JANCAE
Japan Association for Nonlinear CAE

UMMDp

Unified Material Model Driver for Plasticity

User's Guide

Abaqus

Version 2.0
Adapted from original document.

Contents

1 Preface

The Unified Material Model Driver for Plasticity (UMMDp) library is distributed as Fortran source codes. The Fortran compiler specified by the software vender in your analysis environment must be prepared. Please see the manual of Abaqus for the details of the environment for compiling the user subroutines.

In this guide, the sections of Abaqus related to the use of the UMMDp are described below, as well as the details on how to use the UMMDp with Abaqus. In the following procedures, the prompts "%" and ">" represent the examples in UNIX/Linux and Windows, respectively. The common steps for using UMMDp with Abaqus are:

1. Preparation of UMMDp source files.
   - Merge UMMDp source files into one file.
2. Writing procedure to call UMMDp in the input data.
   - To call the material user subroutine, specific keywords are required to be written in the input data. The keywords include the material constants, such as the coefficients of the yield function.
3. Execution of Abaqus software with the UMMDp.
   - When a command is typed to execute Abaqus, options are added to compile the UMMDp and to link it to Abaqus.

2 Abaqus

2.1 Related Section of User's Manual

1. Command to execute with user-subroutine
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3. Keywords for setup of the UMMDp
   - Abaqus Keywords Reference Guide
     - *DEPVAR: define the number of solution-dependent state variables.
     - *ORIENTATION: define local material axis for anisotropy.
     - *USER MATERIAL: define the material constants used in UMAT.
     - *USER OUTPUT VARIABLE: define the number of user output variables.
4. Specification of Abaqus user subroutines used in the UMMDp
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   - Abaqus Analysis User's Guide
     - 26.7.1 User-defined mechanical material behavior.

2.2 Usage

2.2.1 Preparation of Program Source Files

Concatenate the UMMDp source files into one single file with the "plug-in" file first.

- Unix/Linux

```
% cd dir_ummdp
% cp plug_ummdp_abaqus.f jobname_ummdp.f
% cat ummdp*.f >> jobname_ummdp.f
```

- Windows

```
> cd dir_ummdp
> copy plug_ummdp_abaqus.f jobname_ummdp.f
> type ummdp*.f >> jobname_ummdp.f
```

2.2.2 Preparation of the Input File

This section describes the keywords in the input data file for use in the UMMDp.

1. Definition of the principal axis for the material anisotropy (refer to the manual)

```
*ORIENTATION, NAME=ORI-1
1., 0., 0., 0., 1., 0.
3, 0.
```

2. Definition of the material model (the details will be provided later)

```
*MATERIAL, NAME=UMMDp
*USER MATERIAL, CONSTANTS=27
0, 0, 1000.0, 0.3, 2, -0.069, 0.936, 0.079,
1.003, 0.524, 1.363, 0.954, 1.023, 1.069, 0.981, 0.476,
0.575, 0.866, 1.145, -0.079, 1.404, 1.051, 1.147, 8.0,
0, 1.0, 0
```

3. Define the number of internal state variables (SDV)

   Set the number of state variables to 1+NTENS, where NTENS is the number of components of the tensor variables. NTENS=3 for plane stress or a shell element, and NTENS=6 for a solid element. The 1st “1” is reserved for the equivalent plastic strain, and NTENS is reserved for the plastic strain components. The following example corresponds to a solid element without kinematic hardening:

```
*DEPVAR
7,
```

   In the case of kinematic hardening, the number of internal state variables corresponds to the equivalent plastic strain, plastic strain components and components of each partial back-stress tensor.

4. Define the user output variables (UVAR)

   UMMDp can output the following user output variables:

   (a) Equivalent stress computed through the yield criteria.
      - UVAR(1)

   (b) Flow stress computed through the isotropic hardening law.
      - UVAR(2)

(c) Total back-stress tensor components.

- UVAR(3 : 2+NTENS)

(d) Rupture criteria limit and variables.

