UNIVERSITÄT BERN

Digital 3D Geometry Processing
Exercise 11 – Parametrization II and Minimal Surfaces

Handout date: 14.05.2019

Submission deadline: 21.05.2019, 13:00 h

Note

A .zip compressed file renamed to `Exercisen-GroupMemberNames.zip` where n is the number of the current exercise sheet. It should contain:

- Hand in **only** the files you changed (headers and source). It is up to you to make sure that all files that you have changed are in the zip.
- A `readme.txt` file containing a description on how you solved each exercise (use the same numbers and titles) and the encountered problems.
- Other files that are required by your `readme.txt` file. For example, if you mention some screenshot images in `readme.txt`, these images need to be submitted too.
- Submit your solutions to ILIAS before the submission deadline.

1 Parameterization

In this exercise you will implement the *convex combination maps*. A discrete mapping is called a *convex combination mapping* if it satisfies the linear equations

$$\sum_{v_j \in N_1(v_i)} w_{ij}(\mathbf{u}(v_j) - \mathbf{u}(v_i)) = 0 \tag{1}$$

and has positive weights that sum to one, i.e.

$$w_{ij} > 0 \qquad \wedge \qquad \sum_{v_j \in N_1(v_i)} w_{ij} = 1 \tag{2}$$

1.1 Boundary Initialization (3 pts)

The first step you need to do is to map the boundary vertices to a circle centered at `_origin` with radius `_radius`, which is given as the diagnal length of the bounding box divided by 20, in the XY plane. Distribute the boundary vertices on the circle according to the boundary edge lengths. Initialize the texture coordinates for all the interior vertices to the center of the circle. Complete the function `map_suface_boundary_to_circle()` in `ParameterizationII.cc`. Store the texture coordinates by calling the function `mesh_.set_texcoord2D(vh, Vec2d(x, y))`. To see it in the viewer, click on the `Mapping Boundary to Circle` button. The outcome for the provided `Max Head` mesh is shown in Figure1. Note that this only works for mesh of disk-topology.

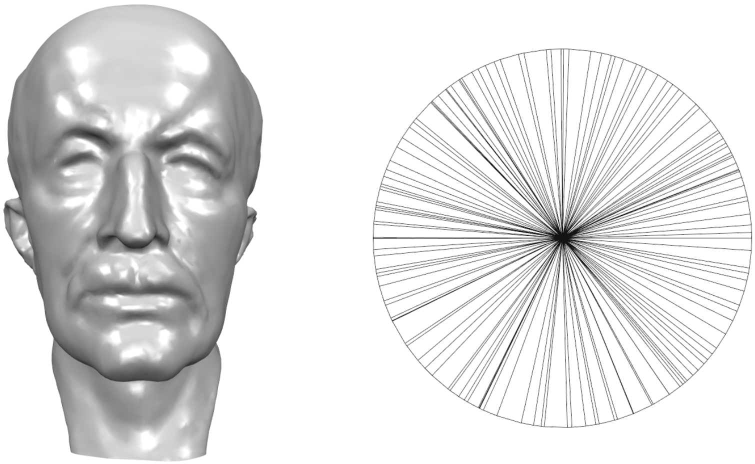

Figure 1: Boundary initialization.

1.2 Iterative Solver (3 pts)

One way of solving the linear system for texture mapping is by simply iterating the following for all vertices:

$$\forall v_i \in S \quad : \quad \mathbf{u}(v_i) \leftarrow \frac{1}{\sum_{v_j \in N_1(v_i)} w_{ij}} \sum_{v_j \in N_1(v_i)} w_{ij} \mathbf{u}(v_j).$$

Complete the function `explicitly_smooth_texture()`. Make sure that the cotan weights you are using satisfy the conditions from equations (2). By clicking on the `Explicit Smooth` button, one can see the result with certain iterations. The texture and texture mesh after running 50 iterations of the explicit smoothing are shown in Figure2.

1.3 Direct Solver (4 pts)

Another way of solving the linear system $\mathbf{Lu} = \mathbf{b}$ is to do the implicit solve. Form the $\mathbf{L}$ matrix with non-boundary cotan weights and store the boundary conditions in matrix $\mathbf{b}$. Complete the function `implicitly_smooth_texture()`. The result is shown in Figure3.

