14 Probability

6.5 Concepts of trial, outcome, equally likely outcomes, sample space (U) and event.
The probability of an event A is

$$P(A) = \frac{n(A)}{n(U)}.$$

The complementary events A and A' (not A);

$$P(A) + P(A') = 1.$$

6.6 Combined events, the formula:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B).$$

$P(A \cap B) = 0$ for mutually exclusive events.

6.7 Conditional probability; the definition

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

Independent events; the definition

$$P(A \mid B) = P(A) = (A \mid B').$$

6.8 Use of Venn diagrams, tree diagrams and tables of outcomes to solve problems.

Suppose a fair die is rolled. Then there are six possible outcomes,

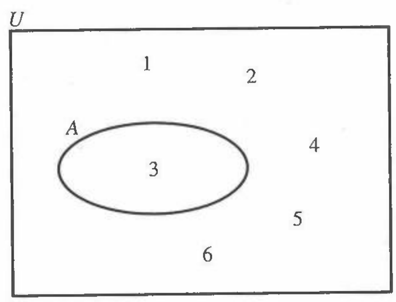
1 2 3 4 5 6

Suppose you want to know the likelihood of getting a 3.
There is only one way of getting a 3.

Therefore the probability of getting a 3 is one out of six, or $\frac{1}{6}$.

The list of all possible outcomes is called the **sample space**, and those outcomes which meet the particular requirement are called the **event**.

In the diagram, the rectangle U represents the sample space, and the oval represents the event A. The probability of A is defined by



$$P(A) = \frac{n(A)}{n(U)}$$

where $n(A)$ is the number of elements in the set A, and $n(U)$ is the number of elements in the whole sample space U. That is

$$\text{The probability of } A = \frac{\text{number of ways in which } A \text{ can happen}}{\text{number of possible outcomes}}$$

In the original example, A is the event that the score is 3, so $n(A) = 1$ and $n(U) = 6$. Therefore

$$P(A) = \frac{n(A)}{n(U)} = \frac{1}{6}$$

as shown earlier.

Example 1

Assuming that births are equally likely on any day of the week, find the probability that the next person you meet was born on a weekday.

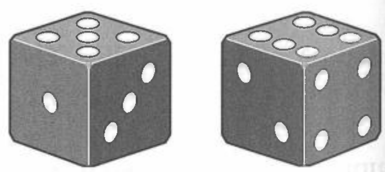
Let A be the event that the day is a weekday – Monday, Tuesday, Wednesday, Thursday or Friday. So $n(A) = 5$.
There are seven days in a week, so $n(U) = 7$.

Hence

$$P(A) = \frac{n(A)}{n(U)} = \frac{5}{7}$$

Example 2

Two fair dice are thrown. Find the probability that the total of the scores on the two dice is five.



There are 36 possible outcomes:

[1, 1]	[1, 2]	[1, 3]	[1, 4]	[1, 5]	[1, 6]
[2, 1]	[2, 2]	[2, 3]	[2, 4]	[2, 5]	[2, 6]
[3, 1]	[3, 2]	[3, 3]	[3, 4]	[3, 5]	[3, 6]
[4, 1]	[4, 2]	[4, 3]	[4, 4]	[4, 5]	[4, 6]
[5, 1]	[5, 2]	[5, 3]	[5, 4]	[5, 5]	[5, 6]
[6, 1]	[6, 2]	[6, 3]	[6, 4]	[6, 5]	[6, 6]

where, for example, [1, 2] means the outcome of getting a 1 on the first die and a 2 on the second die.

Let A be the event that the *total score is five.*

There are four ways of getting a *total score of five*:

[1, 4] [2, 3] [3, 2] [4, 1]

So $n(A) = 4$, and $n(U) = 36$.

Hence, $P(A) = \frac{n(A)}{n(U)} = \frac{4}{36} = \frac{1}{9}$

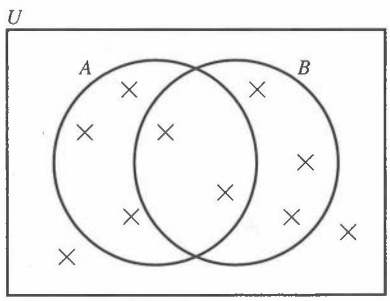
14.1 Combined events

Now consider two events, A and B. Two possible outcomes are

$A \cap B$, which means that A and B **both** occur

and $A \cup B$, which means that A occurs **or** B occurs.

Note that $A \cup B$ includes the case when A and B both occur.



Remember:
$\cap$ means the intersection of two sets.
$\cup$ means the union of two sets.

This is called a Venn diagram, from its inventor, the English logician John Venn (1834–1923).

