Synthetic Control Group Designs: Key Concepts, Recent Extensions, and An Application

Workshop In Methods

Alex Hollingsworth

Coady Wing

Today’s Talk Is Going To Be Great

What is the synthetic control method good for? And how does it work?

Guide to key bits of notation, central concepts, and confusing bits. Examples to make things concrete

An extension to the method that we have been working on.

Practice code for you to take home with you.

Synthetic Control Methods for Comparative Case Studies: Estimating the Effect of California’s Tobacco Control Program

Alberto ABADIE, Alexis DIAMOND, and Jens HAINMUELLER

Building on an idea in Abadie and Gardeazabal (2003), this article investigates the application of synthetic control methods to comparative case studies. We discuss the advantages of these methods and apply them to study the effects of Proposition 99, a large-scale tobacco control program that California implemented in 1988. We demonstrate that, following Proposition 99, tobacco consumption fell markedly in California relative to a comparable synthetic control region. We estimate that by the year 2000 annual per-capita cigarette sales in California were about 26 packs lower than what they would have been in the absence of Proposition 99. Using new inferential methods proposed in this article, we demonstrate the significance of our estimates. Given that many policy interventions and events of interest in social sciences take place at an aggregate level (countries, regions, cities, etc.) and affect a small number of aggregate units, the potential applicability of synthetic control methods to comparative case studies is very large, especially in situations where traditional regression methods are not appropriate.

KEY WORDS: Observational studies; Proposition 99; Tobacco control legislation; Treatment effects.

1. INTRODUCTION

Social scientists are often interested in the effects of events or policy interventions that take place at an aggregate level and affect aggregate entities, such as firms, schools, or geographic or administrative areas (countries, regions, cities, etc.). To estimate the effects of these events or interventions, researchers often use comparative case studies. In comparative case studies, researchers estimate the evolution of aggregate outcomes (such as mortality rates, average income, crime rates, etc.) for a unit affected by a particular occurrence of the event or intervention of interest and compare it to the evolution of the same aggregates estimated for some control group of unaffected units. Card (1990) studies the impact of the 1980 Mariel Boatlift, a large and sudden Cuban migratory influx in Miami, using other cities in the southern United States as a comparison group. In a well-known study of the effects of minimum wages on employment, Card and Krueger (1994) compare the evolution of employment in fast food restaurants in New Jersey and its neighboring state Pennsylvania around the time of an increase in New Jersey’s minimum wage. Abadie and Gardeazabal (2003) estimate the effects of the terrorist conflict in the Basque Country on the Basque economy using other Spanish regions as a comparison group.

Comparing the evolution of an aggregate outcome (e.g., state-level crime rate) between a unit affected by the event or intervention of interest and a set of unaffected units requires only aggregate data, which are often available. However, when data are not available at the same level of aggregation as the outcome of interest, information on a sample of disaggregated units can sometimes be used to estimate the aggregate outcomes of interest (like in Card 1990 and Card and Krueger 1994).

Given the widespread availability of aggregate/macro data (e.g., at the school, city, or region level), and the fact that many policy interventions and events of interest in the social sciences take place at an aggregate level, comparative case study research has broad potential. However, comparative case study research is limited in the social sciences by two problems that affect its empirical implementation. First, in comparative case studies there is typically some degree of ambiguity about how comparison units are chosen. Researchers often select comparison groups on the basis of subjective measures of affinity between affected and unaffected units. Second, comparative case studies typically employ data on a sample of disaggregated units and inferential techniques that measure *only* uncertainty about the aggregate values of the data in the population. Uncertainty about the values of aggregate variables can be eliminated completely if aggregate data are available. However, the availability of aggregate data does not imply that the effect of the event or intervention of interest can be estimated without error. Even if aggregate data are employed, there remains uncertainty about the ability of the control group to reproduce the counterfactual outcome trajectory that the affected units would have experienced in the absence of the intervention or event of interest. This type of uncertainty is not reflected by the standard errors constructed with traditional inferential techniques for comparative case studies.

This article addresses current methodological shortcomings of case study analysis. We advocate the use of data-driven procedures to construct suitable comparison groups, as in Abadie

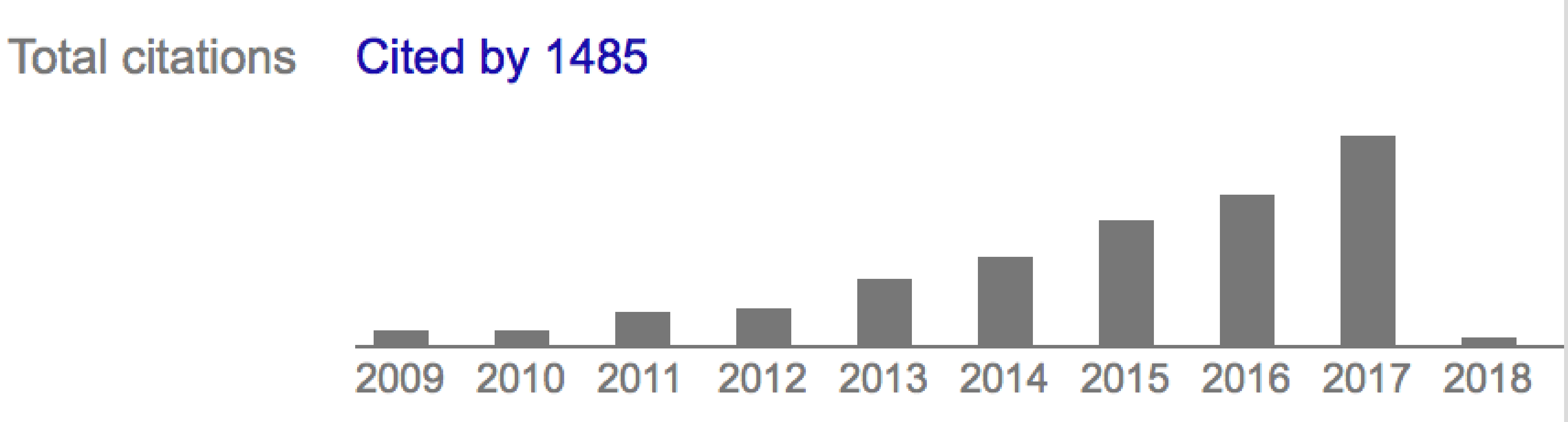
Alberto Abadie is Professor, John F. Kennedy School of Government, Harvard University, 79 John F. Kennedy Street, Cambridge, MA 02138 (E-mail: *alberto_abadie@harvard.edu*). Alexis Diamond is Evaluation Officer, The International Finance Corporation, 2121 Pennsylvania Avenue, Washington, DC 20433 (E-mail: *adiamond@ifc.org*). Jens Hainmueller is Assistant Professor, Department of Political Science, Massachusetts Institute of Technology, 77 Massachusetts Avenue, Cambridge, MA 02139 (E-mail: *jhainm@mit.edu*). In addition, during the course of this study all authors were affiliated with Harvard’s Institute for Quantitative Social Science (IQSS). We thank Joshua Angrist, Jake Bowers, Javier Gardeazabal, Bryan Graham, Dan Hopkins, Rustam Ibragimov, Guido Imbens, Michael Lovenheim, Paul Rosenbaum, Don Rubin, Jas Sekhon, Gary Solon, Chris Winship, and seminar participants at Harvard, Syracuse, Uppsala, the NBER Labor Meetings, and the 2006 APSA Meetings in Philadelphia for helpful comments. Funding for this research was generously provided by NSF grant SES-0350645 (Abadie). The views expressed in this article are not necessarily those of IFC. This article was awarded the 2007 Gosnell Prize for the best work in methods presented at any political science conference during the preceding year.

