1. Let $P(x)$ be $\boxed{x \text{ is a positive real number}}$.

   Then the statement $\exists x \in \mathbb{R}[P(x)]$ is true because there exists at least one positive real number.

   Then the statement $\forall x \in \mathbb{R}[P(x)]$ is false because there are (infinitely many) real numbers that are not positive.

   The quantifiers in front of the predicate $P(x)$ cause these statements to mean different things, and I believe I have accurately shown this here.

2. Let $C(x)$ be $\boxed{x \text{ is a painter}}$. Let S be the set of all students.

- State $\forall x \in S[C(x)]$ in formal mathematical English.
  "For all painters x in the set of all students S, x is a painter."
- State $\forall x \in S[C(x)]$ in plain English. (For this part, think of how you'd talk to a kid.)
  "Every student is a painter."
- State $\exists x \in S[C(x)]$ in formal mathematical English.
  "There exists a painter x in the set of all students S such that x is a painter."
- State $\exists x \in S[C(x)]$ in plain English. (For this part, think of how you'd talk to a kid.)
  "There is a student who is a painter."
- State $\exists x \in S[\neg C(x)]$ in plain English. (For this part, think of how you'd talk to a kid.)
  "There is a student who is not a painter."

3.

$$
\begin{align}
&\forall a \in A[\forall b \in B[\exists c \in C[\forall d \in D[(P(a,b) \wedge Q(c)) \rightarrow R(d)]]]] \tag{1}\\
&\equiv \neg[\forall a \in A[\forall b \in B[\exists c \in C[\forall d \in D[(P(a,b) \wedge Q(c)) \rightarrow R(d)]]]]] \tag{2}\\
&\equiv \exists a \in A \neg[\forall b \in B[\exists c \in C[\forall d \in D[(P(a,b) \wedge Q(c)) \rightarrow R(d)]]]] \tag{3}\\
&\equiv \exists a \in A \exists b \in B \neg[\exists c \in C[\forall d \in D[(P(a,b) \wedge Q(c)) \rightarrow R(d)]]] \tag{4}\\
&\equiv \exists a \in A \exists b \in B \forall c \in C \neg[\forall d \in D[(P(a,b) \wedge Q(c)) \rightarrow R(d)]] \tag{5}\\
&\equiv \exists a \in A \exists b \in B \forall c \in C \exists d \in D \neg[(P(a,b) \wedge Q(c)) \rightarrow R(d)] \tag{6}\\
&\equiv \exists a \in A \exists b \in B \forall c \in C \exists d \in D \neg[\neg(P(a,b) \wedge Q(c)) \vee R(d)] \tag{7}\\
&\equiv \exists a \in A \exists b \in B \forall c \in C \exists d \in D [P(a,b) \wedge Q(c)] \wedge \neg R(d) \tag{8}
\end{align}
$$

4.

$$\forall x \in \mathbb{H}, \exists y \in \mathbb{H}[P(x,y)] \equiv \neg[\forall x \in \mathbb{H}, \exists y \in \mathbb{H}[P(x,y)]] \tag{1}$$

$$\equiv \exists x \in \mathbb{H}, \neg[\exists y \in \mathbb{H}[P(x,y)]] \tag{2}$$

$$\equiv \exists x \in \mathbb{H}, \forall y \in \mathbb{H}[\neg P(x,y)] \tag{3}$$

In plain English, the negation of the statement is, "There is a human who did not send an email to any human."

5.
- Let A be the set of integers.
- Let B be the set of all even integers.
- Let $P(x, y)$ be the predicate $\boxed{x \text{ is less than } y}$ or $\boxed{x < y}$.

Then, we know that $\forall a \in A[\exists b \in B[P(a, b)]]$ is true because for every integer a, there exists an even integer b such that $a < b$.

We also know that $\exists b \in B[\forall a \in A[P(a, b)]]$ is false because there is no even integer b such that every integer a is less than b.