BALL and BEAM

EXPERIMENT and SOLUTION

QUANSER CONSULTING

BALL AND BEAM EXPERIMENT

This experiment is designed specifically for the ball and beam setup BB01 to be used in conjunction with the Plant SRV01 and the Power Module PA0103. The purpose of the experiment is to design a computer controlled feedback system which stabilizes the system and allows you to command the ball to a desired location on the track.

SYSTEM DESCRIPTION

The ball and beam attachment consists of an especially designed track on which a stainless steel ball can roll. One side of the track is a nickel-chromium wire-wound resistor and the other side is a steel rod. When the ball rolls on these two components, it acts as a wiper similar to that of a potentiometer. The position of the ball along the track can be obtained by measuring the voltage at the steel rod when the sensor resistor is properly biased. The two leads from the resistor are brought out to a connecting strip on the support block. These are in turn connected to one 240 Ohm resistor each. A wire connected to the steel rod is also brought to the connecting strip. Keep the sensor and ball clean. Wipe them using the contact cleaner supplied only. **Never use water**. The gears that are supplied with the setup should be used to replace the gears mounted on the plant. This allows the motor to deliver the required torque as well as to increase the range of the angle which the bar can be made to tilt. The entire setup, when assembled, should look like the diagram shown in Figure 1. All system specifications are supplied on the last page of this handout.

MATHEMATICAL MODELLING

Rolling ball

Consider first the dynamics of the rolling ball. The acceleration of the ball is given by the equation (can you derive it?):

$$a = -(5/7)\ g \sin \alpha$$

where α is the tilt angle at the beam and g is the gravitational acceleration.

This equation, when linearized about small values of α, results in the transfer function (double integrator):

$$x(s)/\alpha(s) = -(5g/7)/s^2$$

where x is the position of the ball along the track.

The plant

The tilt angle at the beam is controlled by the angle at the output of the plant θ. The plant consists of a DC motor and a feedback potentiometer. The motor has a built in gearbox of ratio 15:1. The output of the gearbox is connected to the small gear (0.5") which in turn drives the larger gear (2.5"). This results in a further speed reduction ratio of 5:1. The final gear ratio from motor to output is 5x15 = 75:1.

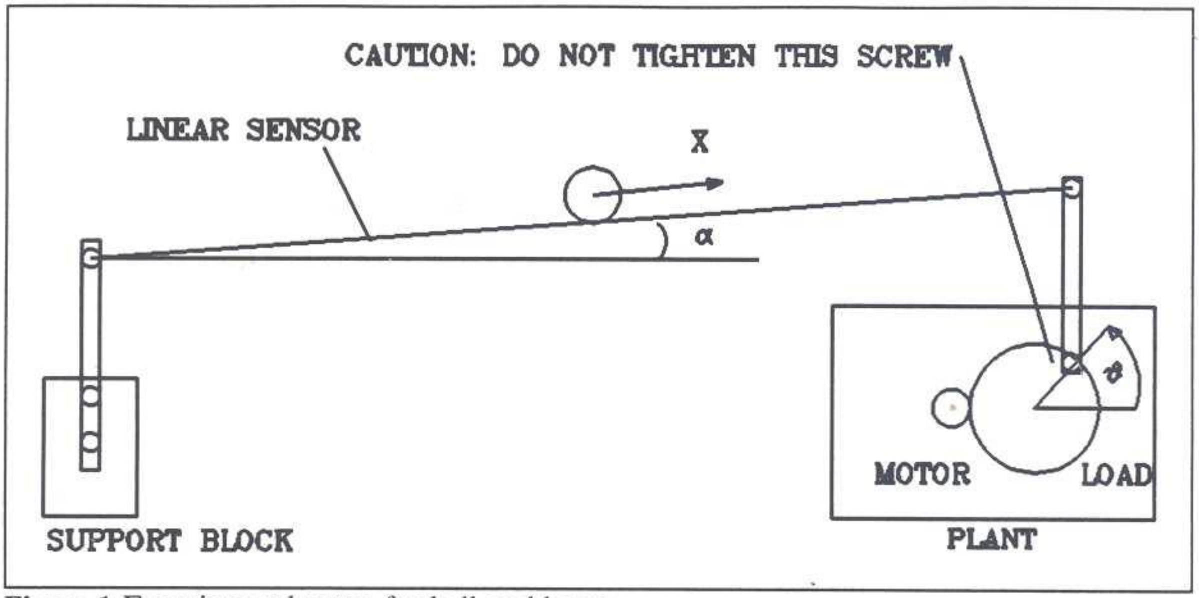

Figure 1 Experimental setup for ball and beam

The system is governed by the following dynamical equations:

Electrically, assuming negligible armature inductance,

$$V_{in} = I\,R_m + K_m\,\omega_m$$

where

V_{in}	Input voltage (Volt)
I	Armature current (Amp)
R_m	Armature resistance (Ohm)
K_m	Motor torque constant (N-m/Amp)
ω_m	Angular velocity of output(rad/sec)

Mechanically, the torque produced at the motor shaft is given by:

$$T_m = K_m\,I$$

This is counteracted by the total load inertia, ie:

$$T_l = T_m = (J_l\,\omega' + B\,\omega)/K_g$$

where

ω	Angular velocity of load (rad/sec) = ω_m/K_g
J	Total load inertia (N-m-sec^2/rad)

B Total load friction (N-m sec/rad)
K_g Gear ratio (dimensionless)

Using the above equations, a transfer function for $\theta(s)/V_{in}(s)$ can be derived.

Finally, the relationship between α and θ is required. This relationship is nonlinear but can be approximated by the following equation:

$\alpha = (1/16)\ \theta$

Using the above equations and the constants specified, a complete linear model of the system can be derived.

CONTROL SYSTEM DESIGN

There are various methods which can be used to design a feedback controller for this system. It could be modeled as a fourth order multivariable system and a state feedback controller could be developed. A simpler method will be used in this experiment. Two second order systems will be designed. It is assumed that the dynamics of the plant are fast enough to be neglected. One closed loop system will be designed that controls the position of the ball through the input α. Another closed loop system will control α by controlling θ through the input V_{in}.

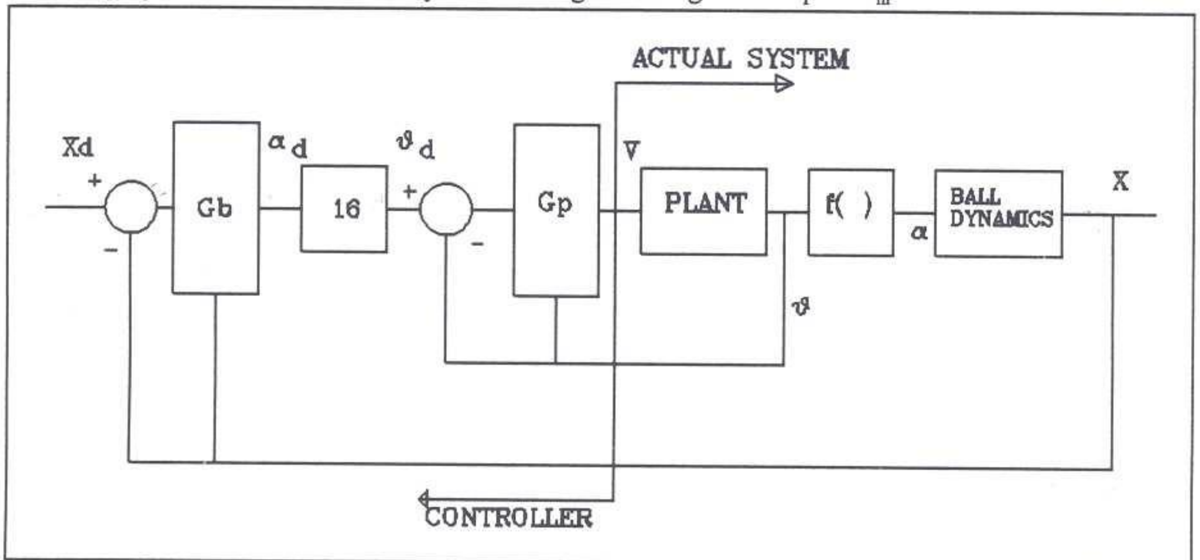

Figure 2 Control system for ball and beam

a) Stabilizing the ball

For the transfer function

$x(s)/\alpha(s) = (7g/8)/s^2$

derive the gains P_x and D_x for the control law:

$\alpha = P_x (x_d - x) + D_x x'$

such that the closed loop system $\alpha(s)/\alpha_d(s)$ has a peak time of 3.0 seconds and a damping ratio of 0.707.

b) Stabilizing the plant

Derive the transfer function for the plant. It should be of the form:

$\theta(s)/V_{in}(s) = K/(s(Ts+1))$

and obtain the gains K_p and K_d of the feedback controller:

$V_{in} = K_p (\theta_d - \theta) + K_d \theta'$

such that the closed loop transfer function $\theta(s)/\theta_d(s)$ has a peak time of 0.2 seconds with a damping ratio of 0.707.

The two closed loop systems you have designed can now be combined to control the position of the ball. A block diagram of the entire system is shown in Figure 2.

c) Digital filtering

The signals measured from the two potentiometers , especially the linear sensor, will be noisy and will deteriorate the performance of the controller you design. A digital filter should be implemented within the controller to filter out high frequency noise. The filters you will implement will be first order low-pass filters of the form:

$y(s)/u(s) = w/(s+w)$

where $f = w/2\pi$ Hz is the cutoff frequency.

In order to derive the equivalent digital filter, use the backward difference approximation for the derivative ie:

$y'(t) = (y(t) - y(t-ts))/ts$

where *ts* is the sampling rate

The filter, in the time domain, is given by:

$y'(t) + w\, y(t) = w\, u(t)$

It can then be approximated digitally by the following:

$(y(t) - y(t-ts))/ts + w\, y(t) = w\, u(t)$

$y(t) - y(t-ts) + w\,ts\,y(t) = w\,ts\,u(t)$

resulting in:

$$\mathbf{y(t) = (y(t\text{-}ts) + w\,ts\,u(t))/(1+ w\,ts)}$$

Write the **pseudo-code** for a program portion that would digitally filter x and θ to obtain x_f and θ_f at each new sample.

EXPERIMENT

a) Connecting the system

Wire up the system as shown in Figure 3. Use the DC Power supply available on the power module to bias the sensors. Make sure that the power op-amp has a feedback path from its output to the negative input. (*If the output of the D/A on your system is not bi-polar, you will have to offset the voltage using a summing network.*). **Do not connect the motor until your controller is running.** The reason for this is that the output of the D/A is undetermined at power-up and if you have the motor connected it will start turning, resulting in unstable behaviour! Use a 2 µF Capacitor to low-pass filter the voltage from the linear sensor.

b) Measurements and calibration

It is assumed you are familiar with the A/D and D/A cards available on your system. If this is not the case, refer to Experiment 1 of the Real Time Control Handbook and develop the required drivers.

Write a C program that obtains the calibration constants for the two sensors. The method used would be to measure two voltages at known positions from each sensor and fitting this data to a straight line. You should obtain two equations:

$x = G_s V_x + G_o$
$\theta = K_s V_\theta + K_o$

where

x = position of ball along the track. Take zero to be the centre.
V_x = voltage from the linear sensor

θ = angle of the plant output. Take zero to be the position when the beam is horizontal.
V_θ = voltage from angle sensor

Note: The two 7K resistors are built in. These were chosen such that the voltage swing obtained from the angle sensor is -5 to 5 volts when biased using +/-12 Volts.

c) Controller

The block diagram for the closed loop system you have designed is shown in Figure 2. Write a C program that implements this controller. The controller should be interrupt driven, running at a sampling frequency of 200 Hz. The operator should be able to enter the desired ball position at any time by entering the letter P. Hitting the letter X should result in program termination. Make sure the input to the motor is zero volts upon exit! For more information on Interrupt Service Routines (ISR) on IBM compatible computers refer to the Laboratory Manual.

