JANCAE
Japan Association for Nonlinear CAE

UMMDp

Unified Material Model Driver for Plasticity

User's Guide

Abaqus

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Adapted from original document.

Contents

1 Preface

The Unified Material Model Driver for Plasticity (UMMDp) library is distributed as Fortran source codes. The Fortran compiler specified by the software vender in your analysis environment must be prepared. Please see the Abaqus manual for details of the environment required for compiling user subroutines.

In this guide, the sections of Abaqus' manual related to the use of the UMMDp are described below, as well as the details on how to use the UMMDp with Abaqus. In the following procedures, the prompts "%" and ">" represent the examples in UNIX/Linux and Windows, respectively. The common steps for using the UMMDp with Abaqus are:

1. Preparation of the UMMDp source files
   - Merge the UMMDp source files into one file.

2. Writing procedure to call tye UMMDp in the input data
   - To call the material user subroutine, specific keywords are required to be written in the input data. The keywords include the material constants, such as the coefficients of the yield criterion.

3. Execution of Abaqus with the UMMDp
   - When a command is typed to execute Abaqus, options are added to compile the UMMDp and to link it to Abaqus.

2 Related Sections of Abaqus User's Manual

1. Command to execute with user-subroutine
   - Abaqus Analysis User's Guide
     - 3.2.2 Abaqus/Standard, Abaqus/Explicit, and Abaqus/CFD execution.

2. Option for execution without compile
   - Abaqus Analysis User's Guide
     - 3.2.18 Making user-defined executables and subroutines.

3. Keywords for setup of the UMMDp
   - Abaqus Keywords Reference Guide
     - *DEPVAR: define the number of solution-dependent state variables.
     - *ORIENTATION: define local material axis for anisotropy.
     - *USER MATERIAL: define the material constants used in UMAT.
     - *USER OUTPUT VARIABLE: define the number of user output variables.

4. Specification of Abaqus user subroutines used in the UMMDp
   - Abaqus User Subroutines Reference Guide
     - 1.1.44 UMAT: user subroutine to define a material's mechanical behavior.
     - 1.1.58 UVARM: user subroutine to generate element output.

5. User defined mechanical properties with UMAT
   - Abaqus Analysis User's Guide
     - 26.7.1 User-defined mechanical material behavior.

3 Setup

3.1 Source Files

Concatenate the UMMDp source files into one single file with the plug-in file first. Simply use the batch files (.sh/.bat) or run each command separately.

- Unix/Linux

```
%  cd  dir_ummdp
%  cp  plug_ummdp_abaqus.f  jobname_ummdp.f
%  cat  ummdp*.f ≫ jobname_ummdp.f
```

- Windows

```
> cd  dir_ummdp
> copy  plug_ummdp_abaqus.f  jobname_ummdp.f
> type  ummdp*.f ≫ jobname_ummdp.f
```

3.2 Abaqus Input File

This section describes the keywords in the input data file for use in the UMMDp.

1. Definition of the principal axis for the material anisotropy (refer to the manual)

```
*ORIENTATION, NAME=ORI-1
1., 0., 0., 0., 1., 0.
3, 0.
```

2. Definition of the material model (the details will be provided later)

```
*MATERIAL, NAME=UMMDp
*USER MATERIAL, CONSTANTS=27
0, 0, 1000.0, 0.3, 2, -0.069, 0.936, 0.079,
1.003, 0.524, 1.363, 0.954, 1.023, 1.069, 0.981, 0.476,
0.575, 0.866, 1.145, -0.079, 1.404, 1.051, 1.147, 8.0,
0, 1.0, 0
```

Note that each line accedpts a maximum of 8 constants.

3. Define the number of internal state variables (SDV)

Set the number of state variables to 1+NTENS, where NTENS is the number of components of the tensor variables. NTENS=3 for plane stress or a shell element, and NTENS=6 for a solid element. The 1st "1" is reserved for the equivalent plastic strain, and NTENS is reserved for the plastic strain components. The following example corresponds to a solid element without kinematic hardening:

```
*DEPVAR
7,
```

In the case of kinematic hardening, the number of internal state variables corresponds to the equivalent plastic strain, plastic strain components and components of each partial back-stress tensor.