Companion software developed by the authors (*Synth* package for MATLAB, R, and Stata) is available at the authors’ webpages.

© 2010 American Statistical Association
Journal of the American Statistical Association
June 2010, Vol. 105, No. 490, Applications and Case Studies
DOI: 10.1198/jasa.2009.ap08746

493

A lot of people are on the synthetic control train!



Total citations Cited by 1485

Maybe you should get on the train too?

What was the economic cost of the 1990 German Reunification?

Reunification in one slide?

After WWII Germany was split into two large blocks:

- West Germany + West Berlin was part of NATO
- East Germany was part of the Warsaw Pact

Then the Berlin Wall fell

In 1990, East and West Germany were re-unified.

East and West Germany had very different economies...

In the late 1980s, GDP per capita was about three times higher in West Germany than in East Germany.

Huge income disparities after being separated for 45 years.

How did reunification affect West Germany’s Economy?

What would West Germany GDP have looked like if it had not merged with East Germany?

Before 1990

West Germany had GDP Per Capita that was a bit higher than the OECD average.

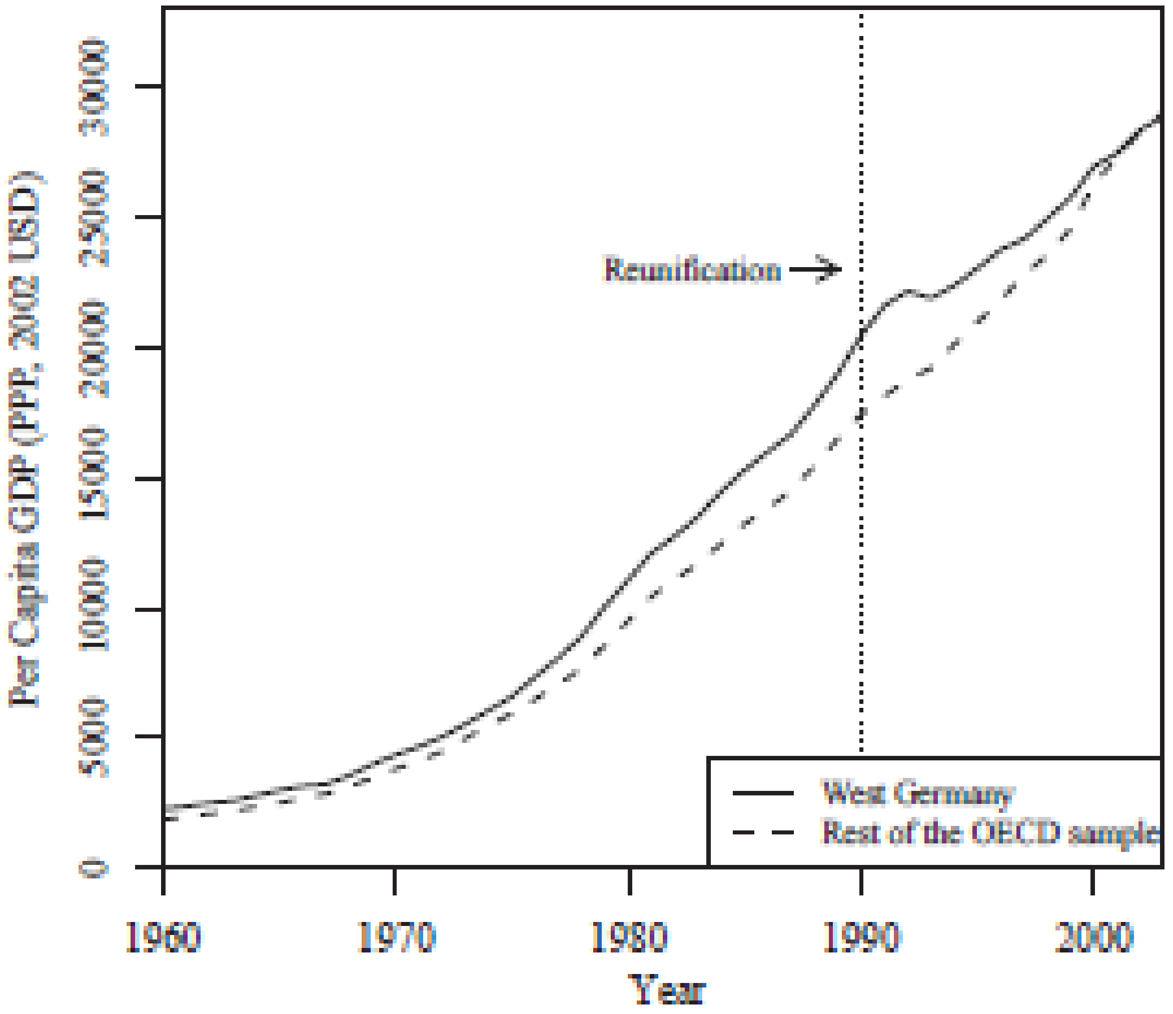
The gap was actually growing a bit.

