

Note that the core of what we do in this class is proving an implication, using an implication, proving a “there exists”, using a “there exists”, proving a “for all”, and using a “for all”. So as you go through the questions below, please make sure you apply those ideas. One step at a time, show your work. Some additional comments/hints/expectations are in the footnotes.

1. Use the following hypotheses:

- H1: For all $r \in M$, if r likes Disneyland, then r is an element in the set S .
- H2: For all $c \in P$, if c walks to school, then c is a rock climber.
- H3: Every element in the set M is an element in the set P .
- H4: For all $b \in M$, if b does not like Disneyland, then b is not an element in the set P .
- H5: For all $s \in S$, the person s walks to school.

to prove the proposition: For all $m \in M$, the person m is a rock climber.¹

2. Prove²: for all $a \in \mathbb{Z}$, for all $b \in \mathbb{Z}$, for all $c \in \mathbb{Z}$, if $a \mid b$, and $b \mid c$, then $a \mid c$.
3. Prove: for all $x \in \mathbb{Z}$, if x is even, then x^2 is even. After processing the “for all”, provide a **direct** proof.
4. Prove: for all $c \in \mathbb{Z}$, if c^2 is even, then c is even. After processing the “for all”, provide an **indirect** proof.
5. Prove: for all $c \in \mathbb{Z}$, if c^2 is even, then c is even. After processing the “for all”, provide a proof by contradiction.

¹As in the previous homework, take things one step at a time. To pick an example that is not from this HW (but apply what I am saying here to the this homework question), say we had a hypothesis For all $j \in T$, if j sings, then j plays baseball and we also knew $b \in T$. Then we'd get if b sings, then b plays baseball. Then, if we also later knew b sings, we can combine this with the implication to get b plays baseball.

²This comment applies to question 1, but also to question 2, and to many of the questions below: do not ignore the “for all” at the beginning of the statement you prove.