- UVAR(3+NTENS : end) in the case of kinematic hardening.
- UVAR(3 : 2+NTENS) in the case of no kinematic hardening.

```
*USER OUTPUT VARIABLES
8,
```

5. Define output variables for post processing

   This keyword controls the output variables (e.g., equivalent plastic strain and equivalent stress) for post processing.

```
*OUTPUT, FIELD
*ELEMENT OUTPUT
SDV, UVARM
```

2.2.3 Execution of Program

To execute the program there are two options: (a) link the user subroutine in source code and (b) link the user subroutine previously compiled.

(a) To execute the program with the user subroutine in source code, execute the command:

```
%> abaqus job=jobname user=jobname_ummdp.f
```

(b) To execute the program with the user subroutine previously compiled, execute the commands:

```
> abaqus job=jobname user=jobname_ummdp.obj
```

```
% abaqus job=jobname user=jobname_ummdp.o
```

The command that compiles the file `jobname_ummdp.obj/o` is:

```
%> abaqus make library=jobname_ummdp.f
```

3 Setup Data Input

The input data in UMMDp is defined as follows:

1. Parameter for Debug and Print
2. Parameters for Elastic Properties
3. Parameters for Yield Function
4. Parameters for Isotropic Hardening
5. Parameters for Kinematic Hardening

The detail of data is shown as follows and in addition, an example of input data is described.

3.1 Options for Debug and Print

The first input parameter corresponds to the definition of debug and print mode, defined by the variable `nvbs0`. It is a mandatory parameter and the options are:

- 0 - Error messages only
- 1 - Summary of Multistage Return Mapping
- 2 - Detail of Multistage Return Mapping and summary of Newton-Raphson
- 3 - Detail of Newton-Raphson
- 4 - Input/Output
- 5 - All status for debug and print

3.2 Options for Elastic Properties

- `prela(1)` - ID for elastic properties
- `prela(2∼)` - Data depends on ID

Only isotropic Hooke elastic properties can be defined. There are 2 ways to set them:

- Young's Modulus and Poisson's Ratio
  - ID: 0
  - Parameters: 2

    ◦ `prela(1) = 0`
    ◦ `prela(2) = 200.0E+3` (Young's modulus E)
    ◦ `prela(3) = 0.3` (Poisson's ratio ν)

- Bulk Modulus and Modulus of Rigidity
  - ID: 1
  - Parameters: 2

  ◦ `prela(1) = 1`
  ◦ `prela(2) = 166666.7` (Bulk modulus $K = E(1-2\nu)/3)$)
  ◦ `prela(3) = 76923.08` (Modulus of rigidity $G = E(1+\nu)/2$)

3.3 Options for Yield Criterion

- `pryld(1)` - ID for yield function

  (Negative value specifies plane stress yield function)

- `pryld(2∼)` - Data depends on ID

Identification number for yield criteria and required parameters are introduced. Please refer to the original papers for more detail on the formulation and meaning of parameters.

- von Mises Isotropic (1913)[1]
  - ID: 0

  ◦ `pryld(1) = 0`

- Hill Quadratic (1948)[2]
  - ID: 1
  - Parameters: 6

  ◦ `pryld(1)   = 1`
  ◦ `pryld(1+1) =` F
  ◦ `pryld(1+2) =` G
  ◦ `pryld(1+3) =` H
  ◦ `pryld(1+4) =` L
  ◦ `pryld(1+5) =` M
  ◦ `pryld(1+6) =` N

  Note: The parameters (FGHLMN) are the same as Hill's original paper. When F=G=H=1 and L=M=N=3, Hill's function is identical to von Mises.

[1] von Mises, R. (1913). Mechanik der festen Korper im plastisch deformablen Zustand. Gottin. Nachr. Math. Phys., 1: 582–592.
[2] Hill, R. (1948). A theory of the yielding and plastic flow of anisotropic metals. Proc. Roy. Soc. London, 193:281–297.

