

# Data Structures and Algorithms

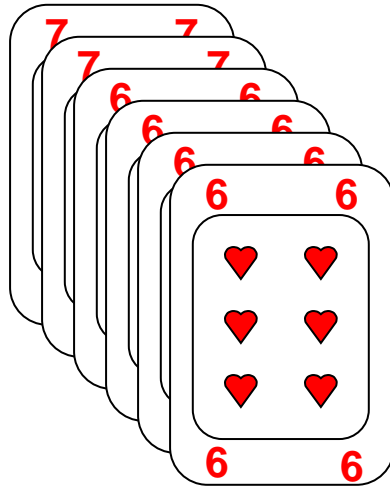
## Objectives

- ◆ In this session, you will learn to:
  - ◆ Identify the features of a stack
  - ◆ Implement stacks
  - ◆ Apply stacks to solve programming problems

# Data Structures and Algorithms

## Stacks

- ◆ Let us play the game of Rummy.



# Data Structures and Algorithms

## Defining a Stack

- ◆ What is a Stack ?
- ◆ A stack is a collection of data items that can be accessed at only one end, called top.
- ◆ Items can be inserted and deleted in a stack only at the top.
- ◆ The last item inserted in a stack is the first one to be deleted.
- ◆ Therefore, a stack is called a Last-In-First-Out (LIFO) data structure.

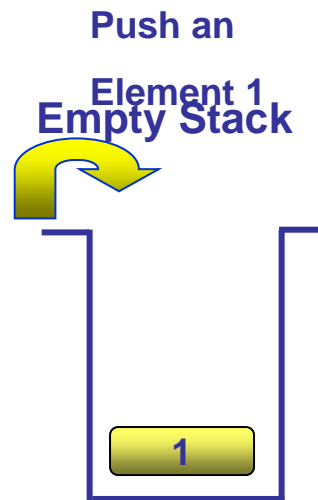
# Data Structures and Algorithms

## Identifying the Operations on Stacks

- ◆ **PUSH** and **POP** are the basic operations that are performed on the top of a stack.

- ◆ PUSH

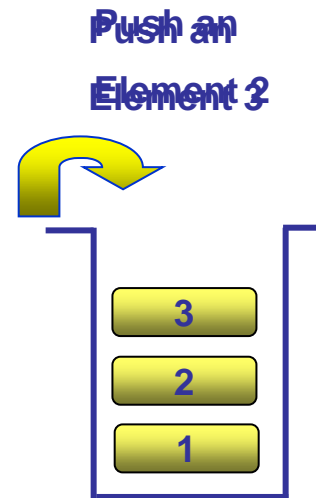
- ◆ POP



# Data Structures and Algorithms

## Identifying the Operations on Stacks (Contd.)

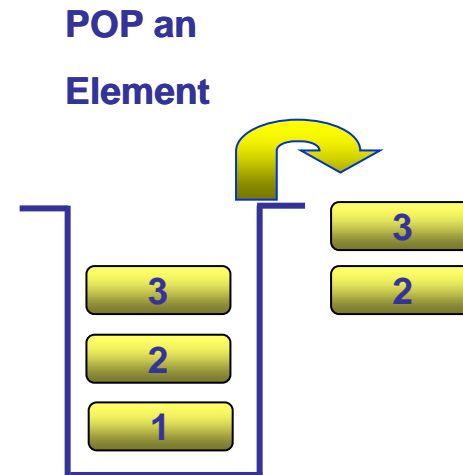
- ◆ **PUSH:** It is the process of inserting a new element on the top of a stack.



# Data Structures and Algorithms

## Identifying the Operations on Stacks (Contd.)

- ◆ **POP:** It is the process of deleting an element from the top of a stack.





# Data Structures and Algorithms

## Just a minute

◆ Elements in stacks are inserted and deleted on a \_\_\_\_\_ basis.

◆ Answer:

◆ LIFO

# Data Structures and Algorithms

## Just a minute

- ◆ List down some real life examples that work on the LIFO principle.
- ◆ Answer:
  - ◆ **Pile of books:** Suppose a set of books are placed one over the other in a pile. When you remove books from the pile, the topmost book will be removed first. Similarly, when you have to add a book to the pile, the book will be placed at the top of the pile.
  - ◆ **Pile of plates:** The first plate begins the pile. The second plate is placed on the top of the first plate and the third plate is placed on the top of the second plate, and so on. In general, if you want to add a plate to the pile, you can keep it on the top of the pile. Similarly, if you want to remove a plate, you can remove the plate from the top of the pile.
  - ◆ **Bangles in a hand:** When a person wears bangles, the last bangle worn is the first one to be removed.



# Data Structures and Algorithms

## Implementing Stacks

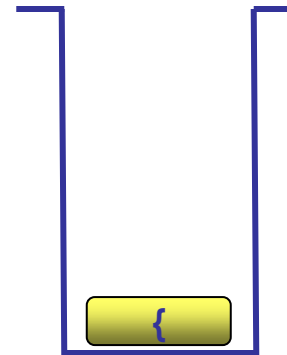
- ◆ You need to develop a method to check if the parentheses in an arithmetic expression are correctly nested.
- ◆ How will you solve this problem?
- ◆ You can solve this problem easily by using a stack.

# Data Structures and Algorithms

## Implementing Stacks (Contd.)

- ◆ Consider an example.
- ◆ Suppose the expression is:  
 $\{(a + b) \times (c + d) + (c \times d)\}$
- ◆ Scan the expression from left to right.
- ◆ The first entry to be scanned is '{', which is a left parenthesis.
- ◆ Push it into the stack.

