1. Find a piece-wise non-recursive formula for the sequence a_n whose first terms are: $1, 1, 1, 2, 2, 2, 3, 3, 3, 4, 4, 4, 5, 5, 5$ and so on.

   Check that your formula works for $n = 1$ up through $n = 15$.

$$a_n = \begin{cases} 1 + \frac{n-1}{3}, & \text{if } n \equiv 1 \pmod 3; \\ 1 + \frac{n-2}{3}, & \text{if } n \equiv 2 \pmod 3; \\ 1 + \frac{n-3}{3}, & \text{if } n \equiv 0 \pmod 3. \end{cases}$$

2. Let p be the proposition "It is below freezing". Let q be the proposition "It is snowing". Write the propositions below symbolically using p and q and logical operation symbols.

   - It is below freezing and snowing.
     $p \wedge q$
   - It is below freezing but not snowing.
     $p \wedge \neg q$
   - It is not below freezing and it is not snowing.
     $\neg p \wedge \neg q$
   - If it is below freezing, it is also snowing.
     $p \rightarrow q$
   - It is below freezing if and only if it is not snowing.
     $p \leftrightarrow \neg q$

3. In the parts to this question, we will examine $\boxed{r \rightarrow s}$. Before starting, review the truth table and the definition of implication. (Note, in the various parts, r might not always mean the same thing, and s might not always be the same proposition.)

   - Determine the truth value of $r \rightarrow s$, given the information that r is $\boxed{\text{Pizza grows on trees}}$. (In this part, we are not told what specific proposition s is. Answer this part based on s is some mystery proposition that we do not know.)

     We can say that the truth value of $r \rightarrow s$ is always $\boxed{\text{True}}$ because s is not dependant on r to be true. According to the truth table for this problem, this entire proposition can only be false when the leftmost proposition is true and the rightmost is false. Therefore, there is no possible case, given the truth value of r, where this proposition is false.

   - State the converse of $r \rightarrow s$ in symbols.

     $s \rightarrow r$

   - State the contrapositive of $r \rightarrow s$ in symbols.

     $\neg s \rightarrow \neg r$

   - State the contrapositive of $r \rightarrow s$ in words, given the information that r is $\boxed{\text{Cats grow on trees}}$ and s is $\boxed{\text{Wisconsin is larger than Alaska}}$.

     If Wisconsin is not larger than Alaska, then cats do not grow on trees.

   - Determine the truth value of the contrapositive of $r \rightarrow s$, given the information that r is $\boxed{\text{Cats grow on trees}}$ and s is $\boxed{\text{Wisconsin is larger than Alaska}}$.

   The proposition is $\boxed{\text{True}}$! Wisconsin is not larger than Alaska, so that part's good, then we evaluate r and find that cats do not grow on trees! Both the premise and conclusion are true, which means the proposition is true!

4. The following logical equivalence is called the distributive law: $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$. Apply the definitions Section 2.3 of the Handbook of Mathematical Proof (the back half of the spiral bound) one step at a time. Write individual sentences: each sentence should process one concept and turn it into another process. After working through the definitions, perform a verification by writing a truth table. (For reference, see the discussion in the first half of Method 105.)

   - We know $q \wedge r$ is true if at least one of q or r is true.
   - We know $p \wedge (q \vee r)$ is true if p is true and $q \vee r$ is true.
   - We know $q \vee r$ is true if at least one of q or r is true.
   - According to the distribuutive law, we can rewrite this expression as $(p \wedge q) \vee (p \wedge r)$. This is true if $p \wedge q$ is true or $p \wedge r$ is true.

p	q	r	$q \vee r$	$p \wedge (q \vee r)$	$p \wedge q$	$p \wedge r$	$(p \wedge q) \vee (p \wedge r)$
T	T	T	T	T	T	T	T
T	T	F	T	T	T	F	T
T	F	T	T	T	F	T	T
T	F	F	F	F	F	F	F
F	T	T	T	F	F	F	F
F	T	F	T	F	F	F	F
F	F	T	T	F	F	F	F
F	F	F	F	F	F	F	F

Table 1: Since the fourth and eighth rows match, we can say these two statements are logically equivalent.

5. Show that the proposition $\neg((p \wedge q) \rightarrow q)$ is a contradiction WITHOUT using truth tables.

- We know that $p \wedge q$ is true if both p and q are true.
- We know that $(p \wedge q) \rightarrow q$ is true if $p \wedge q$ is true and q is true.
- We know that $\neg((p \wedge q) \rightarrow q)$ is true if $(p \wedge q) \rightarrow q$ is false.
- We know that $(p \wedge q) \rightarrow q$ is false if $p \wedge q$ is true and q is false.
- Since $p \wedge q$ is true if both p and q are true, and q is false, we can say that $p \wedge q$ is false.
- Therefore, the proposition $\neg((p \wedge q) \rightarrow q)$ is a contradiction.