- Barlat Yld2004-18p (2005)[3]
  - ID: 2
  - Parameters: 19

    - `pryld(1)      = 2`
    - `pryld(1+1)    =` c'_{12}
    - `pryld(1+2)    =` c'_{13}
    - `pryld(1+3)    =` c'_{21}
    - `pryld(1+4)    =` c'_{23}
    - `pryld(1+5)    =` c'_{31}
    - `pryld(1+6)    =` c'_{32}
    - `pryld(1+7)    =` c'_{44}
    - `pryld(1+8)    =` c'_{55}
    - `pryld(1+9)    =` c'_{66}
    - `pryld(1+10)   =` c''_{12}
    - `pryld(1+11)   =` c''_{13}
    - `pryld(1+12)   =` c''_{21}
    - `pryld(1+13)   =` c''_{23}
    - `pryld(1+14)   =` c''_{31}
    - `pryld(1+15)   =` c''_{32}
    - `pryld(1+16)   =` c''_{44}
    - `pryld(1+17)   =` c''_{55}
    - `pryld(1+18)   =` c'_{66}
    - `pryld(1+19)   =` a (exponent)

  Note: The order of parameters given as input is the same as in the original paper.

[3]Barlat, F., Aretz, H., Yoon, J.W., Karabin, M.E., Brem, J.C., Dick, R.E. (2005). Linear transformation-based anisotropic yield functions., Int. J. Plast. 21:1009–1039

- Cazacu (2006)[4]
    - ID: 3
    - Parameters: 14

    - `pryld(1)     = 3`
    - `pryld(1+1)   =` C_{11}
    - `pryld(1+2)   =` C_{12}
    - `pryld(1+3)   =` C_{13}
    - `pryld(1+4)   =` C_{21}
    - `pryld(1+5)   =` C_{22}
    - `pryld(1+6)   =` C_{23}
    - `pryld(1+7)   =` C_{31}
    - `pryld(1+8)   =` C_{32}
    - `pryld(1+9)   =` C_{33}
    - `pryld(1+10)  =` C_{44}
    - `pryld(1+11)  =` C_{55}
    - `pryld(1+12)  =` C_{66}
    - `pryld(1+13)  =` a (exponent)
    - `pryld(1+14)  =` k (tension-compression ratio)

- Karafillis-Boyce (1993)[5]
    - ID: 4
    - Parameters: 8

    - `pryld(1)    = 4`
    - `pryld(1+1)  =` C
    - `pryld(1+2)  =` α_1
    - `pryld(1+3)  =` α_2
    - `pryld(1+4)  =` γ_1
    - `pryld(1+5)  =` γ_2
    - `pryld(1+6)  =` γ_3
    - `pryld(1+7)  =` c
    - `pryld(1+8)  =` k (k of exponent $2k$)

[4]Cazacu, O., Plunkett, B., Barlat, F., (2006). Orthotropic yield criterion for hexagonal close packed metals., Int. J. Plasticity 22: 1171–1194.

[5]Karafillis, A. P.; Boyce, M. C. (1993). A general anisotropic yield criterion using bounds and a transformation weighting tensor., J. Mech. Phys. Solids, 41:1859-1886.

- Hu (2005)[6]
    - ID: 5
    - Parameters: 10

        - `pryld(1)     = 5`
        - `pryld(1+1)  =` X_1
        - `pryld(1+2)  =` X_2
        - `pryld(1+3)  =` X_3
        - `pryld(1+4)  =` X_4
        - `pryld(1+5)  =` X_5
        - `pryld(1+6)  =` X_6
        - `pryld(1+7)  =` X_7
        - `pryld(1+8)  =` C_1
        - `pryld(1+9)  =` C_2
        - `pryld(1+10) =` C_3

[6] Hu, W. (2005). An orthotropic yield criterion in a 3-D general stress state. Int. J. Plasticity 21:1771–1796.

- Yoshida 6th Polynomial (2011)[7]
  - ID: 6
  - Parameters: 16

    - `pryld(1)     = 6`
    - `pryld(1+1)   =` C_1
    - `pryld(1+2)   =` C_2
    - `pryld(1+3)   =` C_3
    - `pryld(1+4)   =` C_4
    - `pryld(1+5)   =` C_5
    - `pryld(1+6)   =` C_6
    - `pryld(1+7)   =` C_7
    - `pryld(1+8)   =` C_8
    - `pryld(1+9)   =` C_9
    - `pryld(1+10)  =` C_{10}
    - `pryld(1+11)  =` C_{11}
    - `pryld(1+12)  =` C_{12}
    - `pryld(1+13)  =` C_{13}
    - `pryld(1+14)  =` C_{14}
    - `pryld(1+15)  =` C_{15}
    - `pryld(1+16)  =` C_{16}

[7] Yoshida, F., Hamasaki, H., Uemori, T. (2013). A user-friendly 3D yield function to describe anisotropy of steel sheets. Int. J. Plasticity, 45:119-139.

