Statistical Inference Tutorial Questions

1. Sport Shoes

For this question we need to compare two variables with the same response for the same subjects as each athlete in the study tests both brands of trainer.

The options that we have available are to run a paired t-test which requires the assumption of normality of the difference variable or the Wilcoxon signed rank test for non-normal, ranked or scored data.

The output of the distribution analysis is as follows:

Tests for Normality				
Test	**Statistic**		**p Value**	
Shapiro-Wilk	**W**	0.987281	**Pr < W**	0.0259
Kolmogorov-Smirnov	**D**	0.072792	**Pr > D**	<0.0100
Cramer-von Mises	**W-Sq**	0.219548	**Pr > W-Sq**	<0.0050
Anderson-Darling	**A-Sq**	1.221796	**Pr > A-Sq**	<0.0050

As all p-values are less than 0.05, which represents the 5% significance level of the test, the null hypothesis is rejected in favour of the alternative and we can say that there is evidence to suggest that the difference variable is not normally distributed. Hence, we will move forward with the non-parametric alternative.

Difference: branda - brandb

N	**Mean**	**Std Dev**	**Std Err**	**Minimum**	**Maximum**
250	-15.6280	14.3225	0.9058	-55.0000	25.0000

Mean	**95% CL Mean**		**Std Dev**	**95% CL Std Dev**	
-15.6280	-17.4121	-13.8439	14.3225	13.1675	15.7014

By examination of the 95% confidence interval for the difference for each brand it can be seen that the CI does not include 0. In fact, it ranges from -13.8439 to -17.4121. Therefore, this suggests that there is a significant difference between the durability of the trainers.

The hypotheses for the non-parametric test are:

H_o: the distribution of difference is symmetrical around *D0;*

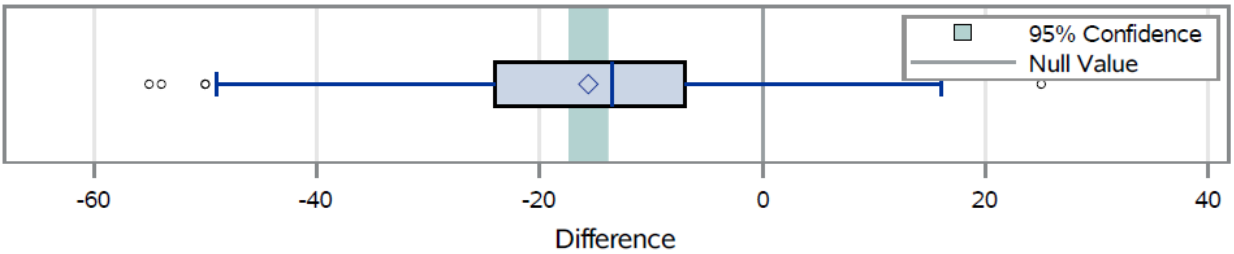
H_1: the distribution of difference tends to be larger or smaller than *D0;*

Variable: _Difference_ (Difference: branda - brandb)

Tests for Location: Mu0=0				
Test	**Statistic**		**p Value**	
Student's t	**t**	-17.2526	**Pr > |t|**	<.0001
Sign	**M**	-92.5	**Pr >= |M|**	<.0001
Signed Rank	**S**	-13400.5	**Pr >= |S|**	<.0001

Furthermore, examining the result of the Wilcoxon Signed Rank test it can be seen that the p-value achieved is < 0.0001, therefore significant at the 5% level. Hence, we reject the null hypothesis and conclude that there is a difference between the two brands of trainer.

The box plot helps us to further visualise this.



It is clear that Brand B trainer is outperforming Brand A in terms of the number of weeks until the trainers were regarded as unusable.

2. Pain Experiment

For this question we are asked to compare two independent variables, as subjects have been assigned to one of two groups, the Placebo or the Drug. Therefore we have to determine if we meet the assumptions to carry out a two-sample t-test, which is that the response in each group is normally distributed, or whether we need to perform the Wilcoxon (Mann Whitney) test, which is for Non-normal, ranked or scored data.

The analysis variable within the data is actually a score variable where subjects were asked to rank their rating of pain, therefore this variable contains ranked data so we need to perform the Wilcoxon Two-Sample (Mann Whitney) test.

The hypotheses for this case are: $H_o: f(x_1) = f(x_2), H_1: f(x_1) \neq f(x_2)$ where $f(x_1)$ is the distribution of the placebo group and $f(x_2)$ is the distribution of the drug group.

The output is as follows.

Wilcoxon Two-Sample Test					
				t Approximation	
Statistic	**Z**	**Pr < Z**	**Pr > |Z|**	**Pr < Z**	**Pr > |Z|**
Z includes a continuity correction of 0.5.					
145.0000	-1.3459	0.0892	0.1783	0.0948	0.1895

The value of interest to us is the **Two-sided t approximation**. We would use the one-sided p-value if we were performing a 1 tailed test.

