A binary search tree

- **A binary search tree is a binary tree with a special property called the BST-property, which is given as follows:**

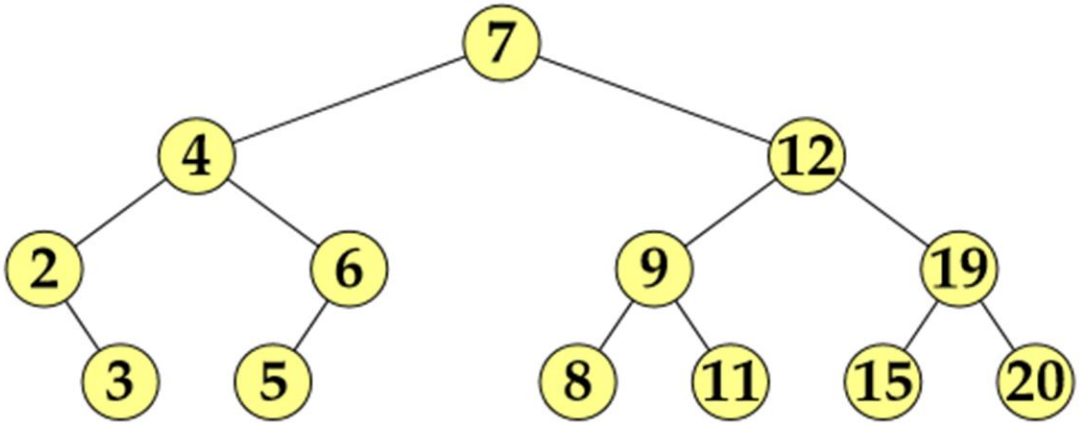
For all nodes x and y, if y belongs to the left subtree of x, then the key at y is less than or equal the key at x, and if y belongs to the right subtree of x, then the key at y is greater than or equal the key at x

Value (Left) <= Value (Root) < Value (Right)

Any one of them if duplication is allowed

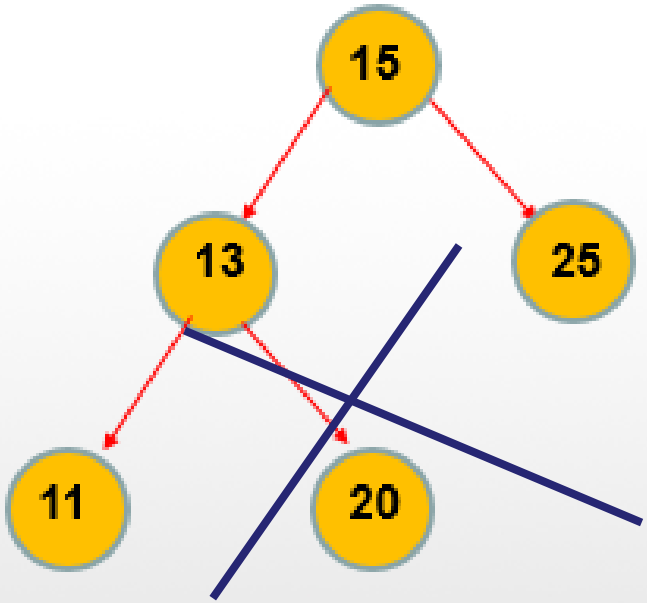
Value (Left) < Value (Root) <= Value (Right)

A binary search tree

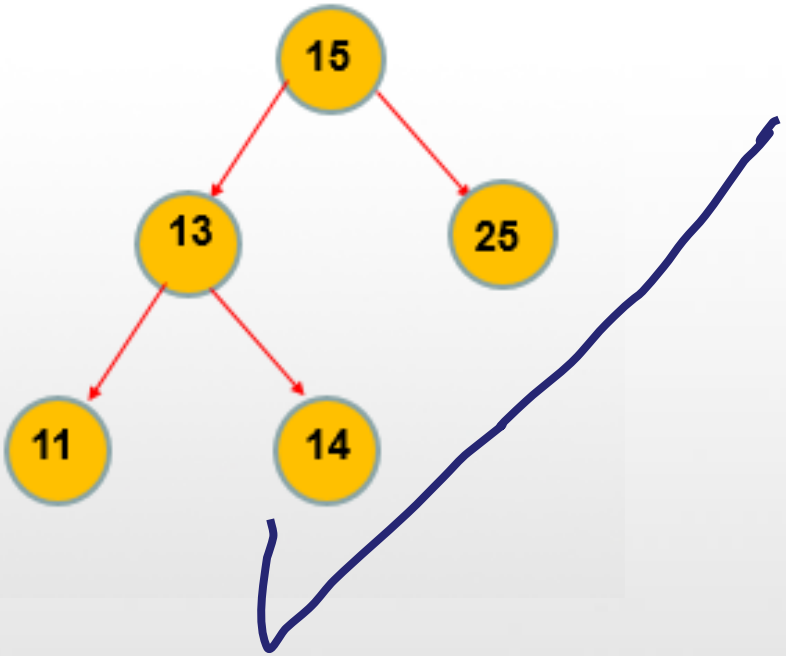


A binary search tree

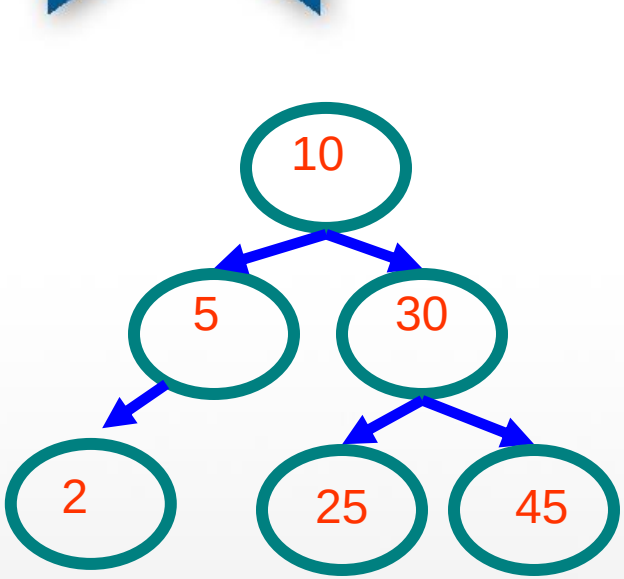
- **Which of the following is a binary search tree ?**

