

# Implementation of anisotropic yield functions into the subroutine library “UMMDp”

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**Abstract.** To implement new yield functions into the user subroutines of advanced FE codes, professional skills and knowledge are required. The Japan Association for Nonlinear Computer Aided Engineering (JANCAE) has been acting as an organization that provides information and knowledge on finite element analysis (FEA) to the researchers and engineers in various positions involved in CAE. Since 2009, as one of the subcommittee activities for applying FEA to practical metal-forming problems, we have not only studied elastoplastic theory, but have also developed the Unified Material Model Driver for Plasticity (UMMDp), a user subroutine library. Basic yield functions, such as the von Mises and Hill's quadratic yield functions, which almost all advanced FE code already has, were implemented in the UMMDp as a basic study at its beginning of the development. Up to now, many anisotropic yield functions, including Yoshida's 6th-order polynomial, Barlat's Yld2000 and Yld2004, Comsa and Banabic's BBC2008, Cazacu's CPB2006, and Vegter's spline yield functions, have been implemented in the UMMDp, and it is possible to add a new yield function. In this presentation, the techniques for implementing yield functions in the UMMDp are explained. Moreover, the problem of BM1 in NUMISHEET 2016 is discussed as an example applying selected yield functions to the implementation procedure.

## 1. Introduction

To complement improvements in metal manufacturing technology, metal-forming simulations using advanced anisotropic yield functions are required in addition to common constitutive laws, and several yield functions have been proposed in the literature [1]. Advanced finite element codes, such as Abaqus, ANSYS, ADINA, LS-DYNA, and Marc, enable application of anisotropic yield functions via user subroutines. However, to implement these yield functions into the user subroutines, professional skills and knowledge are required, so they are not easily used at a practical level.

The Japan Association for Nonlinear Computer Aided Engineering (JANCAE), a non-profit organization [2], has held lectures twice a year for four days each since 2001 to provide training on theories and technology related to nonlinear simulation for researchers and engineers engaged in various CAE businesses. Since 2009, as one of the subcommittee activities, it has developed and verified the user subroutine library the Unified Material Model Driver for Plasticity (UMMDp). Not only researchers but also designers, engineers, and software vendors participated in this project. In the UMMDp, basic

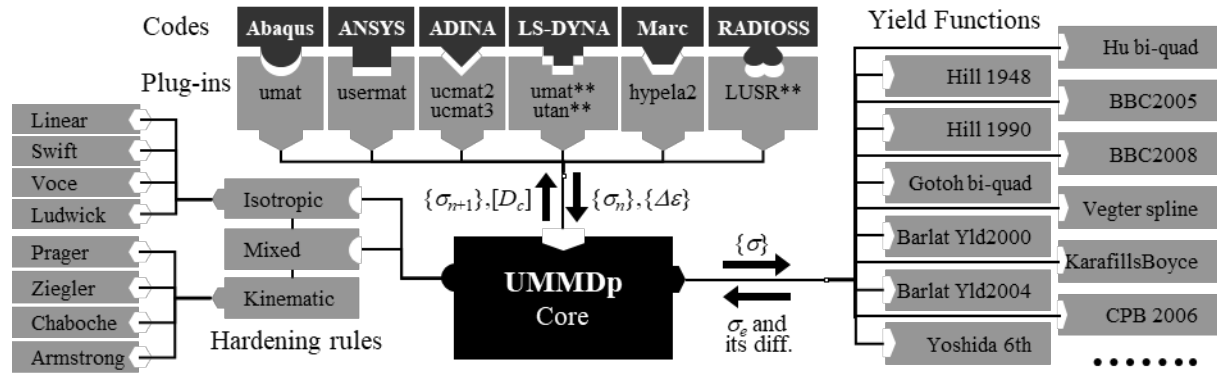


yield functions, such as the von Mises and Hill's quadratic yield functions [3], which almost all the advanced FE code already have, were implemented as a basic study at the beginning of the project. Up to now, many anisotropic yield functions, such as Yoshida's 6th-order polynomial [4], Barlat's Yld89 [5], Yld2000-2d [6], and Yld2004-18p [7], Comsa and Banabic's BBC2008 [8], Cazacu's CPB2006 [9], and Vegter's spline yield functions [10], have been implemented. Moreover, it is possible to add new yield functions to the UMMDp.

In this presentation, the techniques for implementing yield functions in the UMMDp are explained. Moreover, the problem of BM1 in NUMISHEET 2016 is discussed as an example of applying some yield functions to the implementation procedure.