From elementary set theory, you know that

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

So:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

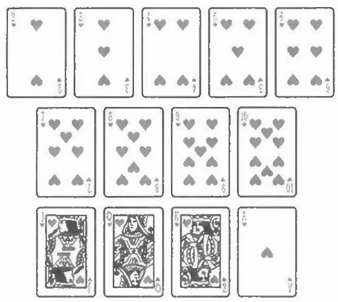
Playing cards

In many countries games are played with playing cards.

An ordinary pack consists of 52 cards.

Cards are in four different suits; hearts, clubs, diamonds, and spades. In each suit there are 13 cards, labelled Ace, 2, 3, 4, …, 10, Jack, Queen, and King.

A typical suit of hearts is illustrated. The diamonds and hearts are usually coloured red, and the clubs and spades are coloured black.



Example 1

A card is selected at random from an ordinary pack of 52 cards. Find the probability that the card is

a) a king b) a heart c) the king of hearts

d) either a king or a heart.

Let K denote the event that the card is a king, and let H denote the event that the card is a heart.

a) $P(K) = \frac{4}{52} = \frac{1}{13}$

b) $P(H) = \frac{13}{52} = \frac{1}{4}$

c) The event 'choosing the king of hearts' is written as $K \cap H$. So:

$$P(K \cap H) = \tfrac{1}{52}$$

d) Choosing the *king* or a *heart* is denoted by the event $K \cup H$,

$$P(K \cup H) = P(K) + P(H) - P(K \cap H)$$

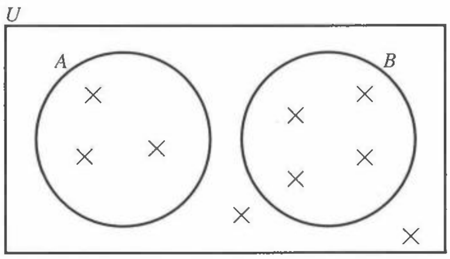
$$\therefore P(K \cup H) = \tfrac{4}{52} + \tfrac{13}{52} - \tfrac{1}{52}$$

$$= \tfrac{16}{52} = \tfrac{4}{13}$$

There is only one king of hearts in the pack.

Mutually exclusive events

Two events, A and B, are said to be **mutually exclusive** when they have *no outcome in common.* In other words, when $n(A \cap B) = 0$. The Venn diagram illustrates such a case.



A and B do not intersect, so $P(A \cap B) = 0$.

When A and B are mutually exclusive,

$$P(A \cup B) = P(A) + P(B)$$

Example 2

Given that the events A and B are mutually exclusive with $P(A) = \tfrac{3}{10}$ and $P(B) = \tfrac{2}{5}$, find the value of $P(A \cup B)$.

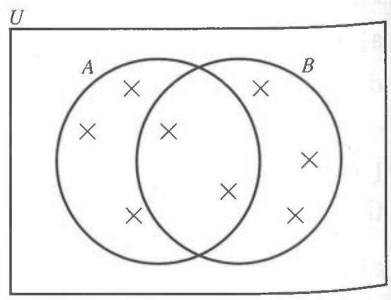
$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

However, $P(A \cap B) = 0$, so:

$$P(A \cup B) = \tfrac{3}{10} + \tfrac{2}{5} = \tfrac{7}{10}$$

Exhaustive events

Two events, A and B, are said to be **exhaustive** if together they include *possible outcomes* in the sample space. In other words, when $A \cup B = U$. The Venn diagram illustrates such a case.



There are no outcomes that are not included in A or B.

When A and B are exhaustive,

$$P(A \cup B) = 1$$

Example 3

Given $\mathrm{P}(X) = \frac{4}{5}$, $\mathrm{P}(Y) = \frac{1}{2}$ and $\mathrm{P}(X \cap Y) = \frac{3}{10}$, show that the events X and Y are exhaustive.

$$\mathrm{P}(X \cup Y) = \mathrm{P}(X) + \mathrm{P}(Y) - \mathrm{P}(X \cap Y)$$

Therefore,

$$\mathrm{P}(X \cup Y) = \tfrac{4}{5} + \tfrac{1}{2} - \tfrac{3}{10} = 1$$

Hence X and Y are exhaustive.

