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Applied Optimization
Exercise 06

6.1 Affine Invariance

Show that Newton methos is invariant under affine transformation:

$$\begin{aligned} & f:\mathbb{R}^n \to \mathbb{R}\ ;\ f''\ exists \\ g(y) &:= f(Ay+b)\ ;\ A\ non\ singular\ const.\ matrix\ \in \mathbb{R}^{n\times n};\ b \in \mathbb{R} \end{aligned}$$

Newton step: $\Delta x_{nt} = -\bigtriangledown^2 f(x)^{-1} \bigtriangledown f(x)$
We have:

$$\begin{aligned} \bigtriangledown g(y) &= A^T \bigtriangledown f(Ay+b) \\ \bigtriangledown^2 g(y) &= A^T \bigtriangledown f(Ay+b)A \end{aligned}$$

Let's suppose that some $x_0 = Ay_0 + b$:

$$\begin{aligned} x_{k+1} &= x_k - t_n \frac{\bigtriangledown f(x_k)}{\bigtriangledown^2 f(x_k)} \\ &= x_k - t_n \bigtriangledown^2 f(x_k)^{-1} \bigtriangledown f(x_k) \\ &= Ay_k + b - t_n \bigtriangledown^2 f(x_k)^{-1} \bigtriangledown f(x_k) \\ &= Ay_k + b - t_n \cdot A^1 \cdot A^{-1} \cdot A^T \cdot (A^T)^{-1} \cdot [\bigtriangledown^2 f(x_k)^{-1} \bigtriangledown f(x_k)] \\ &= A[y_k - t_n[A^T \bigtriangledown^2 f(x_k)A]^{-1} A^T \bigtriangledown f(x_k)] + b \\ &= A[y_k - t_n \bigtriangledown^2 g(y_k)^{-1} \cdot \bigtriangledown g(y_k)] + b \\ &= Ay_{k+1} + b \end{aligned}$$

6.2 Programming