1. Find a piecewise formula for the sequence a_n whose first terms are

   $1, 1, 1, 1, 2, 2, 2, 2, 3, 3, 3, 3, 4, 4, 4, 4, 5, 5, 5, 5, \ldots$

   The piecewise formula for this sequence can be expressed as:

$$a_n = \begin{cases} \frac{n+3}{4} & \text{if } n+3 \text{ is a multiple of 4;} \\ \frac{n+2}{4} & \text{if } n+2 \text{ is a multiple of 4;} \\ \frac{n+1}{4} & \text{if } n+1 \text{ is a multiple of 4;} \\ \frac{n}{4} & \text{if } n \text{ is a multiple of 4.} \end{cases}$$

2. Given the hypotheses:

- H1: If Jax does not make jewlery, then Carl does not go camping. $\boxed{\neg j \rightarrow \neg c}$

- H2: If Bo does not bake cakes, then Jax makes jewelry. $\boxed{\neg b \rightarrow j}$

- H3: If Kofi does not fly kites, then Fiona goes fishing. $\boxed{\neg k \rightarrow f}$

- H4: If Al does not like Archery, then Carl goes camping. $\boxed{\neg a \rightarrow c}$

- H5: If Bo bakes cakes, then Al does not like archery. $\boxed{b \rightarrow \neg a}$

- H6: If Carl goes camping, Kofi flies kites. $\boxed{c \rightarrow k}$

- H7: If Jax makes jewelry, then Matthijs records music. $\boxed{j \rightarrow m}$

- H8: If Kofi flies kites, then Matthijs does not record music. $\boxed{k \rightarrow \neg m}$

Prove: If Al likes archery, then Fiona goes fishing.

Proof. Assume Al likes Archer ("Al does not like Archery" is false).

Using H5, we know that if Bo bakes cakes, then Al does not like Archery. Since Al likes Archery, Bo must not bake cakes. Then, using H2, we know that if Bo does not make cakes, then Jax must make jewelry. Since Bo does not bake cakes, Jax must make jewelry. Since Jax makes jewelrey, we can apply H7 to conclude that Matthijs records music. By H8, if Kofi flies kites, it would contradict our previous finding that Matthijs records music. Therefore, Kofi must not be flying kites. Finally, using H3, we know that if Kofi does not fly kites, then Fiona goes fishing. Since Kofi does not fly kites, we can conclude that Fiona goes fishing.

□

Proof with symbols:

- Let j be the statement, "Jax makes jewelrey".
- Let b be the statement, "Bo bakes cakes".
- Let k be the statement, "Kofi flies kites".
- Let a be the statement, "Al likes Archery".
- Let c be the statement, "Carl goes camping".
- Let m be the statement, "Matthijs records music".

Proof. Assume a is true ($\neg a$ is false).

Using H5, we know that if b is true, then $\neg a$ is true. Since $\neg a$ is false, b must be false ($\neg b$ must be true). Then, using H2, we know that if $\neg b$ is true, then j is true. Since $\neg b$ is true, j must be true. Since j is true, we can apply H7 to conclude that m is true. By H8, if k were true, it would contradict our previous finding that m is true. Therefore, k must be false ($\neg k$ must be true). Finally, using H3, we know that if $\neg k$ is true, then f is true. Since $\neg k$ is true, f must be true.

□

3. Prove: for all $a \in \mathbb{Z}$, for all $b \in \mathbb{Z}$, for all $c \in \mathbb{Z}$, if a divides b, and b divides c, then a divides c.

Symbolic representation: $\forall a \in \mathbb{Z}, \forall b \in \mathbb{Z}, \forall c \in \mathbb{Z}, (a|b \wedge b|c) \rightarrow a|c$.

Proof. Let a, b, and c be integers. Assume a divides b and b divides c. This means there exists integers k and m such that $b = ak$ and $c = bm$. Substituting $b = ak$ into $c = bm$, we get:

$$c = bm = (ak)m = a(km).$$

We know km is an integer because the set of integers is closed under multiplication, so we can conclude that a divides c. Therefore, if a divides b and b divides c, then a divides c. $\square$

4. Prove: for all $c \in \mathbb{Z}$, for all $d \in \mathbb{Z}$, if c is even and d is odd, then $c - d$ is odd.

Proof. Let c and d be integers. Assume c is even and d is odd. This means there exists integers k and m such that

$$c = 2k \text{ and } d = 2m + 1.$$

Substituting $c = 2k$ and $d = 2m + 1$ into $c - d$, we get:

$$c - d = 2k - (2m + 1) = 2k - 2m - 1 = 2(k - m) - 1.$$

Let $w = (k - m)$. We know that $w \in \mathbb{Z}$ because k and m are integers, and the set of integers is closed under subtraction. Thus, we can conclude that $c - d$ is odd. Therefore, if c is even and d is odd, then $c - d$ is odd. $\square$

I have given each of these hypotheses a name, H1 through H7, to more clearly reference them in the proof and preserve conciseness.

5. Using the hypotheses:

   - For all $y \in Y$, if y is a poet, then y does not skateboard.
   - For all $s \in V$, for all $t \in W$, if s gives t chocolate, then t tap dances.
   - Every element in the set C is also in the set V.
   - Every element in the set C is also in the set Y.
   - Every element in the set A is also in the set V.
   - Every element in the set B is also in the set W.
   - For all $h \in B$, if h tap dances, then h is happy.

   Prove: For all $a \in A$, for all $b \in B$, for all $c \in C$, if c skateboards and a gives b chocolate, then b is happy and c is not a poet.

   Proof. Let $a \in A$, $b \in B$, and $c \in C$. Assume c skateboards and a gives b chocolate. Since every element in the set C is also in the set Y, we conclude that $c \in y$. From H1, we note that if c skateboards, then c cannot be a poet. Thus, c is not a poet. Since every element in the set A is also in the set V, we have $a \in V$. According to H2, we know that because a gives b chocolate, it follows that b tap dances. Since every element in the set B is also in the set W, we have $b \in W$. From H7, we know that since b tap dances, b is happy. Therefore, if c skateboards and a gives b chocolate, then b is happy and c is not a poet. $\square$