4. Define the user output variables (UVAR)

   The UMMDp can output some user output variables. The request for user output variables is not mandatory, except when an uncoupled rupture criterion is used. If user output variables are requested, the following variales are output in the presented order:

   - `UVAR(1)`: equivalent stress computed through the yield criteria
   - `UVAR(2)`: flow stress computed through the isotropic hardening law
   - `UVAR(3:2+NTENS)`: total back-stress tensor components (when using a kinematic hardening law)
   - `UVAR(3+NTENS:end)`: rupture criterion parameter and variables (when using a kinematic hardening law)
   - `UVAR(3:2+NTENS)`: rupture criterion parameter and variables (when not using a kinematic hardening law)

```
*USER OUTPUT VARIABLES
8,
```

5. Define output variables for post processing

   This keyword controls the output variables (e.g., equivalent plastic strain and equivalent stress) for post processing.

```
*OUTPUT, FIELD
*ELEMENT OUTPUT
LE, S, SDV, UVARM
```

3.3 Program Execution

To execute the program there are two options: (a) link the user subroutine in source code and (b) link the user subroutine previously compiled.

(a) To execute the program with the user subroutine in source code, execute the command:

```
%> abaqus job=jobname user=jobname_ummdp.f
```

(b) To execute the program with the user subroutine previously compiled, execute the commands:

```
> abaqus job=jobname user=jobname_ummdp.obj
```

```
% abaqus job=jobname user=jobname_ummdp.o
```

The command that compiles the file `jobname_ummdp.obj/o` is:

```
%> abaqus make library=jobname_ummdp.f
```

4 Input

The input data in UMMDp is defined as follows:

1. Parameter for Debug and Print
2. Parameters for Elasticity
3. Parameters for Yield Function
4. Parameters for Isotropic Hardening Law
5. Parameters for Kinematic Hardening Law
6. Parameters for Uncoupled Rupture Criterion

The detail of data is shown as follows and in addition, an example of input data is described.

4.1 Debug and Print

The first input parameter corresponds to the definition of debug and print mode, defined by the variable `nvbs0`. The option selected here will have an effect on the .dat file from Abaqus execution. It is a mandatory parameter and the options are:

- 0 - Error Messages Only
- 1 - Summary of Multistage Return Mapping
- 2 - Detail of Multistage Return Mapping and Summary of Newton-Raphson
- 3 - Detail of Newton-Raphson
- 4 - Input/Output
- 5 - All Status for Debug and Print

4.2 Elasticity

- `prela(1)` - ID for elasticity
- `prela(2~)` - Data depends on ID

Only isotropic Hooke elasticity can be defined. There are 2 ways to define it:

- ▪ Young's Modulus and Poisson's Ratio ✓
  - ID: 0
  - Parameters: 2

    - ○ `prela(1) = 0`
    - ○ `prela(2) = 200.0E+3` (Young's modulus E)
    - ○ `prela(3) = 0.3` (Poisson's ratio ν)

- ■ Bulk Modulus and Modulus of Rigidity ✓
  - ID: 1
  - Parameters: 2

  - ○ `prela(1) = 1`
  - ○ `prela(2) = 166666.7` (Bulk modulus $K = E(1 - 2\nu)/3)$)
  - ○ `prela(3) = 76923.08` (Modulus of rigidity $G = E(1 + \nu)/2$)

4.3 Yield Function

- `pryld(1)` - ID for yield function

  (Negative values specify plane stress yield functions)
- `pryld(2~)` - Data depends on ID

Identification number for yield function and required parameters are introduced. Please refer to the original papers for more detail on the formulation and meaning of parameters.

- ■ von Mises ✓
  - □ von Mises, R., 1913. Mechanik der festen Korper im plastisch deformablen Zustand. Gottin Nachr Math Phys, 1:582–592.

  - ID: 0

  - ○ `pryld(1) = 0`

- ■ Hill 1948 ✓
  - □ Hill, R., 1949. The theory of plane plastic strain for anisotropic metals. Proc Royal Soc Lond Ser Math Phys Sci 198, 428–437.

