Birzeit University

Faculty of Engineering and Technology

Department of Electrical and Computer Engineering

First Semester – 2023/2024

ENCS2340 - Digital Systems

Homework # 1

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Question 1 :

Decimal	Binary	Octal	Hexadecimal	BCD
154_{10}	10011010_2	232_8	$9A_{16}$	$0001\ 0101\ 0100_{BCD}$
29.25_{10}	11101.01_2	35.2_8	$1D.4_{16}$	$0010\ 1001.0010\ 0101_{BCD}$
93.1999_{10}	1011101.0011_2	135.1463_8	5D.333	$10010011.0001100110011001_{BCD}$
17.5_{10}	10001.1_2	21.4_8	11.8_{16}	$0001\ 0111.0101_{BCD}$
0.125_{10}	0.001_2	0.1_8	0.2_{16}	$0000\ .\ 0001\ 0010\ 0101_{BCD}$

Question 2 :

A) A'C' +A'BC + B'C'

= A'C' +A'BC + B'C'

= A' (C'+BC) + B'C'   \\ BY DISTRIBUTIVE : AB+AC= A(B+C)

= A' (B+C` ) +B`C`          \\ by Absorption Law : AB + A` = B+A`

= A`B+A`C`+B`C`   \\ by Distribution

= A'B + B'C' .   \\ by Consensus

B) BC + AC’ + AB + BCD

BC + AC’ + AB + BCD

BC + A.C’ + AB \\ by Absorption Law : A+AB = A

BC + A.C’ \\ by Consensus

C) ABC + A’B’C + A’BC + ABC’ + A’B’C’

BC(A+A’) + A’B’C+ABC’+A’B’C \\ by Distributive : AB+AC= A(B+C)

BC1 + A’B’C+ABC’+A’B’C’ \\ by Complement : A+A`=1

BC(A+A’) + A’B’C+ABC’+A’B’C’ \\ by Identity : A1= A

C(A’B’ + B)+ABC’+A’B’C’ \\ by Distributive : AB+AC= A(B+C)

C(A’ + B) + ABC’ + A’B’C’ \\ by Absorption Law : A`B + A = B + A

CA’ + CB + ABC’ + A’B’C’ \\ by Distribution

CA’ + B(C + AC’) + A’B’C’ \\ by Distributive : AB+AC= A(B+C)

CA’ + B(C+A) + A’B’C’ \\ by Absorption Law : A`B + A = B + A

B(A+C) + A’(B’C’ + C) \\ by Distributive : AB+AC= A(B+C)

B(A+C) + A’(B’+C) \\ by Absorption Law : A`B + A = B + A

BA+BC+A’(B’+C) \\ by Distribution

BA + BC+ A’B + A’C \\ by Distribution

B(A+A’) + BC + A’C \\ by Consensus

B + BC + A’C \\ by Complement : A+A`=1

B + A’C \\ by Absorption Law : A+AB = A

D) ((CD)’+A)’ + A + CD + AB

CD”A’ + A + CD + AB \\ by Demorgan theorm

CDA’ +A + CD + AB \\ by Involution : A``= A

CDA’ + A + CD \\ by Absorption Law : A+AB = A

CDA’ +A \\ by Absorption Law : A+AB = A

CD + A \\ by Absorption Law : A+AB = A

E) (A + C + D)(A + C + D’)(A + C’ + D)(A + B’)

(A+C’+D)(A+B’)(A+C’+D)(A+B’)AD’+(A+C+D’)(A+C’+D)(A+B’)C+(A+C+D’)(A+C’+D)(A+B’)D

A+CDB’+(A+C+D’)(A+C’+D)(A+B’)D \\ by Absorption Law : A+AB = A

A+CDB’+DCA+DCB’ \\ by Distribution

A+CDB’ \\ by Absorption Law : A+AB = A

F) (WX(Y’Z + YZ’) + W’X’(Y’+Z)(Y+Z’))’