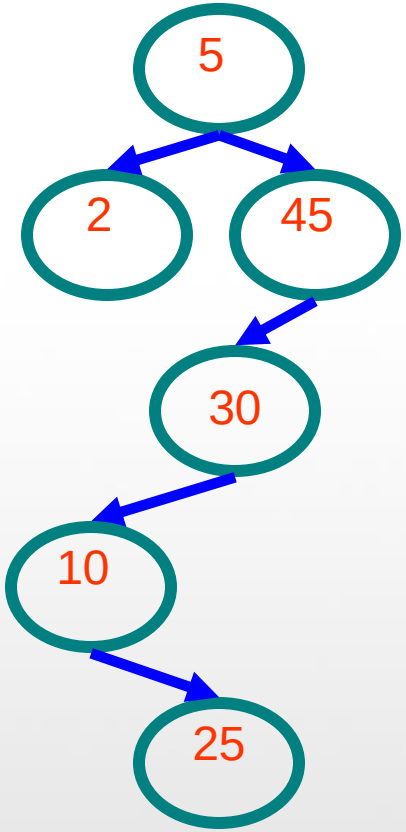


20 15

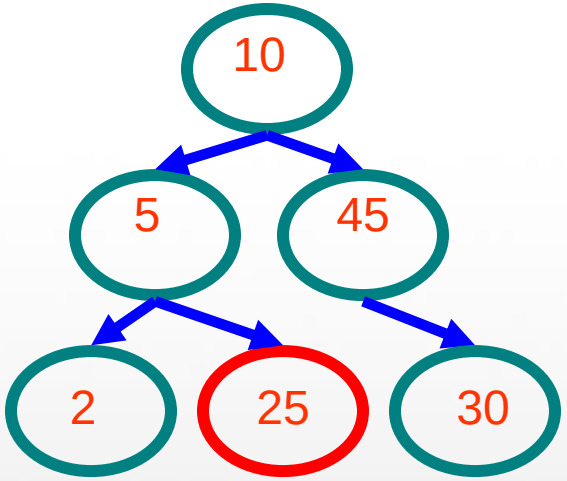


A binary search tree



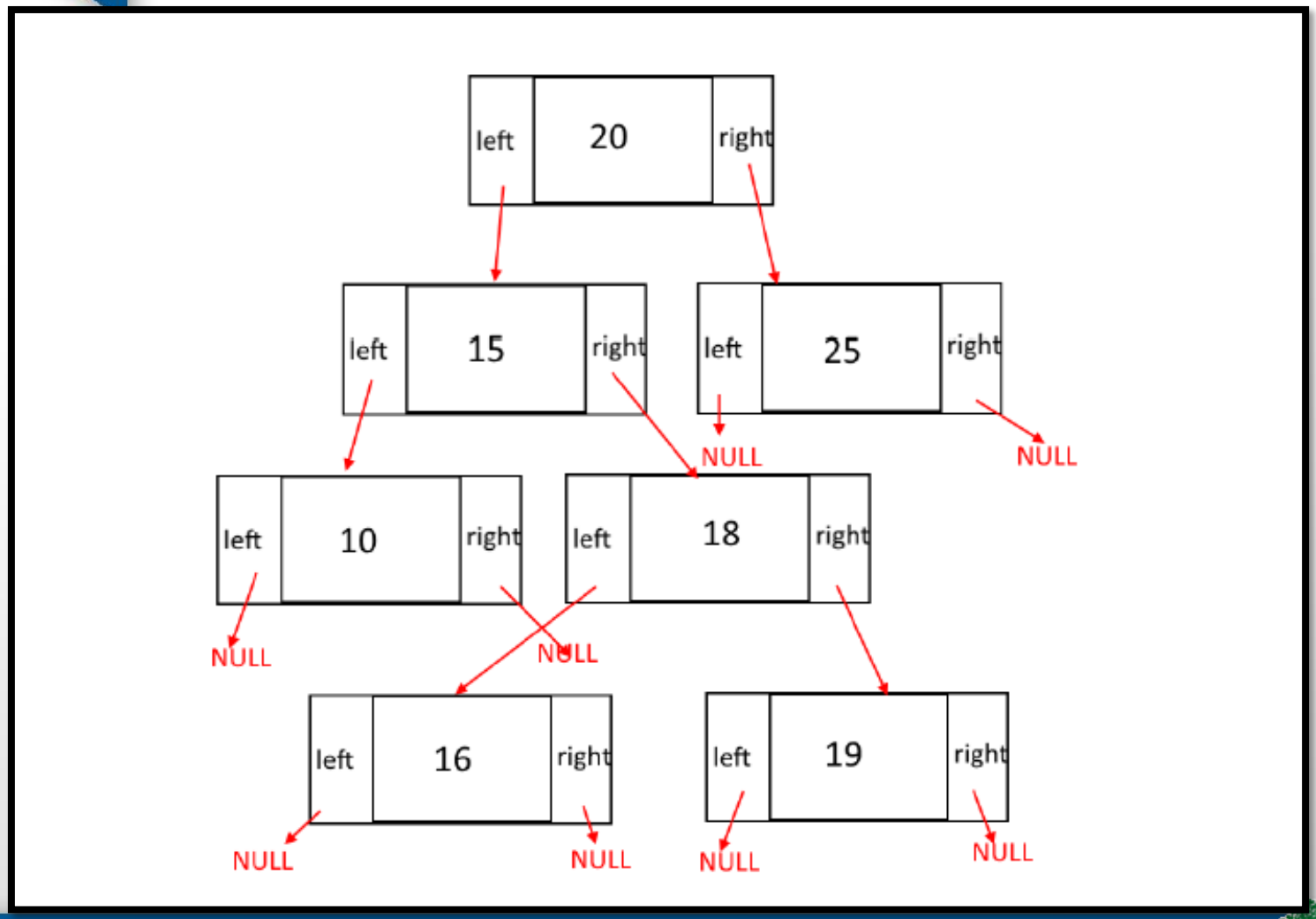


Binary search trees



Non-binary search tree

A binary search tree: Coding



Implementation : Binary Search Tree

```
//Class Node for the Binary Search Tree
public class BSTNode {
//for objects replace int to Object and modify the code
   int element;
   BSTNode left;
   BSTNode right;

   public BSTNode (int element){
      this (element,null,null);
   }
   public BSTNode (int element,BSTNode left,BSTNode right){
      this.element=element;
      this.left=left;
      this.right=right;
   }

}
```

Implementation : Binary Search Tree

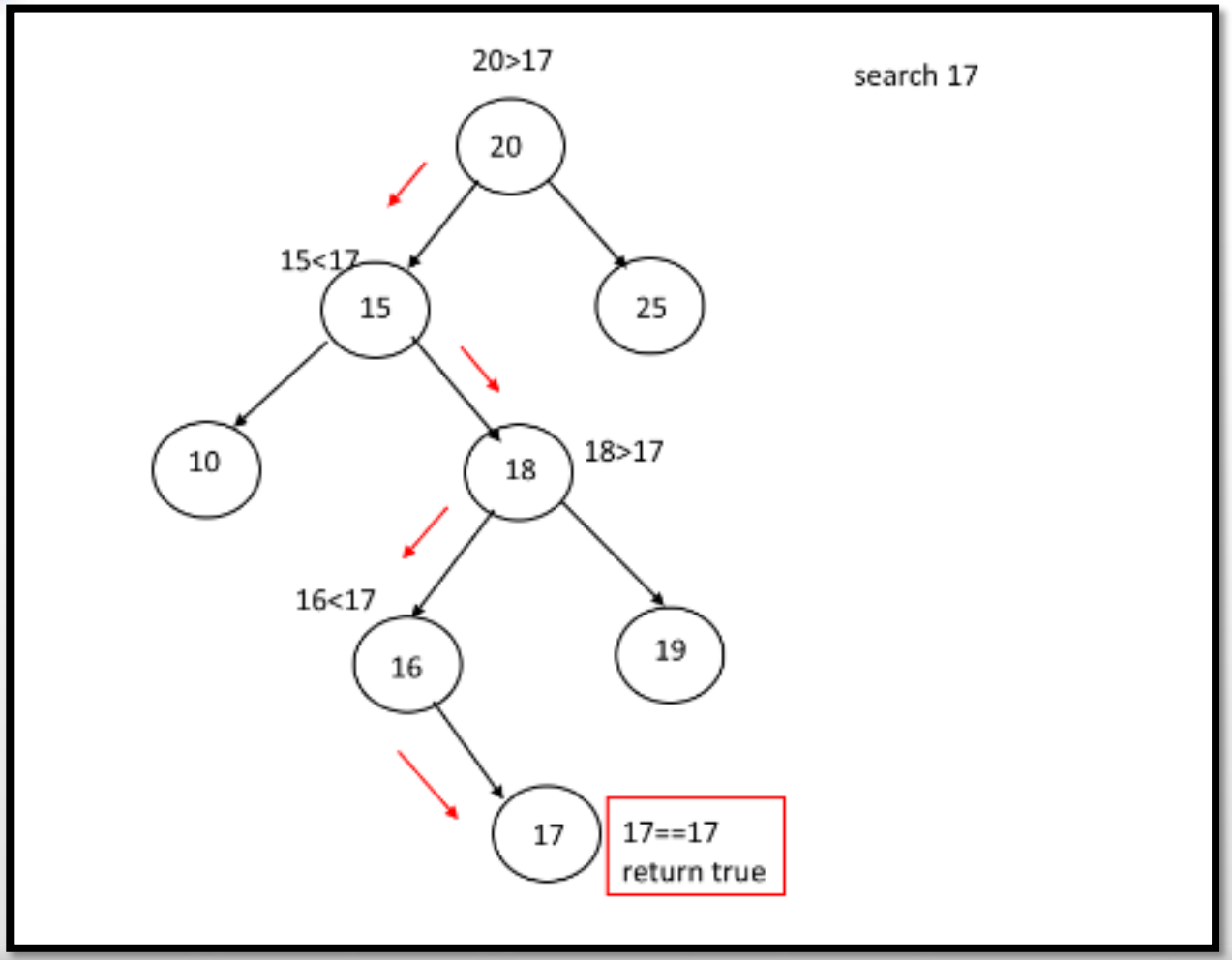
```
// Binary Search Tree Class

public class BST {

   private BSTNode root;

   public BST(){
     root=null;
    }

   /* Methods go here */

 }
```

Binary Search Tree: Contains

Contains(int n):

- Start from the root and compare root.element with n
- if root.element is greater than n that means we need to go to the left of the root.
- if root.element is smaller than n that means we need to go to the right of the root
- if any point of time root.element is equal to the n then we have found the node, return true.
- if we reach to the leaves (end of the tree) return false, we didn't find the element

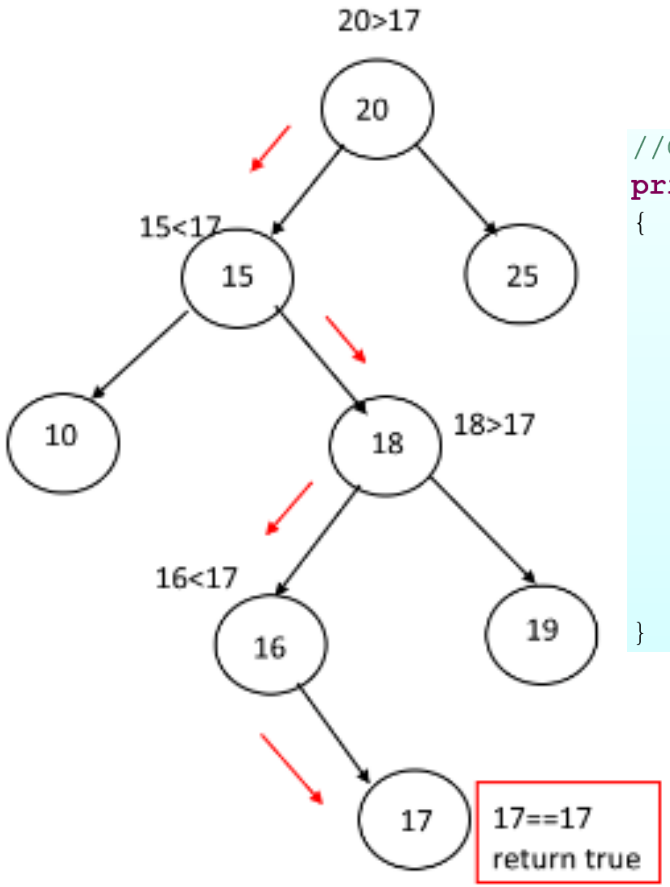
Binary Search Tree: Contains



Binary Search Tree: Conatins

```
//Check whether the element is in a tree or not
private  boolean contains (int e,BSTNode current)
{
   if (current==null)
       return false; // Not found, empty tree.

    else if (e<current.element) // if smaller than root.
       return contains (e,current.left); // Search left subtree
    else if (e>current.element) // if larger than root.
        return contains (e,current.right); // Search right subtree

    return true; // found .

}
```