After 1990

GDP gap widens and then converges.

Is the OECD average a good “counterfactual”

FIGURE 1 Trends in per Capita GDP: West Germany versus Rest of the OECD Sample



Some notation…

There are $j = 1 \ldots J + 1$ units.

Each unit observed in periods $t = 1 \ldots T$

Unit $j = 1$ is treated

- Exposed to control condition from $t = 1 \ldots T_0 - 1$
- Exposed to treatment condition from $t = T_0 \ldots T$

The other J units are untreated for all periods.

- “Donor pool”

German Reunification Example

The units are OECD countries.

Each country’s GDP per capita is observed in each country 1960 to 2003.

West Germany is unit 1. We don’t know it’s GDP after 1990 because it becomes “Germany”.

Treatment Effects

$Y_{jt}^{(1)}$ is the treated potential outcome for unit j in period t.

West Germany’s PC GDP under re-unification.

$Y_{jt}^{(0)}$ is the untreated potential outcome for unit j in period t

West Germany’s PC GDP in the absence of re-unification.

$\alpha_{1t} = Y_{jt}^{(1)} - Y_{jt}^{(0)}$ is the treatment effect for unit j in period t.

The goal is to compute the treatment effect for unit j in each period after T_0.

How much lower or higher was West Germany PC GDP because of re-unification?

Build a synthetic control group…

Y_{st}^{0} is the untreated potential outcome for the synthetic unit in period t.

$$Y_{st}^{0} = \sum_{j=2}^{J+1} \theta_j \times Y_{jt}^{0}$$

The synthetic time series is a weighted average of the J time series in the donor pool.

Example: Counterfactual West German PC GDP is “some” weighted average of PC GDP in the other OECD countries in the donor pool.

Terms that come up when you read about synthetic control weights

- In the pre-treatment period, the treatment unit must be “inside the convex hull” of the donor pool.
  - Interpolation and extrapolation.
  - Non-negative weights that sum to 1.

- How good is pre-treatment fit?
  - RMSE
  - Our idea: cohen’s d

- Trimming rules
  - Trim the donor pool
  - Trim the treatment units in studies with multiple treatment units.

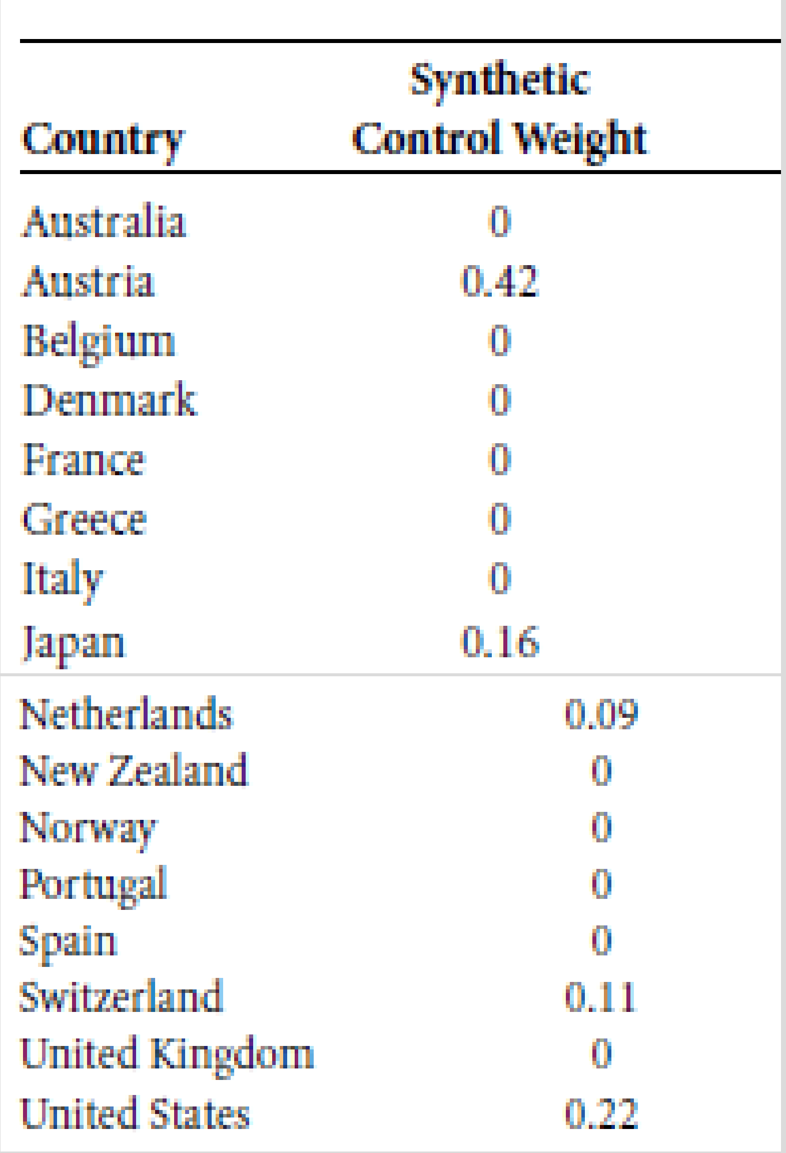
The synthetic control weights...

Don’t worry about how they obtain the weights just yet.

But notice that some of the donor pool countries get zero weight.

The synthetic control is actually a fairly small set of countries.

