Robust Ball & Beam

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INEL 5595 – Robust Control Systems
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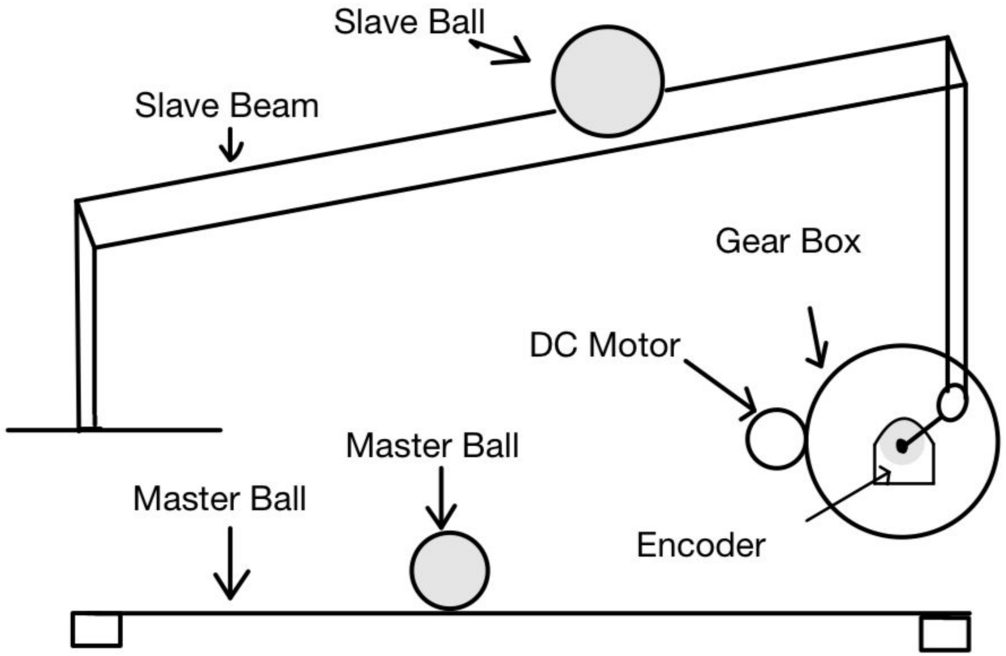
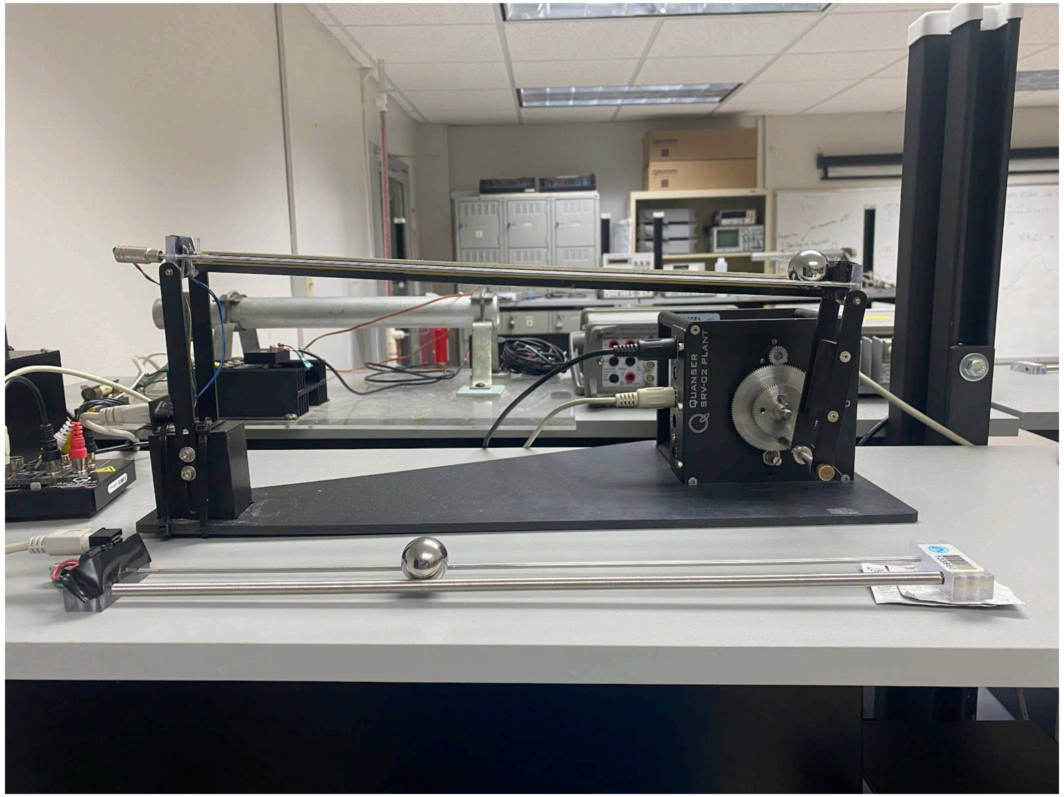
Objective

- Characterize & model the system.
- Identify the variations of the system.
- Determine $l_m(j\omega)$ and $p(j\omega)$.
- Design a controller that meet the performance specifications.
- Simulate the response of the designed controller.
- Implement the controller in a VI with Simulink.

Ball & Beam Description





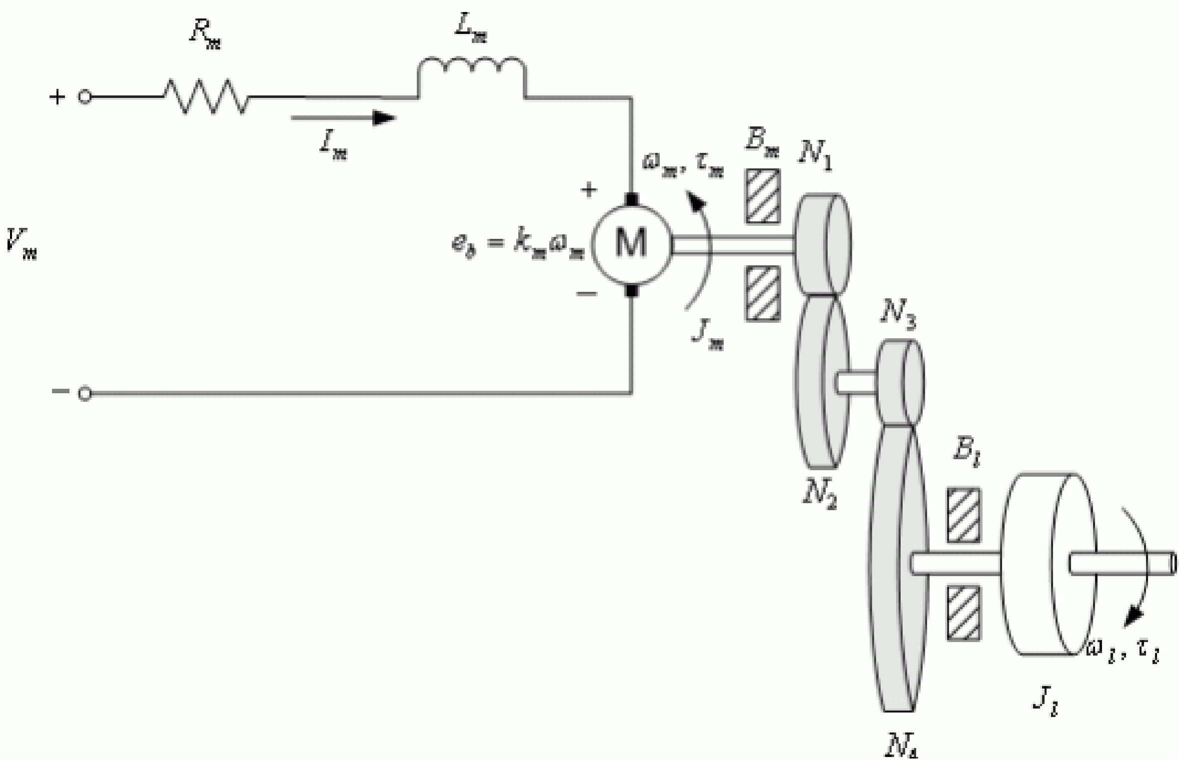
System Model (SRV02)