Binary Search Tree: Contains




```
//Check whether the element is in a tree or not
private  boolean contains (int e,BSTNode current)
{
  if (current==null)
      return false; // Not found, empty tree.

   else if (e<current.element) // if smaller than root.
      return contains (e,current.left); // Search left subtree
   else if (e>current.element) // if larger than root.
       return contains (e,current.right); // Search right subtree

   return true; // found .

}
```

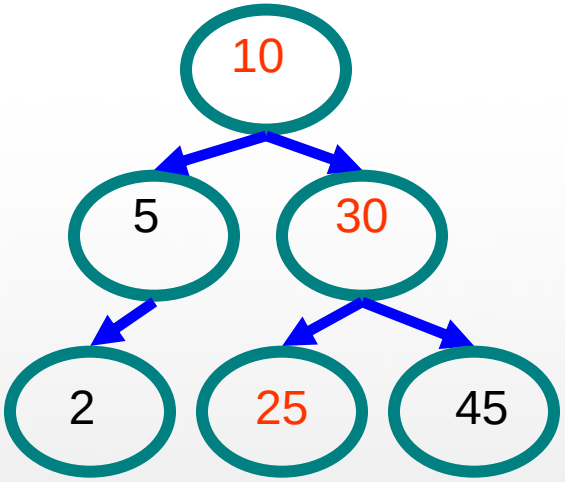
Binary Search Tree: Find

```
//Returns the node contains the given element
private BSTNode find(int element, BSTNode  current)
{
    if (current == null)
       return null;
    if (element < current.element)
        return find(element, current.left);
    else if (element > current.element)
       return find(element, current.right);
    else
       return current;
}
```

Rewrite the above code, using an Iterative way (Loop)?

Binary Search Tree: Find

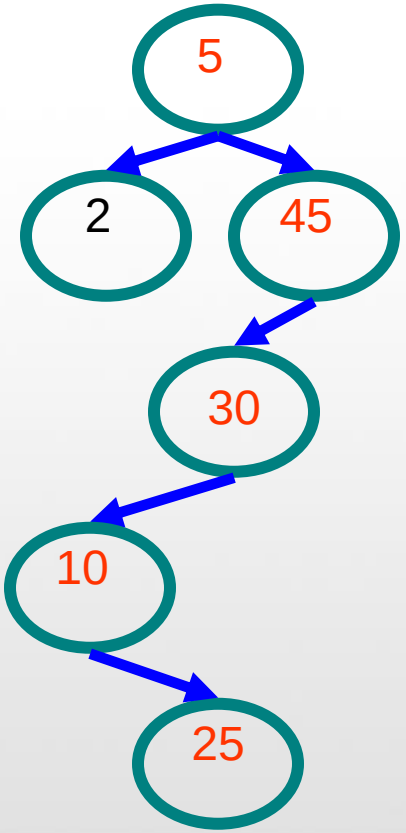
- find (25)



10 < 25, right

30 > 25, left

25 = 25, found



5 < 25, right

45 > 25, left

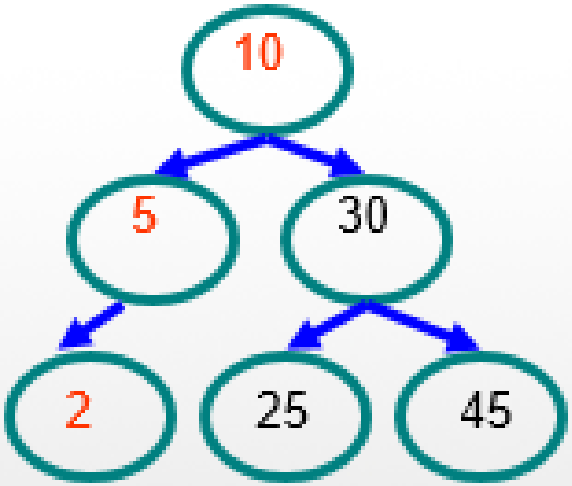
30 > 25, left

10 < 25, right

25 = 25, found

Binary Search Tree: Find

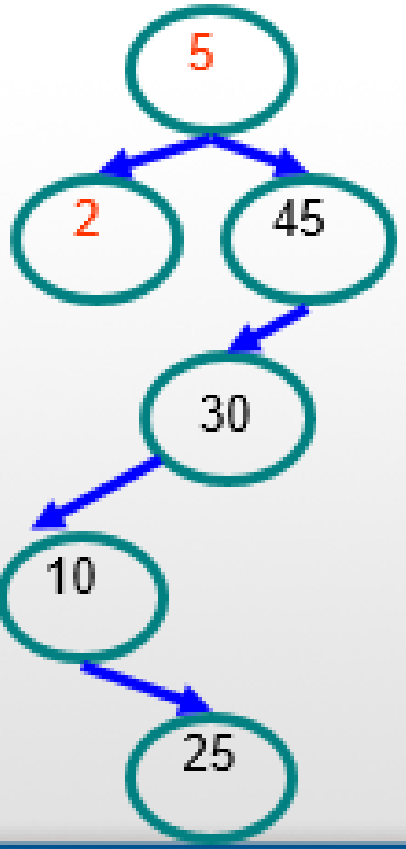
- find (2)



