

Saint Francis Xavier University

# Finding Energy and Energy Difference in Relativistic Cosmic Muons

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for the degree of Bachelor of Science with Honours

in the  
Faculty of Science  
Department of Physics

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## **Declaration of Authorship**

I, Matthew Penner, declare that this thesis titled, Cosmic Muons and the Theory of Special Relativity and the work presented in it are my own. I confirm that:

- This work was done wholly or mainly while in candidature for a research degree at this University.
- Where any part of this thesis has previously been submitted for a degree or any other qualification at this University or any other institution, this has been clearly stated.
- Where I have consulted the published work of others, this is always clearly attributed.
- Where I have quoted from the work of others, the source is always given. With the exception of such quotations, this thesis is entirely my own work.
- I have acknowledged all main sources of help.
- Where the thesis is based on work done by myself jointly with others, I have clarified exactly what was done by others and what I have contributed myself.

Signed: Matthew Penner

Date: March 28<sup>th</sup>, 2023

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## **Abstract**

Muons are subatomic particles that constantly permeate the earth's atmosphere, originating from high-energy protons interaction that decays into a variety of subatomic particles, including muons. Despite being invisible to the naked eye, detectors such as the Teachspin Muon Physics scintillator can be used to measure their properties. While previous studies have measured muon flux and properties in various locations, this thesis investigates cosmic muons in the Antigonish area, aiming to determine their energy, energy change, and flux. Through our investigation, we hope to contribute to a better understanding of muon behaviour in this particular region, with potential implications for broader research in particle physics.

# Contents

<b>1</b>	<b>Introduction</b>	<b>6</b>
<b>2</b>	<b>The Standard Model</b>	<b>8</b>
2.1	History . . . . .	8
2.2	Cosmic Muons . . . . .	11
2.3	Special Relativity . . . . .	13
<b>3</b>	<b>Finding Energy and Flux difference</b>	<b>15</b>
<b>4</b>	<b>Materials and equipment</b>	<b>21</b>
4.1	Muon Detector . . . . .	21
4.1.1	Detector Physics . . . . .	21
4.1.2	Detector Electronics . . . . .	23
4.1.3	Detector Output . . . . .	24
4.2	Other Equipment . . . . .	24
<b>5</b>	<b>Muon flux in the Antigonish area</b>	<b>26</b>
5.1	Theories and Math . . . . .	26
5.2	Data Collection . . . . .	27
5.2.1	Data Table . . . . .	28
5.3	Data Analysis . . . . .	31
5.3.1	Choosing an analysis software . . . . .	31
5.3.2	Analysis Process . . . . .	31
5.4	Error Analysis . . . . .	33
5.4.1	Statistical Uncertainties . . . . .	33

5.4.2	Calibration and Systematic Uncertainties . . . . .	34
5.5	Findings . . . . .	35
<b>6</b>	<b>Finding Muon Energy</b>	<b>37</b>
6.1	Theories and Math . . . . .	37
6.1.1	Math . . . . .	37
6.2	Experimental set up . . . . .	39
6.3	Data Analysis . . . . .	39
6.4	Additional Error Analysis . . . . .	40
6.5	Findings . . . . .	40
<b>7</b>	<b>Conclusion</b>	<b>42</b>
<b>Appendix A:</b>	<b>Ryman Sum Integrator in ROOT</b>	<b>46</b>
<b>Appendix B:</b>	<b>Raw data</b>	<b>51</b>
<b>Appendix C:</b>	<b>Code for Muon Flux in ROOT</b>	<b>58</b>
<b>Appendix D:</b>	<b>Code for Finding Average Muon Energy in ROOT</b>	<b>63</b>

# List of Figures

2.1	The Standard Model [1]	10
2.2	The highlighted particles are the particles affected by the respective field [2]	11
2.3	A diagram created by myself. Of some of the decay modes that lead to muon production. Note that there are many other decay modes not shown because there do not produce muons.	12
2.4	Muon decay Feynman diagrams [3]	12
3.1	Average energy as a function of height. This graph assumes the point (0,4000) meaning it assumes that muons are 4GeV are at sea level and it implies that muons start with an average energy of 5.8GeV.	18
3.2	Average energy as a function of time. This graph also assumes that at sea level muons have an average energy of 4GeV and shows similar results as figure 3.1.	19
3.3	Average height as a function of time. This graph assumes muons are produced at 15km above sea level.	19
3.4	Average number of muons per $cm^2$ as a function of height. You can see that it is fairly linear and this is because the muons are only travelling for 1 $\mu s$ so they don't lose that much energy (shown in figure 3.2).	20
4.1	The muon detector is on the right, and the electronics box on the left.	21
4.2	Image from (Coan & Ye, 2016) of a muon decaying the scintillator	22
4.3	Diagram of the electronics from [4]	23
4.4	Sample raw data represented in an output text file.	24
4.5	The left image is the Pure sine AC power source. The right image to right is the computer used for the experiment.	25
5.1	Histogram with fit produced in methods discussed later.	27
5.2	Pseudo-code for Data analysis	32
5.3	Histogram from teachspins calibration method.	35

