Hypothesis Testing Further Example

Solution

Let X denote the number of patients who survive.
Then $X \sim B(15, p)$, and it is required to test:

$H_0: p = 0.6$ (not effective)

$H_1: p > 0.6$ (effective; one-tailed)

Now assuming H_0 is true, $X \sim B(15, 0.6)$, and so

$$
\begin{aligned}
P(X = 12) &= 0.0633 \\
P(X = 13) &= 0.02194 \\
P(X = 14) &= 0.00470 \\
P(X = 15) &= 0.00047
\end{aligned}
$$

For 5% significance, what is the critical region?

Hence under H_0, the probability of:

15 patients surviving = 0.00047
14 or 15 patients surviving = 0.00517
13, 14 or 15 patients surviving = 0.02711
12, 13, 14 or 15 patients surviving is $= 0.09050$.

From the above (cumulative) probabilities it can be seen that the critical region for say a 5% test is $X \geq 13$

Since $P(X \geq 13) = 0.02711 < 0.05$

But $P(X \geq 12) = 0.09050 > 0.05$

The number of patients who actually survived was 12. This value does not lie in the critical region. Thus there is no evidence, at the 5% significance level, to suggest that the drug is effective.

Remember also that tables of the Cumulative Binomial Distribution Function may often provide an easier and quicker alternative for calculating probabilities or finding critical regions.