Complementary events

The **complement** of an event A (written A') consists of all outcomes in the sample space which are *not* contained in A. Notice that A and A' are both mutually exclusive:

$$\mathrm{P}(A' \cup A) = \mathrm{P}(A') + \mathrm{P}(A)$$

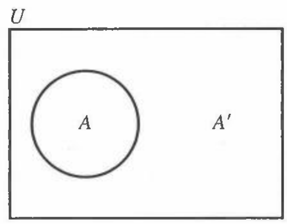
and exhaustive:

$$\mathrm{P}(A' \cup A) = 1$$

Therefore,

$$\mathrm{P}(A') + \mathrm{P}(A) = 1$$

$$\therefore \mathrm{P}(A') = 1 - \mathrm{P}(A)$$



A' is the event 'not A'.
$A \cup A' = U$

Example 4

Given $\mathrm{P}(A) = 0.55$, $\mathrm{P}(A \cup B) = 0.7$ and $\mathrm{P}(A \cap B) = 0.2$, find $\mathrm{P}(B')$.

$$\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$$
$$\therefore \mathrm{P}(B) = 0.7 + 0.2 - 0.55 = 0.35$$
$$\mathrm{P}(B') = 1 - \mathrm{P}(B)$$
$$\therefore \mathrm{P}(B') = 1 - 0.35 = 0.65$$

Example 5

Given $\mathrm{P}(G') = 5x$, $\mathrm{P}(H) = \frac{3}{5}$, $\mathrm{P}(G \cup H) = 8x$ and $\mathrm{P}(G \cap H) = 3x$, find the value of x.

$$\mathrm{P}(G) = 1 - \mathrm{P}(G') = 1 - 5x$$
$$\mathrm{P}(G \cup H) = \mathrm{P}(G) + \mathrm{P}(H) - \mathrm{P}(G \cap H)$$
$$\therefore 8x = (1 - 5x) + \tfrac{3}{5} - 3x$$
$$16x = \tfrac{8}{5}$$
$$\therefore x = \tfrac{1}{10}$$

Exercise 14A

1 A card is selected at random from a pack of 52 cards.
Find the probability that the card is

a) black

b) an honour [aces, kings, queens and jacks are honours]

c) a black honour

d) either black or an honour.

2 In a bag are 100 discs numbered 1 to 100. A disc is selected at random from the bag.
Find the probability that the number on the selected disc is

a) even

b) a multiple of five

c) a multiple of ten

d) either even or a multiple of five.

3 Two fair dice are thrown. Find the probability that

a) at least one of the dice shows a *four*

b) the sum of the scores on the two dice is *nine*

c) one of the dice shows a *four* and the other shows a *five*

d) either at least one of the dice shows a *four* or the total of the scores on the two dice is *nine*.

4 In a class half the pupils study Mathematics, a third study English and a quarter study both Mathematics and English. Find the probability that a pupil selected at random from the class studies either Mathematics or English.

5 Given $\mathrm{P}(A) = \frac{3}{5}$, $\mathrm{P}(B) = \frac{2}{3}$ and $\mathrm{P}(A \cap B) = \frac{1}{2}$, find the value of $\mathrm{P}(A \cup B)$.

6 Given $\mathrm{P}(X) = 0.37$, $\mathrm{P}(Y) = 0.48$ and $\mathrm{P}(X \cup Y) = 0.69$, find the value of $\mathrm{P}(X \cap Y)$.

7 Given $\mathrm{P}(A) = \frac{7}{10}$, $\mathrm{P}(A \cup B) = \frac{9}{10}$ and $\mathrm{P}(A \cap B) = \frac{3}{20}$, find the value of $\mathrm{P}(B)$.

8 Given $\mathrm{P}(F) = 4x$, $\mathrm{P}(G) = \frac{1}{3}$, $\mathrm{P}(F \cap G) = x$ and $\mathrm{P}(F \cup G) = 8x$, find the value of x.

9 Given that the events X and Y are mutually exclusive with $\mathrm{P}(X) = \frac{4}{7}$ and $\mathrm{P}(Y) = \frac{1}{3}$, find the value of $\mathrm{P}(X \cup Y)$.

10 Given $\mathrm{P}(S) = 0.34$, $\mathrm{P}(T) = 0.49$ and $\mathrm{P}(S \cup T) = 0.83$, show that the events S and T are mutually exclusive.

11 Given that the events M and N are mutually exclusive with $\mathrm{P}(M) = 3x$, $\mathrm{P}(N) = 4x$ and $\mathrm{P}(M \cup N) = 1 - x$, find the value of x.

12 Given that the events A and B are exhaustive with $P(A) = \frac{2}{3}$ and $P(B) = \frac{3}{4}$, find the value of $P(A \cap B)$.

13 Given $P(X) = \frac{5}{8}$, $P(Y) = \frac{11}{12}$ and $P(X \cap Y) = \frac{13}{24}$, show that the events X and Y are exhaustive.

14 Given that the events S and T are exhaustive with $P(S) = x$, $P(T) = 3x$ and $P(S \cap T) = 1 - 5x$, find the value of x.