  - ID: 1
  - Parameters: 6

  - ○ `pryld(1)   = 1`
  - ○ `pryld(1+1) =` F
  - ○ `pryld(1+2) =` G
  - ○ `pryld(1+3) =` H
  - ○ `pryld(1+4) =` L
  - ○ `pryld(1+5) =` M
  - ○ `pryld(1+6) =` N

  - □ The parameters are the same as Hill's original paper. When F=G=H=1 and L=M=N=3, Hill's function is identical to von Mises.

- Yld2004-18p ✓
  - Barlat, F., Aretz, H., Yoon, J.-H., Karabin, M.E., Brem, J.C., Dick, R.E., 2005. Linear transfomation-based anisotropic yield functions. Int J Plasticity 21, 1009–1039.

- ID: 2
- Parameters: 19

- `pryld(1)     =` 2
- `pryld(1+1)   =` c'_{12}
- `pryld(1+2)   =` c'_{13}
- `pryld(1+3)   =` c'_{21}
- `pryld(1+4)   =` c'_{23}
- `pryld(1+5)   =` c'_{31}
- `pryld(1+6)   =` c'_{32}
- `pryld(1+7)   =` c'_{44}
- `pryld(1+8)   =` c'_{55}
- `pryld(1+9)   =` c'_{66}
- `pryld(1+10)  =` c''_{12}
- `pryld(1+11)  =` c''_{13}
- `pryld(1+12)  =` c''_{21}
- `pryld(1+13)  =` c''_{23}
- `pryld(1+14)  =` c''_{31}
- `pryld(1+15)  =` c''_{32}
- `pryld(1+16)  =` c''_{44}
- `pryld(1+17)  =` c''_{55}
- `pryld(1+18)  =` c'_{66}
- `pryld(1+19)  =` a (exponent)

- The order of parameters given as input is the same as in the original paper.

- CPB 2006 ✓

  - Cazacu, O., Plunkett, B., Barlat, F., 2006. Orthotropic yield criterion for hexagonal closed packed metals. Int J Plasticity 22, 1171–1194.

  - ID: 3
  - Parameters: 14

  - `pryld(1)` = `3`
  - `pryld(1+1)` = C_{11}
  - `pryld(1+2)` = C_{12}
  - `pryld(1+3)` = C_{13}
  - `pryld(1+4)` = C_{21}
  - `pryld(1+5)` = C_{22}
  - `pryld(1+6)` = C_{23}
  - `pryld(1+7)` = C_{31}
  - `pryld(1+8)` = C_{32}
  - `pryld(1+9)` = C_{33}
  - `pryld(1+10)` = C_{44}
  - `pryld(1+11)` = C_{55}
  - `pryld(1+12)` = C_{66}
  - `pryld(1+13)` = a (exponent)
  - `pryld(1+14)` = k (tension-compression ratio)

- Karafillis-Boyce 1993

  - Karafillis, A.P., Boyce, M.C., 1993. A general anisotropic yield criterion using bounds and a transformation weighting tensor. J Mech Phys Solids 41, 1859–1886.

  - ID: 4
  - Parameters: 8

  - `pryld(1)` = `4`
  - `pryld(1+1)` = C
  - `pryld(1+2)` = α_1
  - `pryld(1+3)` = α_2
  - `pryld(1+4)` = γ_1
  - `pryld(1+5)` = γ_2
  - `pryld(1+6)` = γ_3
  - `pryld(1+7)` = c
  - `pryld(1+8)` = k (k of exponent $2k$)

- Hu 2005

  - Hu, W., 2005. An orthotropic yield criterion in a 3-D general stress state. Int J Plasticity 21, 1771–1796.

  - ID: 5
  - Parameters: 10

  - `pryld(1)     = 5`
  - `pryld(1+1)   =` X_1
  - `pryld(1+2)   =` X_2
  - `pryld(1+3)   =` X_3
  - `pryld(1+4)   =` X_4
  - `pryld(1+5)   =` X_5
  - `pryld(1+6)   =` X_6
  - `pryld(1+7)   =` X_7
  - `pryld(1+8)   =` C_1
  - `pryld(1+9)   =` C_2
  - `pryld(1+10)  =` C_3

- Yoshida 6th Polynomial
  - Yoshida, F., Hamasaki, H., Uemori, T., 2013. A user-friendly 3D yield function to describe anisotropy of steel sheets. Int J Plasticity 45, 119–139.