W`W +W`X + W`Y + WX`+ XX`+X`Y+W`Y+XY`+YY` \\ distribution

0 + W`X + W`Y + WX`+0+X`Y+WY`+XY`+0 \\ complement

W`X+W`Y+WX`+X`Y+WY`+XY` \\ identity

WX` + W`Y + X`Y + W`X + WY` + XY` \\ commutative

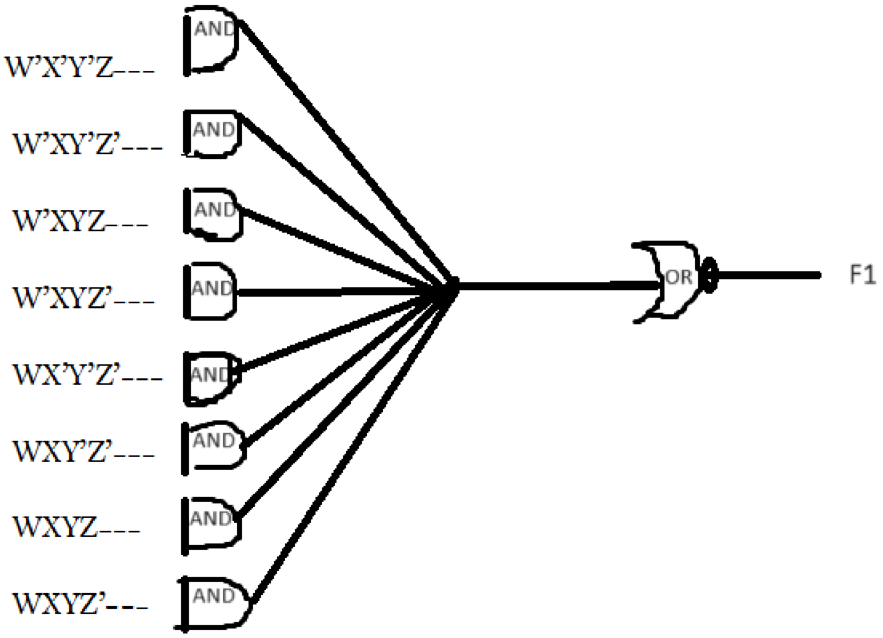
WX`+W`Y+W`X+WY` \\ consensus

WX` + W`(Y`Z+YZ`)+W`X+W(YZ+Y`Z`) \\substitution in place of y

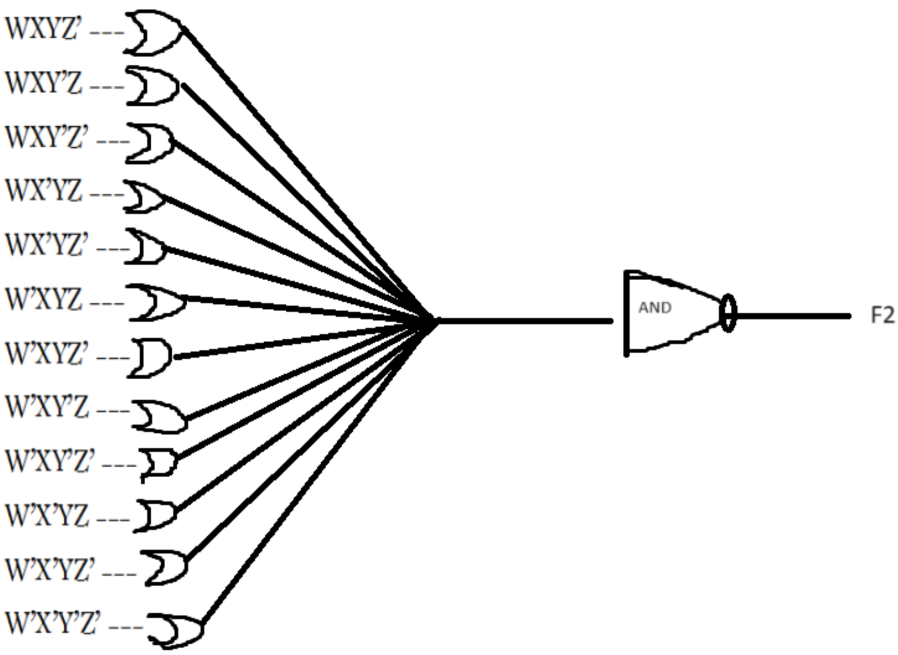
WX`+W`Y`Z+W`YZ`+W`X+WYZ+WY`Z` \\DISTRIBUTIVE

