

# Modal Logic

Thomas Studer

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## **Lecture**

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## **Exercises**

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Prerequisite: propositional logic

Major subject?

Ilias:

Repository → Weitere Institutionen; Weiterbildungen und Studiengänge → BeNeFri Joint Master in Computer Science → HS2019

Lecture notes: Boxes and Diamonds

Exam: Tuesday, December 17, 2019

# Organizational (2)

Lecture will start at **9:20**

Exercises each week, starting in two weeks

Next week: no lecture

## **JMCS students**

Registration for teaching units (see <http://mcs.unibnf.ch/admin>)

Registration for exams (see <http://mcs.unibnf.ch/admin>)

Reimbursement of travel expenses  
(see <http://www.unifr.ch/benefri>)

## **Hosted JMCS students**

Additionally, request for Academia access  
(at <https://mcs.unibnf.ch/organization/request-for-academia-access/>)

Modal logic adds **new connectives**  $\Box$  and  $\Diamond$  to the language of logic

$\Box A$  means  $A$  is necessary

$\Diamond A$  means  $A$  is possible

Semantically, modal logic introduces **possible worlds**

$\Box A$  holds if  $A$  is true in all possible worlds

$\Diamond A$  holds if  $A$  is true in some possible worlds

$\Box$  and  $\Diamond$  are **dual operators**

$\Diamond A$  holds if  $\neg\Box\neg A$  holds

## Epistemic

$\Box A$  means  $A$  **is known** or  $A$  **is believed**

## Temporal

$\Box A$  means **always**  $A$

$\Diamond A$  means **eventually**  $A$

## Deontic

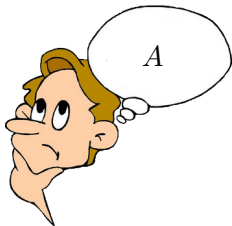
$\Box A$  means  $A$  **is obligatory**

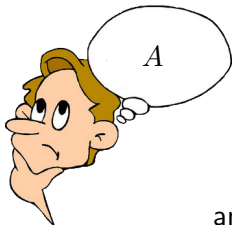
$\Diamond A$  means  $A$  **is permitted**

## Proof Theoretic

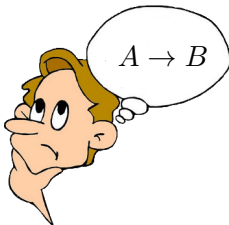
$\Box A$  means  $A$  **is provable**

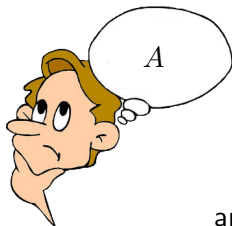




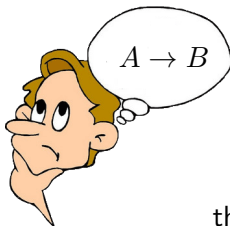


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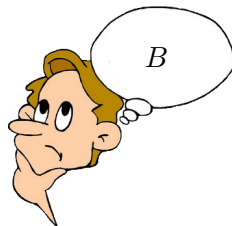


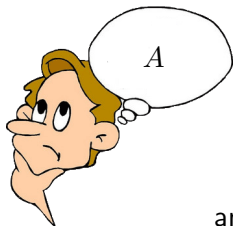


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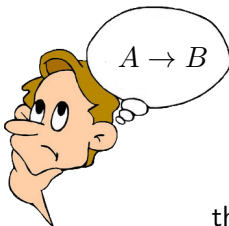


thus

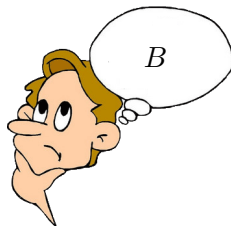




and



thus



$$\Box A \wedge \Box(A \rightarrow B) \rightarrow \Box B$$

# Basic properties of $\Box$

$$\Box(A \rightarrow B) \wedge \Box A \rightarrow \Box B$$

$$\Box A \rightarrow A$$

$$\Box A \rightarrow \Box\Box A$$

If  $A$  is provable, so is  $\Box A$ .