The **pseudo-code** for the program would be:

Calibrate sensors
Initialize sampling frequency, gains, set point
Display the parameters
Set clock speed for desired sampling frequency
Install **control**() as interrupt service routine (ISR)
Loop
{
Get character from keyboard
If 'P' read in new set point
} Until 'X'
Re-install previous ISR
Restore original system
Output zero volts to motor
Stop

The **pseudo-code** for the ISR would be

control()
{
Sample the position of the ball, x
Sample the position of the plant output, θ
Digitally filter x and θ to obtain x_f and θ_f
Numerically differentiate x_f and θ_f to obtain x_f' and θ_f'
Compute the required tilt angle from control law for $\alpha = P_x(x_d-x_f)+D_x x_f'$
Compute the required plant output $\theta_d = \alpha*16$
Software limit θ_d to the range -80 to 80 degrees
Compute the feedback voltage $V_{in} = K_p(\theta_d-\theta_f)+K_d\theta_f'$
Output this voltage to the motor
}

Hints

a) An anti-aliasing filter will be required for the linear sensor. This could simply consist of a 2 μF capacitor as shown in Figure 3.

b) Digital filters are discussed in the Control System Design section. The cutoff frequency should not be *too* low. Start with a cutoff frequency of 10 Hz. for both filters.

c) Differentiation can be performed numerically using the backward difference equation to obtain x'(t) and θ'(t). ie $y'(t) = (y(t)-y(t-ts))/ts$

d) The desired plant output θ_d should be software limited to the range -80 to 80 degrees. Why?

e) The directions in which x, θ and α are defined positive are important! Make sure these are compatible with the signs for the gains you obtained, otherwise instability will result.

d) Servo controller

If your workstation is equipped with the ball sensor SS01, modify the program so that the user can move the ball on this sensor and the plant tracks the desired position. Instead of entering the desired set point from the keyboard, the desired set point is sampled from the second linear sensor.

DEMONSTRATIONS

a) Demonstrate the calibration procedures for the sensors.

b) Demonstrate the successful control system. Disturb the position of the ball and observe the response.

c) Lift the support block and demonstrate the effect of this constant disturbance.

d) Shift the support block **along the direction of the track** and demonstrate the effect of this constant disturbance.

e) If your system is equipped with the ball sensor SS01, demonstrate the controlled ball tracking the position of the ball on the input sensor.

REPORT

a) Show the calibration programs and the calibration constants obtained for both sensors.

b) Show the development of the open loop transfer functions $x(s)/\alpha(s)$ and $\theta(s)/V_{in}(s)$.

c) Show the development of the feedback gains P_x, D_x, K_p, K_d.

d) Show the **pseudo-code** for the digital filter implementation of a first order low pass filter.

e) Discuss the performance of the controller:

- Is the transient response of the system as expected? What are the parameters that affect the transient response?

- What is the effect of varying the cutoff frequency of the digital filters?

- What are the sources of steady state error in the system?

