Please note: answer the first four questions in light of the rules of inference (the "flowcharts") we have seen. The first four questions are very quick to answer, but the point is to have you process and practice the flowcharts we have seen. (Pay attention to "use" versus "prove".)

1. Say we know $\boxed{\text{Olaf likes warm hugs and Anna wants to build a snowman}}$. What (if anything) can we conclude? Briefly explain why.

    - Let p be the statement "Olaf like warm hugs".
    - Let q be the statement "Anna wants to build a snowman".

    We know $p \wedge q$ is true from the above proposition.

    From this, we can safely conclude that both p and q are true independently of each other, seen in the flowchart below.

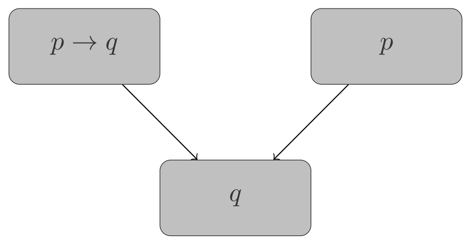
$$\begin{tikzcd} & p \wedge q \arrow[dl] \arrow[dr] & \\ p & & q \end{tikzcd}$$

2. Say we know If Olaf likes warm hugs, then Anna wants to build a snowman. In addition to this, say we also know Olaf likes warm hugs. What (if anything) can we conclude? Briefly explain why.

   - Let p be the statement "Olaf like warm hugs".
   - Let q be the statement "Anna wants to build a snowman".

   We know $p \to q$ is true from the above proposition.

   From this, we can safely conclude that q is true based upon $p \to q$ and p being true, seen in the flowchart below. The rule of inference used to derive this conclusion is *Modus Ponens*.



3. Say we know If Olaf likes warm hugs, then Anna wants to build a snowman. In addition to this, say we also know Olaf does not like warm hugs. What (if anything) can we conclude? Briefly explain why.

   - Let p be the statement, ".”
   - Let q be the statement, ".”
   - Naturally, we can let $\neg p$ be the statement, ".”

   We cannot conclude anything from the information available to us at this time. This is because the two propositions do not form a logically sound argument when used as a rule of inference. I have included a written example of this fallacy and the truth table which highlights my reasoning.

   $p \to q$

   $\neg p$

   We cannot conclude anything from the two above propositions.

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p	q	$\neg p$	$(p \to q)$
~~T~~	~~T~~	~~F~~	~~T~~
~~T~~	~~F~~	~~F~~	~~F~~
F	T	T	T
F	F	T	T
~~T~~	~~T~~	~~F~~	~~T~~
~~T~~	~~F~~	~~F~~	~~F~~
F	T	T	T
F	F	T	T

Table 1: Truth Table

(a) Remove rows where $p \to q$ is false.

(b) Remove rows where $\neg p$ is false (in other words, where p is true).

After this point, we are left with the following truth table:

p	q	$\neg p$	$(p \to q)$
F	T	T	T
F	F	T	T
F	T	T	T
F	F	T	T

Table 2: Truth Table

Given the truth values shown for , we cannot conclude anything about .

4. Say T is the set of all the world's turtles. Say $M(x)$ is the predicate $\boxed{x \text{ has 33 feet}}$. What steps would we have to take to prove $\boxed{\text{There exists } h \in T[M(h)]}$?

    - Let h be the statement, "h is a turtle".
    - Let $M(h)$ be the statement, "h has 33 feet".

    If we can prove that there exists at least one turtle in the set of all the world's turtles that has 33 feet, then we can conclude that $\exists h \in T s.t. M(h)$.

    This statement, if true, proves that there exists a turtle in the set of all the world's turtles that has 33 feet.

    The rule of inference used to derive this conclusion is *Existential Instantiation.*

5.

Theorem 1. *Say that c and d are both integers. Prove: if c is even and d is even, then $c - d$ is even.*

Proof. Let $c, d \in \mathbb{Z}$. Suppose that c is even and d is even. By definition, an integer n is even if there exists an integer k such that $n = 2k$. Since c is even, there exists an integer x such that $c = 2x$. Similarly, since d is even, there exists an integer y such that $d = 2y$. Substituting these values into the expression $c - d$, we have:

$$c - d = 2x - 2y = 2(x - y)$$

The value $(x - y)$ is an integer because the set of integers is closed under subtraction. From this, we can conclude that $c - d$ is even. Therefore, if c is even and d is even, then $c - d$ is even. $\square$