1. Let $R = \{(a, b) \in \mathbb{R} \times \mathbb{R} : 7 \text{ divides } a - b\}$. State five elements of R, with proper notation.

    - $(15, 8) \in \mathbb{R}$, since 7 divides $(15 - 8)$.
    - $(1.25, 8.25) \in \mathbb{R}$, since 7 divides $(1.25 - 8.25)$.
    - $(14, -7) \in \mathbb{R}$, since 7 divides $(14 - (-7))$.
    - $(56, 7) \in \mathbb{R}$, since 7 divides $(56 - 7)$.
    - $(-13.15, 0.85) \in \mathbb{R}$, since 7 divides $(-13.15 - 0.85)$.

2. Let $R = \{(a, b) \in \mathbb{R} \times \mathbb{R} : 7 \text{ divides } a - b\}$. Note that R is a binary relation on $\mathbb{R}$. Prove R is reflexive.

Proof. Want to show that for all $a \in \mathbb{R}$, $(a, a) \in R$. Let $a \in \mathbb{R}$ be arbitrary. Since 7 divides $(a - a)$, we have 7 divides 0. Since 7 divides 0, we have $0 = 7 \cdot 0$, which is true. Therefore, $(a, a) \in R$. Since $a \in \mathbb{R}$ was arbitrary, we have shown that for all $a \in \mathbb{R}$, $(a, a) \in R$. Therefore, R is reflexive. $\square$

3. Let $R = \{(a, b) \in \mathbb{R} \times \mathbb{R} : 7 \text{ divides } a - b\}$. Note that R is a binary relation on $\mathbb{R}$. Prove R is symmetric.

Proof. Want to show that for all $a, b \in \mathbb{R}$, if $(a, b) \in R$, then $(b, a) \in R$. Let $a, b \in \mathbb{R}$ be arbitrary. Suppose $(a, b) \in R$. Then 7 divides $(a - b)$. Since 7 divides $(a - b)$, we have $a - b = 7k$ for some $k \in \mathbb{Z}$. Multiplying both sides by -1 gives $b - a = -7k$. Since $-7k \in \mathbb{Z}$, we have 7 divides $(b - a)$. Therefore, $(b, a) \in R$. Since $a, b \in \mathbb{R}$ were arbitrary, we have shown that for all $a, b \in \mathbb{R}$, if $(a, b) \in R$, then $(b, a) \in R$. Therefore, R is symmetric. $\square$

4. Let $R = \{(a,b) \in \mathbb{R} \times \mathbb{R} : 7 \text{ divides } a-b\}$. Note that R is a binary relation on $\mathbb{R}$. Prove R is transitive.

Proof. Want to show that for all $a,b,c \in \mathbb{R}$, if $(a,b) \in R$ and $(b,c) \in R$, then $(a,c) \in R$. Let $a,b,c \in \mathbb{R}$ be arbitrary. Suppose $(a,b) \in R$ and $(b,c) \in R$. Then 7 divides $(a-b)$ and 7 divides $(b-c)$. Since 7 divides $(a-b)$ and 7 divides $(b-c)$, we have $a-b = 7k$ and $b-c = 7m$ for some $k,m \in \mathbb{Z}$. Adding these equations gives $a-c = 7(k+m)$. Since $k+m \in \mathbb{Z}$, we have 7 divides $(a-c)$. Therefore, $(a,c) \in R$. Since $a,b,c \in \mathbb{R}$ were arbitrary, we have shown that for all $a,b,c \in \mathbb{R}$, if $(a,b) \in R$ and $(b,c) \in R$, then $(a,c) \in R$. Therefore, R is transitive. $\square$

5. Based on the following set definitions:

    - $B = \{j\text{'s cat} : j \in T\}$
    - $L = \{z \in V \mid z \text{ scratches furniture}\}$
    - $S = \{m \in T : m \text{ plays hockey}\}$

    and hypotheses

    - H1: $B \subseteq L$.
    - H2: For all $c \in T$, if c is a baker and c's cat scratches furniture, then c wins GBBO.
    - H3: For all $a \in T$, if a does not wear a helmet, then a does not skateboard.
    - H4: For all $d \in J$, if d scratches furniture, then d does not wear a helmet.
    - H5: $J \subseteq V$.

    prove: for all $x \in S$, for all $y \in T$, if x is a baker and y skateboards, then x wins GBBO and y wears a helmet.

    Proof. Let $x \in S$ and $y \in T$ be arbitrary. Suppose x is a baker and y skateboards. Since $x \in S$, we know x plays hockey. Since x plays hockey, $x \in T$. Since $x \in T$, x's cat scratches furniture. Since x's cat scratches furniture, x wins GBBO by hypothesis H2. Since y skateboards, y must wear a helmet by hypothesis H3. Therefore, for all $x \in S$, for all $y \in T$, if x is a baker and y skateboards, then x wins GBBO and y wears a helmet. □