. This file should contain:

- **Only** the files you changed (headers and source). It is up to you to make sure that all files that you have changed are in the zip.
- A `readme.txt` file containing a description on how you solved each exercise (use the same numbers and titles) and the encountered problems.
- Other files that are required by your `readme.txt` file. For example, if you mention some screenshot images in `readme.txt`, these images need to be submitted too.
- Submit your solutions to ILIAS before the submission deadline.

Vector Composition (2 pts)

A general vector composition rule. Suppose

$$f(x) = h(g_1(x), g_2(x), ..., g_k(x))$$

where $h : \mathbb{R}^k \to \mathbb{R}$ is convex, and $g_i : \mathbb{R}^n \to \mathbb{R}$. Suppose that for each i, one of the following holds:

- h is nondecreasing in the ith argument, and g_i is convex
- h is nonincreasing in the ith argument, and g_i is concave
- g_i is affine.

Show that f is convex.

Linear Programming

Transform (2 pts)

For the following optimization problem

$$\begin{array}{ll} \text{minimize} & ||(2x_1 + 3x_2, -3x_1)^T||_\infty \\ \text{subject to} & |x_1 - 2x_2| \leq 0 \end{array}$$

(1) Express the problem as a a linear program. (2) Convert the LP so that all variables are in $\mathbb{R}_+$ and there is no inequalities.

Transform general LP to standard form (Bonus 2 pts)

A general linear program has the form

$$\begin{array}{ll} \text{minimize} & c^T x + d \\ \text{subject to} & Gx \preceq h \\ & Ax = b \end{array}$$

where $G \in \mathbb{R}^{m \times n} and A \in \mathbb{R}^{p \times n}$. Transform the general LP to its standard form:

$$\begin{array}{ll} \text{minimize} & c^T x' \\ \text{subject to} & Bx' = e \\ & x' \succeq 0 \end{array}$$

Explain in detail the relation between the feasible sets, the optimal solutions, and the optimal values of the standard form LP and the original LP.

Mass Spring System (6 pts)

See exercise 2.