- Gotoh Biquadratic (1978)[8]
  - ID: -1
  - Parameters: 9

  ◦ `pryld(1)   = -1`
  ◦ `pryld(1+1) =` A_1
  ◦ `pryld(1+2) =` A_2
  ◦ `pryld(1+3) =` A_3
  ◦ `pryld(1+4) =` A_4
  ◦ `pryld(1+5) =` A_5
  ◦ `pryld(1+6) =` A_6
  ◦ `pryld(1+7) =` A_7
  ◦ `pryld(1+8) =` A_8
  ◦ `pryld(1+9) =` A_9

- Barlat Yld2000-2d (2003)[9]
  - ID: -2
  - Parameters: 9

  ◦ `pryld(1)   = -2`
  ◦ `pryld(1+1) =` α_1
  ◦ `pryld(1+2) =` α_2
  ◦ `pryld(1+3) =` α_3
  ◦ `pryld(1+4) =` α_4
  ◦ `pryld(1+5) =` α_5
  ◦ `pryld(1+6) =` α_6
  ◦ `pryld(1+7) =` α_7
  ◦ `pryld(1+8) =` α_8
  ◦ `pryld(1+9) =` a (exponent)

[8] Gotoh, M., (1977). A theory of plastic anisotropy based on a yield function of fourth order (plane stress state) - I, Int. J. Mech. Sci. , 19-9 : 505-512. (see also J.JSTP, 19(1978) :337-385)

[9] Barlat, F., Brem, J.C., Yoon, J.W., Chung, K., Dick, R.E., Lege, D.J., Pourboghrat, F., Choi, S.H., Chu, E. (2003). Plane stress yield function for aluminium alloy sheets-part 1: theory, Int. J. Plasticity, 19:1297-1319.

- Vegter (2006)[10]
  - ID: -3
  - Parameters: $3 + 4n$

    - `pryld(1)              = -3`
    - `pryld(1+1)            = n` (max of i)
    - `pryld(1+2)            = f_bi0`
    - `pryld(1+3)            = r_bi0`
    - `pryld(1+3+(i-1)*4+1) = phi_uniaxial(i)`
    - `pryld(1+3+(i-1)*4+2) = phi_shear(i)`
    - `pryld(1+3+(i-1)*4+3) = ph_planestrain(i)`
    - `pryld(1+3+(i-1)*4+4) = omega(i)`

- Banabic BBC2005[11]
  - ID: -4
  - Parameters: 9

    - `pryld(1)   = -4`
    - `pryld(1+1) =` k (k of exponent $2k$)
    - `pryld(1+2) =` a
    - `pryld(1+3) =` b
    - `pryld(1+4) =` L
    - `pryld(1+5) =` M
    - `pryld(1+6) =` N
    - `pryld(1+7) =` P
    - `pryld(1+8) =` Q
    - `pryld(1+9) =` R

[10]Vegter, H., den Boogaard, A.H. van (2006). A plane stress yield function for anisotropic sheet material by interpolation of biaxial stress states, Int. J. Plasticity, 22:557-580.

[11]Banabic, D., Aretz,, D.S. Comsa, H., Paraianu, L.(2005). An improved analytical description of orthotropy in metallic sheets, Int. J. Plasticity 21:493–512.

- Barlat Yld89[12]
    - ID: -5
    - Parameters: 4

        - `pryld(1)   = -5`
        - `pryld(1+1) =` M (exponent)
        - `pryld(1+2) =` a
        - `pryld(1+3) =` h
        - `pryld(1+4) =` p

- Banabic BBC2008[13]
    - ID: -6
    - Parameters: $2 + 8s$

        - `pryld(1)             = -6`
        - `pryld(1+1)           =` s (max of i)
        - `pryld(1+2)           =` k (k of exponent $2k$)
        - `pryld(1+2+(i-1)*8+1) =` l_1
        - `pryld(1+2+(i-1)*8+2) =` l_2
        - `pryld(1+2+(i-1)*8+3) =` m_1
        - `pryld(1+2+(i-1)*8+4) =` m_2
        - `pryld(1+2+(i-1)*8+5) =` m_3
        - `pryld(1+2+(i-1)*8+6) =` n_1
        - `pryld(1+2+(i-1)*8+7) =` n_2
        - `pryld(1+2+(i-1)*8+8) =` n_3

[12] Barlat, F., Lian, J.(1989). Plastic behavior and stretchability of sheet metals. Part I: a yield function for orthotropic sheets under plane stress conditions. Int. J. Plasticity. 5:51–66.