- Austria, Japan, Netherlands, Switzerland, and the United States



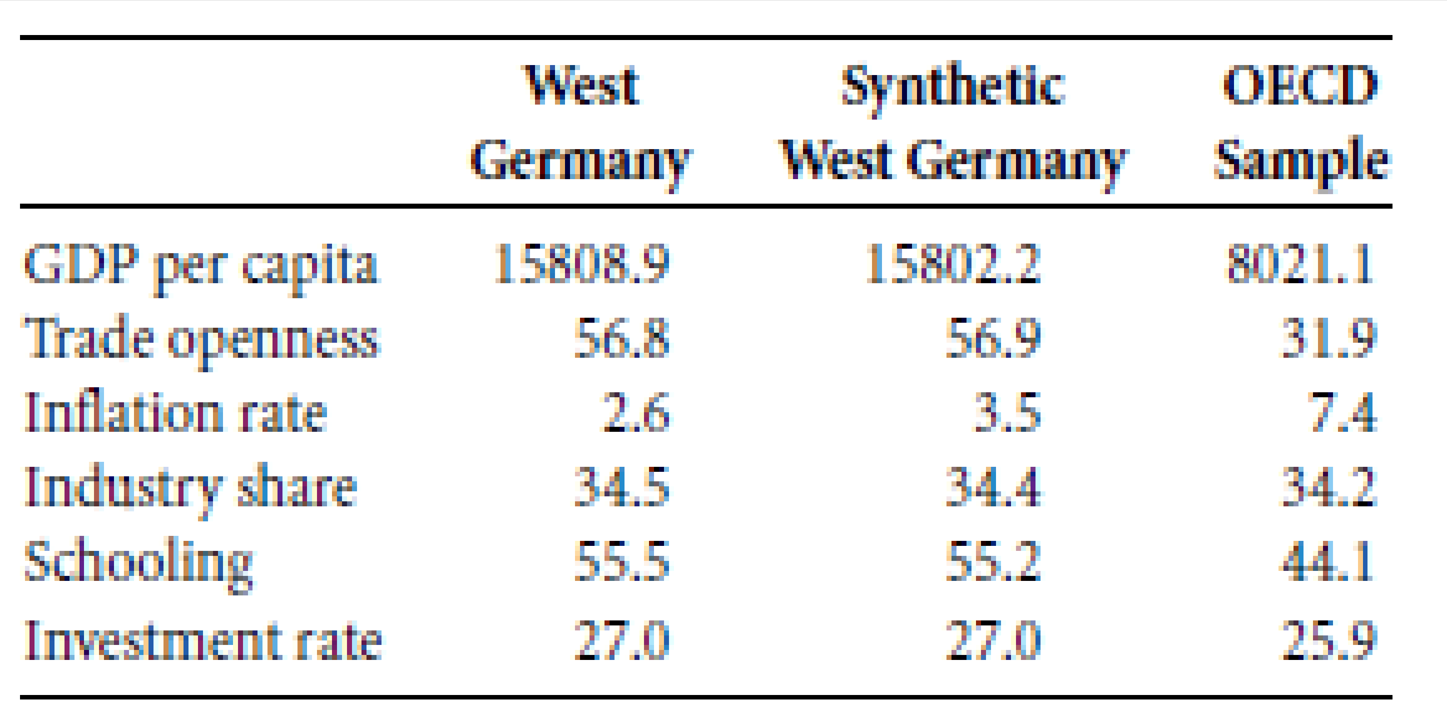
Country	Synthetic Control Weight
Australia	0
Austria	0.42
Belgium	0
Denmark	0
France	0
Greece	0
Italy	0
Japan	0.16
Netherlands	0.09
New Zealand	0
Norway	0
Portugal	0
Spain	0
Switzerland	0.11
United Kingdom	0
United States	0.22

Synthetic West Germany looks a lot like West Germany in the pre-period…

This is a balancing table.

You would see the same thing in a propensity score matching study, or in a randomized experiment.

Remember that the weights are chosen to minimize the imbalance in the pre-treatment time series.



	West Germany	Synthetic West Germany	OECD Sample
GDP per capita	15808.9	15802.2	8021.1
Trade openness	56.8	56.9	31.9
Inflation rate	2.6	3.5	7.4
Industry share	34.5	34.4	34.2
Schooling	55.5	55.2	44.1
Investment rate	27.0	27.0	25.9

Reunification did lower West Germany GDP!