6.1 Pseudo-code for Data analysis . . . . .	39
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## List of Tables

5.1	Total data taken from muon detector. . . . .	28
5.2	Comparison of Experimental Results and Theoretical values.(If there is no error it means it is an approximate global average). The Theoretical values for $\tau$ , $\lambda$ and $G_F$ are from [5]. The expected value for $\mu_+/\mu_-$ is from [4]. The muon rate expected value is from [6] . . . . .	35
5.3	Comparison of experimental results with expected values, taking into account the effect of muons travelling through matter rather than free space. (If there is no error it means it is an approximate global average). The expected value for $\tau$ is from [4] and also derived earlier. $\lambda$ is found from $\tau$ . The expected value for $G_F$ is from [5]. The expected value for $\mu_+/\mu_-$ is from [4]. The muon rate expected value is from [6] . . . . .	36
6.1	The approximate average for Energy is from [6] . $\beta$ , $\gamma$ and KE are found from E. . . . .	40
7.1	Comparison of experimental results with expected values from both parts of this paper. (If there is no error it means it is an approximate global average). The expected value for $\tau$ is from [4] and also derived earlier. $\lambda$ is found from $\tau$ . The expected value for $G_F$ is from [5]. The expected value for $\mu_+/\mu_-$ is from [4]. The expected values for Energy and muon rate are from [6], $\beta$ , $\gamma$ and KE are found from E. . . . .	42

## CHAPTER 1

# Introduction

Muons are fundamental building blocks along with electrons, quarks, and other particles that are part of the Standard Model of particle physics. Muons are quite similar to electrons and are sometimes colloquially referred to as “the electron’s big cousin.” This is because the muon and electron share important properties like spin and charge; however, muons are around 210 times larger than an electron [5], and unlike electrons, they decay. This paper will focus on muons rather than other particles because there is a free muon source where we do not need to invest billions of dollars to create an accelerator. The muon source that is used in this paper is a phenomenon known as cosmic rays. Cosmic rays are mostly high-energy protons produced in outer space by mechanisms that are not fully understood [4]. Recent theories and observations, however, suggest cosmic rays are highly energetic atomic nuclei, (mostly hydrogen) produced in supernova explosions[7][8]. High-energy protons interact with the molecules in the earth’s atmosphere to create a cascade of particles known as cosmic showers [4]. One of the particles created in the cascade process is the muon. Muons have a relatively long average lifetime of  $2.2\mu s$  [5], and because of this, they reach the earth’s surface unlike many other particles created in the cascade process. Muon detection and knowing their properties have become crucial tools in various fields, including particle physics, astrophysics, and geophysics. Scientists have been measuring the properties of cosmic muons for decades using various techniques. One of the most common techniques is to use muon detectors, which are sensitive instruments that can detect the passage of muons through matter. The Teachspin Muon Physics scintillator is one such instrument, which can detect muons by measuring the scintillation light produced when they pass through a material. Our primary goal is to measure the muon flux, energy, and energy change as they pass through the atmosphere. The experiment’s motivation is to understand the behaviour of muons in the atmosphere in the Antigonish area. In analogy with nuclear decay muons decay via the following formula.

$$N = N_0 e^{\frac{-t'}{\tau}} \quad (1.1)$$

where  $\tau$  is the average lifetime and can also be represented by  $\frac{1}{\lambda}$  where  $\lambda$  is the decay constant and  $t'$  is time in the muons reference frame.

It is also possible to design an experiment to find the energy of cosmic muons. To find the energy, measurement of the muon flux at two separate altitudes is needed; after

the flux at the two altitudes is determined using the decay equation 1.1, I will find the time passed in the muons reference frame and compare it to the time passed in the earth's reference frame, assuming that muons are travelling close to the speed of light. By using relativistic kinematics, it is possible to calculate the speed and energy of the average cosmic muon.

This thesis will lay out the theoretical framework behind the experiment, describe the physics and electronics of the muon detector used, discuss the change in energy as a muon travels through the atmosphere, describe the data collection analysis process, and show the preliminary results from the experiment.

## CHAPTER 2

# The Standard Model

Before discussing Muons, an overview of the Standard Model is in order. The Standard Model is the most successful theory devised; it aims to be an accurate description of the fundamental building block of nature. This section will present a brief history of the Standard Model based on Dr. Don Lincoln's description [9].

### 2.1 History

Ever since the ancient Greeks, humans have wondered what the world is made of, and their predictions have kept improving. In 430 BC, Democritus of Abdera believed that matter was made of indivisible particles that he named atoms which literally means indivisible [10]. This idea developed into Chemistry. Over the last century, physicists have made impressive progress in understanding our universe through the development of the Standard Model. In short, the Standard Model is a theory of fundamental particles and their interactions, but that description does not do justice to its significance. The Standard Model can describe a comprehensive range of physical phenomena from the smallest scale (e.g., how does radioactive decay work) to the largest scale (e.g., why do we live in a matter-dominated universe). The first fundamental element to be discovered was the electron in 1897 by J.J. Thomson [11], and the discovery of the nucleus contacting a proton and neutron followed. These three particles make up everything in the periodic table, giving the impression that since these three particles make up an atom, that is it however, physicists discovered more particles in cosmic rays (to be discussed later) and particle accelerators that could not be explained by just these three particles so either some of these particles were not fundamental and could be broken down further or there were many more fundamental particles. In the 1960s, physicists managed to analyze the structure of the proton and neutron and found they were made up of particles called quarks [12] [13]. There are six types of quarks up, down, charmed, strange, top, and bottom (not all discovered at the same time)[14]. The proton is made up of two up quarks and one down quark, and a neutron is made up of two down quarks and one up. While the up and down quarks can explain the nucleus, the others can explain more particles. The electron is not a quark but it is in a different family called leptons. There are six leptons, three charged, and three neutral. The three charged particles are the Electron, Muon, and Tau. The muon and tau are very similar to the electron differing mainly in that they have more mass and they decay; according to the particle data group