15 When a roulette wheel is spun, the score will be a number from 0 to 36 inclusive. Each score is equally likely.
Find the probability that the score is

a) an even number

b) a multiple of 3

c) a multiple of 6

d) an odd number which is not a multiple of 3.

16 As a result of a survey of the households in a town, it is found that 80% have a video recorder and 24% have satellite television. Given that 15% have both a video recorder and satellite television, find the proportion of households with neither a video recorder nor satellite television.

17 The children at a party were asked about their pets.
Two-thirds had a dog and three-quarters had a cat.
Given that half the children had both a cat and a dog, calculate the probability that a child selected at random is found to have neither a cat nor a dog.

18 Given $P(C) = 0.44$, $P(C \cap D) = 0.21$ and $P(C \cup D) = 0.83$, find $P(D')$.

19 Given $P(G') = 3x$, $P(H) = 4x$, $P(G \cap H) = \frac{1}{4}$ and $P(G \cup H) = 9x$, find the value of x.

<u>20</u> Given $P(A) = 0.6$, $P(A \cap B') = 0.4$ and $P(A \cup B) = 0.85$, find the value of $P(B)$.

<u>21</u> Show that $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$.

14.2 Conditional probability

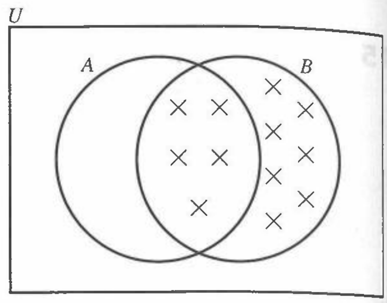
Suppose you are considering two related events, A and B, and you are told that B has occurred. This information may influence the likelihood of A occurring. For example, if you select a card at random from a pack of 52, then the probability that it will be a heart is $\frac{13}{52} = \frac{1}{4}$.

However, if you are given the additional information that the card selected is red, then this probability is increased to $\frac{13}{26} = \frac{1}{2}$.

The probability that event A will occur given that event B has already occurred is given by

$$\frac{n(A \cap B)}{n(B)}$$

This probability, which is denoted by $P(A \mid B)$, is illustrated in the Venn diagram.



$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

Going back to the pack of cards, A is the event *obtain a heart* and B is the event *obtain a red card*, so $A \cap B$ is the event *obtain a heart* (all hearts being red).

Now $P(A \cap B) = \frac{13}{52}$ and $P(B) = \frac{26}{52}$. Therefore,

$$P(A \mid B) = \frac{\frac{13}{52}}{\frac{26}{52}} = \frac{1}{2}$$

Example 1

Two fair dice are thrown. Find the probability that one of the dice shows a *four* given that the total on the two dice is *ten*.

Let F denote the event that one of the dice shows a four and T denote the event that the total on the two dice is ten.

[1, 1]	[1, 2]	[1, 3]	[1, 4]	[1, 5]	[1, 6]
[2, 1]	[2, 2]	[2, 3]	[2, 4]	[2, 5]	[2, 6]
[3, 1]	[3, 2]	[3, 3]	[3, 4]	[3, 5]	[3, 6]
[4, 1]	[4, 2]	[4, 3]	[4, 4]	[4, 5]	[4, 6]
[5, 1]	[5, 2]	[5, 3]	[5, 4]	[5, 5]	[5, 6]
[6, 1]	[6, 2]	[6, 3]	[6, 4]	[6, 5]	[6, 6]

Then $F \cap T$ means [4, 6] or [6, 4], and T means [4, 6] or [5, 5] or [6, 4].

Therefore,

$$P(F \cap T) = \frac{2}{36} \quad \text{and} \quad P(T) = \frac{3}{36}$$

Then: $P(F \mid T) = \dfrac{P(F \cap T)}{P(T)}$

$$\therefore P(F \mid T) = \frac{\frac{2}{36}}{\frac{3}{36}} = \frac{2}{3}$$

Example 2

Given $P(A) = \frac{1}{2}$, $P(A \mid B) = \frac{1}{4}$ and $P(A \cup B) = \frac{2}{3}$, find $P(B)$.

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

$$\therefore \frac{1}{4} = \frac{P(A \cap B)}{P(B)}$$

$$\therefore P(A \cap B) = \frac{1}{4}P(B) \qquad [1]$$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

So: $\frac{2}{3} = \frac{1}{2} + P(B) - P(A \cap B)$

$$\therefore \frac{1}{6} = P(B) - P(A \cap B) \qquad [2]$$

Eliminating $P(A \cap B)$ between [1] and [2] gives

$$\frac{1}{6} = P(B) - \frac{1}{4}P(B)$$

$$\frac{3}{4}P(B) = \frac{1}{6}$$

$$\therefore P(B) = \frac{2}{9}$$

Independence

Two events A and B are said to be **independent** if

$$P(A \mid B) = P(A)$$

In this case:

$$\frac{P(A \cap B)}{P(B)} = P(A)$$

$$\therefore P(A \cap B) = P(A) \times P(B)$$

Example 3

A card is picked at random from a pack of 52 and a fair die is thrown. Find the probability that the card is the ace of spades and the die shows a three.

