

1. Let $f : M \rightarrow N$ and $g : N \rightarrow L$ both be surjective functions. Prove that $g \circ f$ is surjective. (First, look carefully at what the domain and codomain of $g \circ f$ are. Of course, be sure to follow the definition of surjective EXACTLY, and do not ignore quantifiers. Determine which “for all” is being used and which “for all” is being proved, and which “there exists” is being used and which “there exists” is being proved.)

Proof. For

□

2. Let $f : M \rightarrow N$ and $g : N \rightarrow L$ both be injective functions. Prove that $g \circ f$ is injective. (The same types of hints as the previous question, and in fact, at this level of math, the following advice always applies: since the question mentions the word injective, go review the definition first and do NOT ignore quantifiers. Determine which “for all” is being used and which “for all” is being proved.)
3. Let $f : M \rightarrow N$. Prove: if A and B are subsets of N such that $A \subseteq B$, then $f^{-1}(A) \subseteq f^{-1}(B)$.
4. Prove that $[2, 6]$ and $[11, 20]$ are equicardinal. For clarification, both sets/intervals mentioned are subsets of \mathbb{R} .
5. Prove: if A is countably infinite and B is countably infinite and C is countably infinite and $A \cap B = \emptyset$ and $A \cap C = \emptyset$ and $B \cap C = \emptyset$, prove $A \cup B \cup C$ is countably infinite. (Hint: it will be helpful to look at a past HW key where a formula for a sequence was given.)