the muon has a mean lifetime of  $2.1969811 \pm 0.0000022 \mu s$  and a mass of  $105.6583755 \pm 0.0000023$  MeV [5], which is around 200 times that of an electron. Tau is even heavier and has a shorter decay time. There are also three non-charged leptons called neutrinos, and there is one that corresponds to each charged lepton so there is an electron neutrino, muon neutrino and tau neutrino. These particles all have very small masses, and there is ongoing research to find the exact mass. Quarks and leptons are called fermions, and they make up all matter, however, there is still the topic of antimatter. According to the Feynman-Stueckelberg Interpretation, antimatter is identical to matter but moves backward in time [15]. Because time moving backward is a difficult concept, here is a simplified way of looking at it: Antimatter particles have most of the same properties as their matter counterparts, but they have the opposite charge and parity (Pitts, 1998). For example, the anti-particle of the electron is a positron, and it has the same mass as an electron, but it has a charge +1 instead of a charge -1 [5]. Opposite parity means the direction is flipped if it is the outcome of decay, for example. It is important to note that this is a simplified version of antimatter, and there is a charge parity violation in weak force interactions.[16][17]

This overview has so far covered all the matter particles the other component of the standard model is force. There are four fundamental forces: gravity, electromagnetic force, strong nuclear force, and weak nuclear force (the latter two are often abbreviated to the “strong force” and “weak force”). These fundamental forces govern how all the previously discussed particles interact with each other. These forces have different relative strengths. Assuming the strong force has a unit of one, when we compare the other forces to the strong force, the electromagnetic force has a strength of  $\frac{1}{137}$  (we use the symbol  $\alpha$  to denote this). The weak force has a strength of around  $10^{-5}$  and gravity is around  $10^{-40}$ [14]. Notice that gravity is much weaker than the other three forces, and it is therefore extremely difficult to study how gravity works on a quantum scale; as a result, gravity is not included in the Standard Model. In the macroscopic world (in which we live), the only forces one might observe are gravity and electromagnetic force, because the forces differ not only in strength but also in range. While the electromagnetic and gravitational forces have infinite ranges, the range of the weak force is only on the order of  $10^{-18} m$  [5], and the strong force is in the order of  $10^{-15} m$ . At a quantum level, these forces can be described by an exchange of particles. One description from Fermi-Lab says, “imagine standing in a boat and having someone throw you a heavy sack. Your boat would move as if it had felt a force. Similarly, if you throw a heavy sack off the boat, the boat would move. All the subatomic forces work by exchanging a different kind of particle.” For each force, there are different mediator particles known as Gauge Bosons: the photon for the electromagnetic force, the gluon for the strong force, and the W and Z bosons for the weak force.

The only particle left to introduce is the Higgs boson [18]. The Higgs boson was one

of the most sought-after particle. The Higgs boson and the Higgs field are responsible for giving elementary particles their mass. One can think of the Higgs field as a crowd of people and someone unpopular could walk past everyone easily because they do not interact with anyone, in the analogy the unpopular person corresponds to a particle with no mass. On the other hand, someone who is popular would have a harder time walking through the crowd because everyone would want to talk to them, so this person interacts with the crowd. In the analogy, the popular person corresponds to a heavy particle. It is the interaction with the field that gives particles their mass. If it were not for the Higgs field, all particles would be massless [18]. The following diagram organizes the particles in the Standard Model.