  - ID: 6
  - Parameters: 16

  - `pryld(1)     = 6`
  - `pryld(1+1)   =` C_1
  - `pryld(1+2)   =` C_2
  - `pryld(1+3)   =` C_3
  - `pryld(1+4)   =` C_4
  - `pryld(1+5)   =` C_5
  - `pryld(1+6)   =` C_6
  - `pryld(1+7)   =` C_7
  - `pryld(1+8)   =` C_8
  - `pryld(1+9)   =` C_9
  - `pryld(1+10)  =` C_{10}
  - `pryld(1+11)  =` C_{11}
  - `pryld(1+12)  =` C_{12}
  - `pryld(1+13)  =` C_{13}
  - `pryld(1+14)  =` C_{14}
  - `pryld(1+15)  =` C_{15}
  - `pryld(1+16)  =` C_{16}

- Gotoh Biquadratic
  - Gotoh, M., 1977. A theory of plastic anisotropy based on a yield function of fourth order (plane stress state)—I. Int J Mech Sci 19, 505–512.

- ID: -1
- Parameters: 9

- `pryld(1)   = -1`
- `pryld(1+1) =` A_1
- `pryld(1+2) =` A_2
- `pryld(1+3) =` A_3
- `pryld(1+4) =` A_4
- `pryld(1+5) =` A_5
- `pryld(1+6) =` A_6
- `pryld(1+7) =` A_7
- `pryld(1+8) =` A_8
- `pryld(1+9) =` A_9

- Yld2000-2d ✓
  - Barlat, F., Brem, J.C., Yoon, J.-H., Chung, K., Dick, R.E., Lege, D.J., Pourboghrat, F., Choi, S.-H., Chu, E., 2003. Plane stress yield function for aluminum alloy sheets—part 1: theory. Int J Plasticity 19, 1297–1319.

- ID: -2
- Parameters: 9

- `pryld(1)   = -2`
- `pryld(1+1) =` α_1
- `pryld(1+2) =` α_2
- `pryld(1+3) =` α_3
- `pryld(1+4) =` α_4
- `pryld(1+5) =` α_5
- `pryld(1+6) =` α_6
- `pryld(1+7) =` α_7
- `pryld(1+8) =` α_8
- `pryld(1+9) =` a (exponent)

- Vegter
  - Vegter, H., van den Boogaard, A.H., 2006. A plane stress yield function for anisotropic sheet material by interpolation of biaxial stress states. Int J Plasticity 22, 557–580.

  - ID: -3
  - Parameters: $3 + 4n$

  - `pryld(1)                = -3`
  - `pryld(1+1)              = n` (max of i)
  - `pryld(1+2)              = f_bi0`
  - `pryld(1+3)              = r_bi0`
  - `pryld(1+3+(i-1)*4+1) = phi_uniaxial(i)`
  - `pryld(1+3+(i-1)*4+2) = phi_shear(i)`
  - `pryld(1+3+(i-1)*4+3) = ph_planestrain(i)`
  - `pryld(1+3+(i-1)*4+4) = omega(i)`

- BBC 2005
  - Banabic, D., Aretz, H., Comsa, D.S., Paraianu, L., 2005. An improved analytical description of orthotropy in metallic sheets. Int J Plasticity 21, 493–512.

  - ID: -4
  - Parameters: 9

  - `pryld(1)   = -4`
  - `pryld(1+1) =` k (k of exponent $2k$)
  - `pryld(1+2) =` a
  - `pryld(1+3) =` b
  - `pryld(1+4) =` L
  - `pryld(1+5) =` M
  - `pryld(1+6) =` N
  - `pryld(1+7) =` P
  - `pryld(1+8) =` Q
  - `pryld(1+9) =` R

- Yld89

  - Barlat, F., Lian, K., 1989. Plastic behavior and stretchability of sheet metals. Part I: A yield function for orthotropic sheets under plane stress conditions. Int J Plasticity 5, 51–66.

  - ID: -5
  - Parameters: 4

  - `pryld(1)    = -5`
  - `pryld(1+1)  =` M (exponent)
  - `pryld(1+2)  =` a
  - `pryld(1+3)  =` h
  - `pryld(1+4)  =` p

- BBC 2008

  - Comsa, D.-S., Banabic, D., 2008. Plane-stress yield criterion for highly-anisotropic sheet metals, in: NUMISHEET 2008, pp. 43–48.