Question 3:

A) F1 = (W'X'Y'Z + W'XYZ' +W'XYZ + WX'YZ + WX'Y'Z' + WXY'Z' + WXYZ + WXYZ')



B) F2 = (W+X+Y+Z')(W+X+Y'+Z)(W+X+Y'+Z')(W+X'+Y+Z)(W+X'+Y+Z')
(W'+X+Y+Z)(W'+X+Y+Z')(W'+X+Y'+Z)(W'+X+Y'+Z')(W'+X'+Y+Z)
(W'+X'+Y+Z')(W'+X'+Y'+Z')



C) THE DUAL OF F1 IS :

F1 = (W+X+Y+Z’) (W+X’+Y’+Z)(W+X’+Y’+Z’)(W’+X+Y’+Z’)
(W’+X+Y+Z)(W’+X’+Y+Z) (W’+X’+Y’+Z’)(W’+X’+Y’+Z)

Question 4 :

A) f(X,Y,Z) = Σ(0,3,4)

= m0 + m3 + + m4

f’= X’Y’Z’+ X’YZ + XY’Z’

B) f= Σ(0,3,4) and g= Σ(0,2,4,6,7)

f + g = m0 + m2+m3+m4+ m6 + m7

= X’Y’Z’+ X’YZ’+X’YZ+ X’YZ’+ XYZ’ + XYZ

C) g = π(1,3,5) , f` = Σ(1,2,5,6,7) ➔ π(0,3,4)

(f`.g) = M3

= XY`Z`

Question 5 :

a) $(C372)_{16} – (395E)_{16} = (X)_{16}$

The first step convert the hex number to decimal number

$(2*16^0 + 7*16^1 + 3*16^2 + 12*16^3)$ ➔ (2 + 112 + 768 + 49152)➔ (50034)

$(C372)_{16} = (50034)_{10}$

$(14*16^0 + 5*16^1 + 9*16^2 + 3*16^3)$ ➔ (14 + 80 + 2304+ 12288)➔ (14686)

$(395E)_{16} = (14686)_{10}$

Then we need to subtract them from each other :

$50034 – 14686 = (35348)_{10}$

Then convert the decimal number to hex number :

(35348 / 16) = 2209 With 4 rem ➔ (2209/16)= 138 With 1 rem ➔(138/16)=8 with 10 rem ➔(10/16) = 0 with 8 rem ➔ (8A14)

$(35348)_{10} = (8A14)_{16}$

$X = (8A14)_{16}$

b) $(0010\ 1000\ 0000\ 0111)_{BCD} + (0001\ 1001\ 1001\ 0101)_{BCD} = (X)_{BCD}$

The first step convert the BCD number to decimal number

0010 = 2 / 1000= 8 / 0000 =0 / 0111 = 7 ➔ 2807_{10}

0001 =1 / 1001 = 9 / 1001 = 9 / 0101 = 5 ➔ 1995_{10}

And we need to sum two number in decimal :

$1995 + 2807 = 4002_{10}$

Then convert a decimal number to BCD :

4002 ➔ (4=0100 / 0 = 0000 / 0=0000 / 2 = 0010)

