

Bsp.: $a_n = \frac{6+8n-n^3}{n+2n^2-8+3n^3}$

i) Grenzwert für $n \rightarrow +\infty$

$$\lim_{n \rightarrow +\infty} a_n = -\frac{1}{3}$$

ii) Wie viele Glieder liegen außerhalb der ϵ -Umgebung?

$$\epsilon > \frac{1}{100}$$

$$|a_n - a| < \epsilon$$

$$\left| \frac{6+8n-n^3}{n+2n^2-8+3n^3} + \frac{1}{3} \right| < \frac{1}{100}$$

$$\frac{3(6+8n-n^3) + (n+2n^2-8+3n^3)}{3(n+2n^2-8+3n^3)} = \frac{10+25n+2n^2}{3n+6n^2-24+9n^3}$$

$$\left| \frac{10+25n+2n^2}{3n+6n^2-24+9n^3} \right| < \frac{1}{100}$$

$$\hookrightarrow n=0; n=1$$

Fallunterscheidung:

1. Fall: $n \geq 2$: $\frac{10+25n+2n^2}{3n+6n^2-24+9n^3} < \frac{1}{100}$

$$\Leftrightarrow 1000 + 2500n + 200n^2 < 3n + 6n^2 - 24 + 9n^3$$

$$0 < 9n^3 - 194n^2 - 2497n - 1024$$

⋮