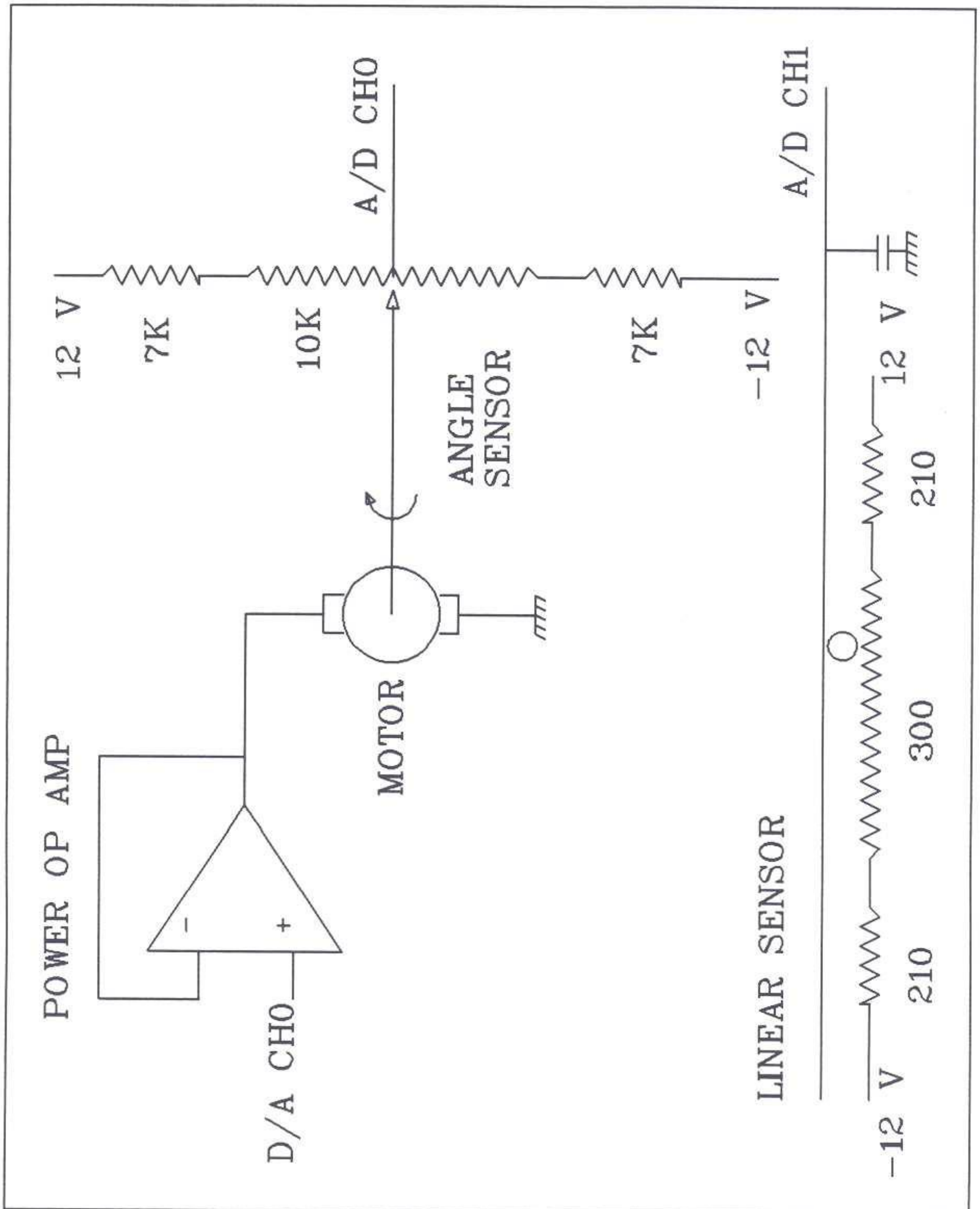

Figure 3 Electrical connections for ball and beam

SPECIFICATIONS

POWER MODULE PA-0103

The power module consists of a regulated dual output DC power supply set at +/- 12 Volts and a linear power operational amplifier. A breadboard is also mounted on the chassis which could be used to implement signal conditioning circuitry. This module is used to drive the DC motors of the experimental setups and can be used for other devices in your laboratory.

One section of the module is labelled "DC Power Supply" and has three binding posts labelled +12V, GND and -12V. These may be used to power active components on the breadboard as well as to bias the sensors.

Another section is labelled "Power Operational Amplifier" and has four binding posts. It can be used in any standard op-amp configuration (summer, inverter, buffer etc...). Its two inputs are labelled (-) for the inverting input and (+) for the non-inverting input. The output is also labelled and a binding post adjacent to it is labelled GND. The load should be connected between the output terminal and GND. The Power Op-Amp must have a feedback path from the output to the negative input otherwise its output will saturate. The module is protected by a 2.0 Ampere slow-blow fuse.

a) DC Power Supply

A input	110/220 Volts
Output ripple	< 1 mV. p-p
DC Output	+12 V. and -12 V.
Stability	+/- 0.03%
Maximum output current	3 Amperes
Short circuit protection	Automatic, self recovering

b) Power operational amplifier

Maximum current output	3 Amperes
Maximum power output	40 Watts
Power bandwidth	60 KHz.
Small signal bandwidth	700 KHz.
Slew rate	9 V/microsec.

BALL and BEAM SETUP

For the plant connected to the ball and beam attachment, the following constants represent the linearized system.

R_m Armature resistance = 9.0 Ohm

K_m Motor torque constant = .0075 N-m/Amp
J_l Total load inertia = 7.35 E-4 N-m-sec^2/rad
B Total load friction = 1.6 E-3 N-m sec/rad
K_g Gear ratio = 75

The output is the angle θ measured at the large gear. It is measured using a 10 KΩ mechanically continuous turn potentiometer. The electrical rotation however is 352 degrees.

The total track length is 16 inches (40.5 cm.). Use this value to calibrate the voltage measured from the track sensor. The total track resistance is approximately 300 Ω. The two bias resistors are 210 Ω each.