  - ID: -6
  - Parameters: $2 + 8s$

  - `pryld(1)                = -6`
  - `pryld(1+1)              =` s (max of i)
  - `pryld(1+2)              =` k (k of exponent $2k$)
  - `pryld(1+2+(i-1)*8+1)    =` l_1
  - `pryld(1+2+(i-1)*8+2)    =` l_2
  - `pryld(1+2+(i-1)*8+3)    =` m_1
  - `pryld(1+2+(i-1)*8+4)    =` m_2
  - `pryld(1+2+(i-1)*8+5)    =` m_3
  - `pryld(1+2+(i-1)*8+6)    =` n_1
  - `pryld(1+2+(i-1)*8+7)    =` n_2
  - `pryld(1+2+(i-1)*8+8)    =` n_3

- Hill 1990

  - Hill, R., 1990. Constitutive modelling of orthotropic plasticity in sheet metals. J Mech Phys Solids 38, 405–417.

  - ID: -7
  - Parameters: 5

  - `pryld(1)   = -7`
  - `pryld(1+1) =` a
  - `pryld(1+2) =` b
  - `pryld(1+3) =` τ
  - `pryld(1+4) =` σ_b
  - `pryld(1+5) =` m

4.4 Isotropic Hardening Laws

- `prihd(1)` - ID for isotropic hardening law
- `prihd(2~)` - Data depends on ID

The equation of flow curve is introduced for each law, where σ_y is the yield stress, σ_{y_0} is the initial yield stress and p the equivalent plastic strain.

- Perfectly Plastic ✓

  - ID: 0
  - Parameters: 1

  - `prihd(1)   = 0`
  - `prihd(1+1) =` σ_y

- Linear Hardening: $\sigma_\mathrm{y} = \sigma_{\mathrm{y}_0} + Hp$ ✓

  - ID: 1
  - Parameters: 2

  - `prihd(1)   = 1`
  - `prihd(1+1) =` σ_{y_0}
  - `prihd(1+2) =` H

- Swift: $\sigma_{\mathrm{y}} = K\,(\varepsilon_0 + p)^n$ ✓
  - ID: 2
  - Parameters: 3

    - `prihd(1)   ` = 2
    - `prihd(1+1)` = K
    - `prihd(1+2)` = ε_0
    - `prihd(1+3)` = n

- Ludwick: $\sigma_{\mathrm{y}} = \sigma_{\mathrm{y}_0} + cp^n$ ✓
  - ID: 3
  - Parameters: 3

    - `prihd(1)   ` = 3
    - `prihd(1+1)` = σ_{y_0}
    - `prihd(1+2)` = c
    - `prihd(1+3)` = n

- Voce: $\sigma_{\mathrm{y}} = \sigma_{\mathrm{y}_0} + Q\,(1 - \exp(-bp))$ ✓
  - ID: 4
  - Parameters: 3

    - `prihd(1)   ` = 4
    - `prihd(1+1)` = σ_{y_0}
    - `prihd(1+2)` = Q
    - `prihd(1+3)` = b

- Voce + Linear: $\sigma_{\mathrm{y}} = \sigma_{\mathrm{y}_0} + Q\,(1 - \exp(-bp)) + Hp$ ✓
  - ID: 5
  - Parameters: 4

    - `prihd(1)   ` = 5
    - `prihd(1+1)` = σ_{y_0}
    - `prihd(1+2)` = Q
    - `prihd(1+3)` = b
    - `prihd(1+4)` = H

- Voce + Swift: $\sigma_\mathrm{y} = a\left[\sigma_{\mathrm{y}_0} + Q\left(1 - \exp(-bp)\right)\right] + (1-a)\left[K(\varepsilon_0 + p)^n\right]$ ✓
  - ID: 6
  - Parameters: 7

  - `prihd(1)   ` = 6
  - `prihd(1+1)` = a
  - `prihd(1+2)` = σ_{y_0}
  - `prihd(1+3)` = Q
  - `prihd(1+4)` = b
  - `prihd(1+5)` = K
  - `prihd(1+6)` = ε_0
  - `prihd(1+7)` = n