$\{(a + b) \times (c + d) + (c \times d)\}$

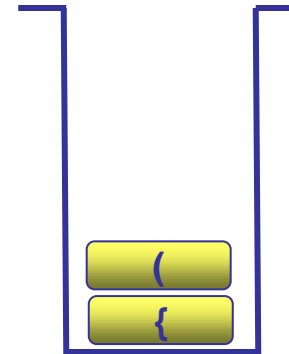


# Data Structures and Algorithms

## Implementing Stacks (Contd.)

$\{(a + b) \times (c + d) + (c \times d)\}$

- ◆ The next entry to be scanned is '(', which is a left parenthesis.
- ◆ Push it into the stack.
- ◆ The next entry is 'a', which is an operand. Therefore, it is discarded.
- ◆ The next entry is '+', which is an operator. Therefore, it is discarded.
- ◆ The next entry is 'b', which is an operand. Therefore, it is discarded.

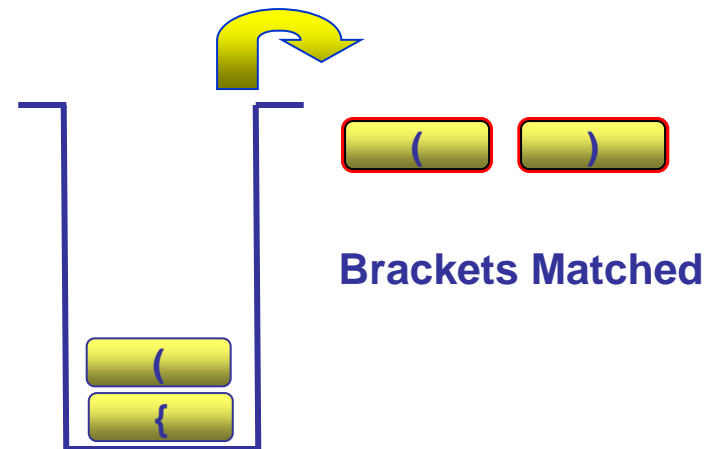


# Data Structures and Algorithms

## Implementing Stacks (Contd.)

- ◆ The next entry to be scanned is ')', which is a right parenthesis
- ◆ POP the topmost entry from the stack.
- ◆ Match the two brackets.

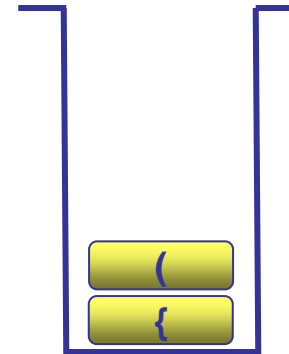
$\{(a + b) \times (c + d) + (c \times d)\}$



# Data Structures and Algorithms

## Implementing Stacks (Contd.)

- ◆ The next entry to be scanned is '×', which is an operator. Therefore, it is discarded.  $\{(a + b) \times (c + d) + (c \times d)\}$
- ◆ The next entry to be scanned is '(', which is a left parenthesis
- ◆ Push it into the stack
- ◆ The next entry to be scanned is 'c', which is an operand. Therefore it is discarded
- ◆ The next entry to be scanned is '+', which is an operator. Therefore it is discarded
- ◆ The next entry to be scanned is 'd', which is an operand. Therefore it is discarded



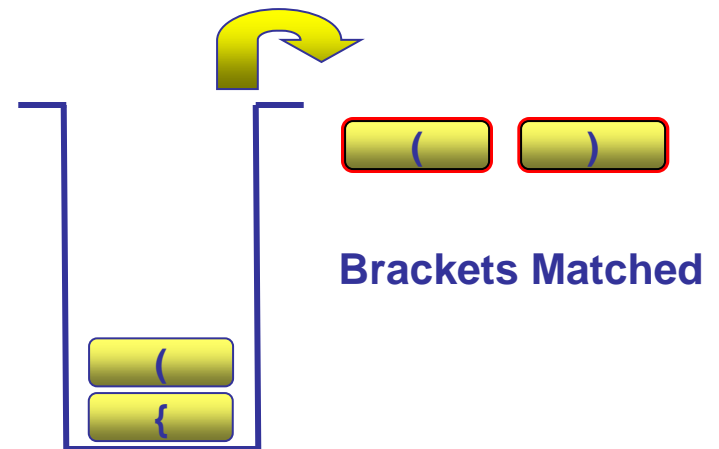


# Data Structures and Algorithms

## Implementing Stacks (Contd.)

- ◆ The next entry to be scanned is ')', which is a right parenthesis.
- ◆ POP the topmost element from the stack.
- ◆ Match the two brackets.

$\{(a + b) \times (c + d) + (c \times d)\}$



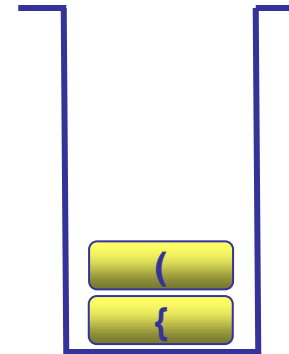


# Data Structures and Algorithms

## Implementing Stacks (Contd.)

- ◆ The next entry to be scanned is '+', which is an operator. Therefore, it is discarded.
- ◆ The next entry to be scanned is '(', which is a left parenthesis.
- ◆ Push it into the stack.
- ◆ The next entry to be scanned is 'c', which is an operand. Therefore, it is discarded.
- ◆ The next entry to be scanned is '×', which is an operator. Therefore, it is discarded.
- ◆ The next entry to be scanned is 'd', which is an operand. Therefore, it is discarded.

