Problem Set 7 Solutions

Computer Vision
University of Bern
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1 Registration, Outlier Rejection

In image registration the corresponding point coordinates are related by homography, $\lambda p' = Hp$, where $p = (x, y, 1)$ and $p' = (x', y', 1)$ are the coordinates on image I and I'. Note that H is equivalent to $H' = \beta H$ for any $\beta > 0$ because all equations can be satisfied by multiplying λ for all matching points by an appropriate number. It is therefore justified to set $\|H\| = 1$ for its estimation. Estimate H by eliminating λ and writing the equations in an appropriate linear system, where the entries of H are the unknowns. Solve the system by enforcing $\|H\| = 1$. What is the minimum number of correspondences needed?

Solution The homography matrix has 9 entries,

$$H = \begin{bmatrix} h_1 & h_2 & h_3 \\ h_4 & h_5 & h_6 \\ h_7 & h_8 & h_9 \end{bmatrix}. \tag{1}$$

With these notations we can express $\lambda = h_7 x + h_8 y + h_9$. We can substitute this into the other 2 equations. Then we can write it in a matrix form.

$$H = \begin{bmatrix} -x & -y & -1 & 0 & 0 & 0 & x'x & x'y & x' \\ 0 & 0 & 0 & -x & -y & -1 & y'x & y'y & y' \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \\ h_3 \\ h_4 \\ h_5 \\ h_6 \\ h_7 \\ h_8 \\ h_9 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \qquad (2)$$

This can be written in the form $Ah = 0$, where $h = (h_1, \cdots, h_9)$. If we have N point pairs, we have $2N$ rows in A instead of 2. We have to minimize $h^T A^T A h$ subject to $\|h\| = 1$. We can solve this by computing the SVD $A = USV^T$ and selecting $h = V(:, 9)$, the vector corresponding to the smallest singular value.

2 Interest Points

Consider the following two images:

$$I^1 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1(*) & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1(**) & 1 & 1 \end{bmatrix}, \quad I^2 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.5(*) & 0.5 & 0.5 \\ 0 & 0 & 0 & 0.5 & 0.5 & 0.5 \\ 0 & 0 & 0 & 0.5(**) & 0.5 & 0.5 \end{bmatrix}. \qquad (3)$$

1. Compute the Harris corner score at the points denoted with $(*)$ and $(**)$ using $k = 0.05$. Approximate the second moment matrix by averaging over a 3×3 neighborhood around the points. Moreover, for boundary pixels assign 0 to their gradients.

Solution Let us first compute the first order derivatives using the filters $D_x = [1\ 0\ -1]$ and $D_y = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$

$$I_x^1 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1(*) & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1(**) & 0 & 0 \end{bmatrix}, \quad I_y^1 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1(*) & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0(**) & 0 & 0 \end{bmatrix}. \tag{4}$$

$$I_x^2 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.5 & 0.5(*) & 0 & 0 \\ 0 & 0 & 0.5 & 0.5 & 0 & 0 \\ 0 & 0 & 0.5 & 0.5(**) & 0 & 0 \end{bmatrix}, \quad I_y^2 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.5 & 0.5 & 0.5 \\ 0 & 0 & 0 & 0.5(*) & 0.5 & 0.5 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0(**) & 0 & 0 \end{bmatrix}. \tag{5}$$

The average of the second moment matrix in a neighborhood $\mathcal{N}$ is

$$A = \sum_{p \in \mathcal{N}} w(p) \begin{bmatrix} I_x(p)I_x(p) & I_x(p)I_y(p) \\ I_y(p)I_x(p) & I_y(p)I_y(p) \end{bmatrix},$$

where $w(p) = \frac{1}{9}$ in our case.
We have the following second moment matrices for the interest point $(*)$:

$$A^{1*} = \frac{1}{9} \begin{bmatrix} 4 & 1 \\ 1 & 4 \end{bmatrix}, \quad A^{2*} = \frac{1}{9} \begin{bmatrix} 1 & 0.25 \\ 0.25 & 1 \end{bmatrix} \tag{6}$$

Then we have

$$\det(A^{1*}) = \frac{15}{81} \qquad \det(A^{2*}) = \frac{1 - 0.25^2}{81}$$
$$\operatorname{tr}(A^{1*}) = \frac{8}{9} \qquad \operatorname{tr}(A^{2*}) = \frac{2}{9}$$

The Harris scores are

$$H^{1*} = \det(A^{1*}) - 0.05 \cdot \operatorname{tr}(A^{1*})^2 = 0.145\ldots$$
$$H^{2*} = \det(A^{2*}) - 0.05 \cdot \operatorname{tr}(A^{2*})^2 = 0.009\ldots$$

Similarly, for the point $(**)$ we get

$$A^{1**} = \frac{1}{9} \begin{bmatrix} 6 & 0 \\ 0 & 0 \end{bmatrix}, \quad A^{2**} = \frac{1}{9} \begin{bmatrix} 1.5 & 0 \\ 0 & 0 \end{bmatrix}. \tag{7}$$

Then we have $\det(A^{1**}) = \det(A^{2**}) = 0, \mathrm{tr}(A^{1**}) = \frac{6}{9}, \mathrm{tr}(A^{2**}) = \frac{1.5}{9}$, and the resulting Harris scores are:

$$H^{1**} = -0.0\bar{2} \qquad H^{2**} = -0.0013\bar{8}$$

2. Use the Hessian detector for the same images of the previous exercise.

Solution We need to compute the second derivatives

$$I^1_{xx} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & -1(*) & -1 & 0 \\ 0 & 1 & 1 & -1 & -1 & 0 \\ 0 & 1 & 1 & -1(**) & -1 & 0 \end{bmatrix}, I^1_{yy} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & -1(*) & -1 & -1 \\ 0 & 0 & 0 & -1 & -1 & -1 \\ 0 & 0 & 0 & 0(**) & 0 & 0 \end{bmatrix}, \tag{8}$$

$$I^1_{xy} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1(*) & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0(**) & 0 & 0 \end{bmatrix}. \tag{9}$$

$$I^2_{xx} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.5 & 0.5 & -0.5(*) & -0.5 & 0 \\ 0 & 0.5 & 0.5 & -0.5 & -0.5 & 0 \\ 0 & 0.5 & 0.5 & -0.5(**) & -0.5 & 0 \end{bmatrix}, I^2_{yy} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.5 & 0.5 & 0.5 \\ 0 & 0 & 0 & 0.5 & 0.5 & 0.5 \\ 0 & 0 & 0 & -0.5(*) & -0.5 & -0.5 \\ 0 & 0 & 0 & -0.5 & -0.5 & -0.5 \\ 0 & 0 & 0 & 0(**) & 0 & 0 \end{bmatrix}, \tag{10}$$

$$I^2_{xy} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.5 & 0.5 & 0 & 0 \\ 0 & 0 & 0.5 & 0.5(*) & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0(**) & 0 & 0 \end{bmatrix}. \tag{11}$$

we have $H^{1*} = I^{1*}_{xx} I^{1*}_{yy} - (I^{1*}_{xy})^2 = 0$, $H^{1**} = I^{1**}_{xx} I^{1**}_{yy} - (I^{1**}_{xy})^2 = 0$, $H^{2*} = I^{2*}_{xx} I^{2*}_{yy} - (I^{2*}_{xy})^2 = -0.25$, $H^{2**} = I^{2**}_{xx} I^{2**}_{yy} - (I^{2**}_{xy})^2 = 0$.