10 > 2, left

5 > 2, left

2 = 2, found



5 > 2, left

2 = 2, found

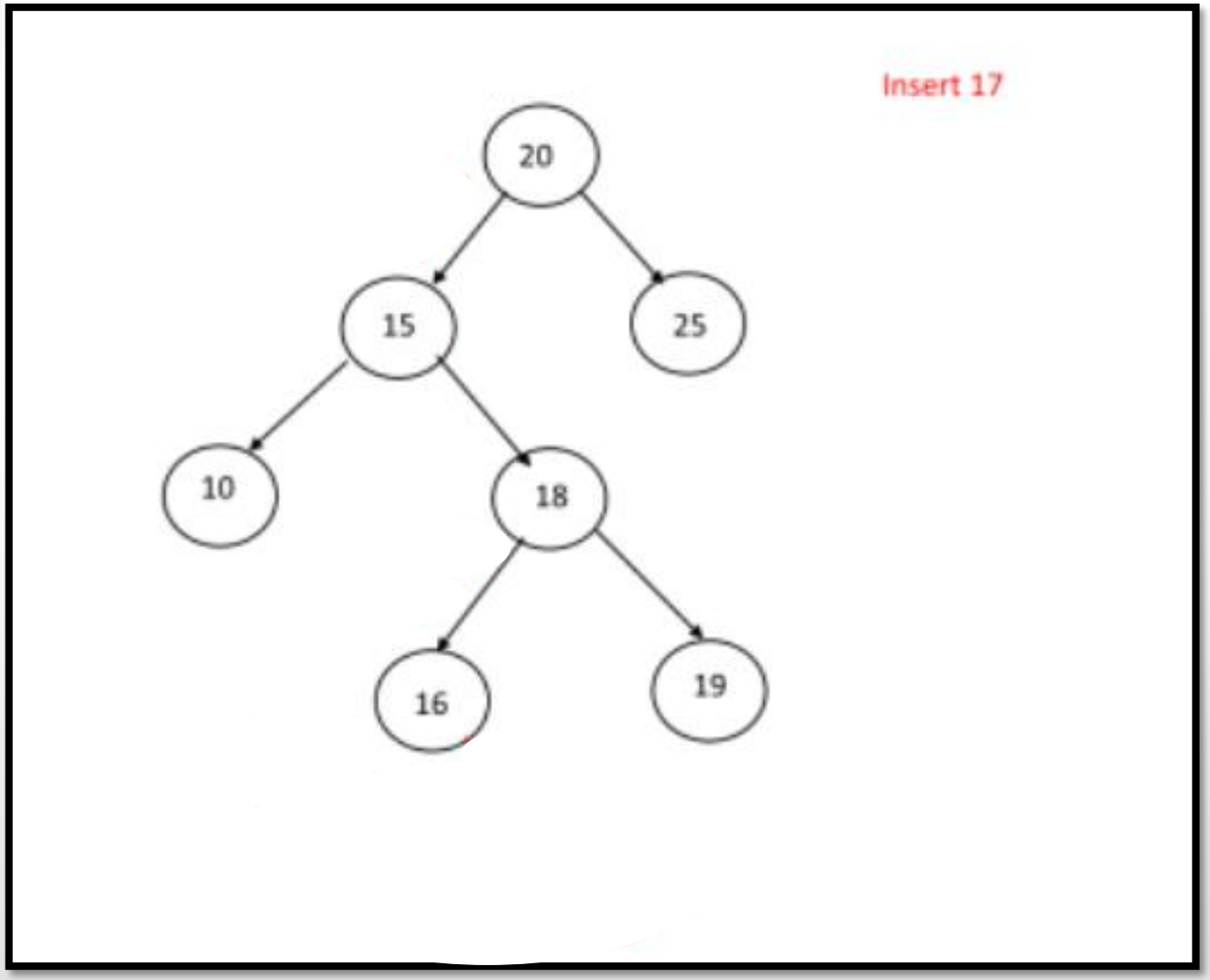
SELF STUDY

Binary Search Tree: Insert

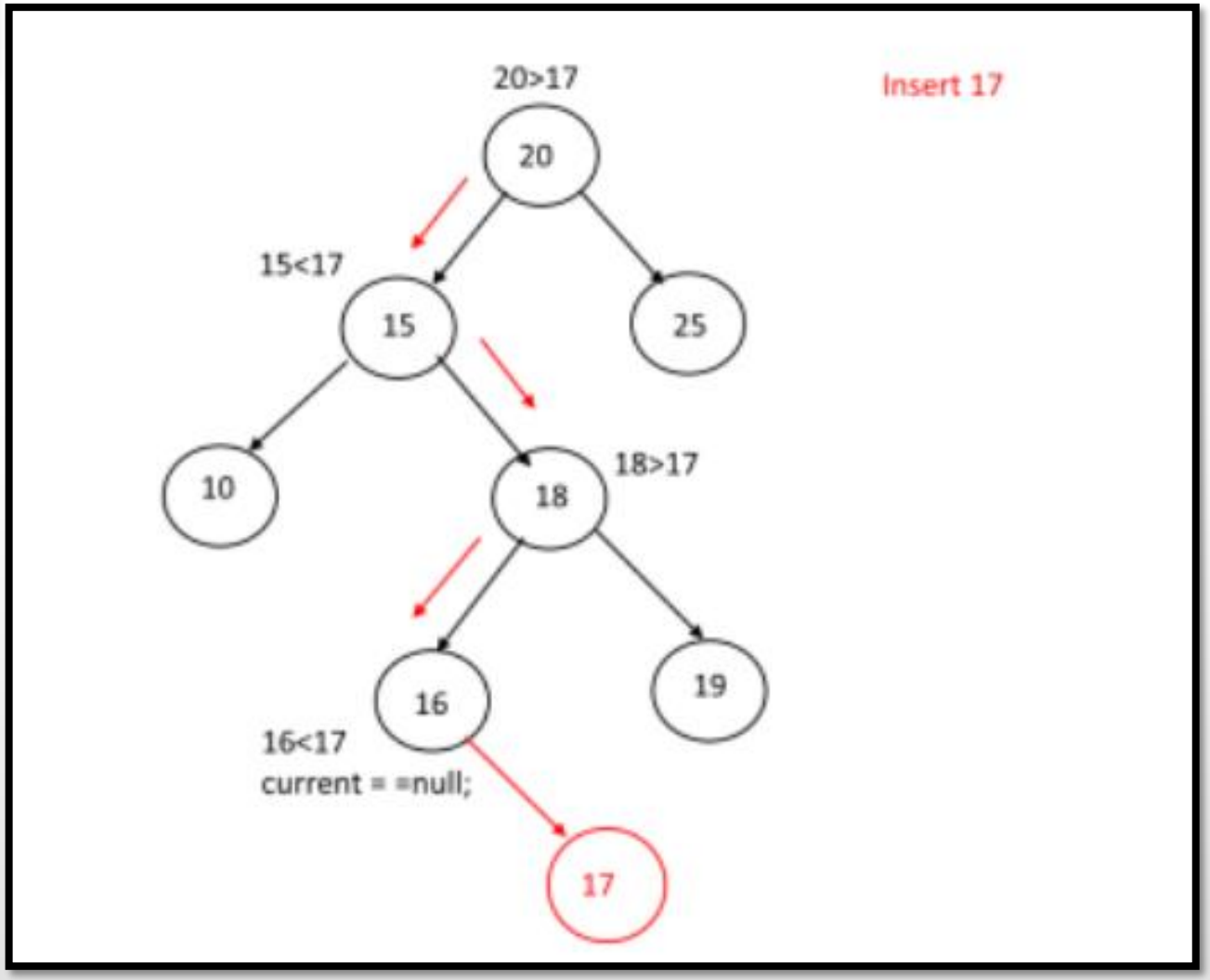
Insert(int n)

- find the **place(location) to insert the node**.
- Let the **current = root** .
- start from the current and compare **current.element with n**
- **if current.element is greater than n** that means we need to **go to the left of the root**.
- **if current.element is smaller than n** that means we need to **go to the right of the root**.
- if any point of **time current is null that means we have reached to the leaf node**, insert your node here with the help of parent node.

Binary Search Tree: Insert



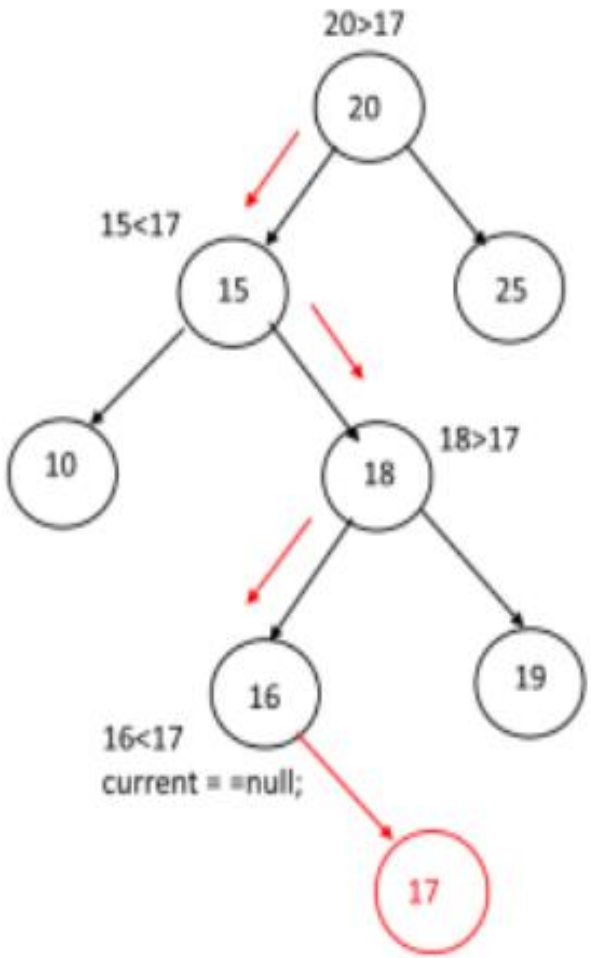
Binary Search Tree: Insert



Binary Search Tree: Insert

```
//Insert element function
private  BSTNode insert (int element, BSTNode current){
    if (current==null)
        current= new BSTNode(element); //create one node tree
    else
    {
          if (element<current.element)
             current.left=insert(element,current.left);
          else
             current.right=insert(element,current.right);

    }

    return current;
}
```

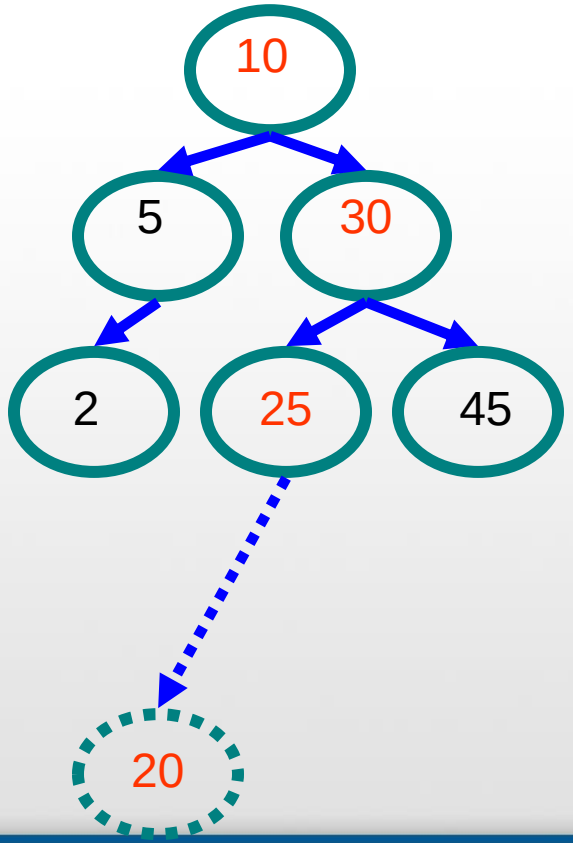
Binary Search Tree: Insert




```
//Insert element function
private  BSTNode insert (int element, BSTNode current){
   if (current==null)
       current= new BSTNode(element); //create one node tree
   else
   {
          if (element<current.element)
             current.left=insert(element,current.left);
          else
             current.right=insert(element,current.right);

   }

   return current;
}
```

Binary Search Tree: Insert

- Insert (20)