$\{(a + b) \times (c + d) + (c \times d)\}$

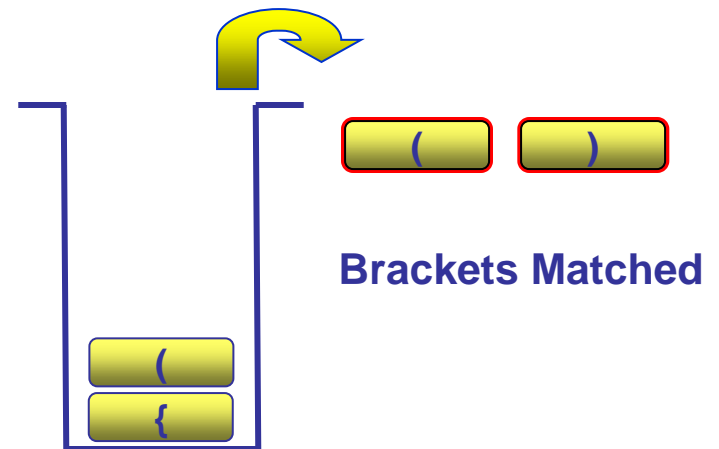


# Data Structures and Algorithms

## Implementing Stacks (Contd.)

- ◆ The next entry to be scanned is ')', which is a right parenthesis.
- ◆ POP the topmost element from the stack.
- ◆ Match the two brackets.

$\{(a + b) \times (c + d) + (c \times d)\}$



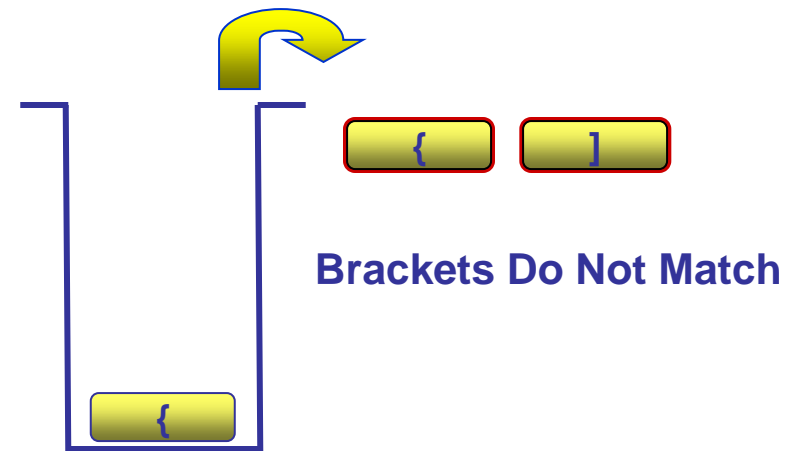
# Data Structures and Algorithms

## Implementing Stacks (Contd.)

- ◆ The next entry to be scanned is ']', which is a right parenthesis.
- ◆ POP the topmost element from the stack.
- ◆ Match the two brackets.

$\{(a + b) \times (c + d) + (c \times d)\}$

The Expression is INVALID



# Data Structures and Algorithms

## Implementing a Stack Using an Array

- ◆ To implement a stack using an array, insertion and deletion is allowed only at one end.
  - ◆ Declare an array:  
`int arr[50];`
- ◆ Therefore, similar to a list, stack can be implemented using both arrays and linked lists.
  - ◆ Declare a variable, top to hold the index of the topmost element in the stacks:  
`int top;`
  - ◆ Initially, when the stack is empty, set:  
`top = -1`

# Data Structures and Algorithms

## Implementing a Stack Using an Array (Contd.)

- ◆ Let us now write an algorithm for the PUSH operation.

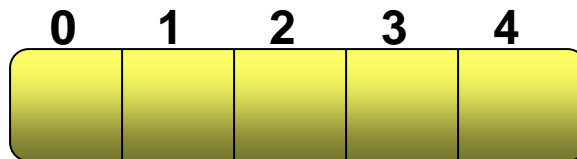
Initially:

$\text{top} = -1$

**PUSH an element 3**

1. Increment top by 1.
2. Store the value to be pushed at index top in the array. Top now contains the index of the topmost element.

**Stack**





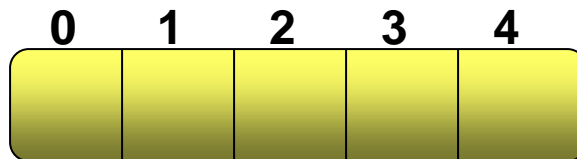
# Data Structures and Algorithms

## Implementing a Stack Using an Array (Contd.)

$\text{top} = -1$

**PUSH** an element 3

**Stack**



1. Increment  $\text{top}$  by 1.
2. Store the value to be pushed at index  $\text{top}$  in the array.  $\text{top}$  now contains the index of the topmost element.



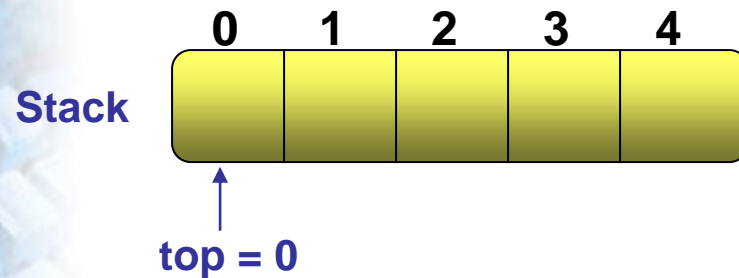
# Data Structures and Algorithms

## Implementing a Stack Using an Array (Contd.)

1. Increment top by 1.
2. Store the value to be pushed at index top in the array. Top now contains the index of the topmost element.