4.5 Kinematic Hardening Laws

- `prkin(1)` - ID for kinematic hardening law
- `prkin(2~)` - Data depends on ID

The equation of backstress is introduced for each law, where $\dot{\boldsymbol{\alpha}}$ is the increment of the total backstress tensor, $\dot{\boldsymbol{\alpha}}_i$ is the increment of the partial backstress tensor, $\boldsymbol{\alpha}$ is the total backstress tensor, $\boldsymbol{\alpha_i}$ is the partial backstress tensor, $\dot{\boldsymbol{\varepsilon}}^\mathrm{p}$ is the increment of plastic strain tensor, and $\dot{p}$ is the increment of equivalent plastic strain. Depending on the law, a specific number of additional state variables (`SDV`) and user output variables (`UVAR`) are required. The variable k represents the number of tensor components (`NTENS`).

- No Kinematic Hardening ✓
  - ID: 0

  - `prkin(1)` = 0

- Prager: $\dot{\boldsymbol{\alpha}} = \dfrac{2}{3}c\dot{\boldsymbol{\varepsilon}}^\mathrm{p}$ ✓
  - ID: 1
  - Parameters: 1
  - State Variables: + k

  - `prkin(1)   ` = 1
  - `prkin(1+1)` = c

- Ziegler: $\dot{\boldsymbol{\alpha}} = c\,(\boldsymbol{\sigma} - \boldsymbol{\alpha})\,\dot{p}$ ✓
  - ID: 2
  - Parameters: 1
  - State Variables: $+\ k$

  - `prkin(1)   = 2`
  - `prkin(1+1) =` c

- Armstrong-Frederick: $\dot{\boldsymbol{\alpha}} = \frac{2}{3} c \dot{\boldsymbol{\varepsilon}}^{\mathrm{p}} - \gamma \boldsymbol{\alpha} \dot{p}$ ✓
  - ID: 3
  - Parameters: 2
  - State Variables: $+\ k$

  - `prkin(1)   = 3`
  - `prkin(1+1) =` c
  - `prkin(1+2) =` γ

- Chaboche I: $\dot{\boldsymbol{\alpha}} = \sum_{i=1}^{n} \dot{\boldsymbol{\alpha}}_i = \sum_{i=1}^{n} \left(\frac{2}{3} c_i \dot{\boldsymbol{\varepsilon}}^{\mathrm{p}} - \gamma_i \boldsymbol{\alpha}_i \dot{p} \right)$ ✓
  - ID: 4
  - Parameters: $1 + 2n$
  - State Variables: $+\ n \times k$
  - User Output Variables: $+\ k$

  - `prkin(1)         = 4`
  - `prkin(1+1)       =` n
  - `prkin(1+1+(i*1)) =` c_i
  - `prkin(1+1+(i*2)) =` γ_i

- Chaboche II: $\dot{\boldsymbol{\alpha}} = \sum_{i=1}^{n} \dot{\boldsymbol{\alpha}}_i = \sum_{i=1}^{n} \left(\frac{c_i}{\bar{\eta}} (\boldsymbol{\sigma} - \boldsymbol{\alpha}) - \gamma_i \boldsymbol{\alpha}_i \right) \dot{p}$ ✓

  - ID: 5
  - Parameters: $1 + 2n$
  - State Variables: $+ \, n \times k$
  - User Output Variables: $+ \, k$

  - `prkin(1)           = 5`
  - `prkin(1+1)         =` n
  - `prkin(1+1+(i*1))   =` c_i
  - `prkin(1+1+(i*2))   =` γ_i

- Yoshida-Uemori

  - ID: 6
  - Parameters: 5

  - `prkin(1)   = 6`
  - `prkin(1+1) =` C
  - `prkin(1+2) =` Y
  - `prkin(1+3) =` a
  - `prkin(1+4) =` k
  - `prkin(1+5) =` b

4.6 Uncoupled Rupture Criteria

- `prrup(1)` - ID for uncoupled rupture criterion
- `prrup(2)` - Flag to complete (0) or terminate (1) analysis if critical value is reached
- `prrup(3∼)` - Data depends on ID

The equation of the uncoupled rupture criterion is introduced for each criterion, where W is the rupture parameter, σ_1 is the maximum principal stress, σ_{h} is the hydrostatic stress, $\bar{\sigma}$ is the equivalent stress, $\dot{p}$ the equivalent plastic strain rate, and W_{L} the user-defined critical value. Depending on the criterion, a specific number of additional user output variables (`UVAR`) are required.