## 2. Implementation of yield functions for a FE code

In an FE code, a material model is assembled in a stress integration algorithm as a constitutive equation. The roles of the material user subroutines are identical for all FE codes, and the numerical procedures in the user subroutines are independent of the yield functions. Therefore, the stress integration method for arbitrary yield functions can be unified as a common numerical library for any FE codes.



**Figure 1.** Framework of the UMMDp (Unified Material Driver for Plasticity).

Figure 1 shows the framework of the UMMDp. In the center, the core that executes the common numerical procedure is called from the user subroutines of FE codes through the plug-ins, and in the right branch, various yield functions are modularized as subroutines. The detail theory for the UMMDp and its verification are in Takizawa et al. [11]. As mentioned in Takizawa et al. [11], the updated stress and the consistent tangent matrix based on the current stress and incremental strain can be calculated if the yield function and its first and second order partial derivatives are given. The basic Hill's functions are shown as follows.

### 2.1. Hill 1948

Hill's quadratic yield functions [3] is the most basic anisotropic yield function and is represented based on von Mises's quadratic yield function, as follows.

$$\sigma_e^2 = \phi = \frac{1}{H+G} \left\{ H(\sigma_x - \sigma_y)^2 + F(\sigma_y - \sigma_z)^2 + G(\sigma_z - \sigma_x)^2 + 2N\tau_{xy}^2 + 2L\tau_{zx}^2 + 2M\tau_{yz}^2 \right\} \quad (1),$$

where  $\sigma_e$  is equivalent stress and  $H, F, G, N, L$ , and  $M$  are material parameters.  $\{\sigma\}$  is stress vector and its components are  $\sigma_x, \sigma_y, \sigma_z, \tau_{xy}, \tau_{zx}$ , and  $\tau_{yz}$ . The first and second order partial derivatives are given with the coefficient matrix  $[C]$  as follows.

$$\frac{\partial \sigma_e}{\partial \sigma_i} = \frac{C_{ij} \sigma_j}{\sigma_e}, \quad \frac{\partial^2 \sigma_e}{\partial \sigma_i \partial \sigma_j} = \frac{1}{\sigma_e} \left( -\frac{1}{\phi} C_{ik} \sigma_k C_{jm} \sigma_m + C_{ij} \right) \quad (2),$$

where

$$[C] = \frac{1}{H+G} \begin{bmatrix} H+G & -H & -G & & & \\ & F+H & -F & & & \\ & & G+F & & & \\ & & & 2N & & \\ & & & & 2L & \\ \text{Sym.} & & & & & 2M \end{bmatrix} \quad (3).$$

The UMMDp core calls the yield function subroutine in the iteration of the stress integration and the yield function subroutines gives the equivalent stress and its first and second order partial derivatives for the current stress. However, before assembling the yield function subroutine into the UMMDp, it is important to debug to ensure that the equivalent stress and its derivatives are properly calculated. For that purpose, the UMMDp provides an independent main program for checking yield locus and derivatives. Any new yield function can be added to the subroutine in the same way.

### 3. Example of using the UMMDp

Figure 2 shows, as an example of using the UMMDp with Abaqus, a cup drawing and reverse redrawing operation, benchmark 1 (BM1) proposed in NUMISHEET 2016 [12].

Figure 3 shows the yield loci calculated using selected yield criteria and the experimental plots of the yield points for a 5352 aluminum alloy sheet provided in the BM1 [12] (Material Constants are in

Table 1). The yield loci of all the yield functions are in good agreement with the experimental results, but each yield function shows different profiles. In particular, the shape of CPB2006 is different from others, because experimental results exist only in the first quadrant.

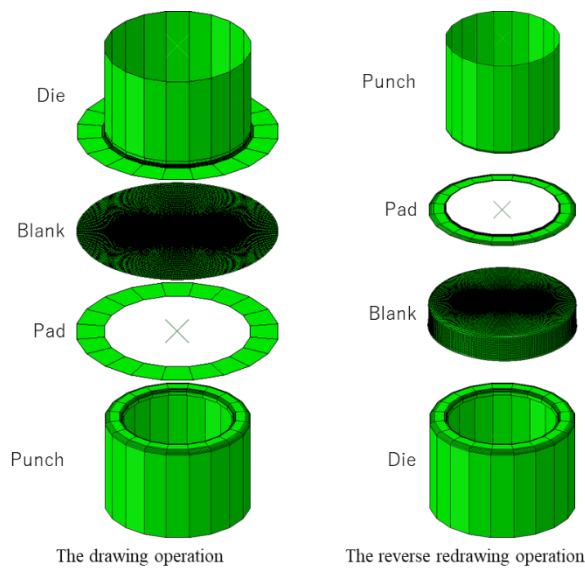
Figure 4 shows the thickness distributions. Figure 5 and Figure 6 show the load vs punch stroke for drawing and reverse redrawing operation. Figure 7 shows the cup height vs angle from rolling direction. The predicted numbers of earing and the angles from the rolling direction are summarized in Table 2. Figure 8 shows the wall thickness vs angle from rolling direction at the height of 25 mm above the cup base after the reverse redrawing operation. The Barlat Yld2004 gives the closest agreement with the experimental result (NUMISHEET 2016 BM1 [12]). For these simulations, the average total CPU time was about 4.5E5 s by using Intel Xeon CPU X5650 2.66 GHz.

