

1. Given $f(n) = 2n^2 + 7n + 3$, answer the following questions by “yes” or “no”. [10 points]

- a. $f(n) = O(n^2)$ Yes
- b. $f(n) = \text{Big Omega}(n^2)$ Yes
- c. $f(n) = \text{Big Theta}(n^2)$ Yes
- d. $f(n) = O(n)$ No
- e. $f(n) = \text{Big Omega}(2^n)$ No

2. Given $f(n) = 7n^2 + 2n + 1$, prove that $f(n) = O(n^2)$

The definition of Big O states A function belongs to Big O if there exists constants c and n_0 s.t.
 $0 \leq f(n) \leq c \cdot g(n)$ for all $n \geq n_0$

Therefore,

$$7n^2 + 2n + 1 \leq c \cdot n^2 \text{ for all } n \geq n_0$$

Let $c = 8$, and $n_0 = 3$

$$7n^2 + 2n + 1 \leq 8 \cdot n^2 \text{ for all } n \geq 3$$

We can test the base case for $n=3$

$$7 \cdot 3^2 + 2 \cdot 3 + 1 \leq 8 \cdot 3^2$$

$70 \leq 72$ which is True therefore $f(n) = O(n^2)$.