- SRV02

$$G_m(s) = \frac{\Theta_l(s)}{V_m(s)}$$

$$= \frac{K_m}{s(\tau s + 1)}$$

$$= \frac{\mathbf{1.5281}}{\boldsymbol{s}(\mathbf{0.0205}\boldsymbol{s} + \mathbf{1})}$$



SRV-02 circuit and Gear Train [2]

System Model (BB01)

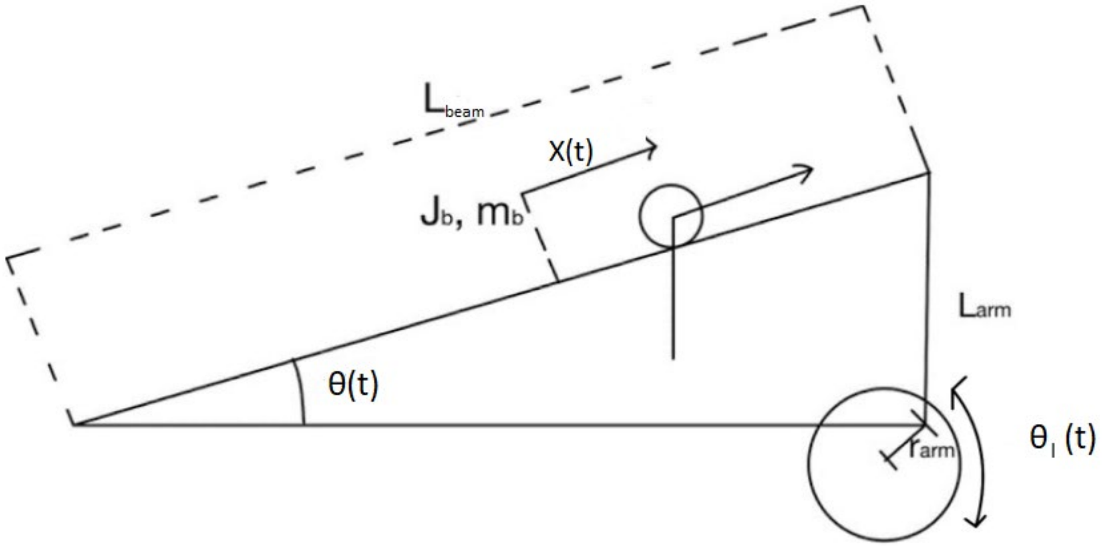
- BB01

$$\frac{d^2x(t)}{dt^2} = \frac{gm_b r_{arm} r_b^2 \sin(\theta(t))}{l_{beam}(m_b r_b^2 + J_b)}$$

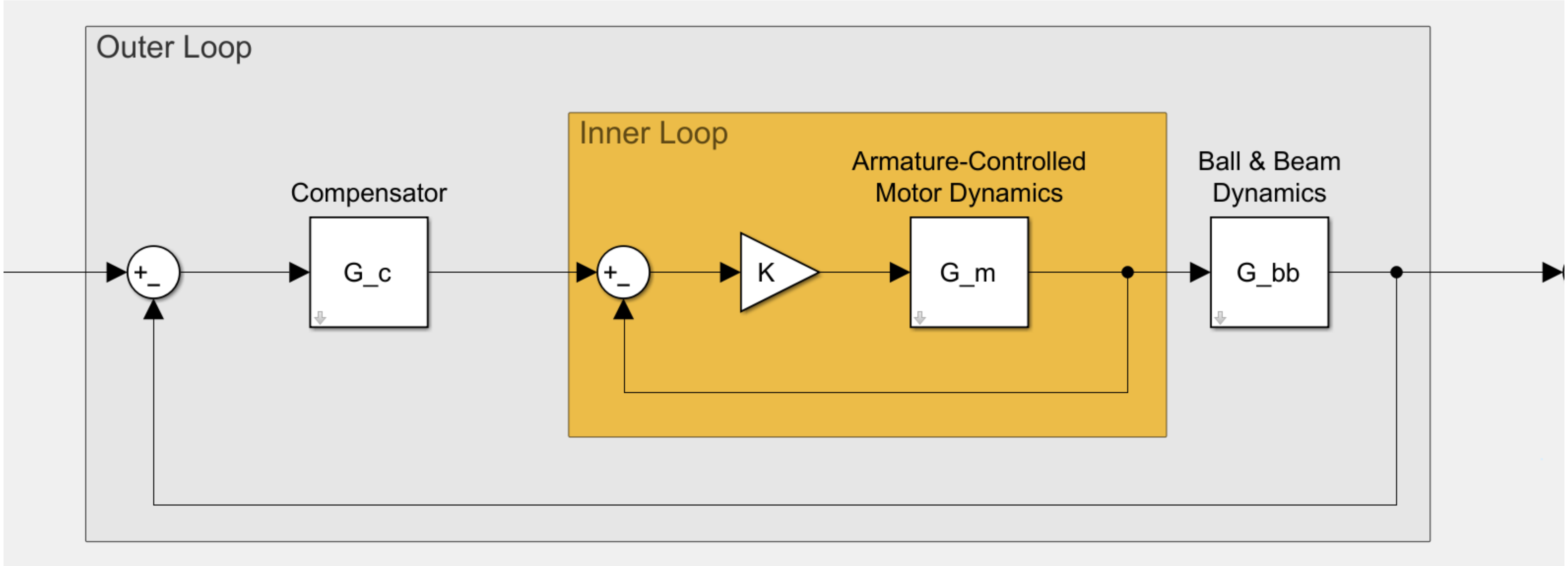
- Linearizing and applying results Laplace transform:

$$G_{bb}(s) = \frac{X(s)}{\Theta_l(s)} = \frac{gm_b r_{arm} r_b^2}{l_{beam}(m_b r_b^2 + J_b)s^2}$$

$$= \frac{5\ g\ r_{arm}}{7\ l_{beam} s^2} = \frac{K_{bb}}{s^2} = \frac{\mathbf{0.42}}{\boldsymbol{s^2}}$$



System Model (Proposed Control Structure)



System Model (SS01 & Upper Rail)

- Neither the SS01 Remote Sensing Module nor the upper position sensing potentiometer do not have manuals
- Low-pass filters smooth the potentiometer voltages
- The following values where measured:

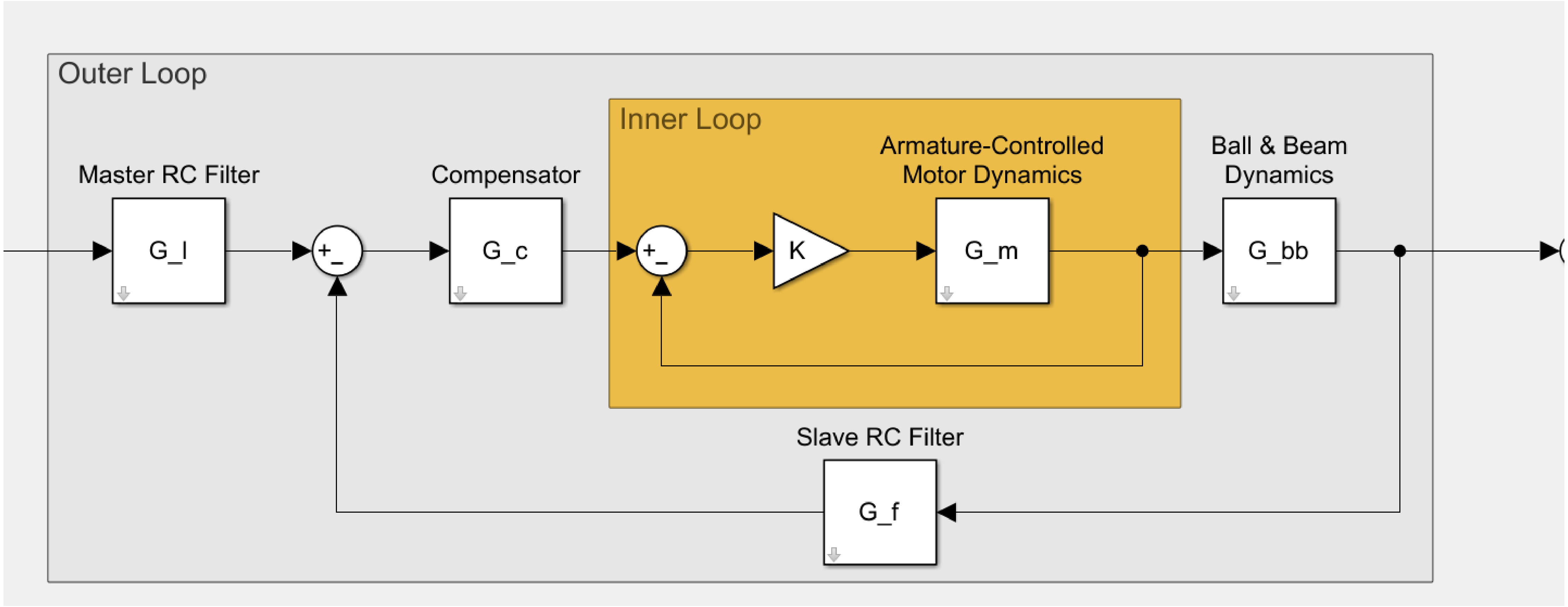
$$\begin{matrix} R_f = 3.6k\Omega \\ C_f = 1\mu F \end{matrix} \Rightarrow \tau_f = R_f C_f = 3.6ms$$

$$G_f(s) = \frac{1}{\tau_f s + 1}$$

$$\begin{matrix} R_l = 270\Omega \\ C_l = 1\mu F \end{matrix} \Rightarrow \tau_l = R_l C_l = 0.27\,ms$$

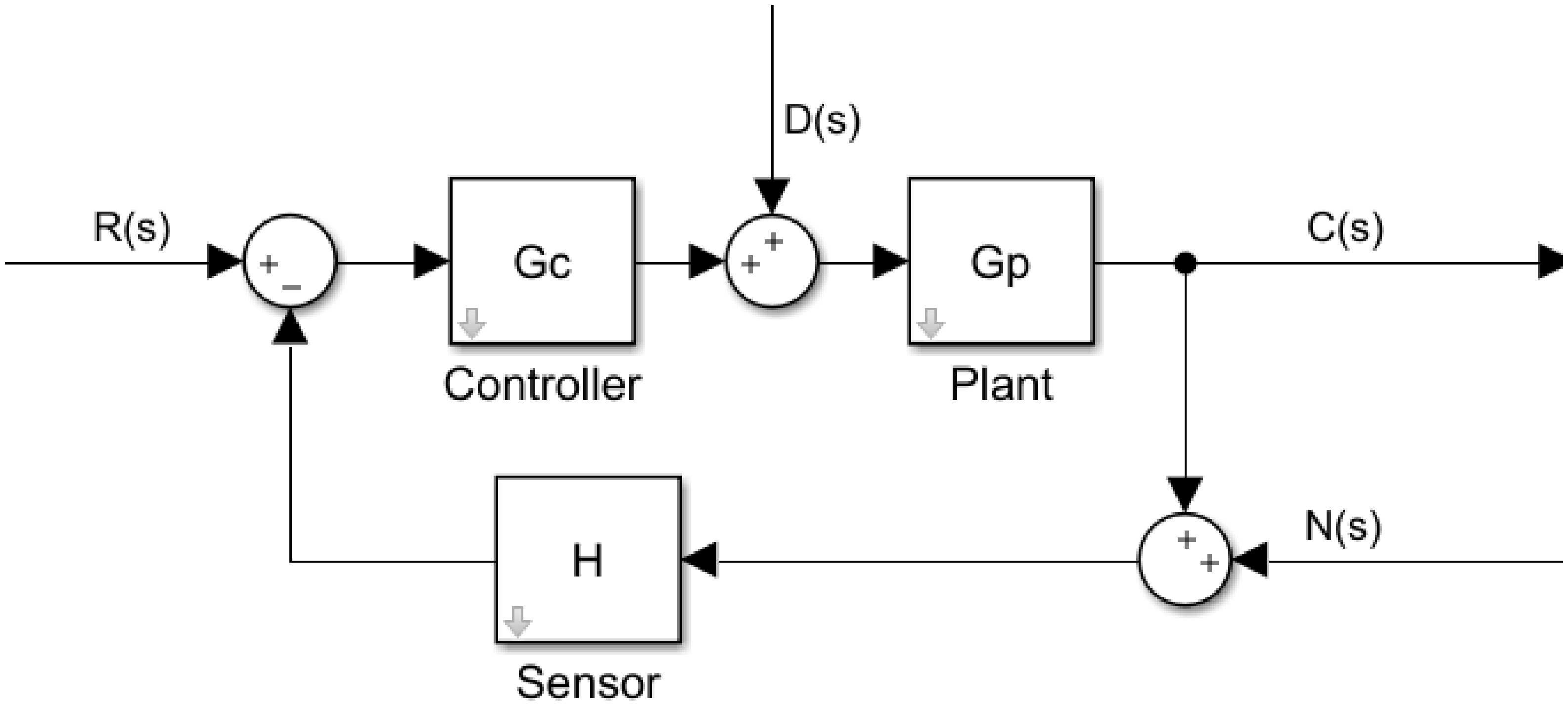
$$G_l(s) = \frac{1}{\tau_l s + 1}$$

System Model (Updated Control Structure)



Disturbances and Noise

- Examples
  - Air conditioner
  - Vibrations
  - Backlash
  - Dirt and stains



Other Effects Considered (Motor Inductor)