**top = 0**

**PUSH an element 3**

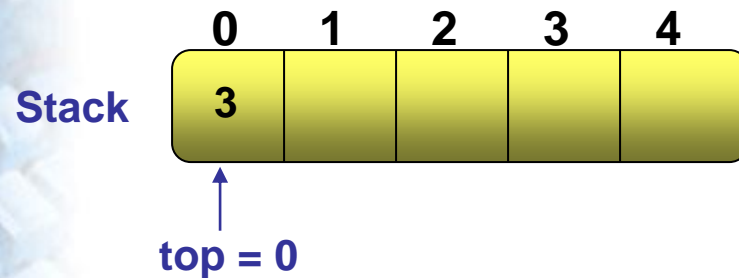


# Data Structures and Algorithms

## Implementing a Stack Using an Array (Contd.)

1. Increment top by 1.
2. Store the value to be pushed at index top in the array. Top now contains the index of the topmost element.

**PUSH an element 3**



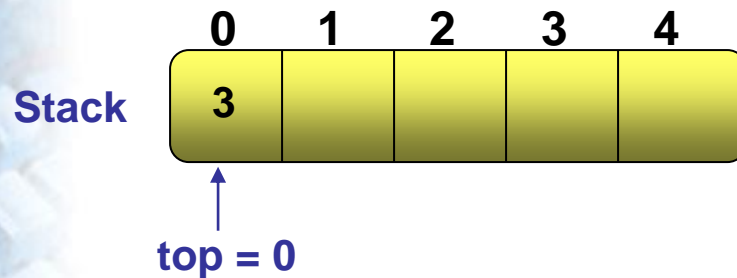
**Item pushed**

# Data Structures and Algorithms

## Implementing a Stack Using an Array (Contd.)

1. Increment top by 1.
2. Store the value to be pushed at index top in the array. Top now contains the index of the topmost element.

**PUSH an element 8**

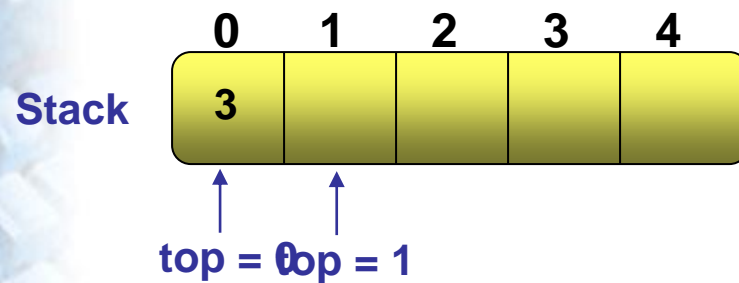


# Data Structures and Algorithms

## Implementing a Stack Using an Array (Contd.)

1. Increment top by 1.
2. Store the value to be pushed at index top in the array. Top now contains the index of the topmost element.

**PUSH an element 8**

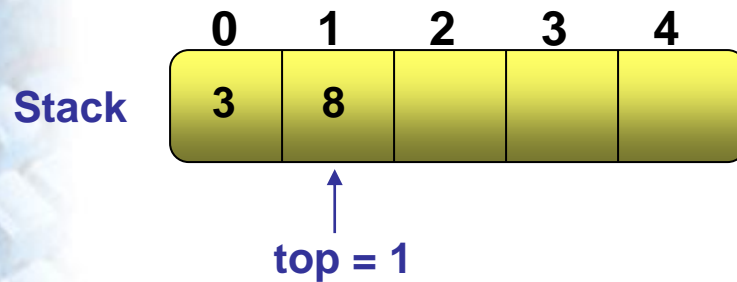


# Data Structures and Algorithms

## Implementing a Stack Using an Array (Contd.)

1. Increment top by 1.
2. Store the value to be pushed at index top in the array. Top now contains the index of the topmost element.

**PUSH an element 8**



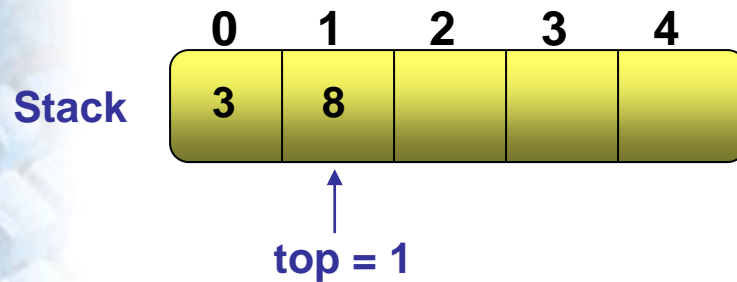
**Item pushed**

# Data Structures and Algorithms

## Implementing a Stack Using an Array (Contd.)

1. Increment top by 1.
2. Store the value to be pushed at index top in the array. Top now contains the index of the topmost element.

**PUSH an element 5**



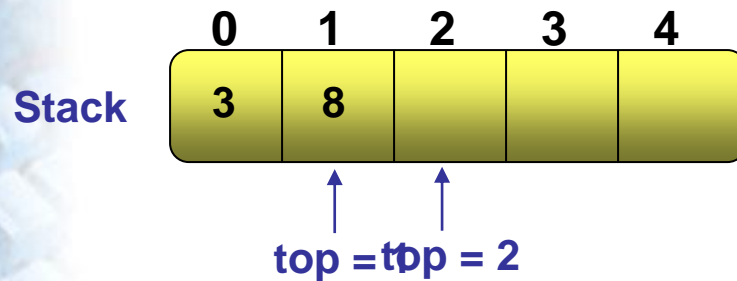


# Data Structures and Algorithms

## Implementing a Stack Using an Array (Contd.)

1. Increment top by 1.
2. Store the value to be pushed at index top in the array. Top now contains the index of the topmost element.