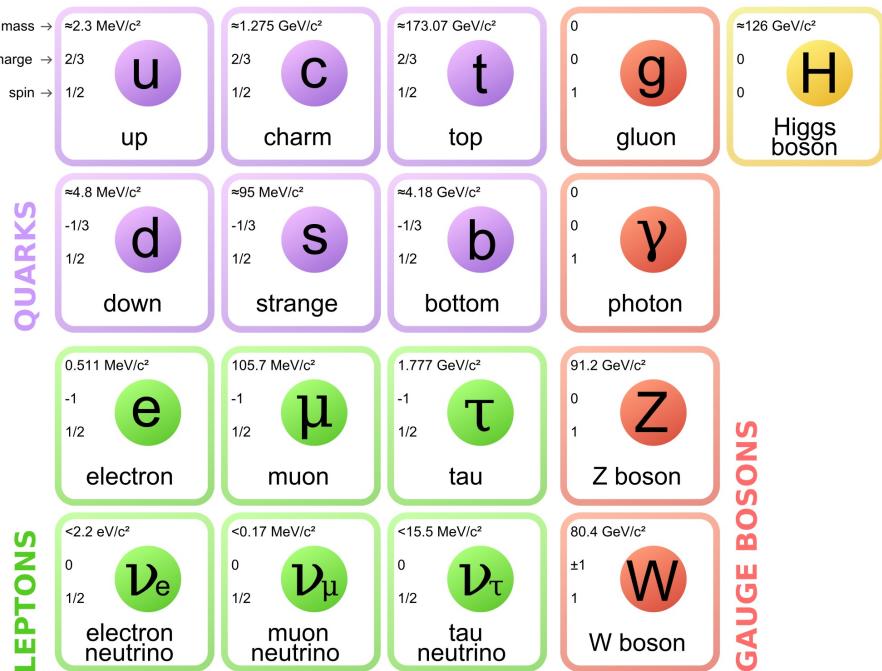


Figure 2.1: The Standard Model [1]

Each force acts on a different set of particles. The Higgs field acts on any particles with mass (or, more accurately, gives the particles mass), the electromagnetic force acts on particles with electric charge, the weak force acts on every particle but the gluon, and the strong force acts on quarks. Dominic Walliman has made the following diagrams to help visualize these interactions:

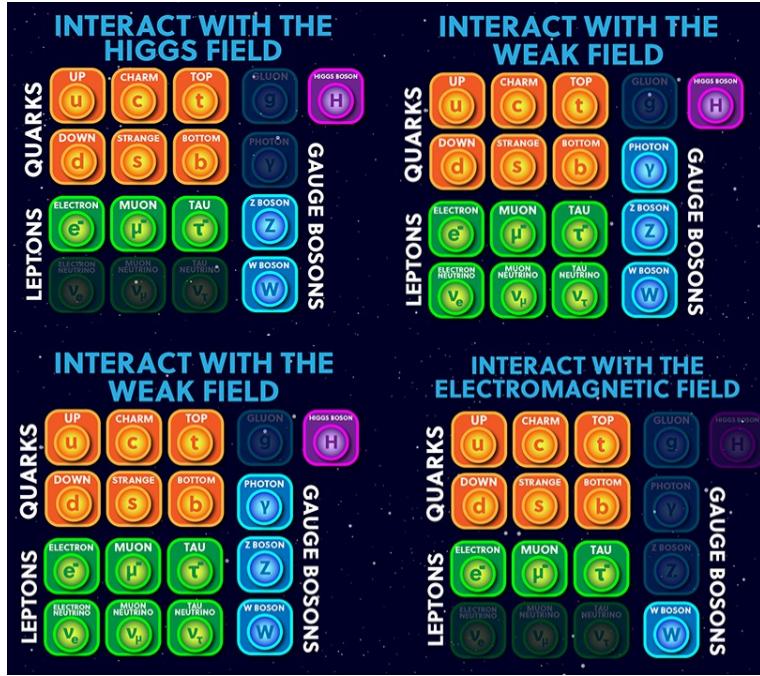


Figure 2.2: The highlighted particles are the particles affected by the respective field [2]

## 2.2 Cosmic Muons

The particle from the standard model that is the subject of this experiment is the muon, specifically cosmic muons, which are a by-product of cosmic rays. Cosmic rays are extremely energetic particles that constantly bombard the earth's atmosphere at close to light speeds. (Around 87% are protons, 12% helium nuclei and some heavier nuclei like iron), and around 1% are electrons [4]. The origins of these cosmic particles are still a mystery, but the most prominent theory proposes that cosmic rays come from supernovas, highly energetic events when a star explodes. The shock wave produced by the explosion can move at speeds on the order of  $10^7 \frac{m}{s}$  [19]. While this is fast, it is not enough to account for the particles moving at near light speeds; to explain the particles' speed, this is where Fermi acceleration comes into play. Fermi acceleration is a process in which supernova shock waves can compress and strengthen magnetic fields. This process can accelerate charged particles to enormous energies to the point that they escape the shock wave and travel through the galaxy [19]. They do not travel in a straight line because the magnetic fields in the galaxy alter their path. As a result, scientists have been unable to find the origins of these cosmic particles. Once these energetic particles reach earth, they collide with the molecules in the atmosphere (mostly proton-nitrogen collisions). This collision has several decay modes creating many more particles, which then decay again, resulting in a cascade, known as cosmic showers. Muons are produced at an altitude of 15 km on average. Below I present a diagram of the decay modes that lead to muon production that have a 5% branching fraction or greater.