- Using equations 1.1.3, 1.1.15, 1.1.20, 1.1.21 from the SRV02 Manual

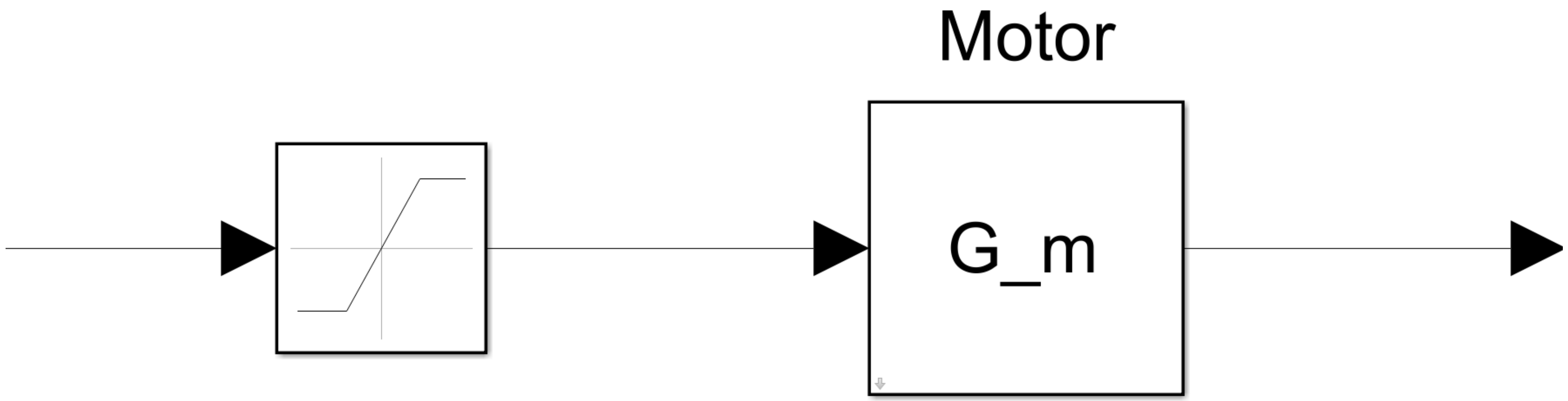
$$G_m(s) \equiv \frac{\Theta_1(s)}{V_m(s)} = \frac{\eta_m \eta_g k_t K_g}{s\left(J_{eq} L_m s^2 + \left(B_{eq} L_m + R_m J_{eq}\right) + \eta_m \eta_g k_m k_t K_g^2 + B_{eq} R_m\right)}$$

- Substituting the model parameters into this equation:

$$G_m(s) = \frac{1{,}078{,}400}{s(s + 48.99)(s + 14{,}400)}$$

Other Effects Considered (Voltage Saturation)

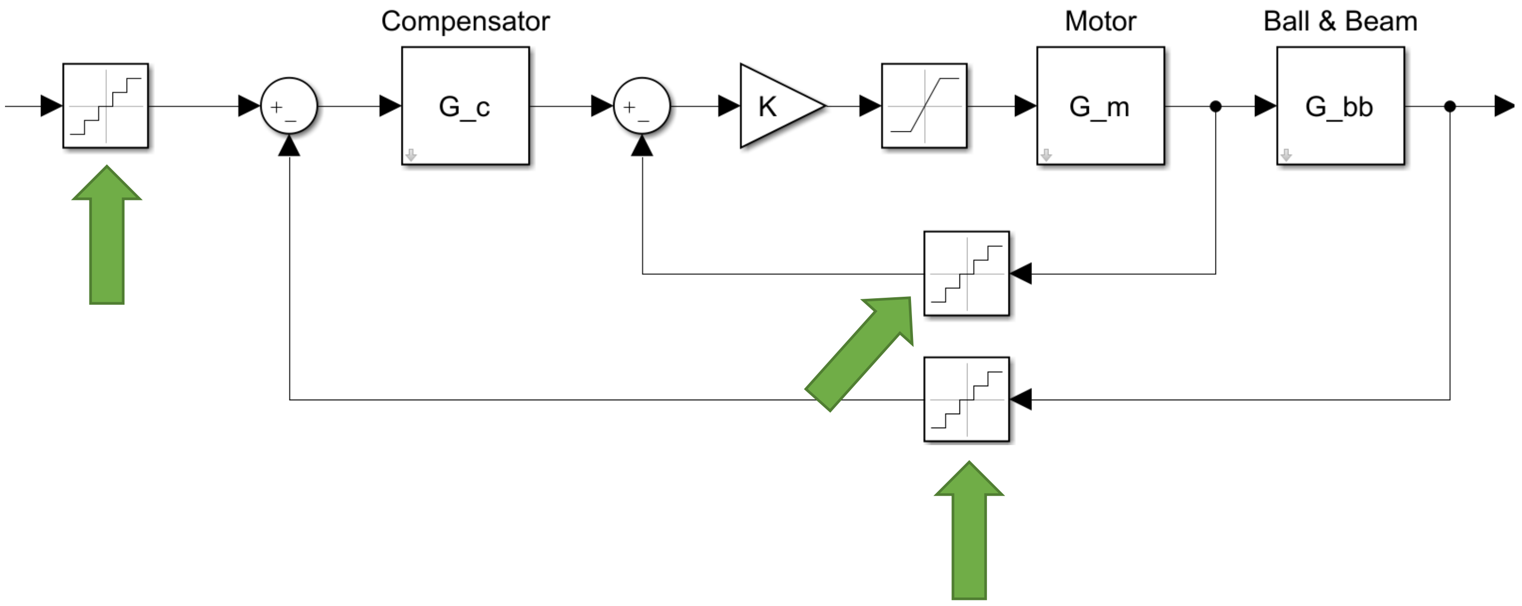
- Voltage Range: $[-5V, +5V]$



Other Effects Considered (Sensor Quantization)

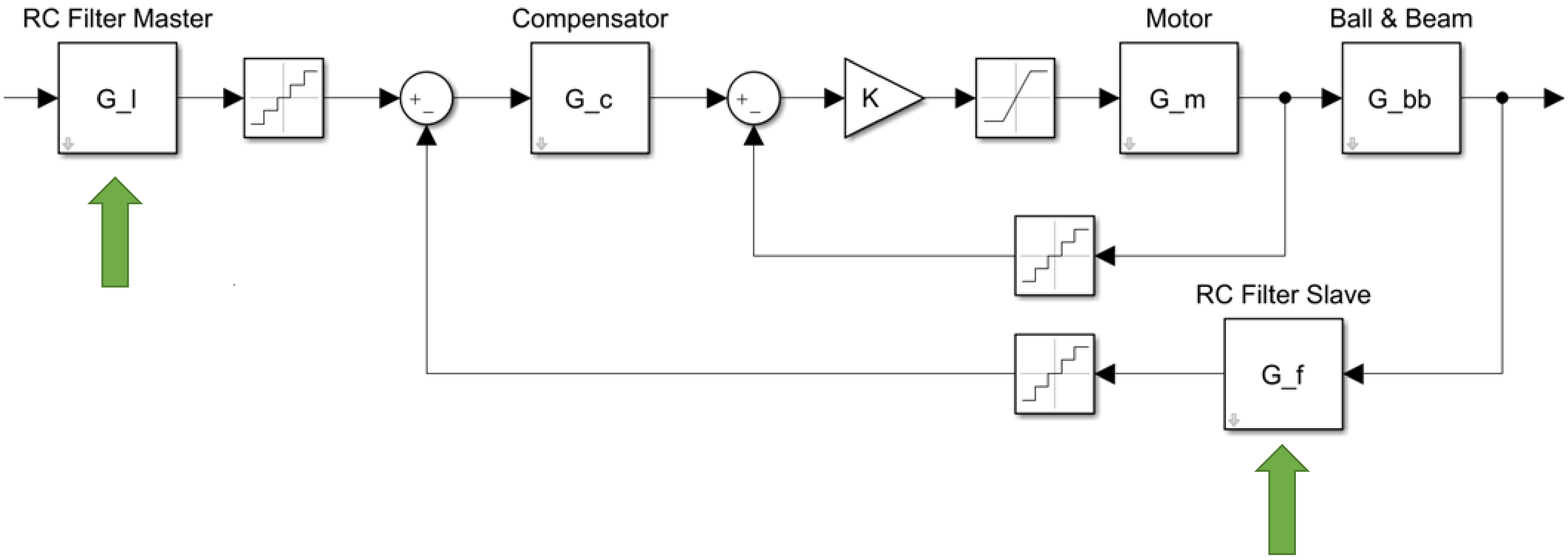
- Q2 DAQ has 12-bit resolution for analog input measurements that range from -$10V$ to $+10V$. Encoders measures 4096 counts per revolution.