A p-value of 0.1895 is not significant at the 5% level indicating that there is no significant difference, so we do not reject the null hypothesis. In conclusion there is no significant difference between the pain experienced by the subjects in the two groups.

3. Eye Experiment

A study compared the use of two varieties of eye droplets on a set of subjects. Therefore we are looking to compare two variables with the same response for the same subjects. As the patients recorded marks between 1 and 4 for four different regions of the eye, this data is therefore scored/ranked data, therefore we need to carry out a Wilcoxon Signed Rank test.

The hypotheses for this test are:

H_o: the distribution of difference is symmetrical around *D0;*

H_1: the distribution of difference tends to be larger or smaller than *D0;*

Difference: LeftEye - RightEye

Mean	95% CL Mean		Std Dev	95% CL Std Dev	
-2.2917	-4.6851	0.1018	5.6682	4.4054	7.9511

By examination of the 95% confidence interval for the difference for each eye it can be seen that the CI just includes 0, however is skewed in favour of a negative difference. Therefore, this suggests that there might be a difference between the 2 eye droplets.

The output on the distribution analysis of this difference variable is as follows.

Tests for Location: Mu0=0				
Test	**Statistic**		**p Value**	
Student's t	t	-1.98066	Pr > |t|	0.0597
Sign	M	-3	Pr >= |M|	0.2863
Signed Rank	S	-62	Pr >= |S|	0.0405

Examining the signed-rank test statistic, the p-value is 0.0405, this is less than 0.05, therefore is significant at 5%. We therefore need to reject the null hypothesis and conclude that there is a significant difference between the results of the two eye droplets.

4. Carbon Monoxide

The level of carbon monoxide emissions of two of the major cigarette brands in a region of South East Asia are investigated and the local authorities wish to determine if there are significant differences between the brands. This requires us to compare two independent variables.

First, we need to check for normality across each of the brands.

brand = Browns

Tests for Normality				
Test	**Statistic**		**p Value**	
Shapiro-Wilk	**W**	0.849156	**Pr < W**	<0.0001
Kolmogorov-Smirnov	**D**	0.129019	**Pr > D**	<0.0100
Cramer-von Mises	**W-Sq**	0.873904	**Pr > W-Sq**	<0.0050
Anderson-Darling	**A-Sq**	5.282653	**Pr > A-Sq**	<0.0050

brand = Witters

Tests for Normality				
Test	**Statistic**		**p Value**	
Shapiro-Wilk	**W**	0.965837	**Pr < W**	0.0009
Kolmogorov-Smirnov	**D**	0.075625	**Pr > D**	0.0351
Cramer-von Mises	**W-Sq**	0.17923	**Pr > W-Sq**	0.0096
Anderson-Darling	**A-Sq**	1.229752	**Pr > A-Sq**	<0.0050

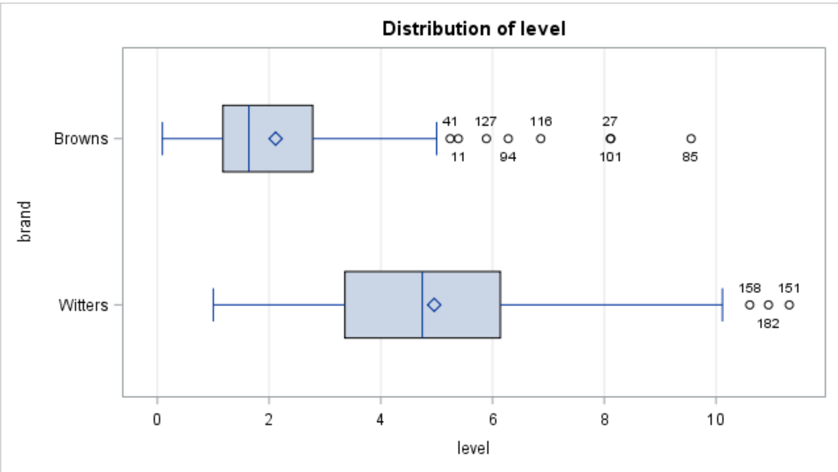
As all p-values are less than 0.05, which represents the 5% significance level of the test, the null hypothesis is rejected in favour of the alternative and we can say that there is evidence to suggest that the variables are not normally distributed. Hence, we will move forward with the non-parametric alternative.

The hypotheses for this case are: $H_o: f(x_1) = f(x_2), H_1: f(x_1) \neq f(x_2)$ where $f(x_1)$ is the distribution of the placebo group and $f(x_2)$ is the distribution of the drug group.

Wilcoxon Two-Sample Test					
				t Approximation	
Statistic	**Z**	**Pr < Z**	**Pr > |Z|**	**Pr < Z**	**Pr > |Z|**
Z includes a continuity correction of 0.5.					
14052.00	-11.3444	<.0001	<.0001	<.0001	<.0001

Examining the results of the Wilcoxon Two-Sample Test, we achieve a p-value of <0.0001. This is significant at 5%, therefore we reject the null hypothesis and conclude that there is a significant difference between the carbon monoxide emissions for the two brands of cigarettes.