- No Uncoupled Rupture Criterion ✓

  - ID: 0

  - `prrup(1) = 0`

- Equivalent Plastic Strain: $W = \displaystyle\int_0^{\varepsilon_\mathrm{f}} \dot{p}\,\mathrm{d}t$ ✓
  - ID: 1
  - Parameters: 1
  - User Output Variables: + 2

  ○ `prrup(1)   = 1`
  ○ `prrup(1+2) =` W_L

  □ `UVAR(_+1) = equivalent plastic strain`
  □ `UVAR(_+2) = rupture parameter normalised by critical value`

- Cockroft and Latham: $W = \displaystyle\int_0^{\varepsilon_\mathrm{f}} \frac{\sigma_1}{\bar{\sigma}}\,\mathrm{d}p$ ✓
  - ID: 2
  - Parameters: 1
  - User Output Variables: + 4

  ○ `prrup(1)   = 2`
  ○ `prrup(1+2) =` W_L

  □ `UVAR(_+1) = equivalent plastic strain`
  □ `UVAR(_+2) = maximum principal stress`
  □ `UVAR(_+3) = rupture parameter`
  □ `UVAR(_+4) = rupture parameter normalised by critical value`

- Rice and Tracey: $W = \displaystyle\int_0^{\varepsilon_\mathrm{f}} \exp\left(\frac{3}{2}\frac{\sigma_h}{\bar{\sigma}}\right)\,\mathrm{d}p$ ✓
  - ID: 3
  - Parameters: 1
  - User Output Variables: + 4

  ○ `prrup(1)   = 3`
  ○ `prrup(1+2) =` W_L

  □ `UVAR(_+1) = equivalent plastic strain`
  □ `UVAR(_+2) = hydrostatic stress`
  □ `UVAR(_+3) = rupture parameter`
  □ `UVAR(_+4) = rupture parameter normalised by critical value`

- Ayada: $W = \displaystyle\int_0^{\varepsilon_\mathrm{f}} \frac{\sigma_h}{\bar{\sigma}} \,\mathrm{d}p$ ✓
  - ID: 4
  - Parameters: 1
  - User Output Variables: + 4

  ○ `prrup(1)   = 4`
  ○ `prrup(1+2) =` W_L

  □ `UVAR(_+1) = equivalent plastic strain`
  □ `UVAR(_+2) = hydrostatic stress`
  □ `UVAR(_+3) = rupture parameter`
  □ `UVAR(_+4) = rupture parameter normalised by critical value`

- Brozzo: $W = \displaystyle\int_0^{\varepsilon_\mathrm{f}} \frac{2\sigma_1}{3\,(\sigma_1 - \sigma_h)} \,\mathrm{d}p$ ✓
  - ID: 5
  - Parameters: 1
  - User Output Variables: + 5

  ○ `prrup(1)   = 5`
  ○ `prrup(1+2) =` W_L

  □ `UVAR(_+1) = equivalent plastic strain`
  □ `UVAR(_+2) = maximum principal stress`
  □ `UVAR(_+3) = hydrostatic stress`
  □ `UVAR(_+4) = rupture parameter`
  □ `UVAR(_+5) = rupture parameter normalised by critical value`

- Forming Limit Diagram: $W = \frac{\varepsilon_1}{\varepsilon_1^{\mathrm{FLD}}}$ ✓

  - ID: 6
  - Parameters: 1
  - User Output Variables: + 5

    - `prrup(1)   = 6`
    - `prrup(1+2) =` W_{L}

    - `UVAR(_+1) = maximum principal strain`
    - `UVAR(_+2) = minimum principal strain`
    - `UVAR(_+3) = projection of maximum principal strain on FLD`
    - `UVAR(_+4) = rupture parameter`
    - `UVAR(_+5) = rupture parameter normalised by critical value`