10 < 20, right

30 > 20, left

25 > 20, left

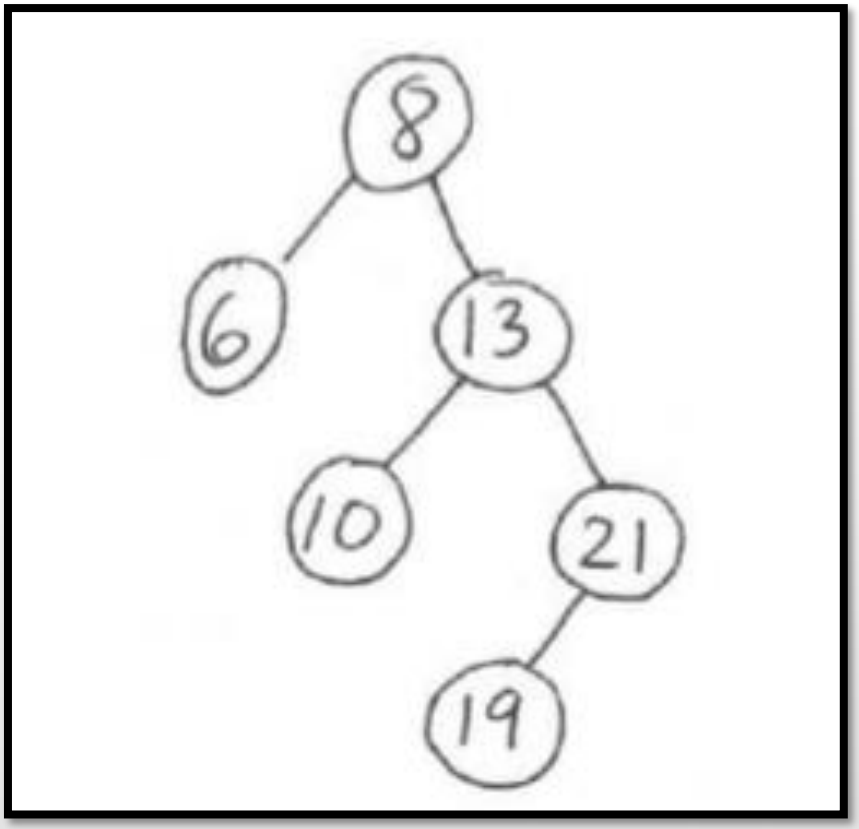
Insert 20 on left

Binary Search Tree: Insert

Beginning with an empty binary search tree, what binary search tree is formed when the following data is inserted in the order given? 8,13,6,10,21,19.

Binary Search Tree: Insert

Beginning with an empty binary search tree, what binary search tree is formed when the following data is inserted in the order given? 8,13,6,10,21,19.



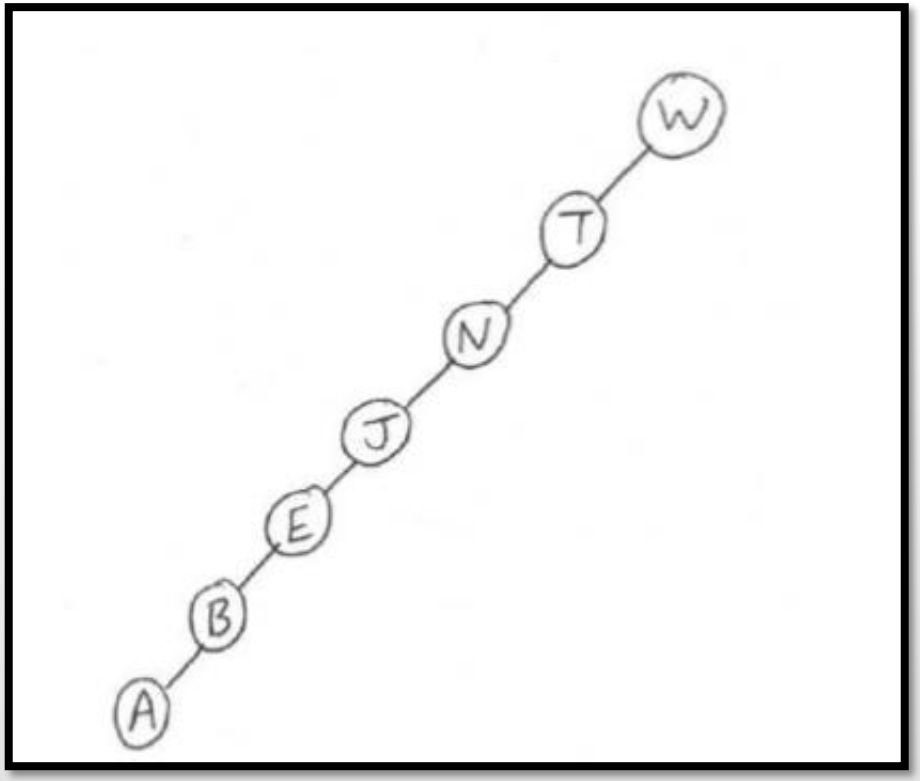
What is the time of search in the formed tree?

Binary Search Tree: Insert

Beginning with an empty binary search tree, what binary search tree is formed when you insert the following values in the order given?

W, T, N, J, E, B, A

What is the time of search in the formed tree?



SELF STUDY

Binary Search Tree : findMin &&findMax

```
private BSTNode findMin (BSTNode current){
 if (current==null)
    return null;
 else if (current.left==null)
     return current;
 else
     return findMin(current.left); //keep going to the left
}
```

```
private BSTNode findMax (BSTNode current){
 if (current==null)
    return null;
 else if (current.right==null)
    return current;
 else
    return findMax(current.right); //keep going to the right
}
```

Binary Search Tree : Delete

Delete(int n):

- **Complicated than Find() and Insert() operations. Here we have to deal with 3 cases.**
- **Node to be deleted is a leaf node (No Children).**
- **Node to be deleted has only one child.**
- **Node to be deleted has two children**

Binary Search Tree : Delete

Delete(int n):

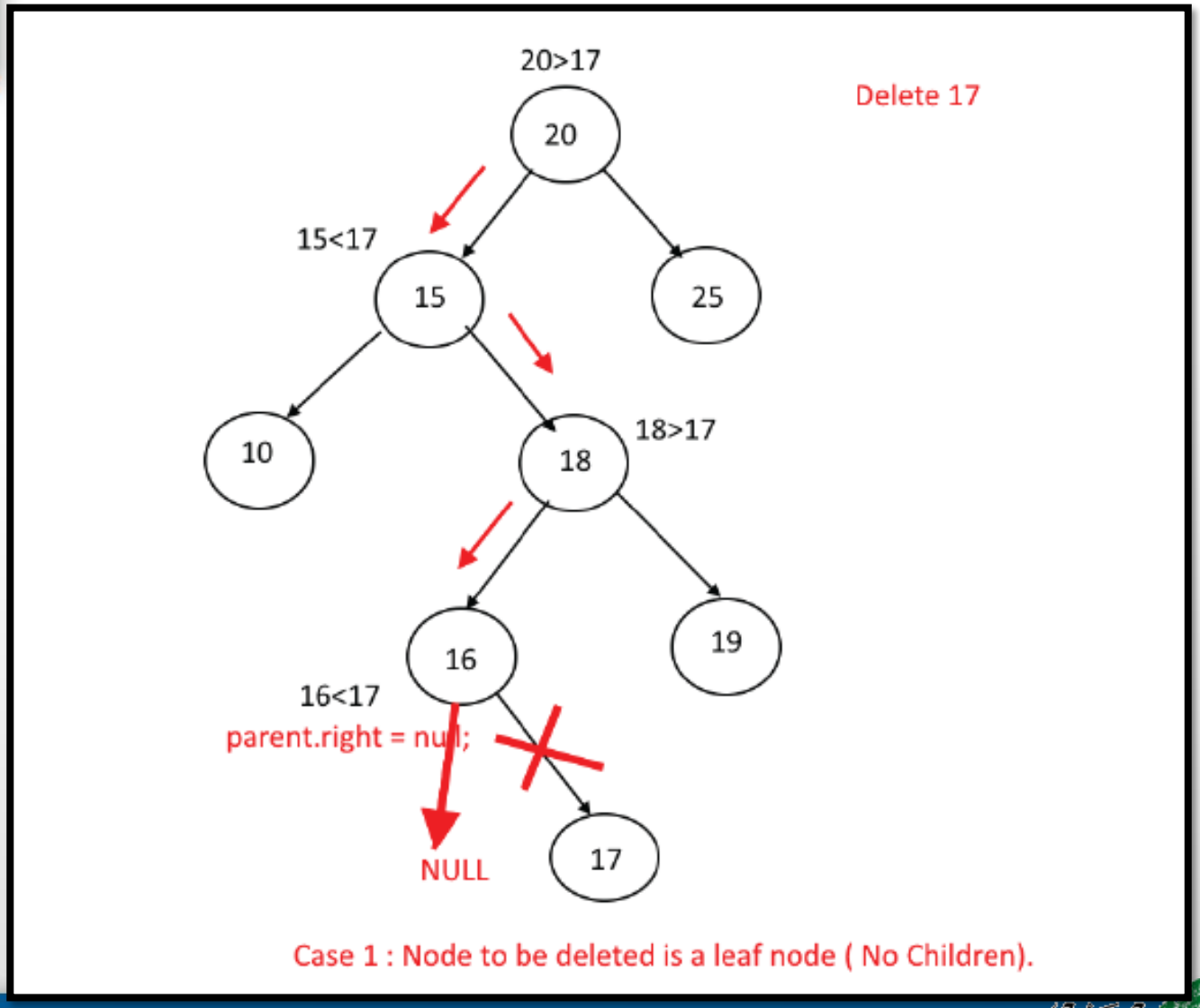
- **Complicated than Find() and Insert() operations. Here we have to deal with 3 cases.**