You could choose to list all the 52×6 possible outcomes, but it is easier to use independence and write

$$\begin{aligned} P(\text{ace of spades and a two}) &= P(\text{ace of spades}) \times P(\text{two}) \\ &= \frac{1}{52} \times \frac{1}{6} \\ &= \frac{1}{312} \end{aligned}$$

Tree diagrams

Sometimes, rather than list all the possible outcomes, it is easier to view probabilities on a **tree diagram**. The next example shows how this is done.

Example 4

A fair coin is thrown twice. Find the probability that the result is a head and a tail, in either order.

You could just calculate this simple example like this:

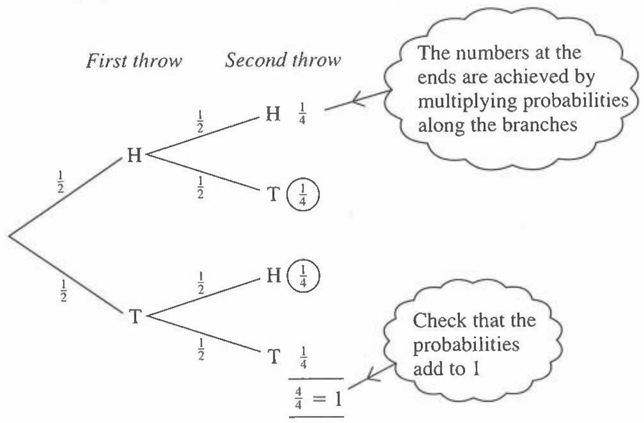
Without using a tree diagram, and using independence to multiply probabilities, and mutual exclusivity to add them,

$$\begin{aligned} \text{P(head and tail)} &= \text{P}(HT) + \text{P}(TH) \\ &= \tfrac{1}{2} \times \tfrac{1}{2} + \tfrac{1}{2} \times \tfrac{1}{2} \\ &= \tfrac{1}{4} + \tfrac{1}{4} \\ &= \tfrac{2}{4} = \tfrac{1}{2} \end{aligned}$$

The outcomes *HT* and *TH* are mutually exclusive.

The events *H* and *T* are independent, so multiply the probabilities.

Alternatively, you can draw a tree diagram like this:



You want all those combinations of branches which include one head and one tail.

To calculate P(*HT*), multiply the probabilities along the branches.

For example,

$$\text{P}(HT) = \tfrac{1}{2} \times \tfrac{1}{2} = \tfrac{1}{4}$$

You can now easily see that the probability you want is the sum of those circled on the diagram: HT and TH. That is:

$$\tfrac{1}{4} + \tfrac{1}{4} = \tfrac{2}{4} = \tfrac{1}{2}$$

Example 4 is a very simple one. However, tree diagrams can be very useful in more complicated problems.

Example 5

A bag contains four red discs and five blue discs.
Three discs are selected at random, without replacing them.
Find the probably that two are red and the other is blue.

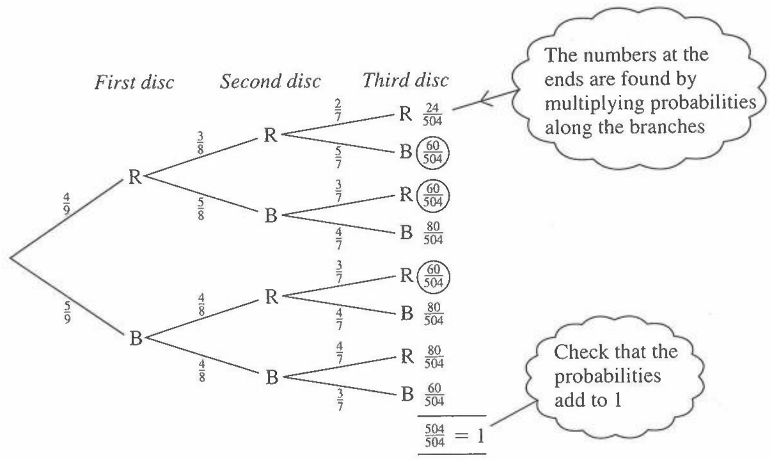
The probability of the first disc being red is $\frac{4}{9}$. That leaves 8 discs, of which 3 are red. The probability of the second disc being red is thus $\frac{3}{8}$. The probability of the third disc being blue is $\frac{5}{7}$, as there are still 5 blue discs left.

