Problem Set 5 Solutions

Computer Vision 2020
University of Bern

1 Optical Flow

Let $I(x, y, t)$ be a video sequence taken by a rigidly moving camera observing a rigid, static and Lambertian scene. Assume that between two consecutive views there is an affine change in the image intensities, i.e. the brightness constancy constraint reads:

$$I(x + u, y + v, t + 1) = aI(x, y, t) + b \tag{1}$$

where $u(x, y)$ and $v(x, y)$ represent the optical flow (motion parameters) and $a(x, y)$ and $b(x, y)$ represent photometric parameters. Propose a linear algorithm for estimating (u, v, a, b) from the image brightness I and its spatial-temporal derivatives I_x, I_y, I_t. What is the minimum size of a window around each pixel that allows one to solve the problem?

Solution After subtracting $I(x, y, t)$ on both sides, using the first order Taylor expansion, we obtain

$$I_x u + I_y v + I_t = (a - 1)I + b, \tag{2}$$

which reduces to the standard BCC when $a = 1$ and $b = 0$. This new brightness constancy constraint can be re-written as

$$I_x u + I_y v + (1 - a)I - b = -I_t \Rightarrow \begin{bmatrix} I_x & I_y & I & 1 \end{bmatrix} \begin{bmatrix} u \\ v \\ 1 - a \\ -b \end{bmatrix} = -I_t. \tag{3}$$

From this equation, we can solve for the parameters (u, v, a, b) in a least squares sense by assuming that such parameters are constant on a neighborhood Ω around each pixel. This leads to the following linear system of equations

$$\sum_{\Omega} \begin{bmatrix} I_x^2 & I_x I_y & I_x I & I_x \\ I_x I_y & I_y^2 & I_y I & I_y \\ I_x I & I_y I & I^2 & I \\ I_x & I_y & I & 1 \end{bmatrix} \begin{bmatrix} u \\ v \\ 1-a \\ -b \end{bmatrix} = -\sum_{\Omega} \begin{bmatrix} I_t I_x \\ I_t I_y \\ I_t I \\ I_t \end{bmatrix} \tag{4}$$

Since there are four unknowns, we need at least 4 pixels, e.g. a 2×2 window.

2 Registration, Outlier Rejection

In image registration the corresponding point coordinates are related by homography, $\lambda p' = Hp$, where $p = (x, y, 1)$ and $p' = (x', y', 1)$ are the coordinates on image I and I'. Note that H is equivalent to $H' = \beta H$ for any $\beta > 0$ because all equations can be satisfied by multiplying λ for all matching points by an appropriate number. It is therefore justified to set $\|H\| = 1$ for its estimation. Estimate H by eliminating λ and writing the equations in an appropriate linear system, where the entries of H are the unknowns. Solve the system by enforcing $\|H\| = 1$. What is the minimum number of correspondences needed?

Solution The homography matrix has 9 entries,

$$H = \begin{bmatrix} h_1 & h_2 & h_3 \\ h_4 & h_5 & h_6 \\ h_7 & h_8 & h_9 \end{bmatrix}. \tag{5}$$

With these notations we can express $\lambda = h_7 x + h_8 y + h_9$. We can substitute this into the other 2 equations. Then we can write it in a matrix form.

$$H = \begin{bmatrix} -x & -y & -1 & 0 & 0 & 0 & x'x & x'y & x' \\ 0 & 0 & 0 & -x & -y & -1 & y'x & y'y & y' \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \\ h_3 \\ h_4 \\ h_5 \\ h_6 \\ h_7 \\ h_8 \\ h_9 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{6}$$

This can be written in the form $Ah = 0$, where $h = (h_1, \cdots, h_9)$. If we have N point pairs, we have $2N$ rows in A instead of 2. We have to minimize $h^T A^T A h$

subject to $\|h\| = 1$. We can solve this by computing the SVD $A = USV^T$ and selecting $h = V(:, 9)$, the vector corresponding to the smallest singular value.