- **Node to be deleted is a leaf node (No Children).**
  - **Reset its parent link to null**

- **Node to be deleted has only one child.**
  - **Replace the node by its single child**

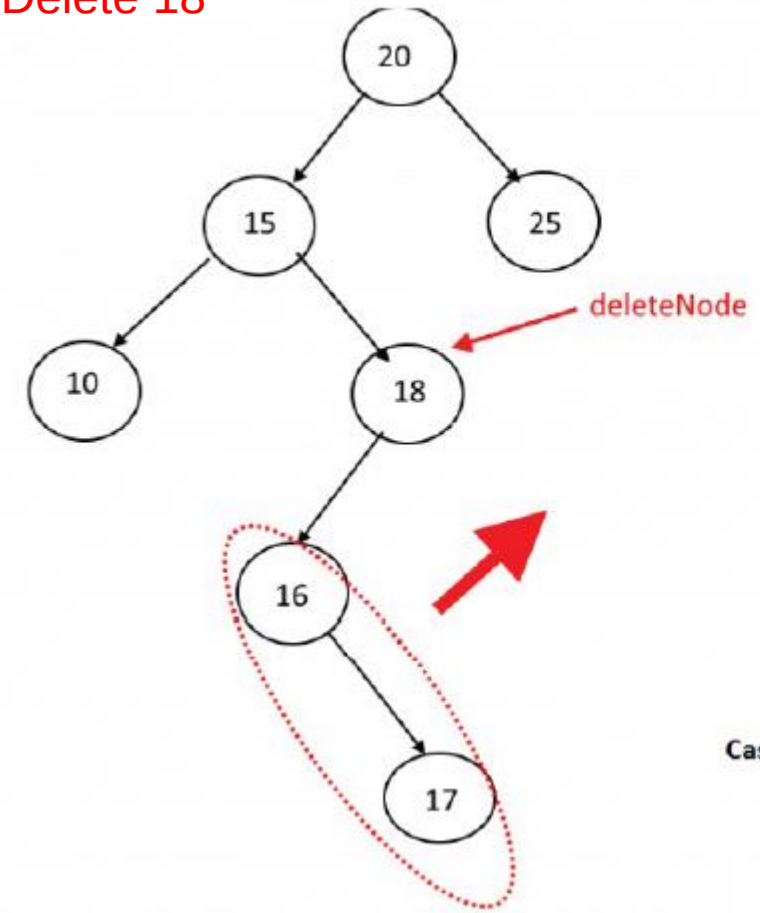
- **Node to be deleted has two children**
  - **Replace the node by the largest one in its left subtree or the smallest one in its right subtree.**
  - **Delete the replacing node from the sub tree.**