Encoders: $\frac{2\pi[rad]}{2^{12}} = \frac{\boldsymbol{\pi}}{\mathbf{2048}}$ Potentiometers: $\frac{(+10V)-(-10V)}{2^{12}} = \frac{\mathbf{1}}{\mathbf{204.8}}$

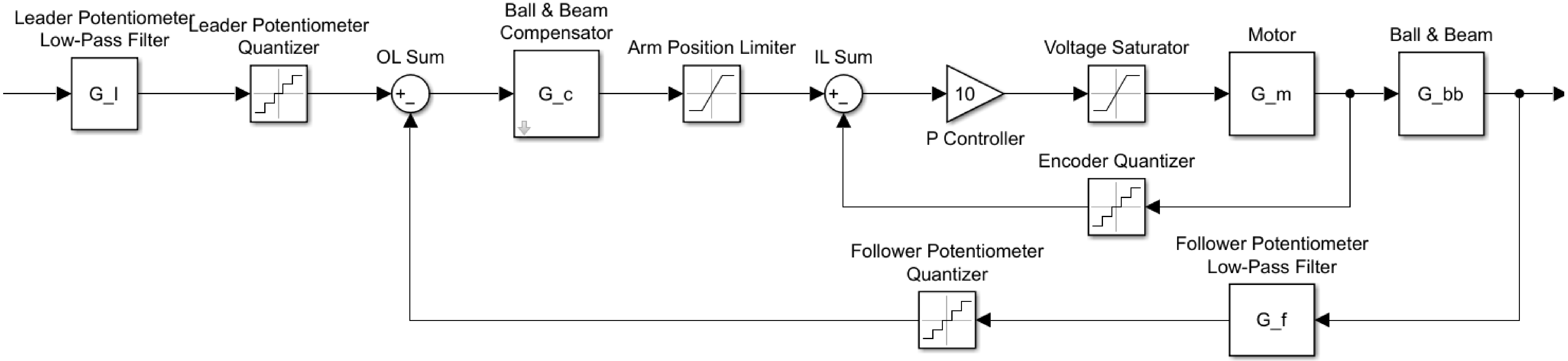


Other Effects Considered (RC Filter)

- SS01 Remote Sensing module and the upper position sensing potentiometer



Other Effects Considered (Result)



Uncertain Linearization of Model (*ulinearize*)

```
% Open Nonlinear Model
open_system("n02_NonlinearModel")

% Retrieve Operating Point (by default 0)
opspec = operspec("n02_NonlinearModel");

% Collect some options for linearization
options = findopOptions("DisplayReport", "off");
op = findop("n02_NonlinearModel", opspec, options);

% Establish input and output of system to be linearized
io(1) = linio("n02_NonlinearModel/Arm Position Limiter", 1, "input");
io(2) = linio("n02_NonlinearModel/Ball & Beam", 1, "openoutput");

% Linearize
ol_linsys = ulinearize("n02_NonlinearModel", io, op);
```

Uncertain Linearization of Model (*ulinearize*)

- The nominal open-loop linearized model is

$$G(s) = \frac{4.5108E6}{s^2(s + 1.44E4)(s^2 + 48.94s + 748.7)}$$

- The nominal simple model (without the other effects included) is

$$G(s) = \frac{312.28}{s^2(s^2 + 48.96s + 746.6)}$$

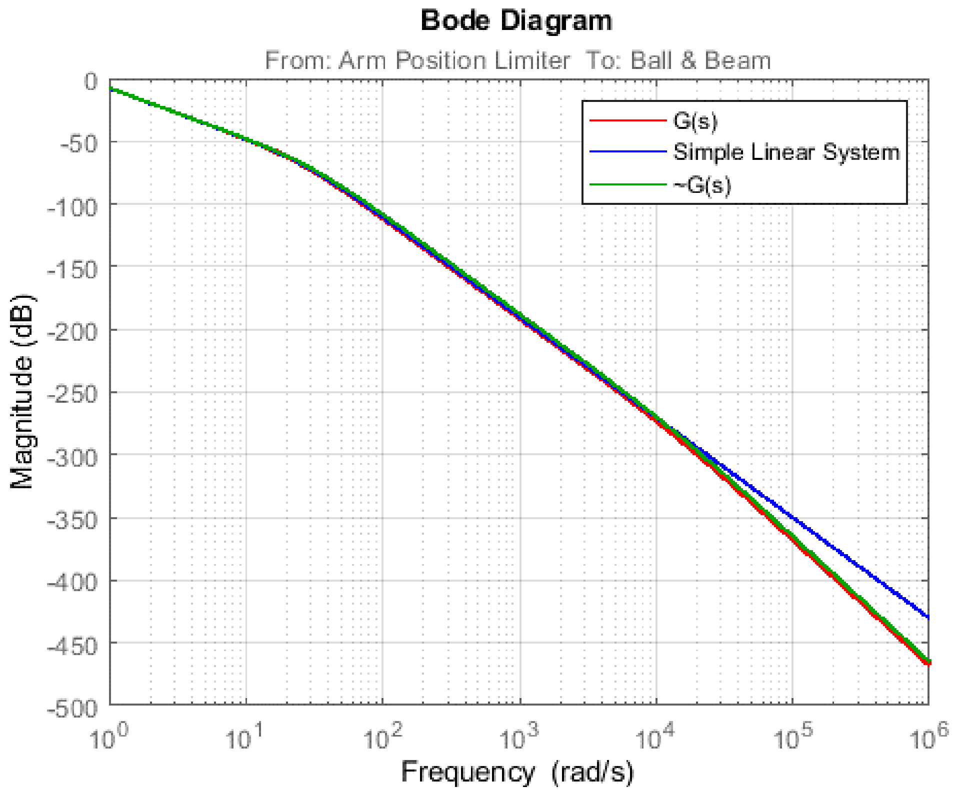
Multiplicative Perturbation $l_m(j\omega)$