So $\mathrm{P}(RRB) = \frac{4}{9} \times \frac{3}{8} \times \frac{5}{7}$ and so on.

So:

$$\begin{aligned} \mathrm{P}(\text{two red and one blue}) &= \mathrm{P}(RRB) + \mathrm{P}(RBR) + \mathrm{P}(BRR) \\ &= \left(\tfrac{4}{9} \times \tfrac{3}{8} \times \tfrac{5}{7}\right) + \left(\tfrac{4}{9} \times \tfrac{5}{8} \times \tfrac{3}{7}\right) \\ &\quad + \left(\tfrac{5}{9} \times \tfrac{4}{8} \times \tfrac{3}{7}\right) \\ &= \tfrac{60}{504} + \tfrac{60}{504} + \tfrac{60}{504} \\ &= \tfrac{5}{14} \end{aligned}$$

Alternatively, you can use a tree diagram:



You want all those combinations of branches which include two red discs and one blue disc: for example, *RRB*.
To calculate $\mathrm{P}(RRB)$, multiply the probabilities along the branches:

$$\mathrm{P}(RRB) = \frac{4}{9} \times \frac{3}{8} \times \frac{5}{7} = \frac{60}{504}$$

The probability you want is the sum of those circled on the diagram: *RRB*, *RBR* and *BRR*. That is:

$$\frac{60}{504} + \frac{60}{504} + \frac{60}{504} = \frac{5}{14}$$

Exercise 14B

In questions **1** to **6** two fair dice are rolled, and the outcome is the total of the scores on each of the two dice. It will help if you first draw a grid to illustrate the 36 possible outcomes for the total score.

1 Find the probability that one of the dice shows a *two* given that the total on the two dice is *six*.

2 Find the probability that one of the dice shows a *three* given that the total on the two dice is *seven*.

3 Find the probability that the total score is *eight* given that at least one of the dice shows a *two*.

4 Find the probability that the scores on each of the two dice are the same given that the total on the two dice is *four*.

5 Find the probability that the total on the two dice is *eight* given that neither die shows a *five*.

6 Find the probability that neither dice shows a *four* given that the total on the two dice is *nine*.

7 Three fair coins are tossed.

a) List the eight possible outcomes.

b) Find the probability that all three coins show *heads* given that there is an odd number of *heads* showing.

8 Four fair coins are tossed.

a) List the 16 possible outcomes.

b) Find the probability that the coins show two *heads* and two *tails* given that there is at least one *tail*.

9 A computer randomly chooses two different numbers from the list 1, 2, 3, 4, 5.

a) Draw a grid to show the 20 possible outcomes.

b) Find the probability that both numbers are odd given that neither is a *four*.

10 Five identical cards are labelled A, B, C, D, E. The cards are placed in a box, and a card is selected at random. This card is replaced in the box, and a second random selection is made.

a) Draw a grid to show the 25 possible outcomes.

b) Given that neither card has a letter which appears in the word VICTORY, calculate the probability that both are vowels.

11 Given $P(A \cap B) = \frac{1}{3}$ and $P(B) = \frac{3}{5}$, find $P(A \mid B)$.

12 Given $P(A \mid B) = \frac{1}{5}$ and $P(B) = \frac{1}{2}$, find $P(A \cap B)$.

13 Given $P(A \mid B) = \frac{5}{6}$, $P(A) = \frac{3}{4}$ and $P(B) = \frac{2}{3}$, find $P(A \cup B)$.

14 Given $P(A) = \frac{2}{5}$, $P(B) = \frac{7}{10}$ and $P(A \cup B) = \frac{4}{5}$, find $P(A \mid B)$.

15 Given $P(A) = \frac{2}{3}$, $P(B) = \frac{4}{9}$ and $P(A \mid B) = \frac{3}{4}$, find $P(A \cup B)$.

16 Given $P(A) = 0.2$, $P(A \mid B) = 0.3$ and $P(A \cup B) = 0.4$, find $P(B)$.

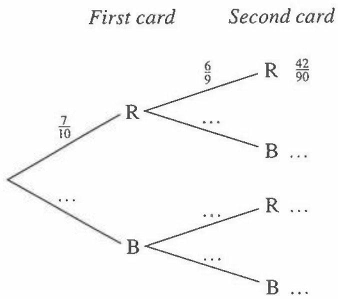
17 Given $P(A) = \frac{1}{3}$, $P(B) = \frac{3}{8}$ and $P(A \cup B) = \frac{7}{12}$, show that A and B are independent.