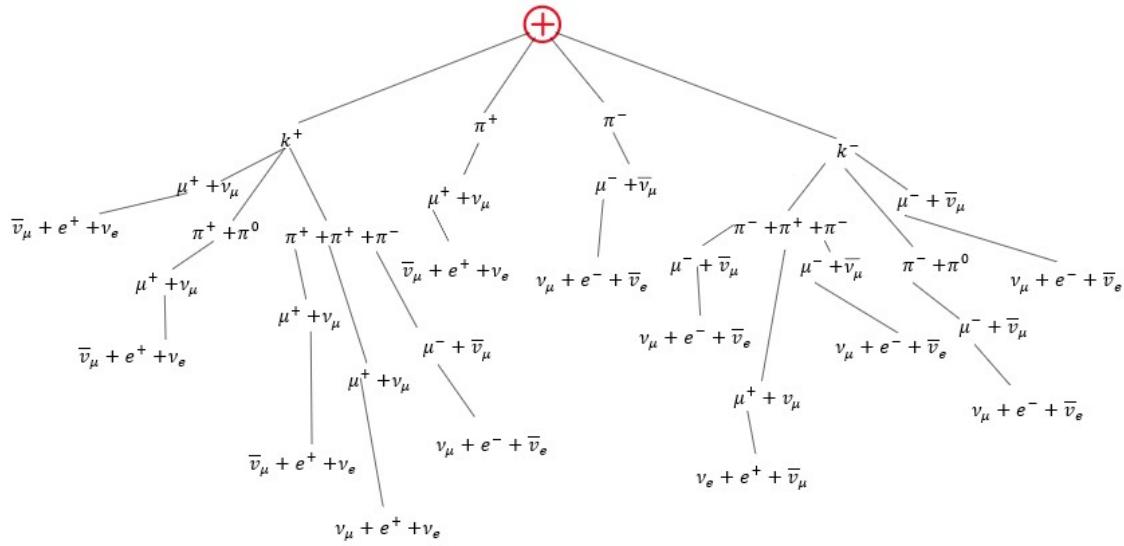


Figure 2.3: A diagram created by myself. Of some of the decay modes that lead to muon production. Note that there are many other decay modes not shown because there do not produce muons.

This diagram shows several mechanisms that can produce a muon, but muons only have one significant decay process, where a muon decays to an electron, muon neutrino, and anti-electron neutrino, the branching fraction for this is approximately 100% [5]. This decay is a weak interaction and mediated by a negative W boson. The same process happens with the positive muon (or anti muon) but with the anti-particle counterparts when a positive muon will decay into a positron, anti muon neutrino, and an electron neutrino, mediated by a positive W boson [5]. Both decays can be seen below.

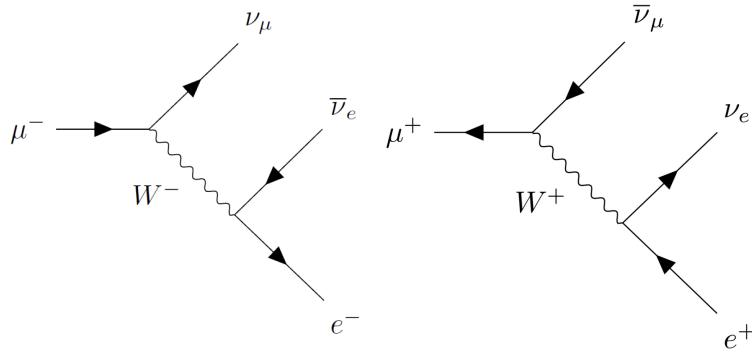


Figure 2.4: Muon decay Feynman diagrams [3]

## 2.3 Special Relativity

The reason why we are so interested in muons is that they are one of the charged particles that decay and actually reach ground level. But the fact that they reach the ground is not trivial. If one were to naively try to find the distance travelled by a muon using  $d = c\tau$  where  $d$  is distance,  $c$  is the speed of light, and  $\tau$  is the average life (approximately  $2.2\mu s$ ) one would not expect muons to be detected at sea level because the resulting distance is only around 700m and they are produced at approximately 15 km above sea level so how are they observed at sea level? It is because muons move close to the speed of light and therefore we need to use relativistic kinematics in our calculations so the 700m is how far the muon can travel in its own reference frame. According to the theory of special relativity, one needs to take time dilation and length contraction into account, which means that the time and distance travelled by the muon in the reference frame of the muon is not the same as the time and space travelled by the muon in the reference frame of an observer on earth. For example, look at a cosmic muon that has a typical energy of 4 GeV, we can easily find the gamma factor or relativistic factor defined as

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{1}{\sqrt{1 - \beta^2}} \quad (2.1)$$

where  $v$  is the speed of the muon,  $c$  is the speed of light (299,792,458 m/s [5]), and  $\beta = v/c$ .

We can easily find  $\gamma$  starting with.

$$E/E_0 = \gamma \quad (2.2)$$

where  $E$  is the energy and  $E_0$  is the rest energy (105.7 MeV or 0.1057 GeV)

The  $\gamma$  can now be calculated as

$$4/0.1057 = 37.8 = \gamma \quad (2.3)$$

The equation for time dilation is

$$t'\gamma = t \quad (2.4)$$

where  $t'$  is time in the muons reference frame [20]. Using the mean muon life as time in the muon's reference frame we can calculate  $t$

$$(2.2\mu s)37.8 = 83.25\mu s = t \quad (2.5)$$

and if we plug that into the classical distance equation

$$d = c \times 83.25\mu s = 24980m \quad (2.6)$$

which is clearly more than 15 km so they can reach a grown level. Now when considering the muons reference frame length contraction must be accounted for, the equation is given by [20]

$$L = \frac{L_0}{\gamma} \quad (2.7)$$

We can now check how much length is contracted in the muons reference frame by using 15,000 as  $L_0$

$$L = \frac{15000}{37.8} = 396m \quad (2.8)$$

This number is less than the 700m calculated for how far muons could travel in their own reference frame while not considering special relativity. This means that the muons should reach sea level no matter what reference frame you look at. This is qualitative proof of special relativity.