Binary Search Tree : Delete



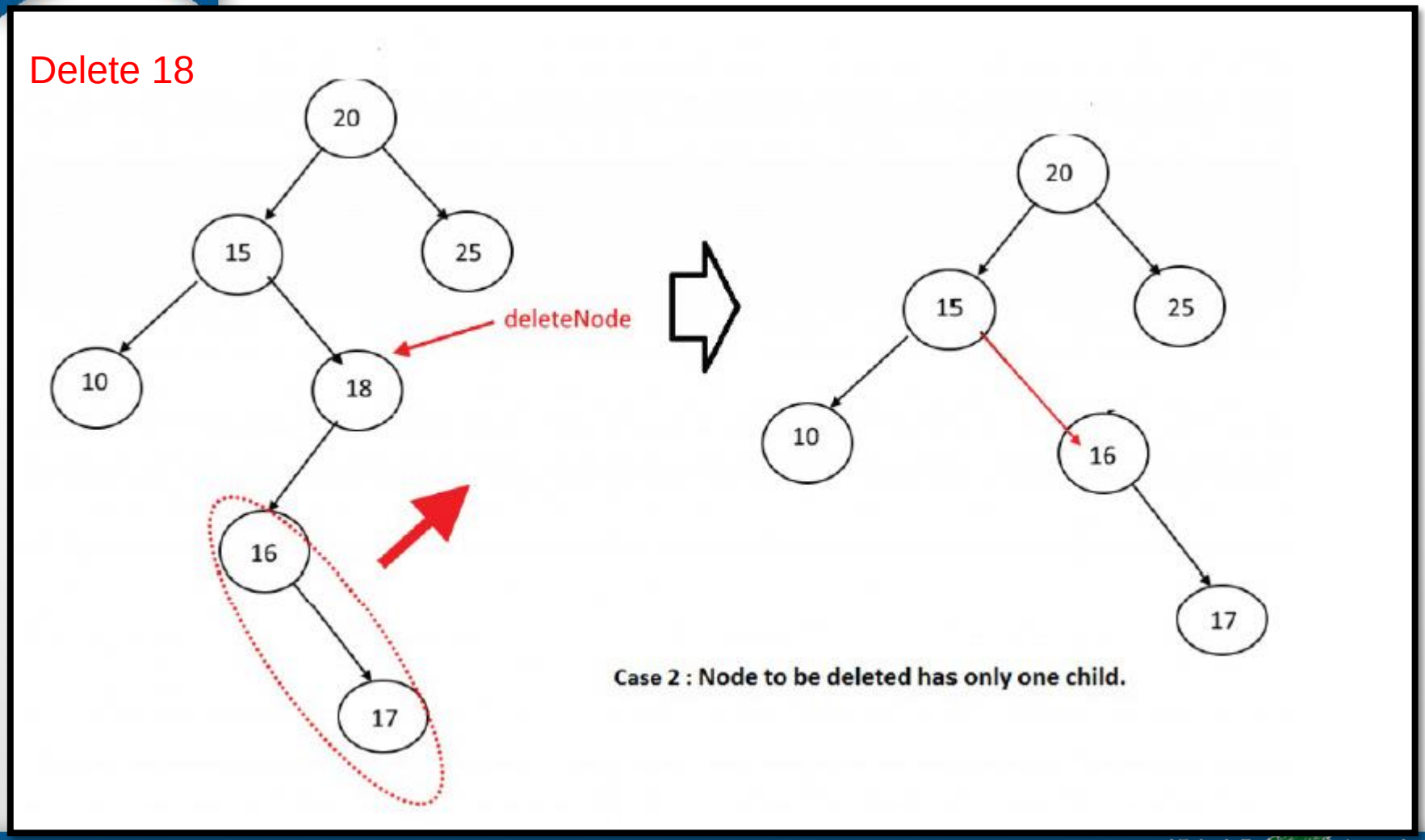
Binary Search Tree : Delete

Delete 18

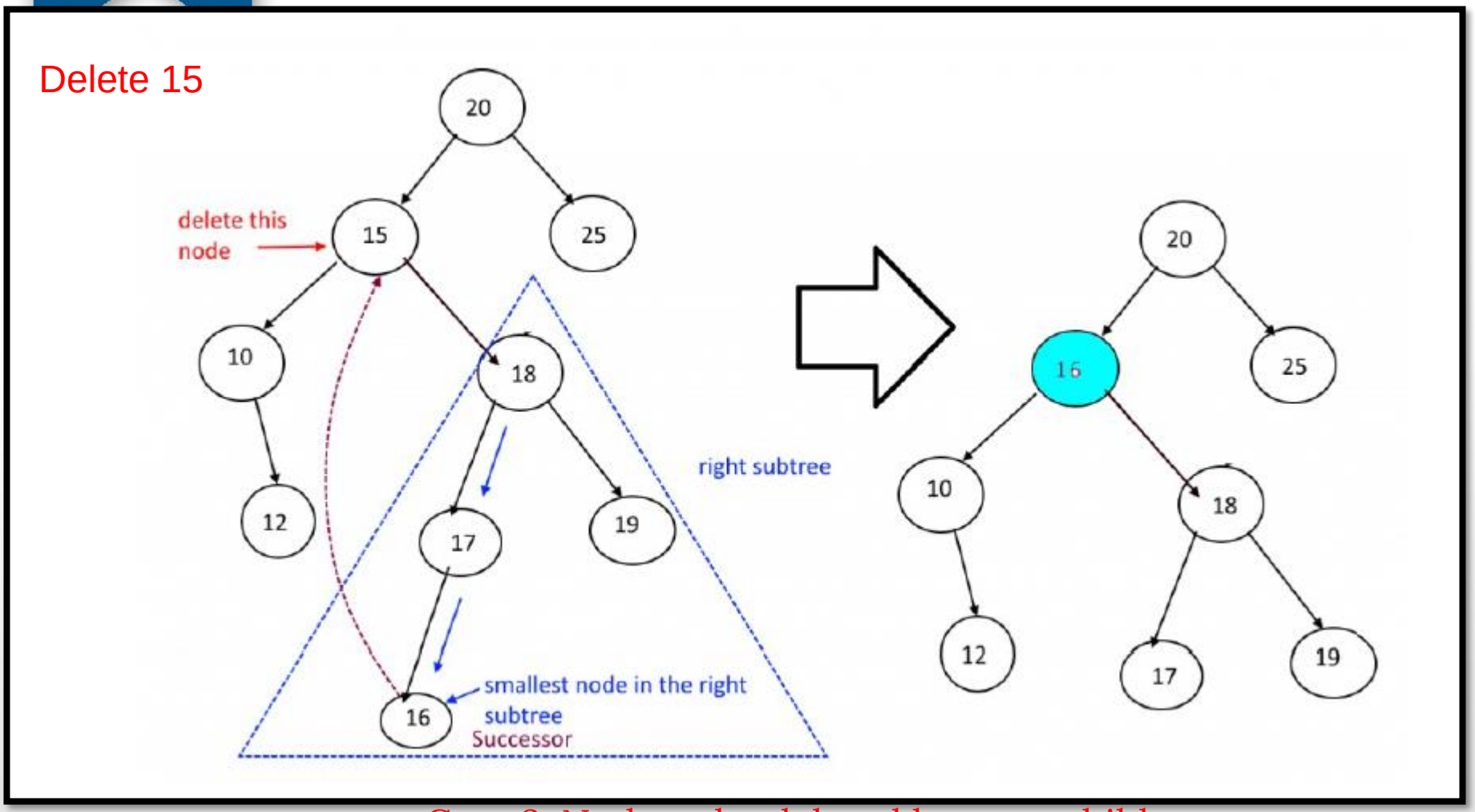


Case 2 : Node to be deleted has only one child.

Binary Search Tree : Delete



Binary Search Tree : Delete



Case 3: Node to be deleted has two children.

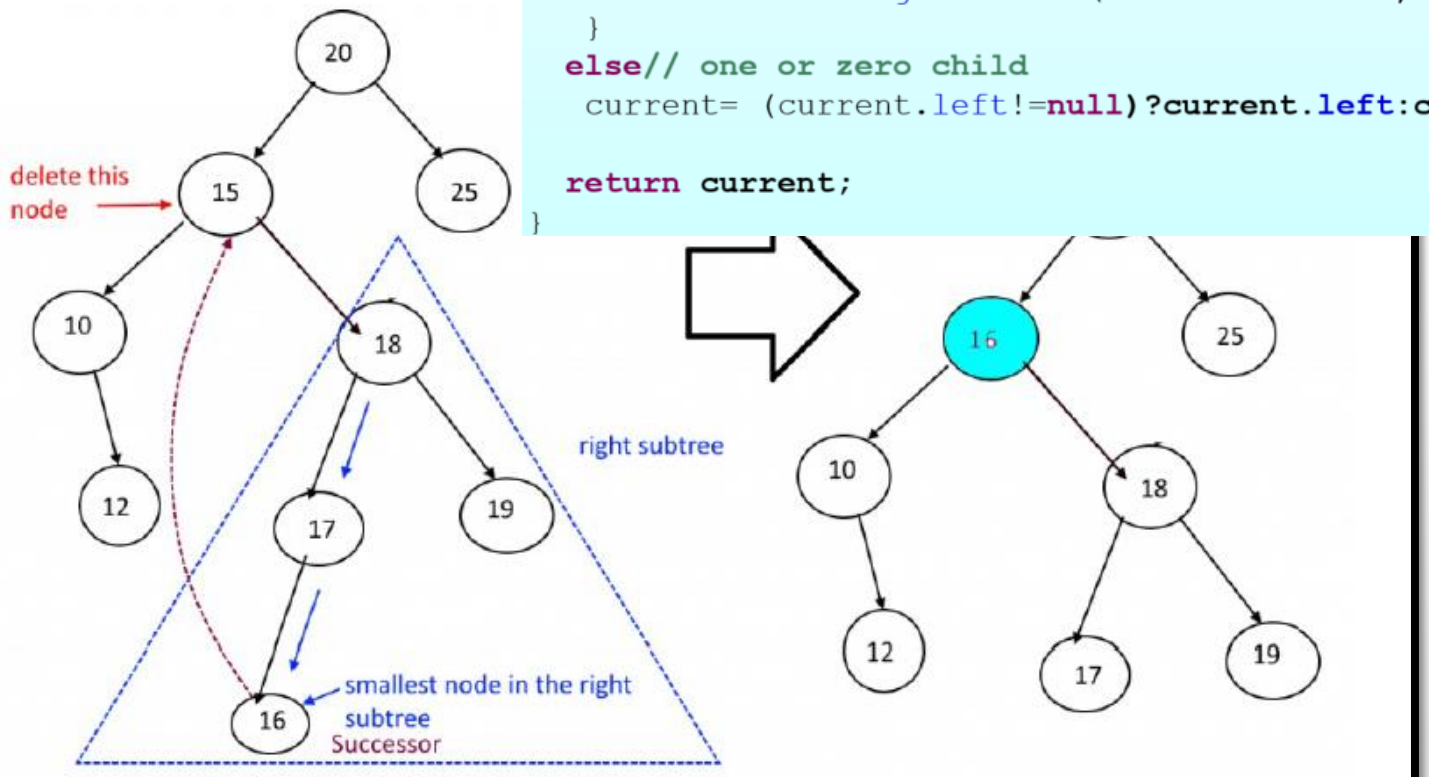
Binary Search Tree : Delete

```
private BSTNode remove (int e,BSTNode current){
  if (current==null) return null; // Item not found,Empty tree
  if (e<current.element)
     current.left=remove(e,current.left);
  else if (e>current.element)
     current.right=remove(e,current.right);
  else // found element to be deleted
  if (current.left!=null && current.right!=null)// two children
   {
         /*Replace with smallest in right subtree */
         current.element=findMin(current.right).element;
         current.right=remove(current.element,current.right);
   }
  else// one or zero child
   current= (current.left!=null)?current.left:current.right;

  return current;
}
```

Binary Search Tree : Delete

Delete 15

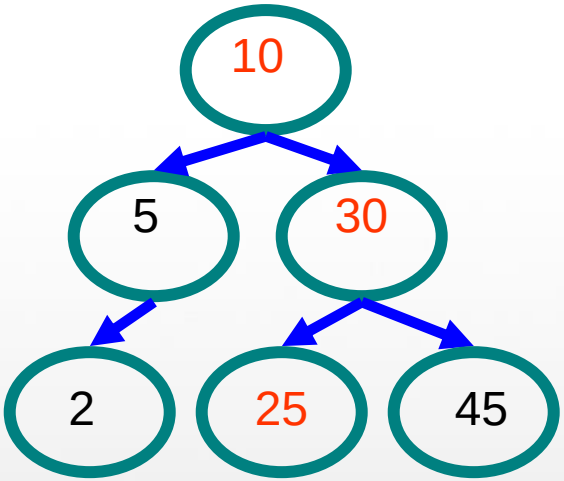
```
private BSTNode remove (int e,BSTNode current){
  if (current==null) return null; // Item not found,Empty tree
  if (e<current.element)
     current.left=remove(e,current.left);
  else if (e>current.element)
     current.right=remove(e,current.right);
  else // found element to be deleted
  if (current.left!=null && current.right!=null)// two children
   {
         /*Replace with smallest in right subtree */
         current.element=findMin(current.right).element;
         current.right=remove(current.element,current.right);
   }
  else// one or zero child
   current= (current.left!=null)?current.left:current.right;

  return current;
}
```



Case 3: Node to be deleted has two children.

Binary Search Tree : Delete

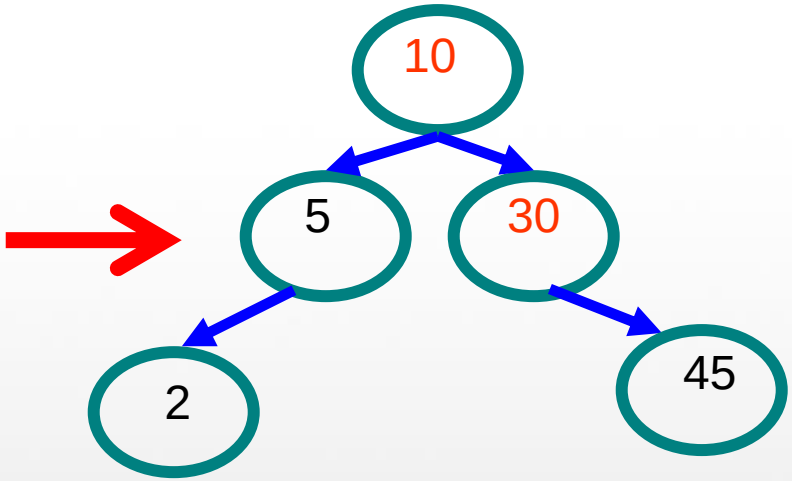
- Delete (25)