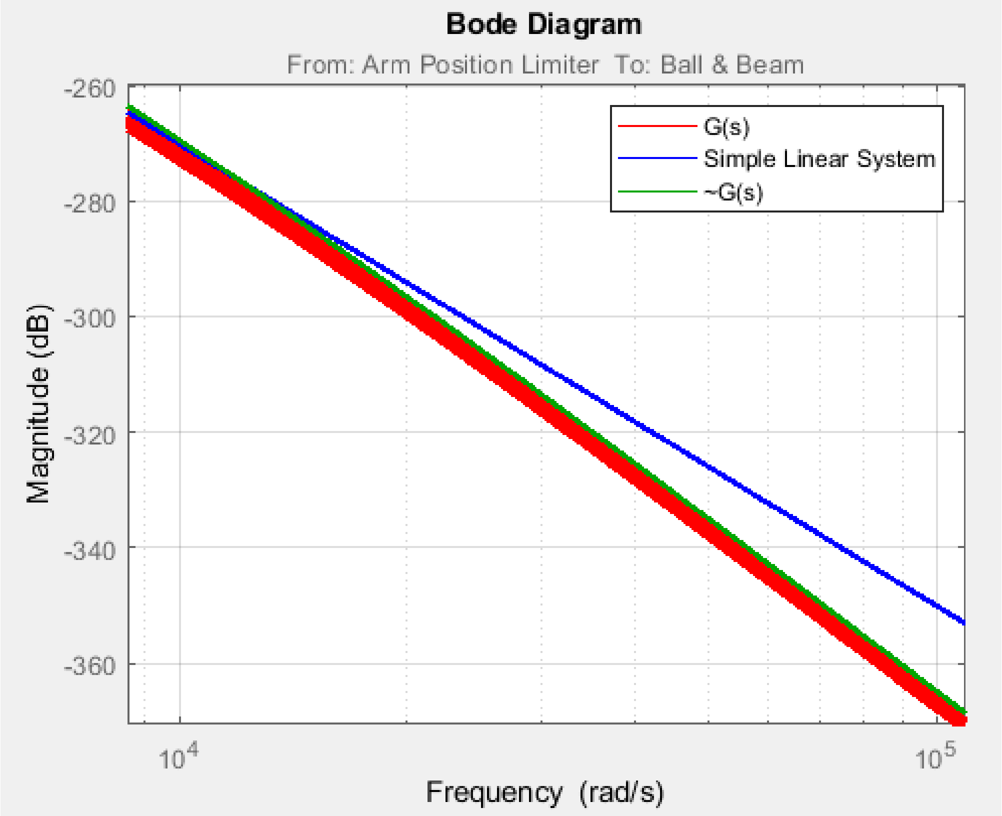
```
% Generating samples of transfer function we want to fit
samples = usample(ol_linsys, 100);

% Fit the function. Find l_m(jω)
[ave, info] = ucover(samples, ol_linsys.NominalValue, 3);
l_m = tf(info.W1);
```

$$l_m(s) = 0.2133\,\frac{(s+1.7E4)(s+109.4)(s+0.4721)}{(s+9590)(s+139.9)(s+31.94)}$$

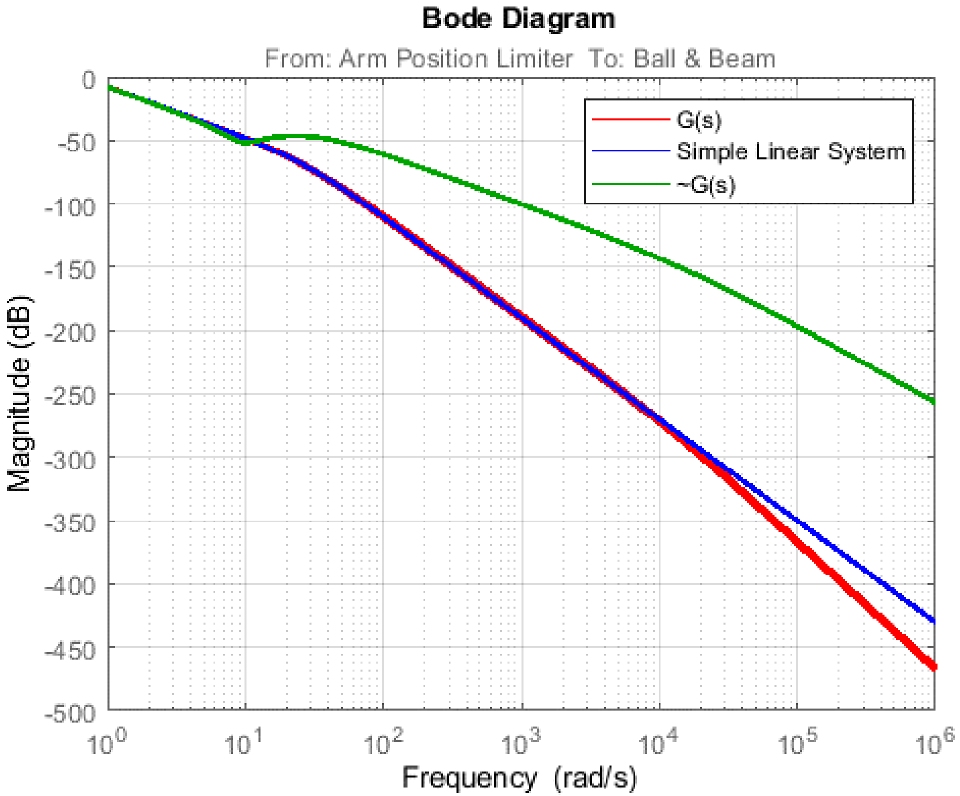
Multiplicative Perturbation $l_m(j\omega)$





$$\tilde{G}(s) = G(s)\left(1 + 0.2133\frac{(s+1.7E4)(s+109.4)(s+0.4721)}{(s+9590)(s+139.9)(s+31.94)}\right)$$

Multiplicative Perturbation $l_m(j\omega)$



$$l_m(s) = 0.0237\,\frac{(s+1.7E4)(s+109.4)(s+0.4721)(s+3)^2}{(s+9590)(s+139.9)(s+31.94)}$$

Performance Characteristics $p(j\omega)$

- Established the following criteria for the temporal performance of the system:

$$\%O.S = 20;\ T_S = 3s$$

- These are translated into poles in the s-plane using the second-order underdamped transfer function:

$$\zeta = 0.459;\ \omega_n = 2.86\ {}^{rad}/_{s}$$

$$\frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} = \frac{8.179}{s^2 + 2.608s + 8.179}$$

Performance Characteristics $p(j\omega)$

- Another necessary constraint for the performance of this system is for it’s steady state error to approximate zero, therefore an integrator is added to the system in the form of a pole in the origin

$$p(s) = \frac{8.179}{s(s^2 + 2.608s + 8.179)}$$

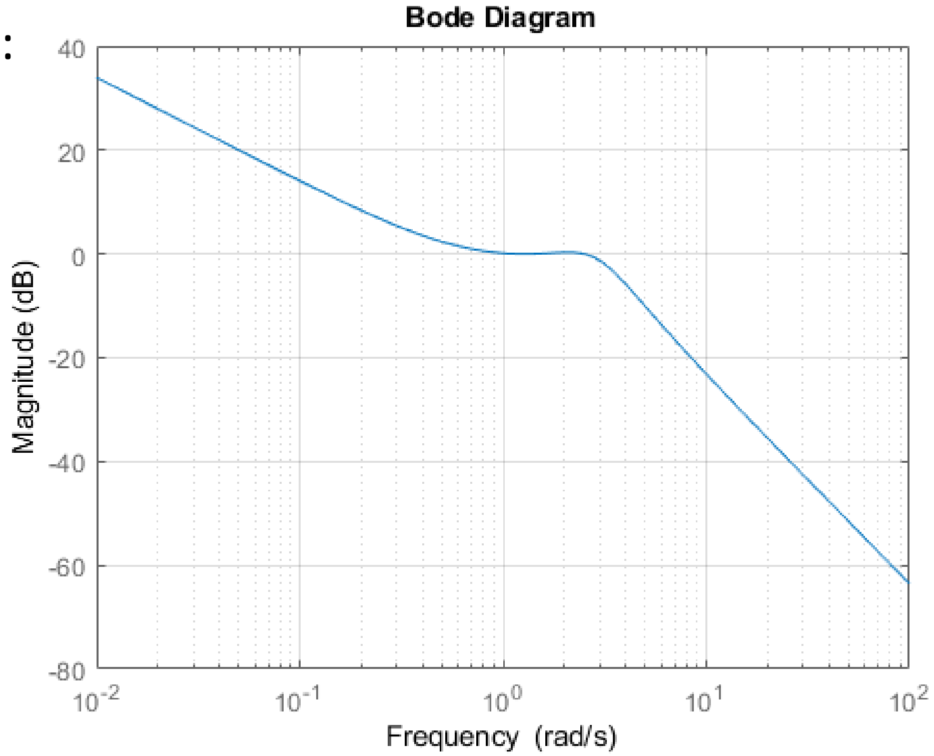
Performance Characteristics $p(j\omega)$