**PUSH an element 5**

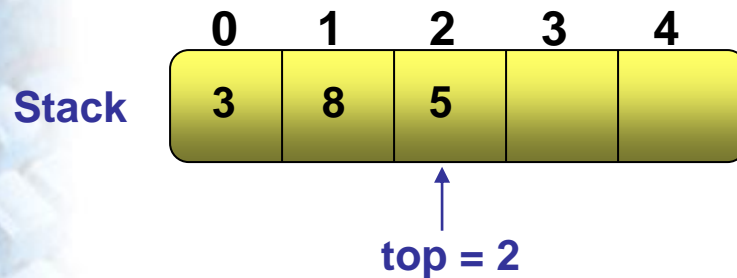


# Data Structures and Algorithms

## Implementing a Stack Using an Array (Contd.)

1. Increment top by 1.
2. Store the value to be pushed at index top in the array. Top now contains the index of the topmost element.

**PUSH an element 5**



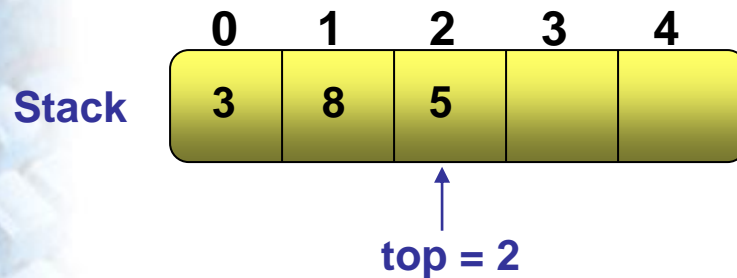
**Item pushed**

# Data Structures and Algorithms

## Implementing a Stack Using an Array (Contd.)

1. Increment top by 1.
2. Store the value to be pushed at index top in the array. Top now contains the index of the topmost element.

**PUSH an element 1**

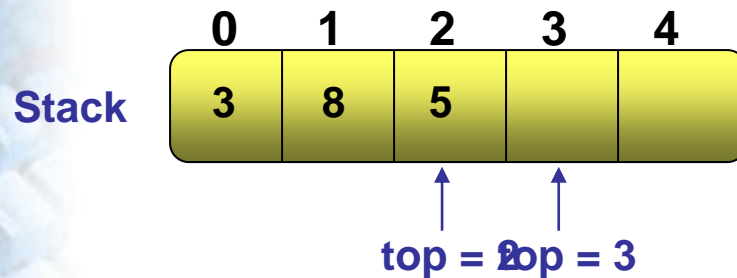


# Data Structures and Algorithms

## Implementing a Stack Using an Array (Contd.)

1. Increment top by 1.
2. Store the value to be pushed at index top in the array. Top now contains the index of the topmost element.

**PUSH an element 1**

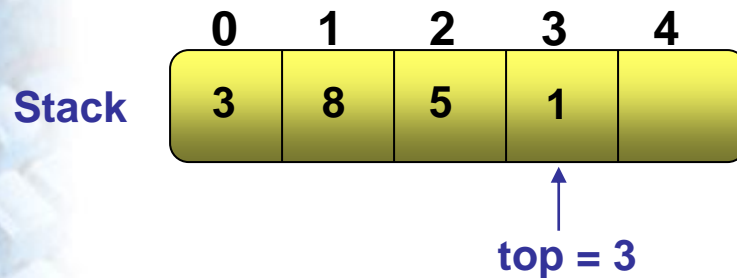


# Data Structures and Algorithms

## Implementing a Stack Using an Array (Contd.)

1. Increment top by 1.
2. Store the value to be pushed at index top in the array. Top now contains the index of the topmost element.

**PUSH an element 1**



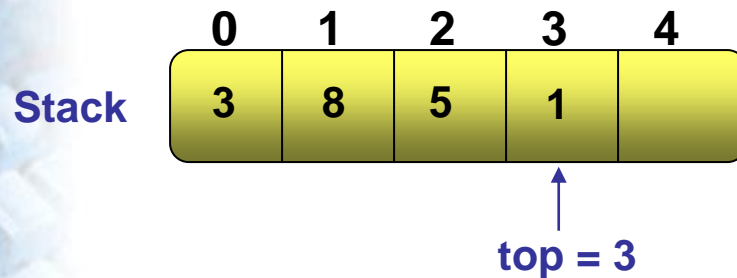
**Item pushed**

# Data Structures and Algorithms

## Implementing a Stack Using an Array (Contd.)

1. Increment top by 1.
2. Store the value to be pushed at index top in the array. Top now contains the index of the topmost element.

**PUSH an element 9**



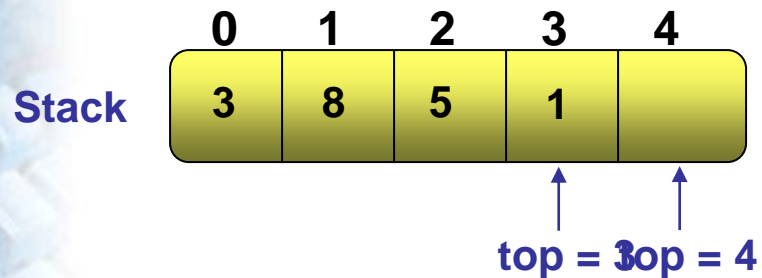


# Data Structures and Algorithms

## Implementing a Stack Using an Array (Contd.)

1. Increment top by 1.
2. Store the value to be pushed at index top in the array. Top now contains the index of the topmost element.

