Machine Learning
Assignment # 1
Universität Bern

Due date: 10/10/2018

Late submissions will incur a penalty. Submit your answers in ILIAS (as a pdf or as a picture of your written notes if the handwriting is very clear). Submission instructions will be provided via email. For any clarification about the problem set ask the teaching assistant. You are not allowed to work with others.

Linear algebra review [Total 100 points]

Solve each of the following problems and show all the steps of your working.

1. Suppose that the matrices A and C satisfy the equations $Ax = b$ and $Cx = b$ with the same set of solutions x for every b. Can we conclude that $A = C$? Justify your answer in detail. **[10 points]**

2. Suppose that the $j - th$ column of a matrix B is a combination of the other columns of B. Show the relationship between the $j - th$ column of the matrix AB and the other columns of AB. **[10 points]**

3. Suppose that matrices A and B are invertible. Show that the inverse of the product AB is **[10 points]**

$$(AB)^{-1} = B^{-1}A^{-1}$$

4. Use the definition of trace to show that $trAB = trBA$, where $A \in \mathbb{R}^{m\times n}, B \in \mathbb{R}^{n\times m}$. **[10 points]**

5. Consider the matrix $G = A^\top A$, where $A \in \mathbb{R}^{m\times n}$ and $E = B^\top B$, where $B \in \mathbb{R}^{m\times n}$. **[20 points]**
   Show that G, E, and $G + E$ are all positive semi-definite.

6. Suppose C is positive definite and A has independent columns. Show that $S = A^\top CA$ is positive definite. **[20 points]**

7. Given two sets of vectors $\{x_1, ...x_n\} \subset \mathbb{R}^n$ and $\{y_1, ..., y_n\} \subset \mathbb{R}^n$, show that rank $\left[\sum_{i=1}^m x_i y_i^\top\right] \leq m$.
   Hint: First show that the square matrix $x_i y_i^\top$ has rank 1. **[20 points]**