Modal Logic
Exercise 03

13-123-922
Elias Wipfli
16-124-836
Marcel Zauder

Problem 3.2 Find derivations in K for the following formulas

3.2.1 $\Box\neg p \to \Box(p \to q)$

1.	$\neg p \to (p \to q)$	Taut
2.	$\Box(\neg p \to (p \to q))$	Nec, 1.
3.	$\Box(\neg p \to (p \to q)) \to (\Box\neg p \to \Box(p \to q))$	K, 2.
4.	$\Box\neg p \to \Box(p \to q)$	MP, 3.

3.2.2 $(\Box p \vee \Box q) \to \Box(p \vee q)$

1.	$p \to (p \vee q)$	Taut
2.	$\Box(p \to (p \vee q))$	Nec, 1.
3.	$\Box(p \to (p \vee q)) \to (\Box p \to \Box(p \vee q))$	K, 2.
4.	$\Box p \to \Box(p \vee q)$	MP, 3.
5.	$q \to (p \vee q)$	Taut
6.	$\Box(q \to (p \vee q))$	Nec, 5.
7.	$\Box(q \to (p \vee q)) \to (\Box q \to \Box(p \vee q))$	K, 6.
8.	$\Box q \to \Box(p \vee q)$	MP, 7.
9.	$(\Box p \vee \Box q) \to \Box(p \vee q)$	PL, 4., 8.

3.2.3 $\Diamond p \to \Diamond(p \vee q)$

1.	$\neg(A \vee B) \to \neg A$	Taut
2.	$\Box\neg(A \vee B) \to \Box\neg A$	RK, 1.
3.	$\neg\Box\neg A \to \neg\Box\neg(A \vee B)$	PL, 2.
4.	$\Diamond A \to \Diamond(A \vee B)$	*rewriting* $\Diamond$ *for* $\neg\Box\neg$.

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Problem 3.4 Show that the following derivability claims holds:

3.4.1 $\mathbf{K} \vdash \Diamond\neg\bot \to (\Box A \to \Diamond A)$

3.4.1 $\mathbf{K} \vdash \Box(A \vee B) \to (\Diamond A \vee \Box B)$

3.4.1 $\mathbf{K} \vdash (\Diamond A \to \Box B) \to \Box(A \to B)$

Problem 3.7 Alternative Proof for *Theorem 3.34* with 3 Worlds

Problem 3.8 Single reflexive transitive model s.t. KT4 $\nvdash B$ **and KT5** $\nvdash B$