**PUSH an element 9**

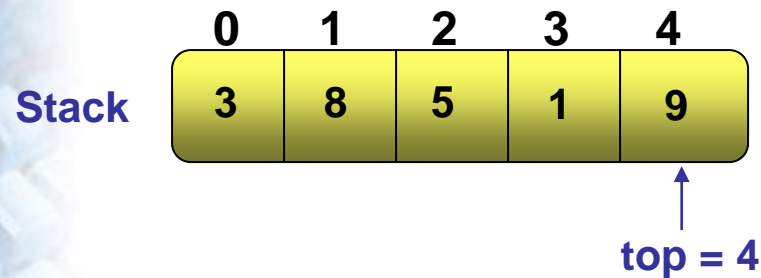


# Data Structures and Algorithms

## Implementing a Stack Using an Array (Contd.)

1. Increment top by 1.
2. Store the value to be pushed at index top in the array. Top now contains the index of the topmost element.

**PUSH an element 9**



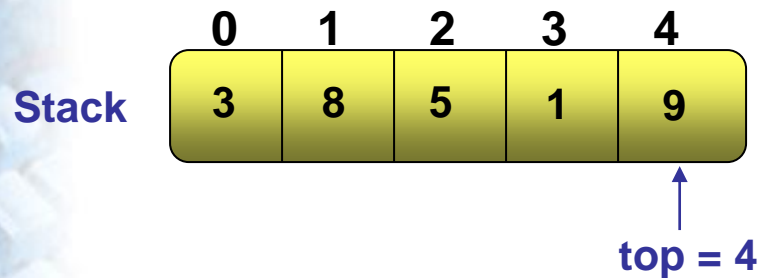
**Item pushed**

# Data Structures and Algorithms

## Implementing a Stack Using an Array (Contd.)

1. Increment top by 1.
2. Store the value to be pushed at index top in the array. Top now contains the index of the topmost element.

**PUSH an element 2**

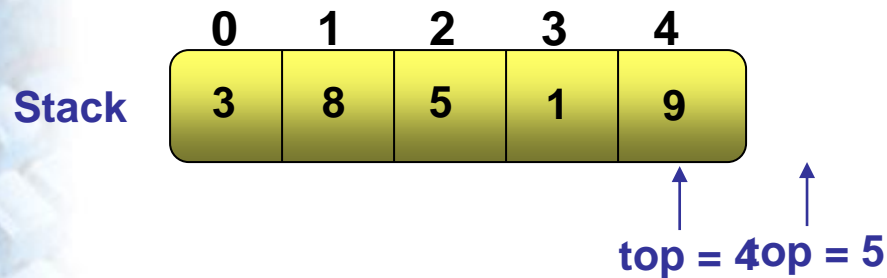


# Data Structures and Algorithms

## Implementing a Stack Using an Array (Contd.)

1. Increment top by 1.
2. Store the value to be pushed at index top in the array. Top now contains the index of the topmost element.

**PUSH an element 2**

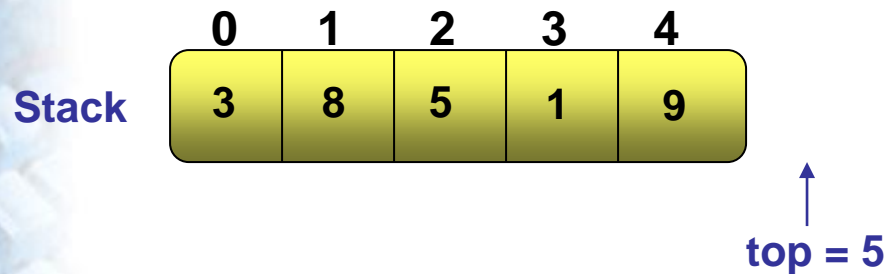


# Data Structures and Algorithms

## Implementing a Stack Using an Array (Contd.)

1. Increment top by 1.
2. Store the value to be pushed at index top in the array. Top now contains the index of the topmost element.

**PUSH an element 2**



**Stack overflow**



# Data Structures and Algorithms

## Implementing a Stack Using an Array (Contd.)

- ◆ The stack has been implemented, you need to check for the stack full condition before pushing an element into the stack.
- ◆ Therefore, you cannot store more than 5 elements in the stack.
- ◆ Let us modify the algorithm to check for this condition.

1. If  $top = MAX - 1$ :
  - a. Display "Stack Full"
2. Store the value to be pushed at index  $top$  in the array.  $top$  now contains the index of the topmost element.
3. Store the value to be pushed at index  $top$  in the array

Stack

0	1	2	3	4
3	8	5	1	9

# Data Structures and Algorithms

## Implementing a Stack Using an Array (Contd.)

- ◆ Write an algorithm to implement the POP operation on a stack.
- ◆ Algorithm for POP operation:
  1. If  $\text{top} = -1$ :
    - a. Display "Stack Empty"
    - b. Exit
  2. Retrieve the value stored at index  $\text{top}$
  3. Decrement  $\text{top}$  by 1

# Data Structures and Algorithms

## Just a minute

- ◆ In a stack, data can be stored and removed only from one end of the stack called the \_\_\_\_\_ of the stack.

- ◆ Answer:

◆ top

# Data Structures and Algorithms

## Implementing a Stack Using a Linked List

- ◆ Write an algorithm for POP operation in the stack as follows:  
operations using a linked list.
  1. Make a variable `top` pointing to the topmost node.
  2. Assign value to the data field of the topmost node.
  3. Make the `next` field of the topmost node point to `top`.
  4. Release memory to the node marked by `tmp`.