[13] Comsa, D.S., Banabic,D. (2008). Plane-stress yield criterion for highly-anisotropic sheet metals, Proc. of NUMISHEET 2008.

- Hill 1990[14]
  - ID: -7
  - Parameters: 5

  - `pryld(1)   = -7`
  - `pryld(1+1) =` a
  - `pryld(1+2) =` b
  - `pryld(1+3) =` τ
  - `pryld(1+4) =` σ_b
  - `pryld(1+5) =` m

3.4 Options for Isotropic Hardening

- `prihd(1)` - ID for isotropic hardening
- `prihd(2~)` - Data depends on ID

The equation of flow curve is introduced for each law, where σ_{y} is the yield stress, σ_{y_0} is the initial yield stress and p the equivalent plastic strain.

- Perfectly Plastic
  - ID: 0
  - Parameters: 1

  - `prihd(1)   = 0`
  - `prihd(1+1) =` σ_y

- Linear Hardening: $\sigma_{\mathrm{y}} = \sigma_{\mathrm{y}_0} + Hp$
  - ID: 1
  - Parameters: 2

  - `prihd(1)   = 1`
  - `prihd(1+1) =` σ_{y_0}
  - `prihd(1+2) =` H

[14]Hill, R (1990). Constitutive modelling of orthotropic plasticity in sheet metals, Journal of the Mechanics and Physics of Solids.

- Swift: $\sigma_{\mathrm{y}} = K\left(\varepsilon_0 + p\right)^n$
  - ID: 2
  - Parameters: 3

  - `prihd(1)   = 2`
  - `prihd(1+1) =` K
  - `prihd(1+2) =` ϵ_0
  - `prihd(1+3) =` n

- Ludwick: $\sigma_{\mathrm{y}} = \sigma_{\mathrm{y}_0} + cp^n$
  - ID: 3
  - Parameters: 3

  - `prihd(1)   = 3`
  - `prihd(1+1) =` σ_{y_0}
  - `prihd(1+2) =` c
  - `prihd(1+3) =` n

- Voce: $\sigma_{\mathrm{y}} = \sigma_{\mathrm{y}_0} + Q\left(1 - \exp(-bp)\right)$
  - ID: 4
  - Parameters: 3

  - `prihd(1)   = 4`
  - `prihd(1+1) =` σ_{y_0}
  - `prihd(1+2) =` Q
  - `prihd(1+3) =` b

- Voce + Linear: $\sigma_{\mathrm{y}} = \sigma_{\mathrm{y}_0} + Q\left(1 - \exp(-bp)\right) + Hp$
  - ID: 5
  - Parameters: 4

  - `prihd(1)   = 5`
  - `prihd(1+1) =` σ_{y_0}
  - `prihd(1+2) =` Q
  - `prihd(1+3) =` b
  - `prihd(1+4) =` H

- Voce + Swift: $\sigma_{\mathrm{y}} = a\left[\sigma_{\mathrm{y}_0} + Q\left(1 - \exp(-bp)\right)\right] + (1-a)\left[K(\varepsilon_0 + p)^n\right]$
  - ID: 6
  - Parameters: 7

  ◦ `prihd(1)   ` = 6
  ◦ `prihd(1+1)` = a
  ◦ `prihd(1+2)` = σ_{y_0}
  ◦ `prihd(1+3)` = Q
  ◦ `prihd(1+4)` = b
  ◦ `prihd(1+5)` = K
  ◦ `prihd(1+6)` = ε_0
  ◦ `prihd(1+7)` = n

3.5 Options for Kinematic Hardening

- `prkin(1)` - ID for kinematic hardening
- `prkin(2∼)` - Data depends on ID

The equation of backstress is introduced for each law, where $\dot{\boldsymbol{\alpha}}$ is the total increment of backstress tensor, $\dot{\boldsymbol{\alpha}}_i$ is the a term of the total increment of backstress tensor, $\boldsymbol{\alpha}$ is the backstress tensor, $\dot{\boldsymbol{\epsilon}}^{\mathrm{p}}$ is the increment of plastic strain tensor and $\dot{p}$ is the increment of equivalent plastic strain.