10 < 25, right

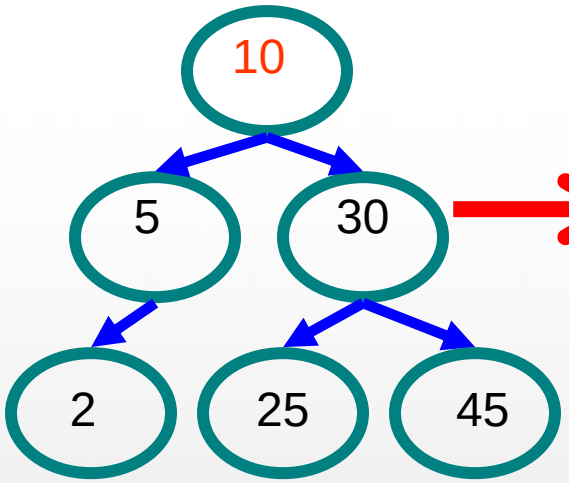
30 > 25, left

25 = 25, delete



Binary Search Tree : Delete

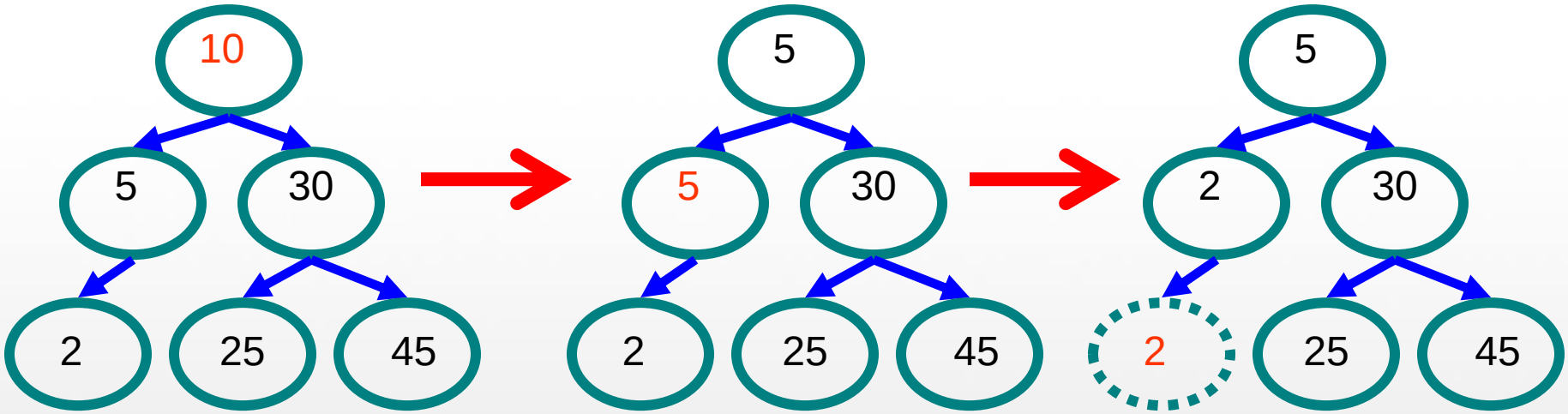
- Delete (10)



Replacing 10 with largest value in left subtree

Binary Search Tree : Delete

- Delete (10)



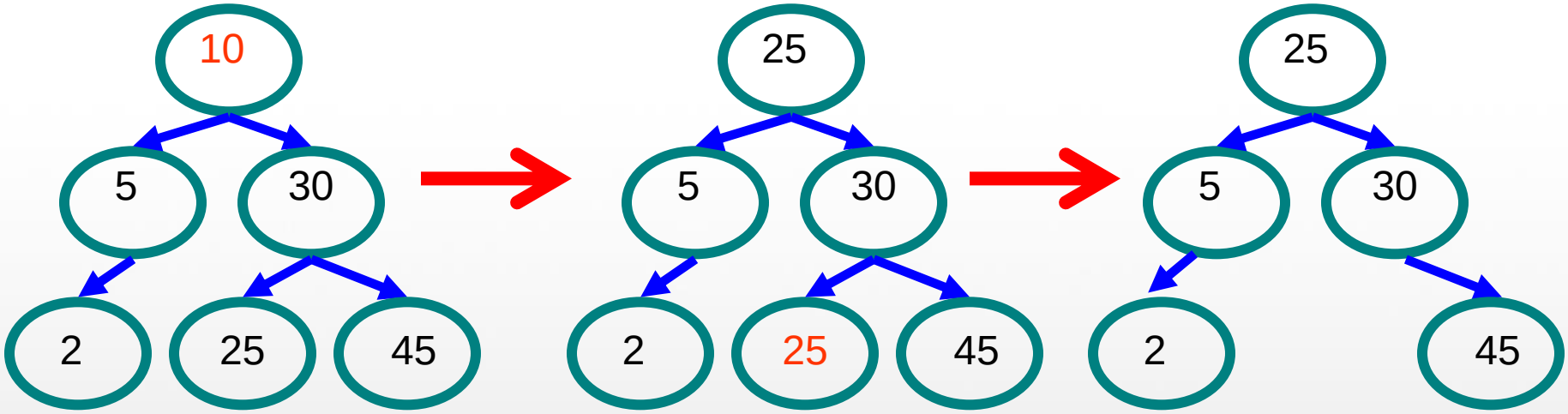
Replacing 10 with largest value in left subtree

Replacing 5 with largest value in left subtree

Deleting leaf

Binary Search Tree : Delete

- Delete (10)

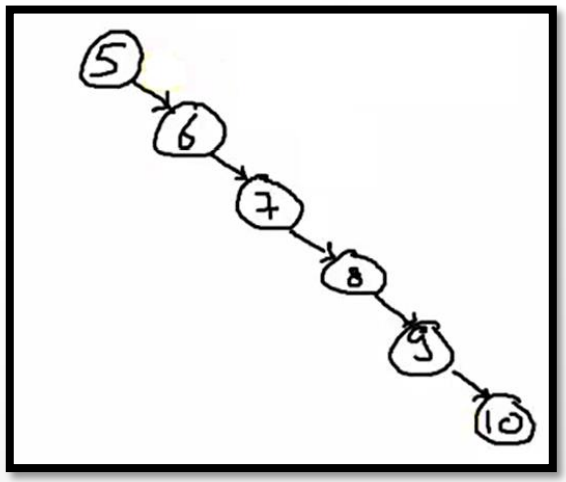


Replacing 10 with smallest value in right subtree

Deleting leaf

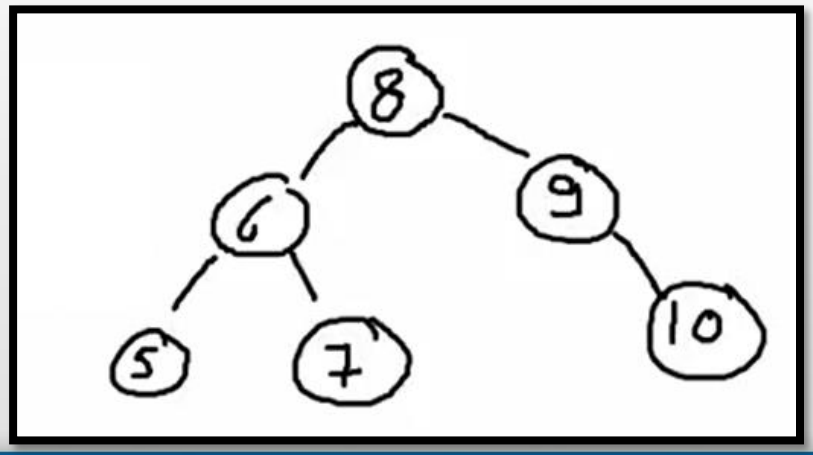
Resulting tree

Time complexity of the binary search tree.



T(n) = O (n) for Search, Insert, and Delete

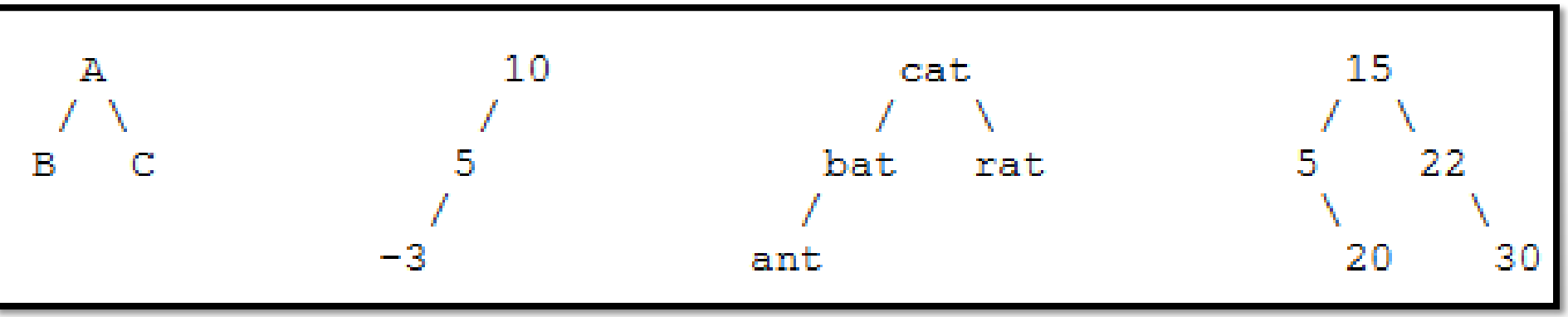
Like linked list



T(n) = O (log n) for Search, Insert, and Delete

Extra Exercises

- Which of the following binary trees are BSTs? If a tree is **not** a BST, say why.



- Using which kind of traversal (preorder, postorder, inorder, or level-order) visits the nodes of a BST in sorted order?

Question?

“Success is the sum of small efforts, repeated day in and day out.”
Robert Collier