- Approximating the cut-off frequency of the system ω_c to the ω_n, we then seek to force the response of the system to have a -20dB slope in the resulting bode plot around 2/3 of a decade before ω_n, therefore a zero added in that region would achieve this for the current system:

$$p(s) = \frac{8.179\left(s+\frac{\omega_n}{10^{2/3}}\right)}{s(s^2+2.608s+8.179)} = \frac{6.6139(s+0.6162)}{s(s^2+2.608s+8.179)}$$

Performance Characteristics $p(j\omega)$

- The current bode plot is:



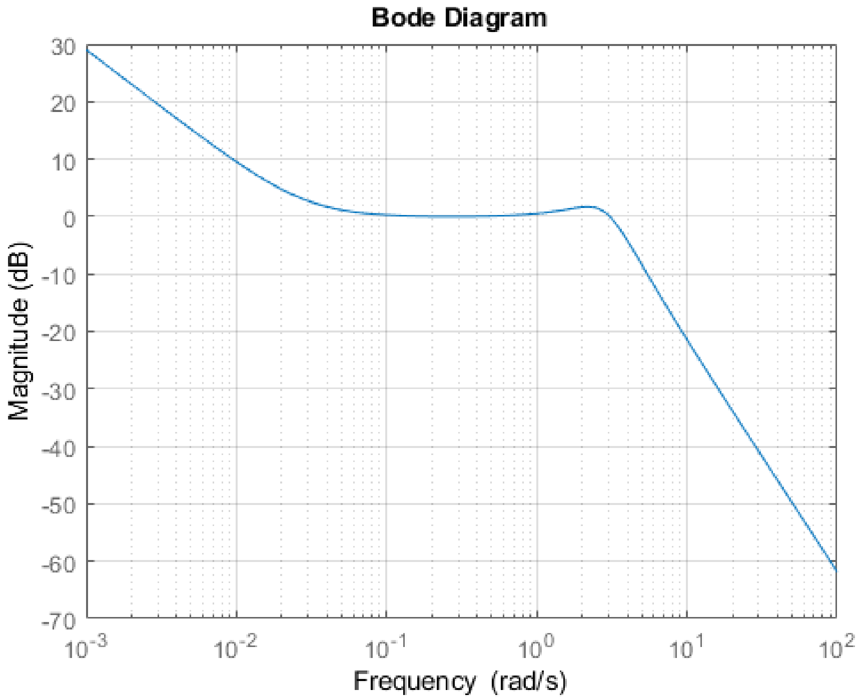
- The exhibited behavior of the plot complies with most of the specifications established initially. This response can be adjusted and optimized to suit the desired response even more.

Performance Characteristics $p(j\omega)$

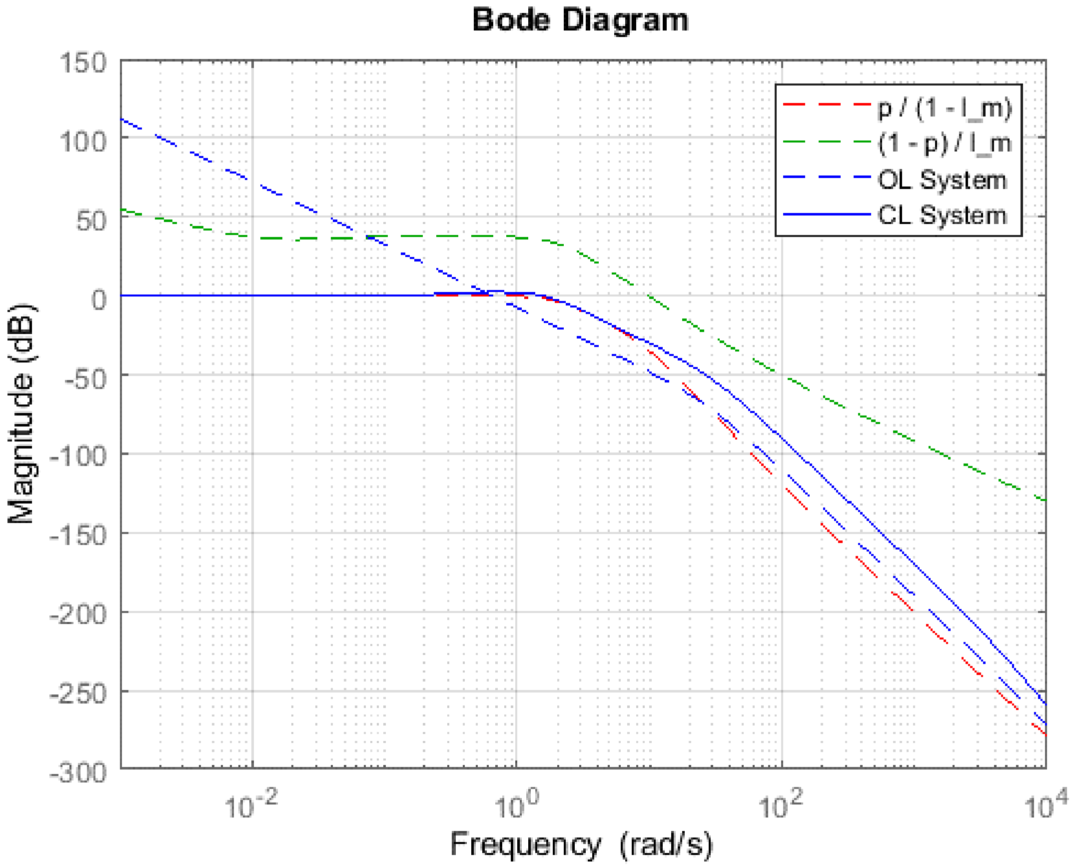
- In this case the zero was selected to be 2 decades before ω_n:

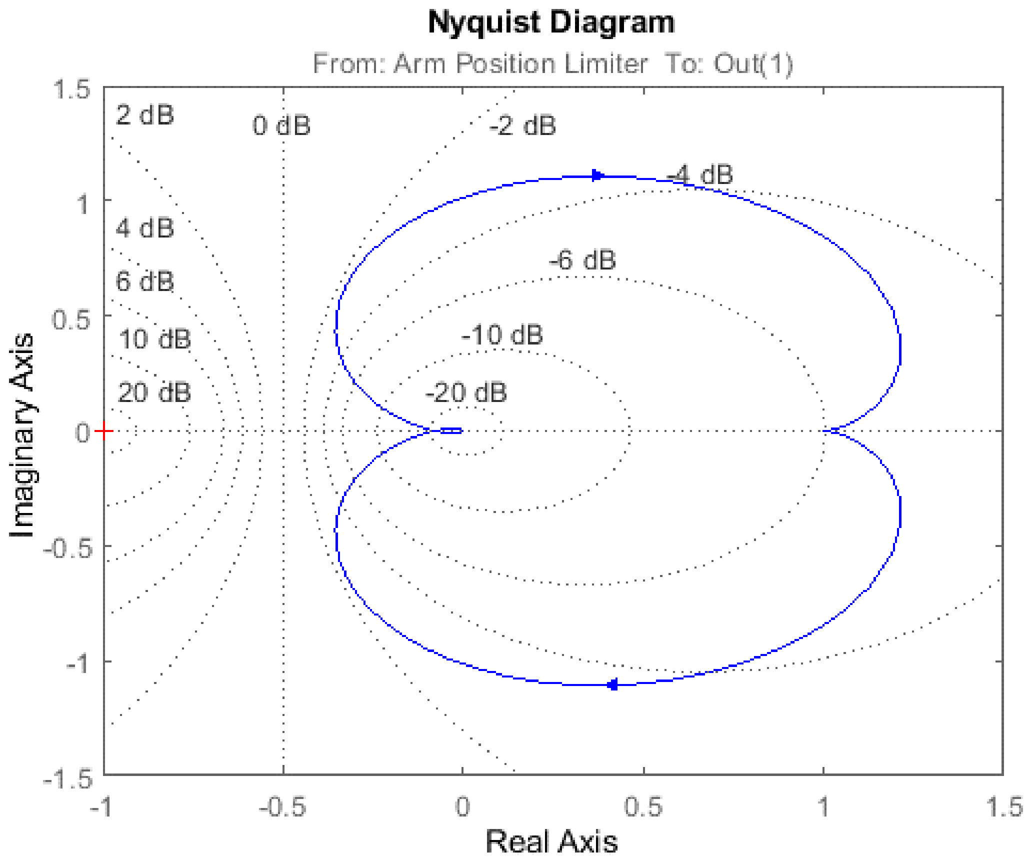
$$p(j\omega) = \frac{8.179\left(s+\frac{\omega n}{100}\right)}{s(s^2+2.608s+8.179)} = \frac{8.179\,(s+0.0286)}{s(s^2+2.608s+8.179)}$$

- Bode plot of $p(j\omega)$:

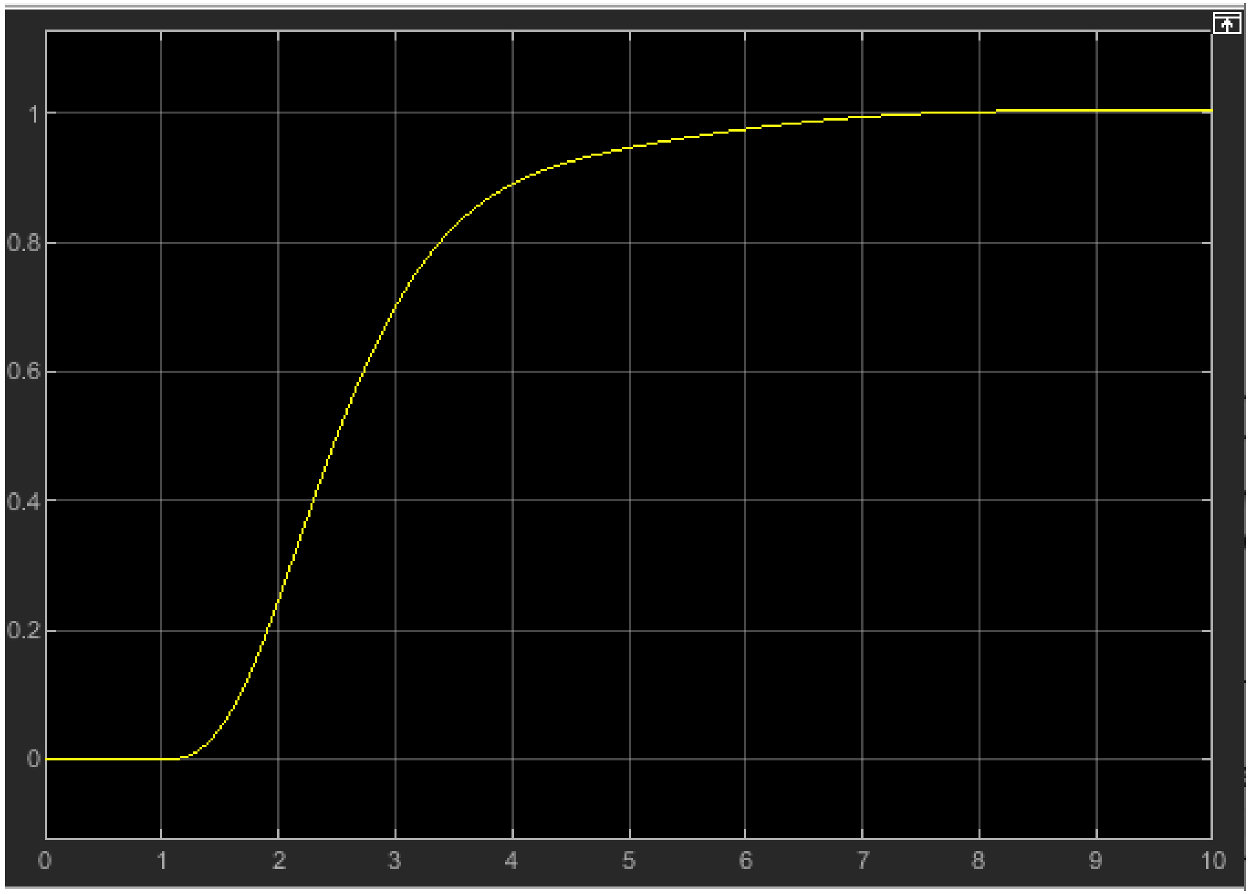


Result


Bode Diagram
Magnitude (dB)
150
100
50
0
-50
-100
-150
-200
-250
-300
10^{-2}
10^{0}
10^{2}
10^{4}
Frequency (rad/s)
p / (1 - l_m)
(1 - p) / l_m
OL System
CL System


Nyquist Diagram
From: Arm Position Limiter To: Out(1)
Imaginary Axis
Real Axis
2 dB
0 dB
-2 dB
-4 dB
4 dB
-6 dB
6 dB
10 dB
-10 dB
20 dB
-20 dB
1.5
1
0.5
0
-0.5
-1
-1.5
-1
-0.5
0
0.5
1
1.5

Result Simulation



Simulation graph

Result

References

[1] C. E. Rohrs, J. L. Melsa, D. G. Schultz, and J. L. Melsa, *Linear Control Systems*. New York: McGraw-Hill, 1993.

[2] *Student Workbook: SRV02 Base Unit Experiment for Matlab®/Simulink® Users*, Quanser, Inc., 2011. Accessed: Oct. 17, 2022. [Online]. Available https://github.com/M4rqu1705/Robust-Ball-and-Beam/blob/7728c946d3f10be49682aa277e40e3844ce7e2e5/manuals/Student%20Workbook%20-%20SRV02.pdf

[3] C. G. Bolívar-Vicenty and G. Beauchamp, “Modelling the Ball-and-Beam System From Newtonian Mechanics and from Lagrange Methods,” 2014. Accessed: Oct. 17, 2022 [Online]. Available: http://www.laccei.org/LACCEI2014-Guayaquil/RefereedPapers/RP176.pdf

[4] C. G. Bolivar_Vicenty, K. Z. Rosa-Medina, and G. Beauchamp-Baez, “Control Robusto del Sistema de Bola y Viga,” thesis, LACCEI, Ecuador, 2014.

[5] Student Workbook: Ball and Beam Experiment for Matlab®/Simulink® Users, Quanser, Inc., 2011. Accessed: Oct. 17, 2022. [Online]. Available https://github.com/M4rqu1705/Robust-Ball-and-Beam/blob/7728c946d3f10be49682aa277e40e3844ce7e2e5/manuals/Student%20Workbook%20-%20BB01.pdf