- No Kinematic Hardening
  - ID: 0

  ◦ `prkin(1)` = 0

- Prager (1949): $\dot{\boldsymbol{\alpha}} = \frac{2}{3}c\dot{\boldsymbol{\varepsilon}}^{\mathrm{p}}$
  - ID: 1
  - Parameters: 1

  ◦ `prkin(1)   ` = 1
  ◦ `prkin(1+1)` = c

- Ziegler (1959): $\dot{\boldsymbol{\alpha}} = c\,(\boldsymbol{\sigma} - \boldsymbol{\alpha})\,\dot{p}$
    - ID: 2
    - Parameters: 1
    - `prkin(1)   = 2`
    - `prkin(1+1) =` c

- Armstrong-Frederick (1966): $\dot{\boldsymbol{\alpha}} = \dfrac{2}{3}c\dot{\boldsymbol{\varepsilon}}^{\mathrm{p}} - \gamma\boldsymbol{\alpha}\dot{p}$
    - ID: 3
    - Parameters: 2

    - `prkin(1)   = 3`
    - `prkin(1+1) =` c
    - `prkin(1+2) =` γ

- Chaboche (1979): $\dot{\boldsymbol{\alpha}} = \sum_{i=1}^{n} \dot{\boldsymbol{\alpha}}_i = \sum_{i=1}^{n} \left(\dfrac{2}{3}c_i\dot{\boldsymbol{\varepsilon}}^{\mathrm{p}} - \gamma_i\boldsymbol{\alpha}_i\dot{p} \right)$
    - ID: 4
    - Parameters: $1 + 2n$

    - `prkin(1)           = 4`
    - `prkin(1+1)         =` n
    - `prkin(1+1+(i*1))   =` c_i
    - `prkin(1+1+(i*2))   =` γ_i

- Chaboche (1979) - Ziegler Type: $\dot{\boldsymbol{\alpha}} = \sum_{i=1}^{n} \dot{\boldsymbol{\alpha}}_i = \sum_{i=1}^{n} \left(\dfrac{c_i}{\bar{\eta}}\,(\boldsymbol{\sigma} - \boldsymbol{\alpha}) - \gamma_i\boldsymbol{\alpha}_i \right) \dot{p}$
    - ID: 5
    - Parameters: $1 + 2n$

    - `prkin(1)           = 5`
    - `prkin(1+1)         =` n
    - `prkin(1+1+(i*1))   =` c_i
    - `prkin(1+1+(i*2))   =` γ_i

- Yoshida-Uemori
    - ID: 6
    - Parameters: 5

    - `prkin(1)   = 6`
    - `prkin(1+1) =` C
    - `prkin(1+2) =` Y
    - `prkin(1+3) =` a
    - `prkin(1+4) =` k
    - `prkin(1+5) =` b

3.6 Parameters for Uncoupled Rupture Criterion

- `prrup(1)` - ID for uncoupled rupture criterion
- `prrup(2∼)` - Data depends on ID
- No Uncoupled Rupture Criterion
    - ID: 0

    - `prrup(1) = 0`
- Equivalent Plastic Strain: $W = \int_0^{\varepsilon_f} \dot{p}\,\mathrm{d}t$
    - ID: 1
    - Parameters: 1
    - User Output Variables: + 2

    - `prrup(1)   = 1`
    - `prrup(1+1) =` W_L
- Cockroft and Latham: $W = \int_0^{\varepsilon_f} \frac{\sigma_1}{\bar{\sigma}}\,\mathrm{d}p$
    - ID: 2
    - Parameters: 1
    - User Output Variables: + 4

    - `prrup(1)   = 2`
    - `prrup(1+1) =` W_L

- Rice and Tracey: $W = \int_0^{\varepsilon_f} \exp\left(\frac{3}{2}\frac{\sigma_h}{\bar{\sigma}}\right) \mathrm{d}p$

