

### 3.1 Relations among Failure Detectors

### 3.2 Perfect Failure Detector

### 3.3 Quorum Systems

- **SINGLETON**

There mustn't be any failed process. Because:

$$\nexists p \in \Pi : (\nexists Q \in \mathbb{Q} : p \in Q)$$

- **MAJORITY:**

MAXIMUM/MINIMUM:  $\lfloor \frac{n-1}{2} \rfloor$ , because:

$$\begin{aligned} \forall Q \in \mathbb{Q} : |Q| &= \lceil \frac{n+1}{2} \rceil \\ \Rightarrow |\Pi| - |Q| &= n - \lceil \frac{n+1}{2} \rceil = \lfloor \frac{n-1}{2} \rfloor \end{aligned}$$

- **GRID:**

MINIMUM:

We take the  $Q$  with the fewest elements, which would be equal to the last row of the grid. Therefore we must have  $k$  correct processes and at most  $k^2 - k$  faulty processes.

MAXIMUM:

We take the  $Q$  with the most elements which would be equal to the last row with  $k - 1$  additional processes. Therefore we must have in total  $2k - 1$  correct processes, so at most  $k^2 - 2k + 1$  faulty processes.