MTH 225 — Homework 01

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I chose to denote my responses to each question with a "•" to hopefully make things easier to digest. Hopefully I didn't make any mistakes with the truth table and that the notation for my responses is acceptable. Responses begin on the next page.

1. Find a piece-wise non-recursive formula for the sequence a_n whose first terms are: $2, 2, 5, 5, 8, 8, 11, 11, 14, 14, 17, 17$ and so on.

   Clarification: When $x = 1$ we have $y = 1$. When $x = 2$ then $y = 2$. When $x = 3$ then $y = 5$. However, I am just using x and y for convenience. Change the xs to ns, and the ys to a_ns when you turn in your homework.

   • The formula for this question can be expressed as:

$$a_n = \begin{cases} 2 + \frac{3(n-1)}{2} \text{ for all } n \text{ if } n \text{ is odd;} \\ 2 + \frac{3(n-2)}{2} \text{ for all } n \text{ if } n \text{ is even.} \end{cases}$$

2. For this homework question, let C be the set of all car brands.

   (a) With correct notation, state three elements of C.

   • Let r be "Rolls-Royce", let f be "Ford", and let t be "Toyota". Then $r, f, t \in C$.

   (b) Write what $\boxed{r \in C}$ means in plain English.

   • In plain english and applying my example for r, $r \in C$ means that Rolls-Royce is an element of the set of all car brands. More generally speaking, it just means that r is an element of the set C. Therefore, we can deduce that r is a car brand!

3. Let p be the proposition "Madison is the capital of Wisconsin", and let q be the proposition "Tables are a type of food", and let r be "Dogs are not animals". State each of the following in words:

   (a) $p \wedge q$
   - Madison is the capital of Wisconsin and tables are a type of food.

   (b) $p \vee q$
   - Madison is the capital of Wisconsin or tables are a type of food.

   (c) $(r \wedge p) \rightarrow q$
   - If dogs are not animals and Madison is the capital of Wisconsin, then tables are a type of food.

   (d) $q \rightarrow \neg r$
   - If tables are a type of food, then dogs are animals.

   (e) $r \wedge r$
   - Dogs are not animals and dogs are not animals.

4. Let p be the proposition "Madison is the capital of Wisconsin", and let q be the proposition "Tables are a type of food", and let r be "Dogs are not animals". For each proposition below, state in symbols and ALSO determine if the proposition is true or false (and provide some explanation for why).

   (a) Madison is the capital of Wisconsin and dogs are not animals.
   - Symbolic representation: $p \wedge r$
   - Truth value: False
   - Explanation: We know conjunction of two propositions is true, in this case, only if both p and r are true. Since r ("Dogs are not animals") is clearly false, the entire statement is false.

   (b) Madison is the capital of Wisconsin or dogs are animals.
   - Symbolic representation: $p \vee \neg r$
   - Truth value: True
   - Explanation: We also know the disjunction of two propositions is true if either p or $\neg r$ (the negation of r) is true. Since p ("Madison is the capital of Wisconsin") is true, the entire statement is true, regardless of the truth value of $\neg r$.

   (c) If dogs are animals, then tables are a type of food.
   - Symbolic representation: $\neg r \rightarrow q$
   - Truth value: True
   - Explanation: This is a conditional statement, which is true unless the antecedent ($\neg r$) is true and the consequent (q) is false. Since $\neg r$ ("Dogs are not animals") is false, the entire statement is true, regardless of the truth value of q.

   (d) If Madison is not the capital of Wisconsin, then dogs are not animals.
   - Symbolic representation: $\neg p \rightarrow r$
   - Truth value: False
   - Explanation: This is also a conditional statement. It's true unless the antecedent ($\neg p$) is true and the consequent (r) is false. Since both $\neg p$ ("Madison is not the capital of Wisconsin") and r ("Dogs are not animals") are false, the entire statement is false.

   (e) Dogs are not animals or dogs are animals.
   - Symbolic representation: $r \vee \neg r$
   - Truth value: True
   - Explanation: This statement is a tautology, meaning it is always true regardless of the truth values of r and $\neg r$. This is because this equation represents a disjunction (OR) of a statement and its negation. At least one of these must be true, therefore the statement is always true.

5. Provide a complete truth table for $(p \to \neg q) \wedge (q \vee r)$.

   Clarification: for a complete truth table, I am expecting you to "show your work" by giving relevant columns that lead up to the final column. For example, one of the columns you should display work for is $q \vee r$.

p	q	r	$\neg q$	$(p \to \neg q)$	$(q \vee r)$	$(p \to \neg q) \wedge (q \vee r)$
T	T	T	F	F	T	F
T	T	F	F	F	T	F
T	F	T	T	T	T	T
T	F	F	T	T	F	F
F	T	T	F	T	T	T
F	T	F	F	T	T	T
F	F	T	T	T	T	T
F	F	F	T	T	F	F

Table 1: Truth Table for $(p \to \neg q) \wedge (q \vee r)$