18 Given $P(A) = x$, $P(B) = 2x$, $P(A \cup B) = 1 - x$ and $P(A \mid B) = \frac{1}{5}$, find the value of x.

19 A bag contains seven red cards and three blue cards. Two cards are selected at random.

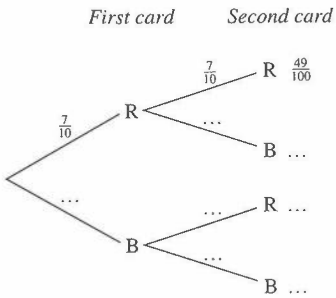
a) Copy and complete the tree diagram of possible outcomes.

b) Find the probability that both cards are the same colour.



20 Another bag also contains seven red cards and three blue cards. This time a card is selected and replaced. A second card is then selected.

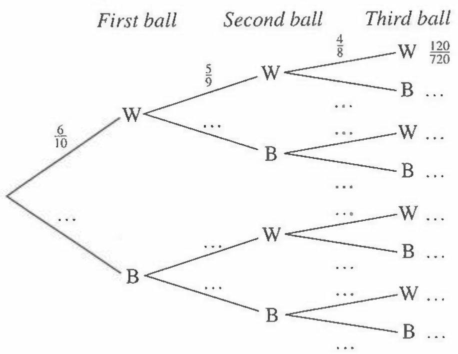
a) Copy and complete the tree diagram of possible outcomes.



b) Find the probability that the cards are a different colour.

21 A box contains six white balls and four black balls. Three balls are selected at random.

a) Copy and complete the tree diagram of possible outcomes.



b) Find the probability that two of the selected balls are white and the other is black.

22 Two sacks, *A* and *B*, each contain a mixture of plastic and leather rugby balls. Sack *A* contains four plastic balls and two leather balls, and sack *B* contains three plastic balls and five leather balls. A sack is selected at random and a ball is taken from it.

a) Represent this information on a tree diagram.

b) Calculate the probability that the ball is leather.

c) Given that the ball is leather, calculate the probability that it came from sack *A*.

23 A girl cannot decide what to wear for a party. She will wear either trousers or a skirt. She has two wardrobes. In wardrobe *A* are two pairs of trousers and three skirts, and in wardrobe *B* are five pairs of trousers and one skirt. She decides to select a wardrobe at random and then randomly select one item from that wardrobe.
Draw a tree diagram and find the probability that

a) she wears trousers

b) she selected wardrobe *A*, given that she wears trousers.

24 An eccentric mathematics teacher decides to award a prize to a pupil selected from one of his three classes. Class 1 has five boys and seven girls; class 2 has eight boys and two girls; and class 3 has three boys and three girls. He decides to award the prize by selecting a class at random and then randomly selecting a pupil from that class.
Use a tree diagram to find the probability that

a) the selected pupil is a boy

b) the selected pupil is from class 3, given that the selected pupil is a boy.

25 A bag contains eleven discs numbered 1 to 11. Two discs are selected at random from the bag. Given that the sum of the selected numbers is even, use a tree diagram to find the probability that the number on each of the selected discs is odd.

26 All of the pupils in two classes sit the same examination. In class A, 15 out of 20 pupils pass; in class B, 8 out of 12 pupils pass. A pupil is selected at random from the 32 pupils who have taken the examination. Given that she passes, use a tree diagram to find the probability that she was in class A.

<u>27</u> In a given week, 200 people take a driving test in a given centre. Of the 120 who pass, 50 were taking the test for the first time; and of those who failed, 60 were taking the test for the first time. Estimate the probability of passing the driving test at a first attempt at this centre.

Summary

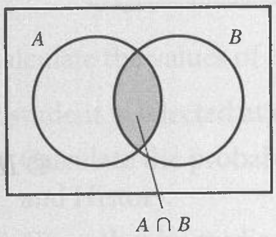
You should know how to ...

▶ Calculate the probability of an event.

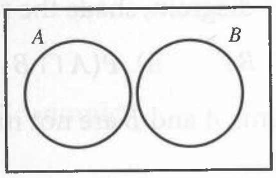
▷ For an event A, $P(A) = \dfrac{n(A)}{n(U)}$ where $n(A)$ is the set of outcomes comprising event A, and $n(U)$ is the total set of all possible outcomes.

▶ Calculate probabilities associated with more than one event.

▷ $P(A \cup B) = P(A) + P(B) - P(A \cap B)$



▷ If A and B are mutually exclusive, $P(A \cup B) = P(A) + P(B)$



▷ $P(A) + P(A') = 1$, where A' is the complementary event to A (not A).