The box plot further emphasizes this.



5. Concrete

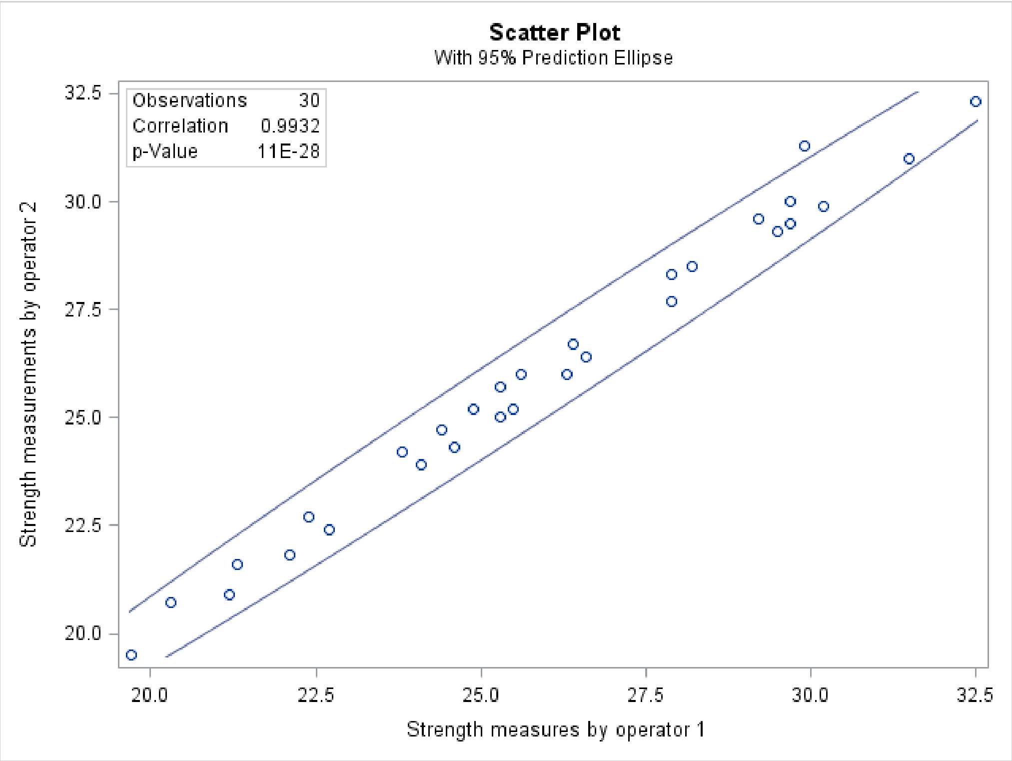
We are asked to determine whether a relationship exists between the measured strength of concrete by operator 1 and operator 2, therefore we need to determine if there is a relationship between the two variables. A correlation needs to be performed between each of the strength variables and the correlation coefficient used to interpret the analysis should either be Pearson's, if both variables are independently normally distributed and Spearman's rank, for non-normal, ranked or scored data.

After testing both variables independently it is found that they are both normally distributed, therefore a Pearson's correlation coefficient can be used.

The hypothesis in this case are: $H_0: \rho = 0, H_1: \rho \neq 0$ .The output was as follows.

Pearson Correlation Coefficients, N = 30 Prob > |r| under H0: Rho=0	
Strength measurements by operator 2	0.99319
	<.0001

By first examining the p-value of the test it can be seen that it is <0.0001, therefore significant at the 5% level. Therefore we reject the null hypothesis and conclude that there is a significant relationship between the strength of operator 1 and operator 2. The strength of the relationship is determined by the correlation coefficient which is 0.99319 indicating a strong positive relationship.



6. Salary

We are asked to determine if the collective outcome of a pupils GCSE exams has an association with the salary of the teacher, therefore a correlation is required.

We are told that the salary of the teacher is given by the rank of the average wage of the A-level teachers, therefore this data will require us to perform a correlation using Spearman's correlation coefficient to interpret the results.

The hypothesis in this case are: $H_0: \rho = 0, H_1: \rho \neq 0$ .The output was as follows.

Spearman Correlation Coefficients, N = 10 Prob > |r| under H0: Rho=0	
	Salary
Points	-0.63636 0.0479

By first examining the p-value of the test it can be seen that it is 0.0476, therefore significant at 5% level. Therefore we reject the null hypothesis and conclude that there is a significant relation between the salary of the teacher and a pupils GCSE results.

The strength of the relationship is determined by the correlation coefficient which is -0.63636 indicating a moderate to strong negative relationship, this indicates that the teachers ranked 1, have the best outcomes of pupils at GCSE.