    - ID: 3
    - Parameters: 1
    - User Output Variables: + 4

    ∘ `prrup(1)   = 3`
    ∘ `prrup(1+1) =` W_L

- Ayada: $W = \int_0^{\varepsilon_f} \frac{\sigma_h}{\bar{\sigma}} \mathrm{d}p$

    - ID: 4
    - Parameters: 1
    - User Output Variables: + 4

    ∘ `prrup(1)   = 4`
    ∘ `prrup(1+1) =` W_L

- Brozzo: $W = \int_0^{\varepsilon_f} \frac{2\sigma_1}{3\left(\sigma_1 - \sigma_h\right)} \mathrm{d}p$

    - ID: 5
    - Parameters: 1
    - User Output Variables: + 5

    ∘ `prrup(1)   = 5`
    ∘ `prrup(1+1) =` W_L

4 Example of Data Input

- Elastic Properties: E=200 GPa, ν=0.3
- Yield Criterion: Barlat Yld2004-18p (coefficients of AA6111-T4 given in the original paper)
- Isotropic Hardening: Swift
- Kinematic Hardening: Armstrong-Frederick (1966)
- Uncoupled Rupture Criterion: Ayada

The material parameters and IDs are given as input to the UMMDp in one dimensional array, named by default `props` in the program. In the beginning of UMMDp, this array is copied to a new variable `prop`, and the variable to define debug and print mode is excluded from this new array.

- `props(1)  = 0` Debug and Print ID = 0
- `props(2)  = 0` Elastic Property ID = 0
- `props(3)  = 200000` Young's Modulus E
- `props(4)  = 0.3` Poisson's Ratio ν
- `props(5)  = 2` Yield Criterion ID (Barlat Yld2004-18p)
- `props(6)  = 1.241024` c'_{12}
- `props(7)  = 1.078271` c'_{13}
- `props(8)  = 1.216463` c'_{21}
- `props(9)  = 1.223867` c'_{23}
- `props(10) = 1.093105` c'_{31}
- `props(11) = 0.889161` c'_{32}
- `props(12) = 0.501909` c'_{44}
- `props(13) = 0.557173` c'_{55}
- `props(14) = 1.349094` c'_{66}
- `props(15) = 0.775366` c''_{12}
- `props(16) = 0.922743` c''_{13}
- `props(17) = 0.765487` c''_{21}
- `props(18) = 0.793356` c''_{23}
- `props(19) = 0.918689` c''_{31}

- ○ `props(20) = 1.027625` c''_{32}
- ○ `props(21) = 1.115833` c''_{44}
- ○ `props(22) = 1.112273` c''_{55}
- ○ `props(23) = 0.589787` c''_{66}
- ○ `props(24) = 8` a
- ○ `props(25) = 2` Isotropic Hardening ID (Swift)
- ○ `props(26) = 541.0` K
- ○ `props(27) = 0.0036` ϵ_0
- ○ `props(28) = 0.249` n
- ○ `props(29) = 3` Kinematic Hardening ID (Armstrong-Frederick)
- ○ `props(30) = 1018.4245` c
- ○ `props(31) = 22.85` γ
- ○ `props(32) = 4` Uncoupled Rupture Criterion ID (Ayada)
- ○ `props(33) = 0.5` W_L

This `prop(i)` array is divided into each properties in UMMDp as follow:

- `prela(i)` - Elastic Properties
- `pryld(i)` - Yield Criterion
- `prihd(i)` - Isotropic Hardening
- `prkin(i)` - Kinematic Hardening
- `prrup(i)` - Uncoupled Rupture Criterion

The model ID is stored in the top of each of these arrays. Here, it is shown the input example of the material data for the abaqus input file. The red letter indicates ID of each properties.

```
*MATERIAL, NAME=UMMDp
*USER MATERIAL, CONSTANTS=33
0, 0, 200000.0, 0.3, 2, 1.241024, 1.078271, 1.216463,
1.223867, 1.093105, 0.889161, 0.501909, 0.557173, 1.349094, 0.775366, 0.922743,
0.765487, 0.793356, 0.918689, 1.027625, 1.115833, 1.112273, 0.589787, 8.0,
2, 541.0, 0.0036, 0.249, 3, 1018.4245, 22.85, 4,
0.5
```