▷ $P(A \mid B) = \dfrac{P(A \cap B)}{P(B)}$ where $P(A \mid B)$ is a conditional probability

▷ If A and B are independent events

$P(A \mid B) = P(A)$, and $P(A \cap B) = P(A) \times P(B)$

Revision exercise 14

1 In a survey, 100 students were asked, 'Do you prefer to watch television or play sport?' Of the 46 boys in the survey, 33 said they would choose sport, while 29 girls made this choice.

By copying and completing this table, find the probability that

	Boys	Girls	Total
Television			
Sport	33	29	
Total	46		100

a) a student selected at random prefers to watch television

b) a student prefers to watch television, given that the student is a boy.

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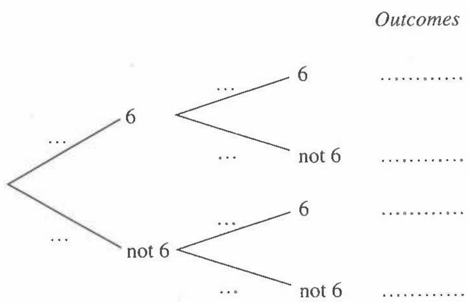
2 Two ordinary, six-sided dice are rolled and the total scored is noted.

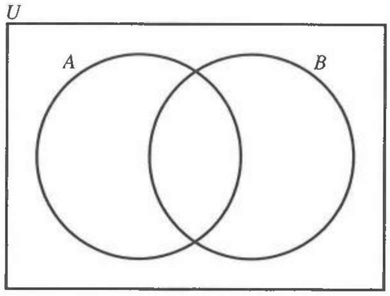
a) Copy and complete the tree diagram by entering probabilities and listing outcomes.

b) Find the probability of getting one or more sixes.



© *IBO[2000]*

3 This Venn diagram shows a sample space U and events A and B.



$n(U) = 36$, $n(A) = 11$, $n(B) = 6$ and $n(A \cup B)' = 21$.

a) On a copy of the diagram, shade the region $(A \cup B)'$.

b) Find i) $n(A \cap B)$ ii) $P(A \cap B)$.

c) Explain why events A and B are not mutually exclusive. © *IBO [2000]*

4 A bag contains 10 red balls, 10 green balls and 6 white balls. Two balls are drawn at random from the bag without replacement. What is the probability that they are different colours? © *IBO [2001]*

5 A box contains 22 red apples and 3 green apples. Three apples are selected at random, one after the other without replacement.

a) The first two apples are green. What is the probability that the third apple is red?

b) What is the probability that exactly two of the three apples are red? © *IBO [2002]*

6 The events A and B are independent and $P(A \cap B) = 0.6$, $P(B) = 0.8$. Find

a) $P(A | B)$ b) $P(A | B')$ © *IBO [1998]*

7 For the events A and B, $P(A) = 0.3$ and $P(B) = 0.4$.

a) Find $P(A \cup B)$ if A and B are independent events.

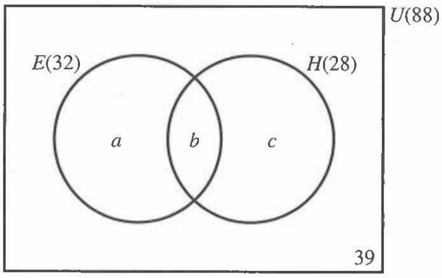
b) Find $P(A' \cap B')$ if A and B are mutually exclusive events. © *IBO [1996]*

8 In a certain country, 40% of the population own a car and of these 90% are male. The two sexes are equally represented in the population.

a) Find the probability that a randomly selected person is a male car owner.

b) What percentage of women do not own a car? © *IBO [1998]*

9 In a school of 88 boys, 32 study Economics (E), 28 study History (H) and 39 do not study either subject.
This information is represented in the Venn diagram.



a) Calculate the values of a, b and c.

b) A student is selected at random.

i) Calculate the probability that he studies both Economics and History.

ii) Given that he studies Economics, calculate the probability that he does not study History.

c) A group of three students is selected at random from the school.

i) Calculate the probability that none of these students studies Economics.

ii) Calculate the probability that at least one of these students studies Economics.

© *IBO[2003]*

10 A student sits three examinations in Physics, Chemistry and French. From past experience she estimates that the probability of passing Physics is 0.4. If she passes Physics the probability of passing Chemistry is 0.6, otherwise it is 0.3. Passing French is independent of her performance in the other two subjects and has a probability of 0.45.

a) Draw a tree diagram to represent this information.

b) Find the probability that she

i) passes all three subjects

ii) passes exactly one subject

iii) passes at least one subject.