Note that the core of what we do in this class is proving an implication, using an implication, proving a "there exists", using a "there exists", proving a "for all", and using a "for all". So as you go through the questions below, please make sure you apply those ideas. One step at a time, show your work. Some additional comments/hints/expectations are in the footnotes.

1. Use the following hypotheses:

   - H1: For all $r \in M$, if r likes Disneyland, then r is an element in the set S.
   - H2: For all $c \in P$, if c walks to school, then c is a rock climber.
   - H3: Every element in the set M is an element in the set P.
   - H4: For all $b \in M$, if b does not like Disneyland, then b is not an element in the set P.
   - H5: For all $s \in S$, the person s walks to school.

   to prove the proposition: For all $m \in M$, the person m is a rock climber.[1]

   Proof. From H3, we know that every element in the set M is an element in the set P. So, for all $m \in M$, $m \in P$. From H2, we know that for all $c \in P$, if c walks to school, then c is a rock climber. So, for all $m \in M$, if m walks to school, then m is a rock climber. From H5, we know that for all $s \in S$, the person s walks to school. So, for all $m \in M$, if m likes Disneyland, then m is an element in the set S. From H1, we know that for all $r \in M$, if r likes Disneyland, then r is an element in the set S. So, for all $m \in M$, if m likes Disneyland, then m is an element in the set S. From H4, we know that for all $b \in M$, if b does not like Disneyland, then b is not an element in the set P. So, for all $m \in M$, if m does not like Disneyland, then m is not an element in the set P. Therefore, for all $m \in M$, the person m is a rock climber. □

[1] As in the previous homework, take things one step at a time. To pick an example that is not from this HW (but apply what I am saying here to the this homework question), say we had a hypothesis $\boxed{\text{For all } j \in T, \text{ if } j \text{ sings, then } j \text{ plays baseball}}$ and we also knew $\boxed{b \in T}$. Then we'd get $\boxed{\text{if } b \text{ sings, then } b \text{ plays baseball}}$. Then, if we also later knew $\boxed{b \text{ sings}}$, we can combine this with the implication to get $\boxed{b \text{ plays baseball}}$.

2. Prove[2]: for all $a \in \mathbb{Z}$, for all $b \in \mathbb{Z}$, for all $c \in \mathbb{Z}$, if $a \mid b$, and $b \mid c$, then $a \mid c$.

Proof. Let $a, b, c \in \mathbb{Z}$ be arbitrary. Suppose $a \mid b$ and $b \mid c$. Then, by definition of divisibility, there exists an integer k such that $b = ak$ and an integer m such that $c = bm$. Substituting $b = ak$ into $c = bm$ gives $c = akm$. Since k and m are integers, km is an integer. Therefore, $c = a(km)$, which implies that $a \mid c$. Thus, for all $a, b, c \in \mathbb{Z}$, if $a \mid b$ and $b \mid c$, then $a \mid c$. $\square$

[2]This comment applies to question 1, but also to question 2, and to many of the questions below: do not ignore the "for all" at the beginning of the statement you prove.

3. Prove: for all $x \in \mathbb{Z}$, if x is even, then x^2 is even. After processing the "for all", provide a **direct** proof.

   Proof. Let $x \in \mathbb{Z}$ be arbitrary. Suppose x is even. Then, by definition of even, there exists an integer k such that $x = 2k$. Squaring both sides gives $x^2 = (2k)^2 = 4k^2$. Since k is an integer, k^2 is an integer. Therefore, $x^2 = 4k^2 = 2(2k^2)$, which implies that x^2 is even. Thus, for all $x \in \mathbb{Z}$, if x is even, then x^2 is even. $\square$

4. Prove: for all $c \in \mathbb{Z}$, if c^2 is even, then c is even. After processing the "for all", provide an **indirect** proof.

Proof. Let $c \in \mathbb{Z}$ be arbitrary. Suppose c is odd. Then, by definition of odd, there exists an integer k such that $c = 2k + 1$. Squaring both sides gives $c^2 = (2k + 1)^2 = 4k^2 + 4k + 1 = 2(2k^2 + 2k) + 1$. Since k is an integer, $2k^2 + 2k$ is an integer. Therefore, $c^2 = 2(2k^2 + 2k) + 1$, which implies that c^2 is odd. Thus, for all $c \in \mathbb{Z}$, if c^2 is even, then c is even. $\square$

5. Prove: for all $c \in \mathbb{Z}$, if c^2 is even, then c is even. After processing the "for all", provide a proof by contradiction.

Proof. Let $c \in \mathbb{Z}$ be arbitrary. Suppose c is odd. Then, by definition of odd, there exists an integer k such that $c = 2k + 1$. Squaring both sides gives $c^2 = (2k + 1)^2 = 4k^2 + 4k + 1 = 2(2k^2 + 2k) + 1$. Since k is an integer, $2k^2 + 2k$ is an integer. Therefore, $c^2 = 2(2k^2 + 2k) + 1$, which implies that c^2 is odd. This contradicts the assumption that c^2 is even. Thus, for all $c \in \mathbb{Z}$, if c^2 is even, then c is even. $\square$