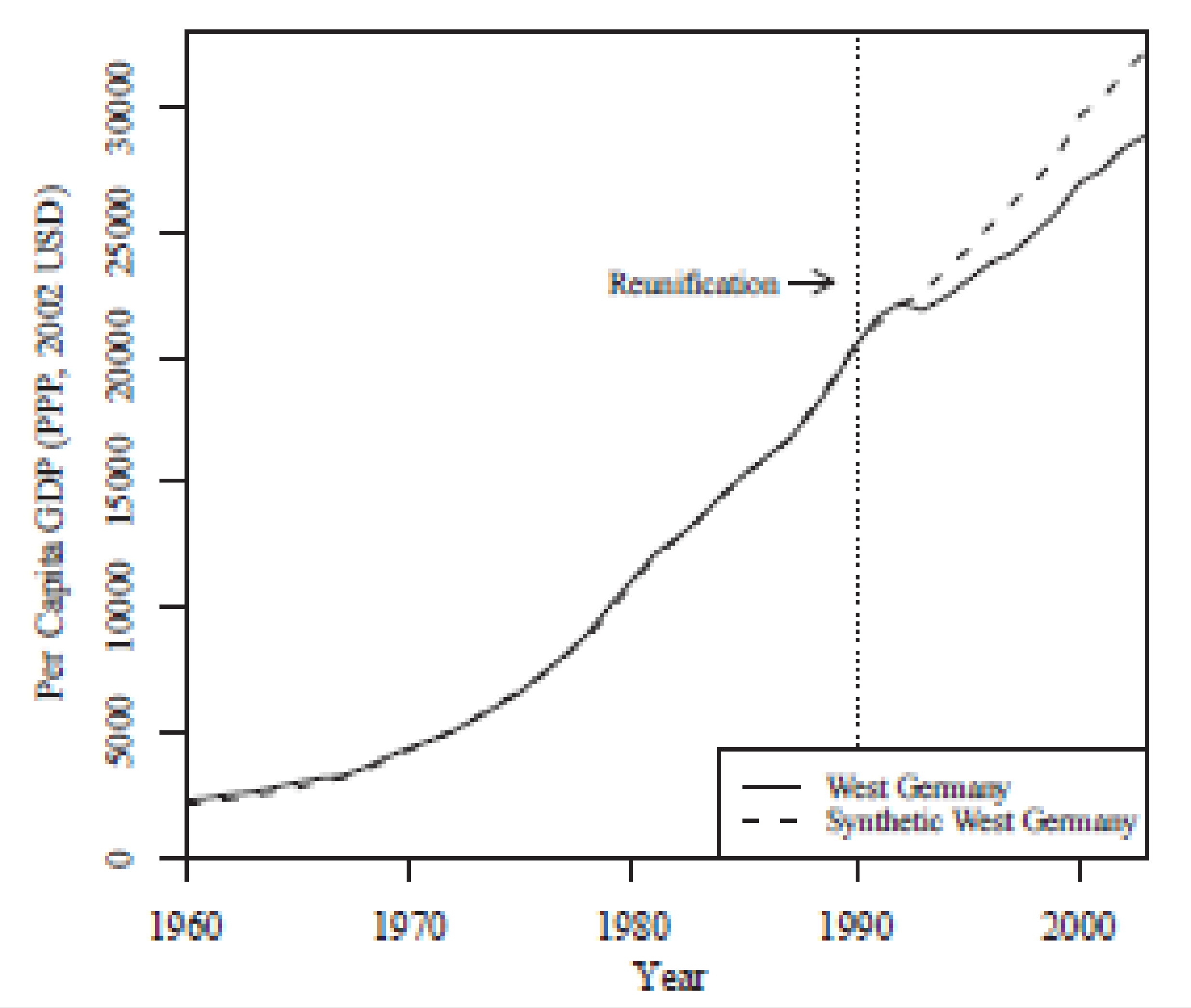
Before 1990

Close match between synthetic control and real West Germany.

After 1990

Synthetic West Germany grew faster than real West Germany.

The gap implies a negative treatment effect of reunification

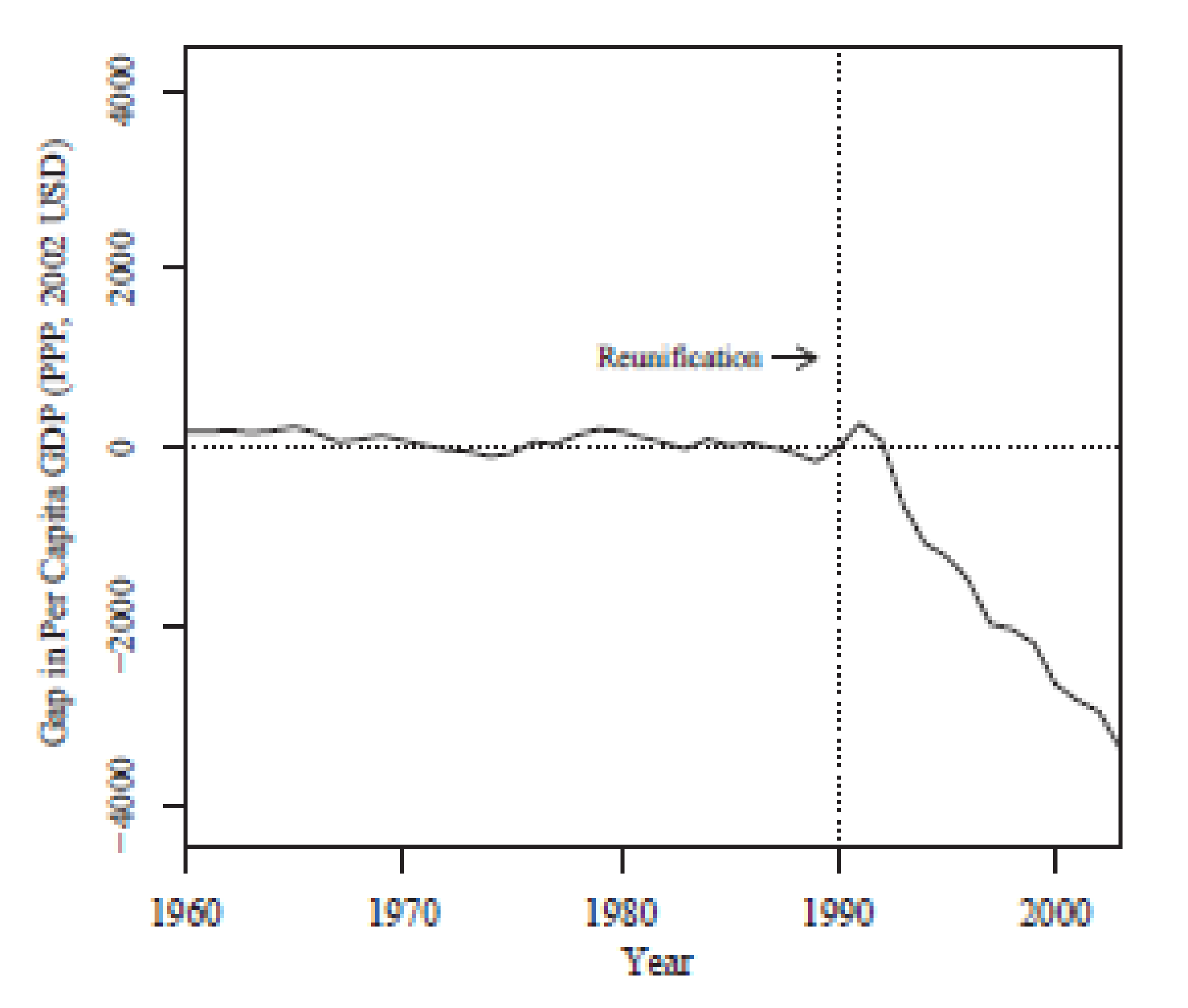


Plot the difference between the two lines...

Over the 1990 to 2003 period, West German PC GDP was reduced by about $1600 per year, on average.

That’s about 8% of the 1990 level.

