Problem Set 8

Computer Vision
University of Bern
Fall 2021

1 Fitting

1. Show the procedure to fit a line $y = c_1 x + c_0$ to some observed points (x_j, y_j), $j = 1, ..., n$ by minimizing the least squares error:

$$\epsilon(c_1, c_0) = \sum_{j=1}^{n} (c_1 x_j + c_0 - y_j)^2 \tag{1}$$

2. Show that the Prewitt gradient operator can be obtained by fitting the least-squares plane through the 3×3 neighborhood of the intensity function.

   Hint: Fit a plane to the nine points $(x + \delta x, y + \delta y, I[x + \delta x, y + \delta y])$ where δx and δy range through $-1, 0, +1$. Then, having the planar model $z = ax + by + c$ that best fits the intensity surface (i.e. minimizes the least-squares error in the z-direction), show that the two Prewitt masks actually compute a and b.

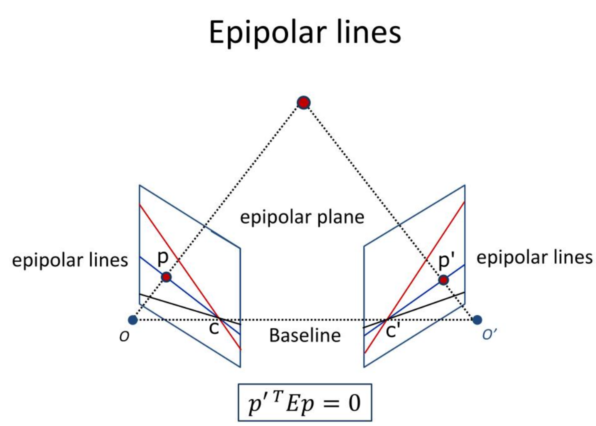
3. **Bonus:** Derive the expression for the shortest distance between a line l expressed with equation $ax + by = d$ and the point (x_0, y_0) . Hint: It's the distance of a perpendicular line segment between the line l and the point (x_0, y_0).

4. Show the procedure to fit a line $ax + by = d$ to some observed points (x_j, y_j), $j = 1, ..., n$ by minimizing the total least squares error:

$$\epsilon(a, b, d) = \sum_{j=1}^{n} (a x_j + b y_j - d)^2, \tag{2}$$

where $N = (a, b)$ is the unit normal, so $\|N\|^2 = a^2 + b^2 = 1$. The line equation is $ax + by = d$. Compute the parameters a, b and d of the best line through the points (0,-7), (2,-1) and (4,5).

2 Multiview Stereo



1. The camera projection matrices of two cameras (given in the coordinate system attached to the first camera) are

   $$\mathbf{C} = [\mathbf{I}\ \mathbf{0}] \text{ and } \mathbf{C}' = [\mathbf{R}\ \mathbf{t}],$$

   where $\mathbf{R}$ is a rotation matrix and $\mathbf{t} = (t_1, t_2, t_3)^T$ describes the translation between the cameras. Hence, the cameras have identical internal parameters and the image points are given in the normalized image coordinates (the origin of the image coordinate frame is at the principal point and the focal length is 1).

   The epipolar constraint implies that if p and p' are corresponding image points then the vectors $\overrightarrow{Op}$, $\overrightarrow{O'p'}$ and $\overrightarrow{OO'}$ are coplanar,i.e.

   $$\overrightarrow{O'p'} \cdot (\overrightarrow{O'O} \times \overrightarrow{Op}) = 0, \tag{3}$$

   where O and O' are the camera centers.

   Let $\mathbf{p} = (x, y, 1)^T$ and $\mathbf{p}' = (x', y', 1)^T$ denote the homogeneous image coordinate vectors of p and p'. Show that the equation (3) can be written in the form

   $$\mathbf{p}'^T E \mathbf{p} = 0, \tag{4}$$

where matrix $\mathbf{E}$ is the essential matrix $\mathbf{E} = [\mathbf{t}]_\times \mathbf{R}$, and

$$[\mathbf{t}]_\times = \begin{bmatrix} 0 & -t_3 & t_2 \\ t_3 & 0 & -t_1 \\ -t_2 & t_1 & 0 \end{bmatrix} \tag{5}$$