# Data Structures and Algorithms

## Activity: Implementing a Stack Using an Array

### ◆ Problem Statement:

- ◆ Write a program to implement a stack by using an array that can store five elements.

# Data Structures and Algorithms

## Activity: Implementing a Stack Using a Linked List

### ◆ Problem Statement:

- ◆ Write a program to implement a stack by using a linked list.



# Data Structures and Algorithms

## Just a minute

- ◆ What will be the condition for stack full in a stack implemented as a linked list?

- ◆ Answer:

- ◆ When a stack is implemented as a linked list, there is no upper bound limit on the size of the stack. Therefore, there will be no stack full condition in this case.

# Data Structures and Algorithms

## Just a minute

- ◆ If a stack is represented in memory by using a linked list, then insertion and deletion of data will be done \_\_\_\_\_.
  1. At the end of the list
  2. At the beginning of the list
  3. Anywhere in the list
  4. At the beginning and at the end of the list respectively
  
- ◆ Answer:
  2. At the beginning of the list

# Data Structures and Algorithms

## Applications of Stacks

- ◆ Some of the applications of stacks are:
  - ◆ Implementing function calls
  - ◆ Maintaining the UNDO list for an application
  - ◆ Checking the nesting of parentheses in an expression
  - ◆ Evaluating expressions

# Data Structures and Algorithms

## Implementing Function Calls

- ◆ Implementing function calls:
  - ◆ Consider an example. There are three functions, F1, F2, and F3. Function F1 invokes F2 and function F2 invokes F3, as shown.

# Data Structures and Algorithms

## Implementing Function Calls (Contd.)

```
        void F1 ()
        {
1100            int x;
1101            x = 5;
1102            F2 ();
1103            print(x) ;
        }
        void F2(int x)
        {
1120            x = x + 5;
1121            F3(x) ;
1122            print(x) ;
        }
        void F3(int x)
        {
1140            x = x * 2;
1141            print x;
        }
```

Assuming these instructions at the given locations in the memory.

# Data Structures and Algorithms

## Implementing Function Calls (Contd.)

```
void F1()  
{  
1100    int x;  
1101    x = 5;  
1102    F2();  
1103    print(x);  
}  
void F2(int x)  
{  
1120    x = x + 5;  
1121    F3(x);  
1122    print(x);  
}  
void F3(int x)  
{  
1140    x = x * 2;  
1141    print x;  
}
```

The execution starts from  
function F1



# Data Structures and Algorithms

## Implementing Function Calls (Contd.)

```
        void F1 ()
        {
1100            int x;
1101            x = 5;
1102            F2 ();
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        }
        void F2(int x)
        {
1120            x = x + 5;
1121            F3(x) ;
1122            print(x) ;
        }
        void F3(int x)
        {
1140            x = x * 2;
1141            print x;
        }
```

# Data Structures and Algorithms

## Implementing Function Calls (Contd.)

**x = 5**

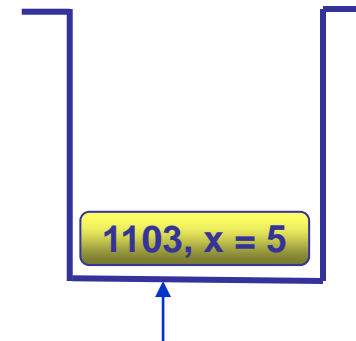
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{  
1140     x = x * 2;  
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```

# Data Structures and Algorithms

## Implementing Function Calls (Contd.)

```
void F1()  
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1100    int x;  
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}  
void F3(int x)  
{  
1140    x = x * 2;  
1141    print x;  
}
```

**x = 5**



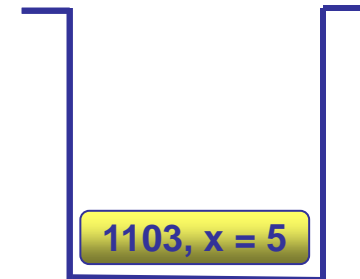
Address and the local variable of F1

# Data Structures and Algorithms

## Implementing Function Calls (Contd.)

```
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{  
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1102    F2();  
1103    print(x);  
}  
void F2(int x)  
{  
1120    x = x + 5;  
1121    F3(x);  
1122    print(x);  
}  
void F3(int x)  
{  
1140    x = x * 2;  
1141    print x;  
}
```

**x = 50**

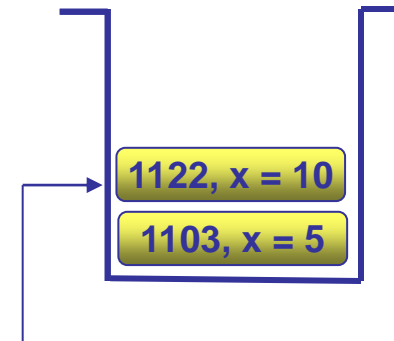


# Data Structures and Algorithms

## Implementing Function Calls (Contd.)

```
void F1()  
{  
1100    int x;  
1101    x = 5;  
1102    F2();  
1103    print(x);  
}  
void F2(int x)  
{  
1120    x = x + 5;  
1121    F3(x);  
1122    print(x);  
}  
void F3(int x)  
{  
1140    x = x * 2;  
1141    print x;  
}
```