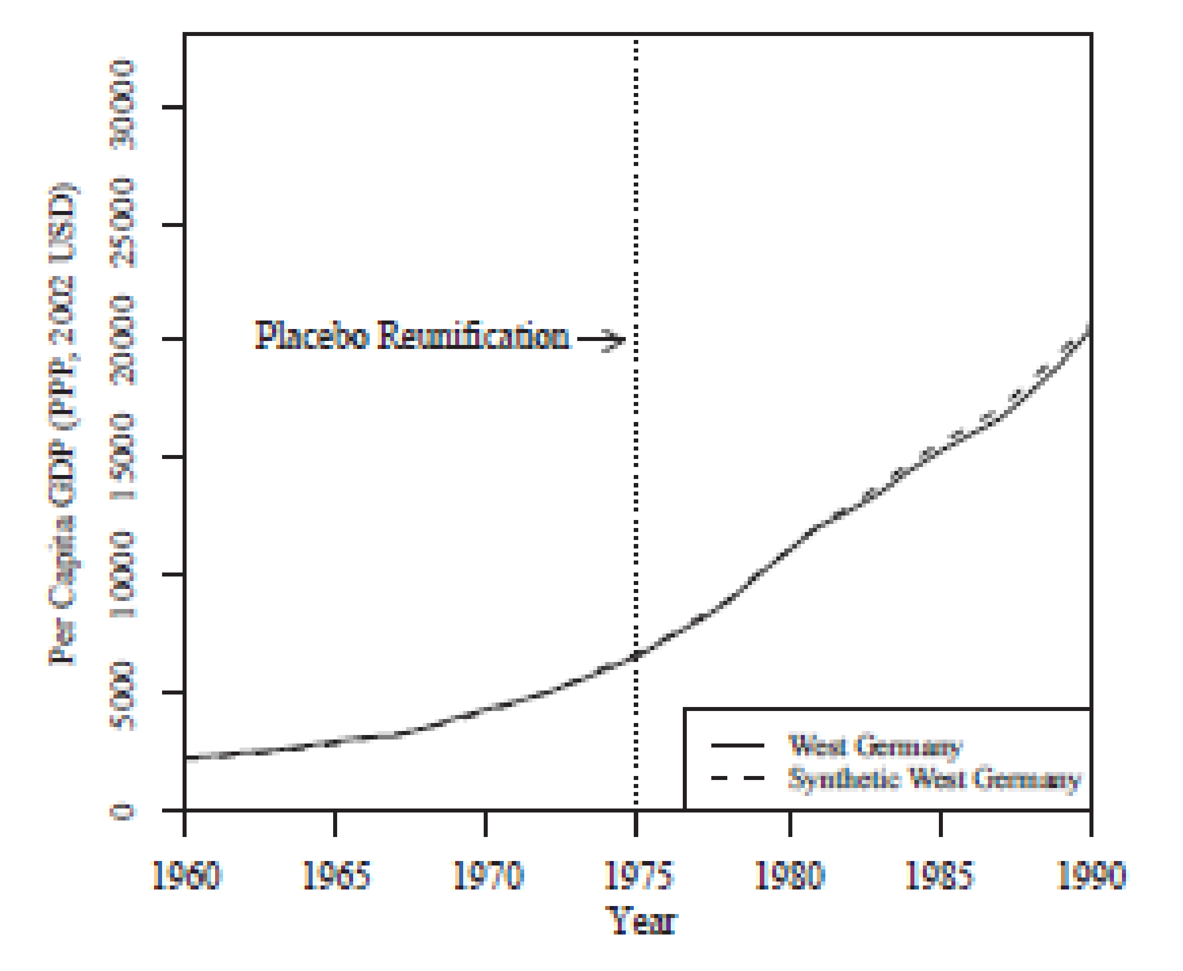
In 2003, synthetic West Germany’s PC GDP was about 12% higher than real West Germany.



Timing Based Placebo Test

Suppose we do the whole thing over again but pretend that re-unification happened in 1975.

If the synthetic control "discovers" an effect in 1975...then the whole thing is a bit unimpressive.



Country Based Placebo Tests

Pretend each of the control countries is the treated unit.

Use the same method to construct a synthetic comparison group for Norway, Greece, Italy,...

In each case, we pretend that “something happened” in 1990.

Summary Statistic to Gauge Performance

Compute the gap between the lines in each period.

Square the gap. (It’s a residual.)

Compute the average of the squared residuals over the pre-period. Take the square root.

Repeat for post period.

Compute the ratio: $\frac{RMSE_{post}}{RMSE_{pre}}$

Did the gap between the lines get bigger in the post period?

$$RMSE_{pre} = \sqrt{\left(\frac{1}{T}\sum_{t=1960}^{1989}\left(Y_{jt} - Y_{st}\right)^2\right)}$$

$$RMSE_{post} = \sqrt{\left(\frac{1}{T}\sum_{t=1960}^{1989}\left(Y_{jt} - Y_{st}\right)^2\right)}$$