### 4. Conclusion

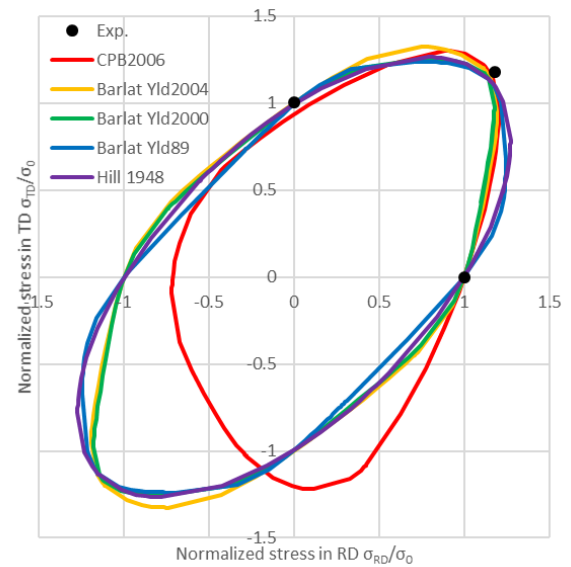
The techniques for implementing yield functions in the UMMDp are explained. As an example, selected yield functions were applied to the reverse cup drawing simulation [12] using the UMMDp, the UMMDp enables users to easily implement various yield functions to commercial FE codes.

### Acknowledgments

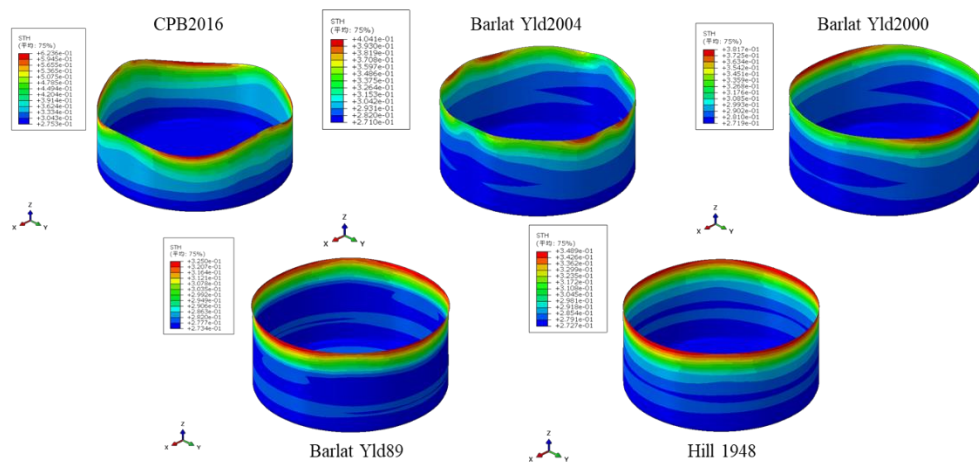
All programs were developed by participants of the Material Modeling Working Group of the JANCAE. The authors would like to thank all the people who supported the present project.



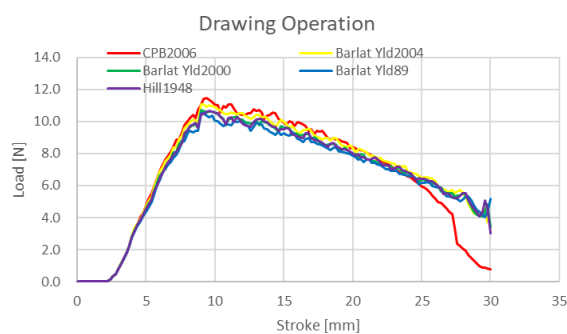
**Figure 2.** The example model for the cup drawing and reverse redrawing operations in the BM1 of NUMISHEET 2016[12]