## CHAPTER 3

### Finding Energy and Flux difference

Muons do not travel at a constant velocity; they slow down as they move through the atmosphere. This section determines how much energy they lose. We start with the equations from [4].

“Like all charged particles, a muon loses energy through coulombic interactions with the matter it traverses. The average energy loss rate in the matter for singly charged particles travelling close to the speed of light is approximately  $2\text{MeV/g/cm}^2$ , where we measure the thickness  $s$  of the matter in units of  $\text{g/cm}^2$  ”

$$s = \rho x \quad (3.1)$$

where  $\rho$  is the mass density of the material through which the particle is passing, measured in  $\text{kg/m}^3$ , and the  $x$  is the particle’s path length, measured in m. [4]

In the following equations, Muon height is  $H$ .  $\beta$  and  $\gamma$  have their usual meanings introduced in equation 2.1. The equations from special relativity tell us

$$t' = \frac{t}{\gamma} \quad (3.2)$$

where  $t'$  is time in the muons reference frame and  $t$  is the earth’s reference frame Using the basic kinematic equation of  $t = d/v$ , we find

$$t = \frac{H}{\beta c} \quad (3.3)$$

subbing that into the equation 3.2 gives

$$t' = \frac{H}{\gamma \beta c} \quad (3.4)$$

However because the muon slows down the further it moves through the atmosphere  $\beta$  and  $\gamma$  are not constants. To deal with this we will use equation 3.5 found in [4]

$$t' = \int_H^0 \frac{dh}{c\beta(h)\gamma(h)} \quad (3.5)$$

where we say the sea level is 0 height and H is the altitude in meters.

[4] also gives us equation 3.6 that is the energy lost for a charged particle.

$$\frac{dE}{ds} = C_0 \quad (3.6)$$

we can use equation 3.6 to get energy as a function of height. We need this so we can have  $\gamma$  and  $\beta$  as a function of height.

where  $C_0$  is  $0.2MeV/Kg/m^2$ . Now use equation 3.1 in equation 3.6.

$$dE = \rho C_0 dh \quad (3.7)$$

Here I differ from [4] because they approximate the density of air to be constant with height. Instead, acknowledge the change in density, this is an important distinction because air density at sea level and 15km above sea level can differ by 3 orders of magnitudes. To do this I use the equation below from NASA and converted it to metric units.

$$\rho(h) = \left( \frac{-\alpha h + t_0}{t_0} \right)^{n-1} \rho_0 \quad (3.8)$$

Where  $\alpha = 0.0065$ ,  $n=5.2561$  ,  $\rho_0 = 1.225$  and  $t_0 = 288.16$  [21]. now energy can be presented as a function of height.

$$E(h) = \int_H^0 \left( C_0 \left( \frac{-\alpha h + t_0}{t_0} \right)^{n-1} \rho_0 \right) dh + E_H \quad (3.9)$$

where  $E_H$  is energy at height H. Solving that integral with maple results in

$$E(h) = -\frac{Crt \cdot \left( \left( \frac{t+Ha}{t} \right)^n - 1 \right)}{\alpha n} + E_H \quad (3.10)$$

now that we have  $E(h)$  we can easily determine  $\gamma(h)$  and  $\beta(h)$  to find  $\gamma$ , divide by the rest energy.

$$\gamma(h) = \frac{E(h)}{E_0} = \frac{1}{\sqrt{1 - (\beta(h))^2}} \quad (3.11)$$

Since

$$\gamma = \frac{1}{\sqrt{1 - \beta^2}} \quad (3.12)$$

Rearrange this to get  $\beta(h)$  and we get

$$\beta(h) = \sqrt{-\left(\frac{1}{\gamma(h)}\right)^2 + 1} = \sqrt{-\left(\frac{E_0}{E(h)}\right)^2 + 1} \quad (3.13)$$

now time can be found as.

$$t' = \int_H^0 \frac{dh}{c\beta(h)\gamma(h)} \quad (3.14)$$

or

$$t' = \int_H^0 \left( c \sqrt{\left( -\left( \frac{E_0}{\frac{C_0\rho_0 t_0 \cdot \left( \left( \frac{t_0+h\alpha}{t_0} \right)^n - 1 \right)} + E_H}{n\alpha} + E_H \right) + 1 \right)^2} \left( \frac{C_0\rho_0 t_0 \cdot \left( \left( \frac{t_0+h\alpha}{t_0} \right)^n - 1 \right)}{n\alpha} + E_H \right) \right)^{-1} dh \quad (3.15)$$