2 Minimal Surfaces (4 pts)

In this exercise we will implement a technique for obtaining the minimal surface given an initial mesh and boundary constraints. The minimal surface is the solution to the $\mathbf{LX} = 0$ equation. For this exercise the boundary condition is that the vertices on the boundary of the mesh are kept fixed. This is done by modifying the L matrix accordingly.
The implementation needs to be done in the `minimal_surface()` function in `ParametrizationII.cc` and is called by pressing the `Compute Minimal Surface` button. An example result is shown in Figure 4.

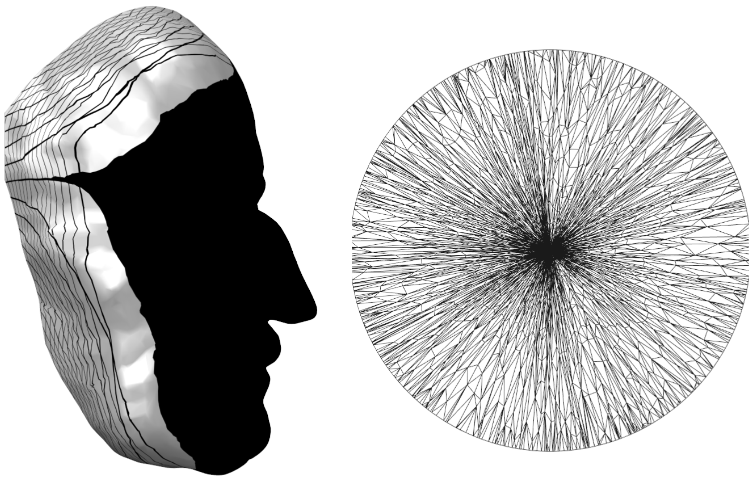

Figure 2: Texture after 50 iterations of iterative solve.

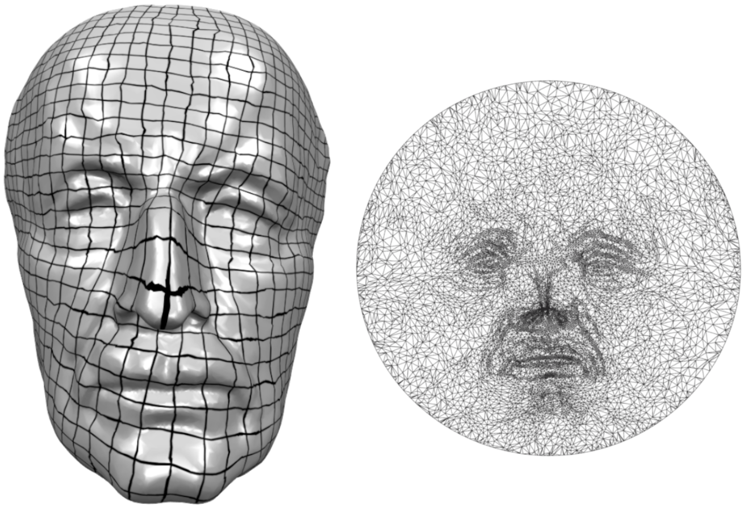

Figure 3: The result of the direct solver.

Iterate your method on the three provided cylinders. One of them shows a behavior that is different from the other two. Can you explain what is happening? Is the result consistent with the goal of the minimal surface optimization?

Does the same effect happen if you replace the cotan Laplacian with the uniform Laplacian? Elaborate your answer.

Please include screenshots of your results, accompanied by explanations in the `readme.txt` file.

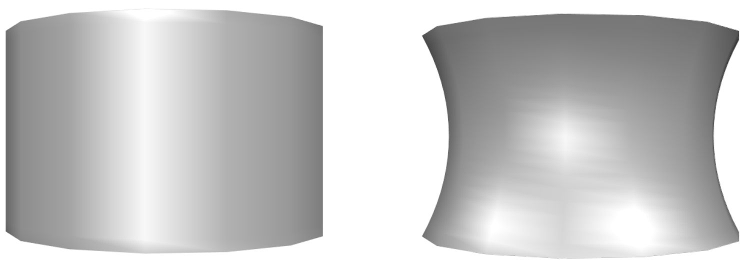

Figure 4: The initial *cylinder1* model and its minimal surface variant when keeping the lower and upper circle boundaries fixed.