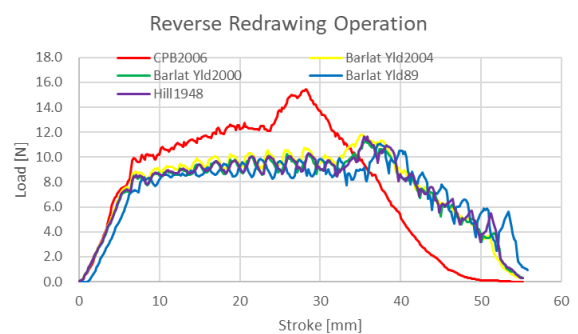
**Figure 3.** Yield loci calculated using selected yield criteria and the experimental plots of yield points for a 5352 aluminum alloy sheet provided in the BM1 of NUMISHEET 2016 [12]



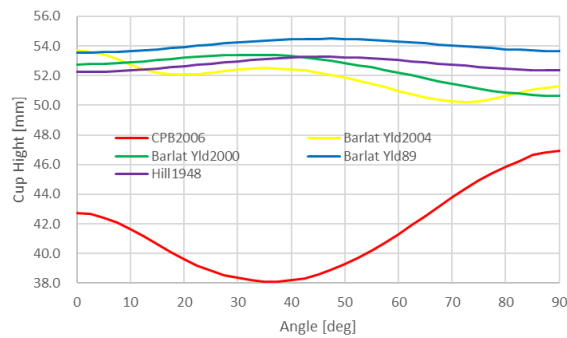
**Figure 4.** Thickness distributions after the reverse redrawing operation.



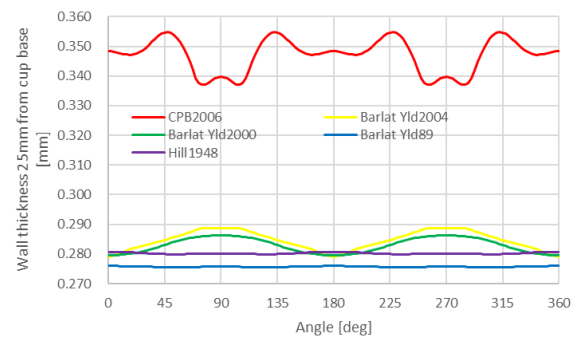
**Figure 5.** Load vs draw punch stroke.



**Figure 6.** Load vs reverse redraw punch stroke.



**Figure 7.** Cup height vs angle from rolling direction after the reverse redrawing operation.



**Figure 8.** Wall thickness vs angle from rolling direction at a height of 25mm above cup base after the reverse redrawing operation.

**Table 1.** Material Constants for the yield functions.

CPB06 ( $a=2.0$ )									
$C_{11}$	$C_{12}$	$C_{13}$	$C_{22}$	$C_{23}$	$C_{33}$	$C_{44}$	$C_{55}$	$C_{66}$	$k$
0.7704	-0.3253	-0.3426	1.2732	0.1305	2.1750	1	1	1.3296	-0.4150
Yld2004-18p ( $a=8.0$ )									
$A_1$	$A_2$	$A_3$	$A_4$	$A_5$	$A_6$	$A_7$	$A_8$	$A_9$	$A_{10}$
0.6402	0.7462	0.5014	0.0636	1.0439	0.2379	0.9089	0.9140	0.8654	0.6105
$A_{11}$	$A_{12}$	$A_{13}$	$A_{14}$	$A_{15}$	$A_{16}$	$A_{17}$	$A_{18}$		
1.0386	1.1726	1.3065	0.3464	1.4753	0.9812	0.9410	1.0763		
Yld2000-2d ( $a=8.0$ )									
$\alpha_1$	$\alpha_2$	$\alpha_3$	$\alpha_4$	$\alpha_5$	$\alpha_6$	$\alpha_7$	$\alpha_8$		
0.6547	1.2883	0.9050	0.8641	0.9710	0.6820	1.0091	1.2084		
Yld89 ( $m=8.0$ )									
$a$	$c$	$h$	$p$						
0.3401	1.6599	1.0052	1.0315						
Hill (1948)									
$F$	$G$	$H$	$L$	$M$	$N$				
0.7782	0.7572	1.2428	3.0	3.0	3.4687				

**Table 2.** The predicted earing after the reverse redrawing operation.

Yield Function	Number of earing	Angle from the rolling direction							
CPB2006	4	0	90	180	270				
Barlat Yld2004	8	0	35	90	145	180	215	270	325
Barlat Yld2000	4	32.5	147.5	212.5	327.5				
Barlat Yld89	4	45	135	225	315				
Hill 1948	4	45	135	225	315				
Experimental	8	0	51	92	131	183	231	272	311

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