BALL AND BEAM EXPERIMENT

SOLUTION

a) Calibration

i) Linear sensor.

The linear track sensor should be biased by + and - 12 volts. The two 200 Ω resistors are placed in series with the track resistance in order to ensure that voltages measured at the ends do not exceed 5 volts in magnitude. This is the typical range for A/D inputs.

The right end is defined as +20 cm. and the left end is defined as -20 cm. Calibration is performed by measuring the voltage at the right end V_r and the voltage at the left end V_l. The slope is then obtained as

G_s = 40/(Vr-Vl) = typically 4 cm/V

The offset G_o may vary about zero.

ii) Angular sensor

Similarly, the plant output potentiometer is calibrated by measuring the voltage at two known points. One would usually choose + and - 90 degrees for simplicity. The resistor is 10 KΩ and should be biased by two 7 KΩ resistors in order to obtain a range of +/- 5 Volts from the potentiometer if the entire circuit is biased by +/- 12 Volts.

The slope should be approximately:

K_s = 35.2 deg/V

The offset K_o depends on the proper centring of the potentiometer. It should be made as close to zero as possible by adjusting the coupling between potentiometer and gear.

b) Open loop transfer functions

i) Rolling ball

The rolling ball equation is

$s^2 x(s) = -5g/7\ \alpha(s)$

or

$x(s)/\alpha(s) = -7/s^2$

ii) Plant

Joining the two equations given for the plant, the open loop transfer for the plant is

$\omega(s)/V_{in}(s) = 1/(sR_mJ_l/R_mK_m + R_mB/K_mK_g + K_mK_g))$

Substituting the values given in the system specifications we obtain:

$\theta(s)/V_{in}(s) = 1/s(0.01176\ s + 0.58823)$

which is the open loop transfer function

c) Control system design

i) Rolling ball

The desired response of the system is characterised by:

$t_p = 3.0$
$\zeta = 0.707$

which means $W_n = \pi/(t_p(1-\zeta^2)^{1/2}) = 1.48$ rad/sec

and the desired closed loop characteristic equation is:

$s^2 + 2.09\ s + 2.19$

For the feedback law : $\alpha = P_x\ (x_d - x) + D_x\ s\ x$

The actual closed loop characteristic polynomial is

$s^2 + 7\ D_x\ s - 7\ P_x$

Equating terms and solving we obtain:

$P_x = -0.312$ rad/metre or -0.17 deg/cm
$D_x = 0.299$ rad/m/sec or 0.17 deg/cm/sec

i) Plant

The desired response of the system is characterised by:

$t_p = 0.2$
$\zeta = 0.707$

which means $W_n = \pi/(t_p(1-\zeta^2)^{1/2}) = 22.2$ rad/sec

and the desired closed loop characteristic equation is:

$s^2 + 31.3\, s + 492.84$

For the feedback law : $V_{in} = K_p(\theta_d - \theta) + K_d\, s\, \theta$

The actual closed loop characteristic polynomial is

$s^2 + s\,(0.58823 - K_d)/.01176 + K_p/.01176$

Equating terms and solving we obtain:

$K_p = 5.79$ V/rad = 0.1 V/deg
$K_d = 0.22$ V/rad/sec = .003 V/deg/sec

d) Digital filter implementation

The **pseudo-code** for the digital filter should be something like this:

```
f = 2.0 /* let's say a cutoff of 2 Hz */
ts = 0.1 /*sampling at 100 Hz */
w = 2.0*pi*f;
wts = w*ts;
wtsp1 = 1.0+wts;
xf = 0.0;
thetaf = 0.0;

loop
{
get the measurements of theta and x /* A/D operation and calibration equation*/
thetaf = (thetaf+ theta*wts)/wtsp1;
xf = (xf+ x*wts)/wtsp1;
use the values xf and thetaf from here on...
}
```

e) Discuss the performance of the controller

- Is the transient response as expected?

The transient response will be slightly **underdamped** as compared to the expected results. The reason for this are several:

a) The motor dynamics are not negligible thus introducing a delay in the loop.

b) The digital filters will also introduce delays in the loop.

c) Nonlinear behaviour at large angles.

- *Steady state errors* are mainly due to:

a) Altered geometry which introduces constant disturbances.

b) Improper calibration of the sensors.

c) Static friction in the plant dynamics.