I wrote a numeric integrator using C++ and root to evaluate this integral reported in appendix A. Once the time in the muons rest frame is computed the basic decay equation (equation 1) can provide the muon flux as a function of height if we have  $N_0$  (flux at height H). This flux can be determined by rearranging equation 1 and using total time, given that the flux at  $h = 0$  is experimentally measured as 1 per minute per  $cm^2$

$$N_0 = N_g e^{-t_g/\tau} \quad (3.16)$$

where  $N_g$  is the flux on the ground (sea level) and  $t_g$  is the time it takes for the muon to get to the ground. With  $N_0$  we can calculate flux as a function of height

$$N(h) = N_0 e^{-t'(h)/\tau} \quad (3.17)$$

$$N(h) = N_0 e^{-\int_H^0 \left( c \sqrt{\left( -\left( \frac{E_0}{\frac{C_0\rho_0 t_0 \cdot \left( \left( \frac{t_0+h\alpha}{t_0} \right)^n - 1 \right)} + E_H}{n\alpha} + E_H \right) + 1 \right)^2} \left( \frac{C_0\rho_0 t_0 \cdot \left( \left( \frac{t_0+h\alpha}{t_0} \right)^n - 1 \right)}{n\alpha} + E_H \right) \right)^{-1} dh} \tau} \quad (3.18)$$

The resulting relations of time, height, energy, and events are graphed on the next page.

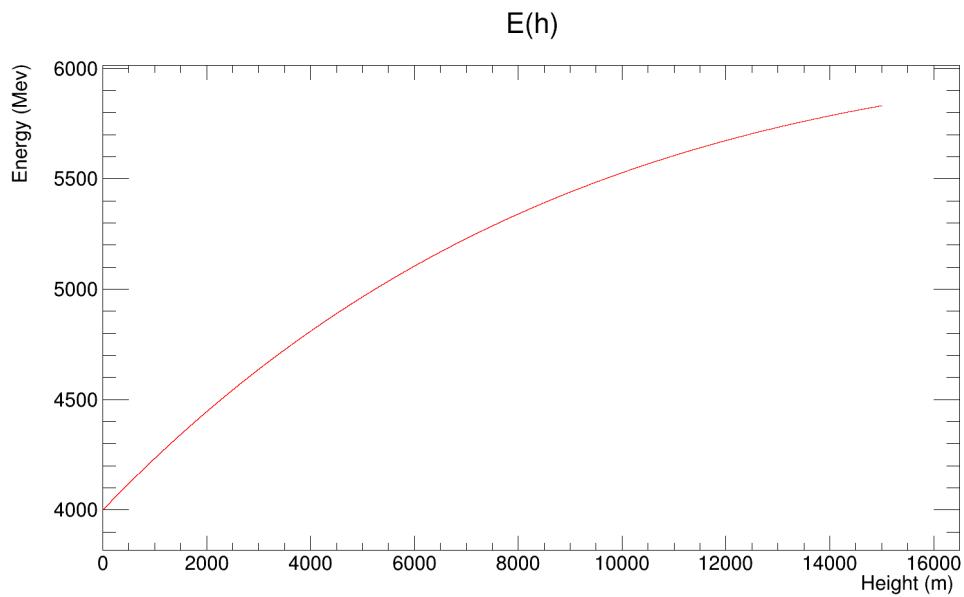


Figure 3.1: Average energy as a function of height. This graph assumes the point (0,4000) meaning it assumes that muons are 4GeV are at sea level and it implies that muons start with an average energy of 5.8GeV.

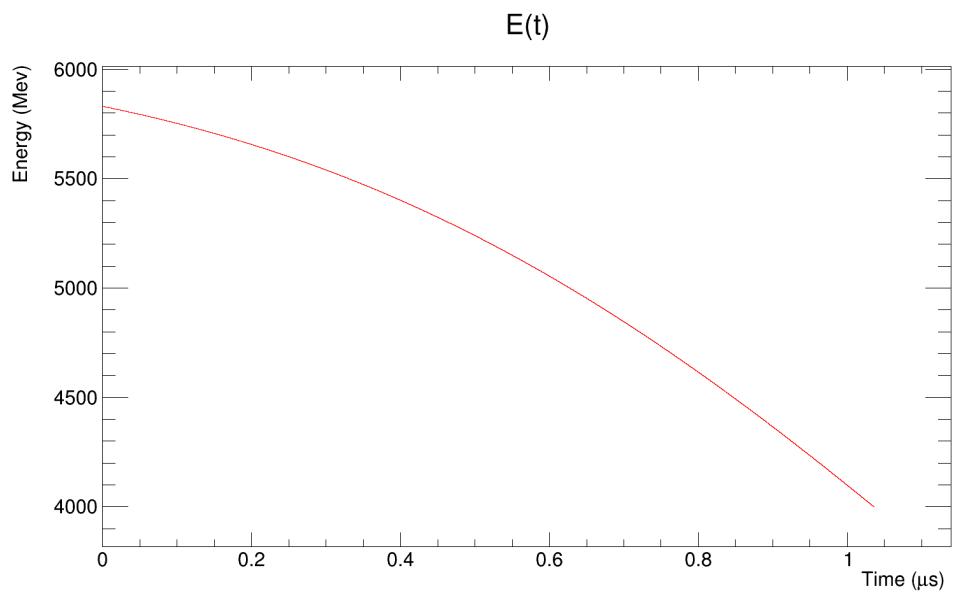


Figure 3.2: Average energy as a function of time. This graph also assumes that at sea level muons have an average energy of 4GeV and shows similar results as figure 3.1.