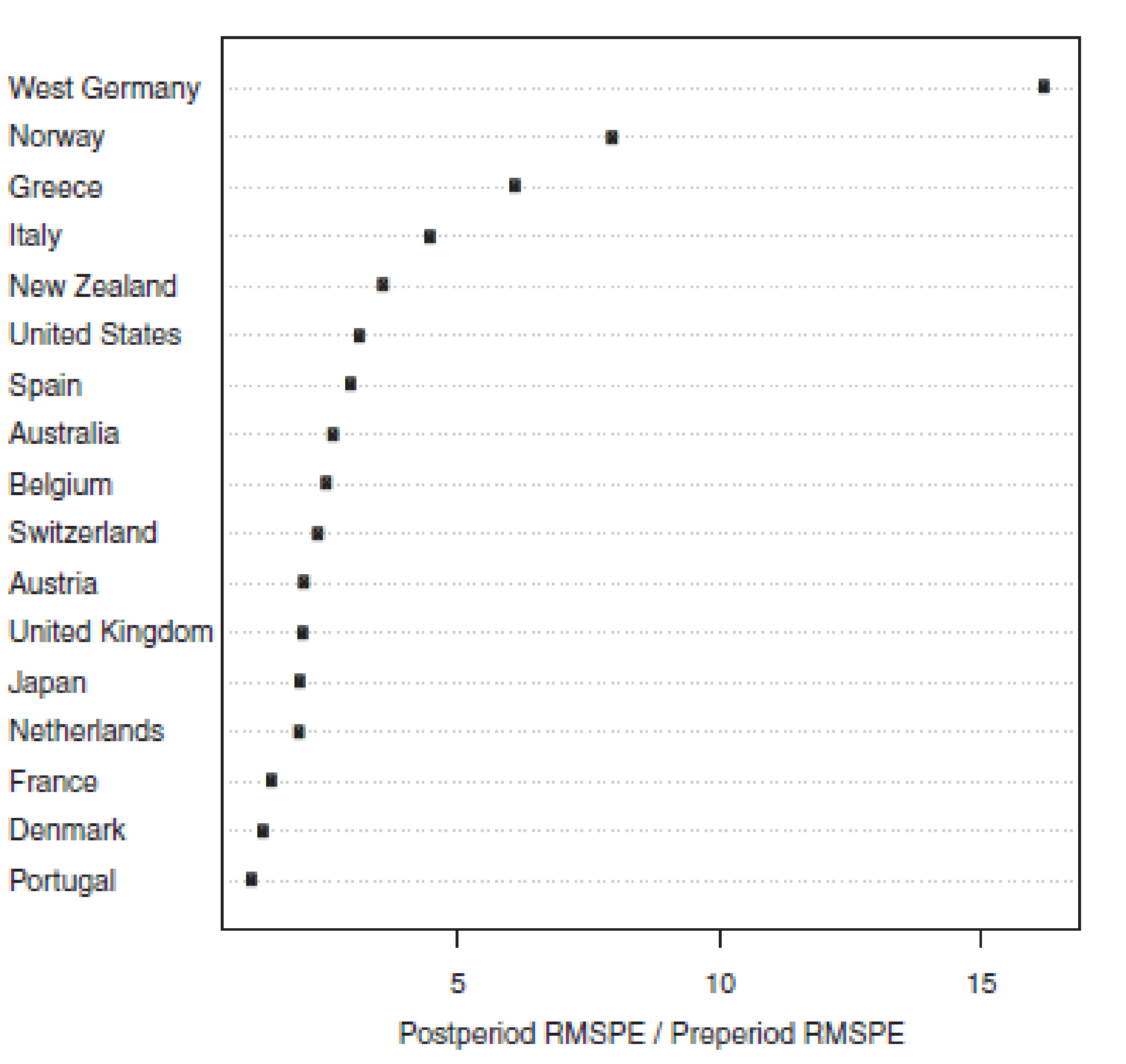
Placebo Test Results

West Germany's gap was much worse in the post period. (Because there is a big treatment effect.)

In the placebo countries, the fit is always a bit worse.

But West Germany is a big outlier.

This makes you think that it's not just statistical noise. Germany



How do they choose the weights?

Nitty Gritty: how do they make the synthetic West Germany?

Two Kinds of Weights:

Country weights

Importance weights

Pre-unification Attributes:

PC GDP
Inflation Rate
Industry Share of Value Added
Investment Rate
Schooling Level
Measure of Trade Openness.

Use country weights to make the synthetic time series

$$Y_{st}^{0} = \sum_{j=2}^{J+1} \theta_j \times Y_{jt}^{0}$$

Choose country weights to minimize differences in pre-unification attributes.

But what if you are well matched on inflation rates and badly matched on trade openness?

Solution: minimize an importance weighted average of differences on pre-unification attributes.

Technical Version

X_1 is a *(k x 1)* vector of pre-treatment statistics in West Germany.

PC GDP, Inflation Rate, Industry Share of Value Added, Investment Rate, Schooling Level, Measure of Trade Openness.

X_0 is a *(k x J)* matrix of the same pre-treatment statistics from the donor pool

θ is a J x 1 vector of country weights

$X_0\theta$ is the k x 1 vector of country weighted pre-unification statistics.

Weighted sum of differences

Choose country weights to minimize:

$$\min_{w} ||X_1 - X_0\theta||$$

Where: $\left||X_1 - X_0\theta|\right| = \sqrt{(X_1 - X_0\theta)^T V (X_1 - X_0\theta)}$

V is a diagonal matrix of importance weights.

How important is it that the synthetic control and the treated unit “match” on each of the covariates.

Synthetic Control Using Lasso

An extension to regular Synthetic Control

Synthetic Control Using Lasso

$\widehat{\alpha_{1t}} = Y_{jt}^{(1)} - Y_{st}^{(0)}$ is the treatment effect for unit j in period t.

How do we determine the synthetic control $Y_{st}^{(0)}$?

What kind of “things” can be in the “donor pool”?

Adopt a regression framework

Y_{st}^{0} is the untreated potential outcome for the synthetic unit in period t.

$$Y_{st}^{0} = \sum_{j=2}^{J+1} Y_{jt}^{0} \times \theta_j$$

Equivalent to write it like:

$$Y_{st}^{0} = Y_{jt}^{0}\theta$$

The synthetic control looks a lot like the predicted value from a regression of the treated unit on the vector of donor pool units.

Could simply choose the synthetic control weights using OLS regressions…

$$WGGDP_t = USGDP_t\theta_1 + UKGDP_t\theta_2 + \cdots + AustriaGDP_t\theta_J + \epsilon_t$$

This looks so easy. The synthetic control is the predicted value from the regression.

Coefficients are a mix of importance weights and country weights…But so what.

Relaxes some constraints: regular synth weights have to be non-negative, sum to 1, etc.

Problem with OLS approach

Very easy to over fit the time series and then there will be poor out of sample performance.

OLS can’t accommodate situations where the donor pool has more candidates than there are time periods to analyze.

Solution: LASSO

Choose weights with lasso

Choose weights to satisfy:

$$argmin_{\beta}\left\{\frac{1}{2N}\sum_{t=1}^{T_0}\left(Y_{1t}-\sum_{s=2}^{S}\beta_s Y_{st}\right)^2+\lambda\left(\sum_{s=2}^{S}|\beta_s|\right)\right\}$$

The first term is just regular OLS.

Second term is a penalty for complexity.

How does this work?

$$argmin_{\beta}\left\{\frac{1}{2N}\sum_{t=1}^{T_0}\left(Y_{1t}-\sum\nolimits_{s=2}^{S}\beta_s Y_{st}\right)^2+\lambda\left(\sum\nolimits_{s=2}^{S}|\beta_s|\right)\right\}$$

- λ is a parameter that controls the penalty.
  - When $\lambda = 0$ you have OLS.
  - When $\lambda > 0$ you shrink the coefficients towards zero and sometimes you set some coefficients to zero. (Sparsity)
- Choose λ using cross-validation
  - Split the pre-period into training and evaluation sets

Why not just do simple regression?

