Enrollment No: ______________

END-TERM EXAMINATION, EVEN SEMESTER MAY 2025

Course Code : CSET106
Course Name : Discrete Mathematical Structures
Program Name: B. Tech.
Semester: 2nd
Time: 02:00 Hours **Max Marks: 40**

General Instruction: -Do not write anything on the question paper except enrollment number.
Note: - Attempt the questions as per instruction given in each section.
Use of Scientific Calculators are allowed.

Section A: All the questions in this section are compulsory. ***Attempt either A or B part of each question***. Each question carries 03 marks. **[5QX3= 15 Marks]**

Q1.

1A. Discuss with all the properties that G is an abelian group given by a*b = a + b + ab for every a, b ∈ Z -{1}.

OR

1B. Discuss with all the properties that G is an abelian group given by a*b = a + b + 1 for every a, b ∈ Z.

$a + a^{-1} + 1 = e$

Q2.

2A. Discuss with all the properties that a ≡ b (mod m) forms an equivalence relation for all m ∈ N and a,b ∈ Z.

$a + a^{-1} + 1 = -1$

$a + a^{-1} = -2$

OR

2B. If set A has x number of elements and set B has y number of elements, then calculate the total number of one-to-one functions from set A to B.

Q3.

3A. In a competition, a school awarded medals in different categories. 36 medals in dance, 12 medals in dramatics and 18 medals in music. If these medals went to a total of 45 persons and only 4 persons got medals in all the three categories, how many have received medals in exactly two of these categories?

$n(A \cup B \cup C) = 45$

OR

$n(A) + n(B) + n(C) - n(A \cup B) -$

3B. Explain handshaking theorem with suitable example.

Q4.

4A. Explain truth table for: $[(p \vee q) \wedge \sim p] \rightarrow q$.

OR

4B. Explain the truth table for: $[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow p$.

Q5.

5A. A graph contains 21 edges and 3 vertices of degree 4, and all the other vertices of degree 2. Find the total number of vertices.

OR

5B. Define Bipartite graphs, Lattice and Planer graphs with examples.

Section B: All questions in this section are compulsory. **Attempt either A or B part of each question.** Each question carries 05 marks. [3QX5=15 Marks]

$(n+1)^3 = (n^3+1+3n^2+3n)$

Q6.

6A. Prove that $(n^3 - n + 3)$ is divisible by 3, for all $n \in Z^+$ using the principle of mathematical induction.

OR

6B. Draw the Haase diagrams for the posets:

a) $(P(\{a, b, c\}), \subseteq)$.

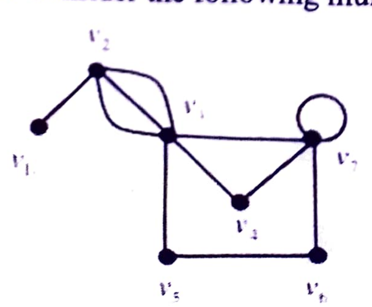
b) $(P(\{1, 2, 3, 6, 9, 18, 36, 72\}, |)$, where $a|b$ is a divides b.

Q7.

7A. Calculate the ~~CGD~~ GCD of 34 and 126 using Euclidean method. Express this GCD as linear combination of 34 and 126.

OR

7B. Consider the following multigraph G.



a) Find the adjacency matrix of above graph.

b) Explain if the obtained adjacency matrix symmetric?

c) Calculate the degree of vertex: V_1, V_2, V_3, V_6 and V_7.

Q8.

8A. Using Chinese remainder theorem find an integer x such that:

a) x gives a remainder of 3 when divided by 7.

b) x gives a remainder of 3 when divided by 13.

c) x is divisible by 12.

OR

8B. Solve the following linear congruence: $9X \equiv 21 \pmod{30}$.

Section C: All questions in this section are Compulsory. ***Attempt either A or B part of the question***

[1X10=10 Marks]

Q9. Answer the following questions

9A. In the realm of cryptography, the security of digital communications relies heavily on the mathematical properties of certain algebraic structures. Two such structures are $(Z_6, +_6)$ and $(Z_7 - \{0\}, \times_7)$, which are utilized in various cryptographic algorithms. This case study explores these structures to determine their suitability as Abelian groups and identifies their generators, providing insight into their role in secure communication systems.

Structure Overview

- Z_6 under Addition Modulo 6 $(Z_6, +_6)$:

  Set Definition: $Z_6 = \{0, 1, 2, 3, 4, 5\}$

  Operation: Addition modulo 6, denoted as $+_6$.

- $Z_7 - \{0\}$ under Multiplication Modulo 7 $(Z_7 - \{0\}, \times_7)$:

  Set Definition: $Z_7 - \{0\} = \{1, 2, 3, 4, 5, 6\}$

  Operation: Multiplication modulo 7, denoted as $\times_7$.

a) Analyze with all the properties if $+_6$ and $\times_7$ forms an abelian group on Z and Z - $\{0\}$ respectively or not.

b) Find all the generators of $(Z_6, +_6)$ and $(Z_7 - \{0\}, \times_7)$.

OR

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9B. In the design of quantum computing algorithms, complex numbers and unitary matrices play a key role. A quantum engineer is analysing the behaviour of a specific gate used in a quantum algorithm. This gate performs rotations based on multiplication with the elements of the set:

$$G = \{1, -1, i, -i\} \text{ and } H = \{1, \omega, \omega^2\}.$$

The operation is standard multiplication, and these elements represent roots of unity commonly used in quantum Fourier transforms (where i is the imaginary unit such that $i^2 = -1$ and ω is complex cube root of unity $\omega^3 = 1$). The operation on G and H is multiplication. The engineer wants to validate whether the behavior of this set follows desirable algebraic properties such as being Abelian and Cyclic, which are important for predictable and reversible operations in quantum circuits.

a) Verify that G and H forms a group under multiplication.
b) Evaluate whether the group G and H is Abelian Group.
c) Examine with all the possible generators whether G and H forms a Cyclic group.

*************************All the Best****************************

No. of Edge = 2 × Degree of Vertices