  - Additionaly, this rupture criterion requires the definition of data points, representative of the forming limit curve major and minor strains. The following lines should be added to the input file below the material definition, where in red are indicated the fields to be modified. Do not change anything else. The number of properties should be modified according to the number of data points. Below `*PROPERTY TABLE, TYPE=FLD1` insert the major strain data points, and below `*PROPERTY TABLE, TYPE=FLD2` insert the minor strain data points. Please note that each line only accepts a maximum of 8 properties, while multiple lines are accepted.

```
*PROPERTY TABLE TYPE, NAME=FLD1, PROPERTIES=5
*PROPERTY TABLE TYPE, NAME=FLD2, PROPERTIES=5
*TABLE COLLECTION, NAME=FLD
*PROPERTY TABLE, TYPE=FLD1
0.3, 0.2, 0.1, 0.2, 0.3
*PROPERTY TABLE, TYPE=FLD2
-0.2, -0.1, 0.0, 0.1, 0.2
```

5 Example

- Debug and Print: Error messages only
- Elasticity: Young's Modulus and Poisson's Ratio
- Yield Function: Yld2000-2d
- Isotropic Hardening Law: Voce + Linear
- Kinematic Hardening Law: Chaboche II
- Uncoupled Rupture Criterion: Cockroft and Latham

The material parameters and IDs are given as input to the UMMDp in one dimensional array, named by default `props` in the program. In the beginning of the UMMDp, this array is copied to a new variable `prop`, and the variable to define debug and print mode is excluded from this new array.

- `props(1)  = 0` Debug and Print ID
- `props(2)  = 0` Elasticity ID
- `props(3)  = 200000.0` E
- `props(4)  = 0.3` ν
- `props(5)  = -2` Yield Function ID
- `props(6)  = 1.0` α_1
- `props(7)  = 1.0` α_2
- `props(8)  = 1.0` α_3
- `props(9)  = 1.0` α_4
- `props(10) = 1.0` α_5
- `props(11) = 1.0` α_6
- `props(12) = 1.0` α_7
- `props(13) = 1.0` α_8
- `props(14) = 8.0` a
- `props(15) = 5` Isotropic Hardening Law ID
- `props(16) = 180.0` σ_{y_0}
- `props(17) = 5.06` Q
- `props(18) = 2.11` b
- `props(19) = 52.2` H

- `props(20) = 5` Kinematic Hardening Law ID
- `props(21) = 3` n
- `props(22) = 824.9` c_1
- `props(23) = 56.5` γ_1
- `props(24) = 135.4` c_2
- `props(25) = 0.0684` γ_2
- `props(26) = 27509.4` c_3
- `props(27) = 415.55` γ_3
- `props(28) = 2` Uncoupled Rupture Criterion ID
- `props(29) = 1` Flag of Analysis Completion/Termination
- `props(30) = 0.5` W_{L}

This `prop(i)` array is divided in the UMMDp as follow:

- `prela(i)` - Elasticity
- `pryld(i)` - Yield Function
- `prihd(i)` - Isotropic Hardening Law
- `prkin(i)` - Kinematic Hardening Law
- `prrup(i)` - Uncoupled Rupture Criterion

The model ID is stored in the top of each of these arrays. Here, it is shown some parts of a plane stress example for the Abaqus input file. The red letter indicates ID of each properties.

```
*MATERIAL, NAME=UMMDp
*USER MATERIAL, CONSTANTS=30
0, 0, 200000.0, 0.3, -2, 1.0, 1.0, 1.0
1.0, 1.0, 1.0, 1.0, 1.0, 8.0, 5, 180.0,
5.06, 2.11, 52.2, 5, 3, 824.9, 56.5, 135.4,
0.0684, 27509.4, 415.55, 2, 1, 0.5
**
*DEPVAR
13,
**
*USER OUTPUT VARIABLES
9,
```

6 Error Codes

- 100 - Element Type
- 20x - Material Properties ID
  - 201 - Elasticity
  - 202 - Yield Function
  - 203 - Isotropic Hardening Law
  - 204 - Kinematic Hardening Law
  - 205 - Uncoupled Rupture Criterion
- 30x - Variables Size
  - 301 - Partial Back Stress
  - 302 - Internal State Variables
  - 303 - Material Properties
  - 304 - Normal or Shear Components
- 40x - Computation
  - 401 - Matrix Determinant
  - 402 - Singular Matrix
  - 403 - Multistage Convergence
- 500 - Uncoupled Rupture Criterion Termination