Set $\lambda = 0$ to obtain OLS Weights

Over-fitting to the pretest data may lead to poor out of sample forecasts.

Every candidate control gets “some” weight. It might be nice/interpretable to form the synthetic control a smaller number of states.

Regression doesn’t seem to let you “match” many pretest outcomes at once.

- That is, you may have more combinations of “donor units” and regressors than observations

Advantages From Lasso Weights

Useful when more regressors than observations

Regularization helps avoid overfitting, which aids extrapolation.

Allows for non-outcome of interest products to contribute to the prediction

Allows for negative weights

Interpretable- It’s still a regression, which is easy to understand.

It’s easy to extend to multivariate case using seemingly unrelated regression with Lasso.

Removes the researcher degree of freedom in model selection for both specification of interest **and** placebo goods that form the basis of statistical inference

Why would you have so many controls?

Donor pool could include:

- Time series of the dependent variable in other territories.
  - Example: Annual Per capita GDP in a bunch of other countries.
- Time series of other variables in other territories.
  - Examples:
    - Annual unemployment rate in a bunch of other countries.
    - Annual defense budget in a bunch of other countries.
    - Annual measures of the value of a stock index.
    - Etc, Etc, Etc.

Once you start adding things up, it’s easy for $J \gg T$

Application to Recreational Marijuana Laws In Colorado

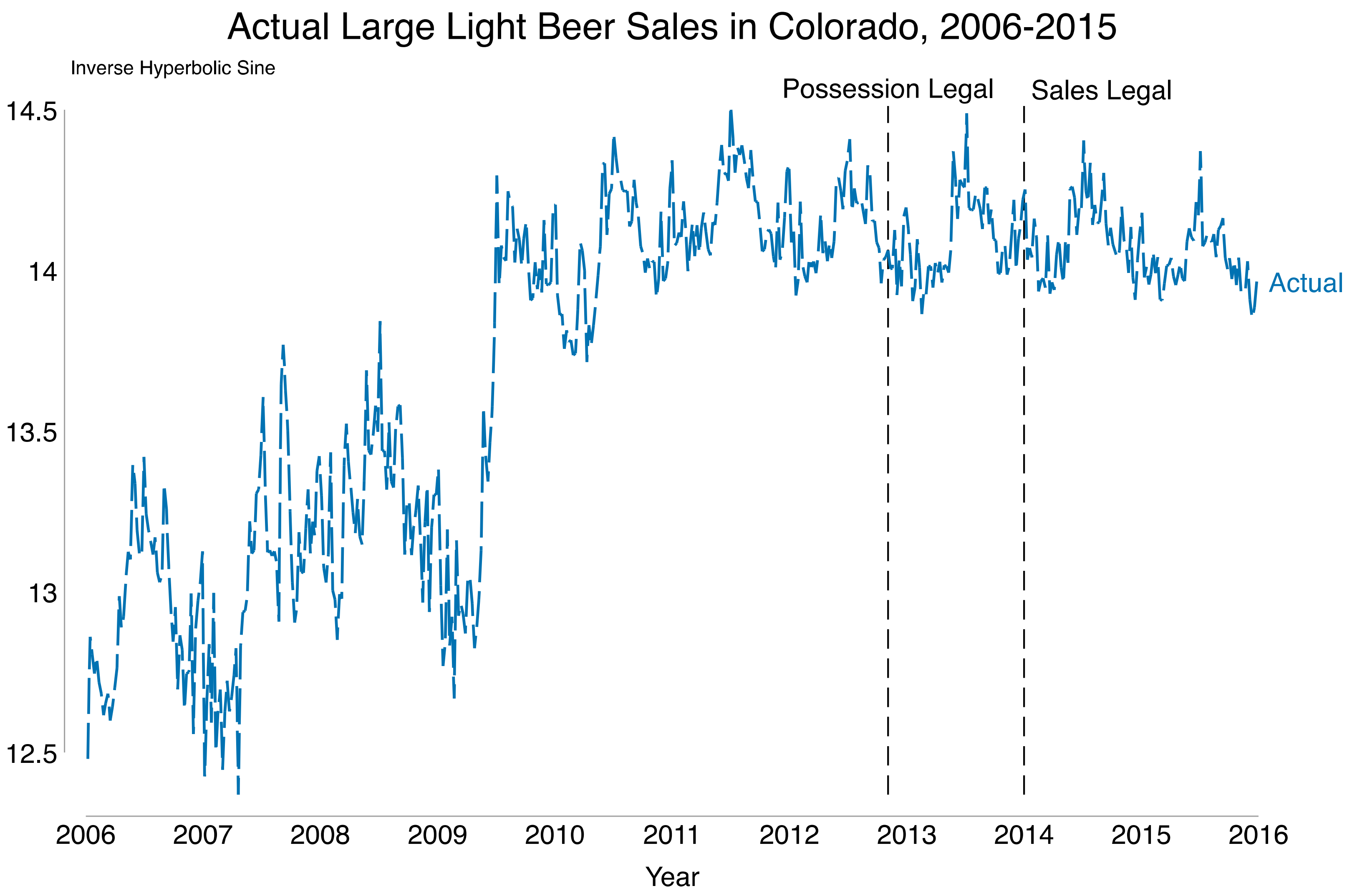
What happens to Beer Sales when you legalize marijuana?

Use retail scanner data to measure the quantity of products sold every month in Colorado.

Scanner data are organized by UPC codes. There are so many types of alcohol and tobacco products!

And what is the best donor pool of comparison time series for each possible “type” of product?

What could possibly be a good synthetic control for this crazy time series?



Light beer in other states?

Wine in other states?

Milk in other states?

Shampoo, razor blades, diapers in other states?

The list is very large and the chance of overfitting is very high.

Important problem for a lot of newly available data sets.

Things we want to learn from the marijuana alcohol study...

What is the effect of the policy on substitution to other products?

Can we implement a synthetic control strategy in a very high dimensional setting and still make sense of the results?

Can we find effective ways to avoid researcher degrees of freedom, over fitting, etc.