**x = 10**



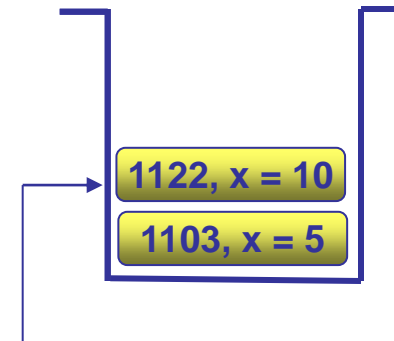
Address and the local variable of F2

# Data Structures and Algorithms

## Implementing Function Calls (Contd.)

```
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void F2(int x)  
{  
1120    x = x + 5;  
1121    F3(x);  
1122    print(x);  
}  
void F3(int x)  
{  
1140    x = x * 2;  
1141    print x;  
}
```

**x = 20**



Address and the local variable of F2

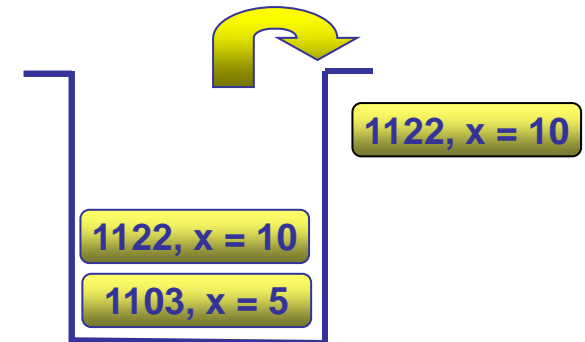


# Data Structures and Algorithms

## Implementing Function Calls (Contd.)

```
void F1()  
{  
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1101    x = 5;  
1102    F2();  
1103    print(x);  
}  
void F2(int x)  
{  
1120    x = x + 5;  
1121    F3(x);  
1122    print(x);  
}  
void F3(int x)  
{  
1140    x = x * 2;  
1141    print x;  
}
```

**x = 20**



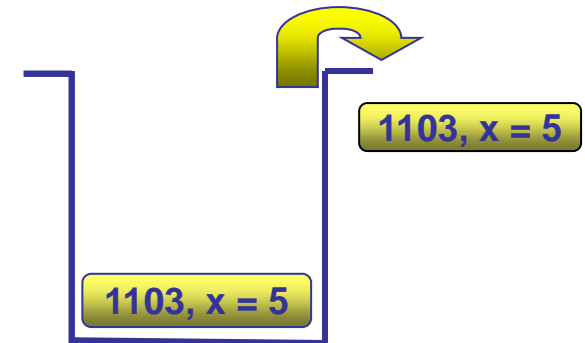
**20**

# Data Structures and Algorithms

## Implementing Function Calls (Contd.)

```
void F1()  
{  
1100    int x;  
1101    x = 5;  
1102    F2();  
1103    print(x);  
}  
void F2(int x)  
{  
1120    x = x + 5;  
1121    F3(x);  
1122    print(x);  
}  
void F3(int x)  
{  
1140    x = x * 2;  
1141    print x;  
}
```

**x = 50**



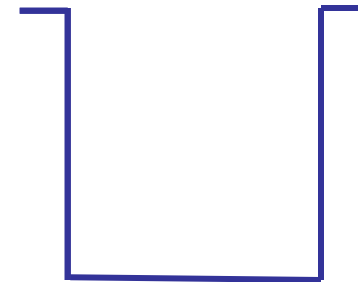
**20 10**

# Data Structures and Algorithms

## Implementing Function Calls (Contd.)

```
void F1()  
{  
1100     int x;  
1101     x = 5;  
1102     F2();  
1103     print(x);  
}  
void F2(int x)  
{  
1120     x = x + 5;  
1121     F3(x);  
1122     print(x);  
}  
void F3(int x)  
{  
1140     x = x * 2;  
1141     print x;  
}
```

**x = 5**



**20    10    5**

- ◆ Maintaining the UNDO list for an application:
  - ◆ Consider that you made some changes in a Word document. Now, you want to revert back those changes. You can revert those changes with the help of an UNDO feature.
  - ◆ The UNDO feature reverts the changes in a LIFO manner. This means that the change that was made last is the first one to be reverted.
  - ◆ You can implement the UNDO list by using a stack.

# Data Structures and Algorithms

## Checking the Nesting of Parentheses in an Expression

- ◆ Checking the nesting of parentheses in an expression:
  - ◆ You can do this by checking the following two conditions:
    - ◆ The number of left parenthesis should be equal to the number of right parenthesis.
    - ◆ Each right parenthesis is preceded by a matching left parenthesis.

# Data Structures and Algorithms

## Evaluating Expressions

- ◆ Evaluating an expression by using stacks:
  - ◆ Stacks can be used to solve complex arithmetic expressions.
  - ◆ The evaluation of an expression is done in two steps:
    - ◆ Conversion of the infix expression into a postfix expression.
    - ◆ Evaluation of the postfix expression.



# Data Structures and Algorithms

## Activity: Implementing a Stack using a Linked List

### ◆ Problem Statement:

- ◆ Write a program that accepts an infix expression, and then converts it into a postfix expression. You can assume that the entered expression is a valid infix expression.

# Data Structures and Algorithms

## Summary

- ◆ In this session, you learned that:
  - ◆ A stack is a collection of data items that can be accessed at only one end, called top. The last item inserted in a stack is the first one to be deleted.
  - ◆ A stack is called a LIFO data structure.
  - ◆ There are two operations that can be performed on stacks. They are:
    - ◆ PUSH
    - ◆ POP
  - ◆ Stacks can be implemented by using both arrays and linked lists.

# Data Structures and Algorithms

## Summary (Contd.)

- ◆ Stacks are used in many applications. Some of the application domains of stacks are as follows:
  - ◆ Implementing function calls
  - ◆ Maintaining the UNDO list for an application
  - ◆ Checking the nesting of parentheses in an expression
  - ◆ Evaluating expressions