$X = 0100\ 0000\ 0000\ 0010_{BCD}$

c) $(35)_X + (18)_X = (51)_X$
$(5*X^0 + 3*X^1) + (8*X^0 + 1*X^1) = (1*X^0+5*X^1)$
3X+5 + X+8 = 5X +1
4X + 13 = 5X +1
X =12

d) $(10110.11)_5 = (X)_{15}$
First step we need to convert the base5 to base 15 , we should convert it to decimal then convert it to base 15 .
The integer number: $(0 * 5^0 + 1 * 5^1 + 1*5^2 + 0*5^3 + 1*5^4)$ ➔ (0+5+25+0+625) ➔ (655)
The fractional number : $(1*5^{-1} + 1*5^{-2})$ ➔ (0.2 +0.04) ➔ (0.24)
$(10110.11)_5$ ➔ $(655.24)_{10}$
Now we should convert a decimal to 15 base , for the integer number :
(655/15 = 43 and 10 rem) / (43/15 = 2 and 13 rem) / (2/15=0 and 2 rem) ➔ (2DA)
For the fractional number : (0.24 * 15 = 3.6 // 0.6 * 15 =9) ➔ (39)
So $X = (2DA.39)_{15}$

e) $(2404)_{10} = (C3A)_X$
$2404 = (A*X^0 + 3*X + C*X^2)$
$2404 = 12X^2 + 3X + 10$
$0 = 12X^2 + 3X - 2394$
(a=4 , b=1 , c= -798) , after divide it and a long math
X = 14

f) X = the 15’s complement of $(2B070)_{15}$
To find the 15’s complement of a base 15 number , you would subtract each digit from 14 (14 is the big digit in base 15) and then add 1 to the result .
(14-2 =12 // 14 – B = 3 // 14 -0 = E // 15– 7 =8) // and adding 1 to 7
$(2B070)_{15} = (12C80)_{15}$
$X = (C3E80)_{15}$

g) X = the Gray code for the binary value $(101100)_2$
We take the XOR of the subsequent binary of digit
(1 o 0 =1 // 0 o 1=1 // 1 o 1 =0 // 1 o 0 = 1 // 0 o 0 =0)
X = 111010

Question 6 :

a) (2^0 =1 , ….. , 2^{11} = 2048 , 2^{12} = 4096)
So we would need ≈ 12 bits

b) Number = 3500 * $2^{20/5}$
= 3500 * 2^4
= 56000
(2^0=1 , …. , 2^{15} = 32768 , 2^{16} = 65536)
So we would need ≈ 16 bits

c) 1) Now : you need 3 hex digits (12/4 =3) for 3500 students .
2) after 20 years : you need 4 hex digits (16/4 = 4) for 56,000 students.
3) comment : Hexadecimal provides a more compact and readable representation compared to binary, as each hex digit corresponds to 4 binary bits.
The use of hexadecimal is efficient for documenting binary codes, offering a concise and manageable way to represent large amounts of binary data.
Now = 3 bits
After 20 years = 4 bits

Question 7 :

a) 11001101 + 01101011
11001101 + 01101011 = 00111000_2
In decimal = 56 \\ carry = 1
And this is Correct (no overflow)

b) 01110010 – 10010111
We need convert the subtraction operation into adding operation (by find 2’s complement of the subtrahend) .
01110010 – 10010111 ➔ 01110010 + 01101001 = 11011011_2
Result is = 219 \\ no carry
And this is Overflow Occurred

c) 11111011 – 10000
We need convert the subtraction operation into adding operation (by find 2’s complement of the subtrahend). ++ (we ask a doctor for if we need to add a bits for a negative number , we add 1 or 0 , he say’s adding 0 for it by 2’s comp)
11111011 – 00010000 ➔ 11111011 + 11110000 = 11101011
Result is = 107\\ carry = 1
And this is Correct (no Overflow)

d) 01101 – 11101101
We need convert the subtraction operation into adding operation (by find 2’s complement of the subtrahend).
00001101 – 11101101 ➔ 00001101 + 00010011 = 00100000
Result is = 32 \\ no carry
And this is Correct (no Overflow)

e) 010011 – 01101
We need convert the subtraction operation into adding operation (by find 2's complement of the subtrahend).
00010011 – 00001101 ➔ 00010011 + 11110011 = 00000110
Result is = 6 \\ carry = 1
And this is Correct (no Overflow)

f) 10011 + 101101
00010011 + 00101101 = 01000000
Result is = 64 \\ carry = 1   And this is Correct (not Overflow)

Question 8 :

a) Unsigned numbers :
A = (1011001)
$A_{decimal} = (1 * 2^0) + (0*2^1) + (0*2^2) + (1*2^3) + (1*2^4) + (0*2^5) + (1*2^6)$
$A_{decimal} = (1+0+0+8+16+0+64)$
$A_{decimal} = 89$

B = (0111010)
$B_{decimal} = (0 * 2^0) + (1*2^1) + (0*2^2) + (1*2^3) + (1*2^4) + (1*2^5) + (0*2^6)$
$B_{decimal} = (0+2+0+8+16+32+0)$
$B_{decimal} = 58$

b) Signed-magnitude numbers.
A=(1011001), but the magnitude is given by the remaining bits (011001)
$A_{magnitude} = (1 * 2^0) + (0*2^1) + (0*2^2) + (1*2^3) + (1*2^4) + (0*2^5)$
$A_{magnitude} = (1+0+0+8+16+0+0)$
$A_{magnitude}$=25,but the decimal value of A as a signed-magnitude number is
$A_{magnitude} = -25$

B=(0111010) , but the magnitude is given by the remaining bits(110010)
$B_{magnitude} = (0 * 2^0) + (1*2^1) + (0*2^2) + (1*2^3) + (1*2^4) + (1*2^5)$
$B_{magnitude} = (0+2+0+8+16+32)$
$B_{magnitude} = 58$

c) Signed 1's complement numbers.
A=(1011001), but the 1's complement is given by the flip bits (100110)
$A_{1\text{'s complement}} = (0 * 2^0) + (1*2^1) + (1*2^2) + (0*2^3) + (0*2^4) + (1*2^5)$
$A_{1\text{'s complement}} = 0+2+4+0+0+32$
$A_{1\text{'s complement}} = 38$, Since a negative , it's 1's complement value is
$A_{1\text{'s complement}} = -38$

B = (0111010) , Since B is positive , we do not need to flip the bits, direct convert the magnitude to decimal .
$B_{1\text{'s complement}} = (0 * 2^0) + (1*2^1) + (0*2^2) + (1*2^3) + (1*2^4) + (1*2^5) + (0*2^6)$
$B_{1\text{'s complement}} = (0+2+0+8+16+32+0)$ ➔ $B_{1\text{'s complement}} = 58$.

d) Signed 2's complement numbers. :
First , we need find the 1's complement to A (because it's negative) by flipping the bits , and then we need add to it 1 bits to get 2's complement
A = (011001) ➔ A = (100111)
$A_{2\text{'s complement}} = (1*2^0) + (1*2^1) + (1*2^2) + (0*2^3) + (0*2^4) + (1*2^5)$
$A_{2\text{'s complement}} = 1+2+4+0+0+32$
$A_{2\text{'s complement}} = 39$, Since a negative , it's 2's complement value is
$A_{2\text{'s complement}} = -39$

Question 9 :

a) F(W, X, Y, Z) = WX'Y' + WXZ' + W'XZ + YZ'
Sum of Minterms (SOM) :
$F(W,X,Y,Z) = \Sigma m(2,5,6,7,8,9,10,12,14)$
Product of Maxterms (POS) :
$F(W,X,Y,Z) = \pi M(0,1,3,4,11,13,15)$

b) F(A, B, C, D) = D(A' + B) + B'D
Sum of Minterms :
$F(W,X,Y,Z,) = \Sigma m (1,3,5,7,9,11,13,15)$
Sum of Maxterms :
$F(W,X,Y,Z) = \pi M(0,2,4,6,8,10,12,14)$

c) F(A, B, C, D) = (A + B' + C)(A + B')(A + C' + D')(A + B + C + D')(B + C' + D')
Sum of Minterms :
$F(W,X,Y,Z,) = \Sigma m (0,2,8,9,10,12,13,14,15)$
Sum of Maxterms :
$F(W,X,Y,Z) = \pi M(1,3,4,5,6,7,11)$

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