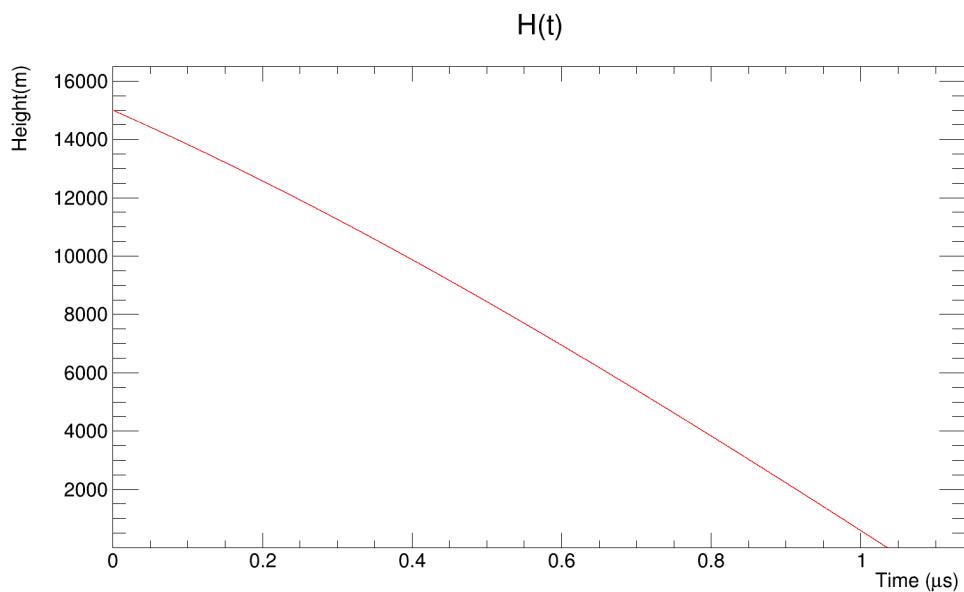


Figure 3.3: Average height as a function of time. This graph assumes muons are produced at 15km above sea level.

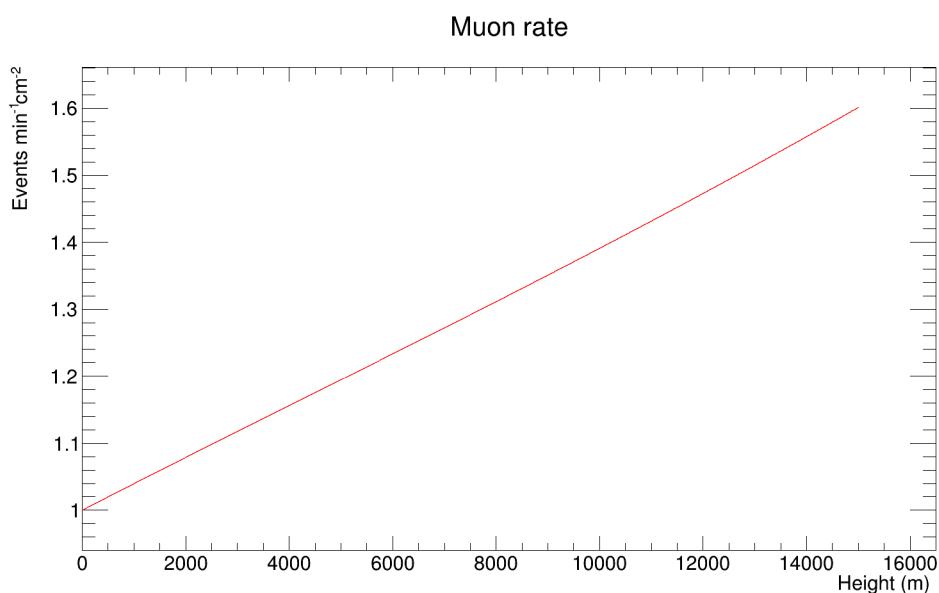


Figure 3.4: Average number of muons per  $cm^2$  as a function of height. You can see that it is fairly linear and this is because the muons are only travelling for 1  $\mu s$  so they don't lose that much energy (shown in figure 3.2).

## CHAPTER 4

# Materials and equipment

### 4.1 Muon Detector

The main instrument used for this experiment was Teachspin’s “Muon Physics” scintillator detector [4]. This section will mostly discuss the components and physics of the detector along with the accompanying equipment. Most of the information for this section will be taken from the manual that came with the detector [4] along with more detail added.



Figure 4.1: The muon detector is on the right, and the electronics box on the left.

#### 4.1.1 Detector Physics

The detector is a plastic scintillator made of organic material made by mixing together one or more fluors with a solid plastic solvent with an aromatic ring structure. Charged particles such as muons and electrons lose kinetic energy through ionization[4][22]. Part of the energy lost from the charged particle gets absorbed by a fluor molecule which promotes the electrons to an excited state. In this excited state the electron emits a photon in the spectrum from blue light to ultraviolet, with a typical photon yield of 1 photon per 100eV. These photons will eventually hit the photo multiplier tube (PMT). The first part of the PMT is a photocathode; a photocathode is a piece of metal that when hit by a photon will emit an electron due to the photoelectric effect[23] [24]. A simple way of viewing the photoelectric effect is that a photon will hit an electron with enough energy to knock the electron out of its orbital. The photocathode is followed by

an electron multiplier made up of dynodes. The electron is accelerated towards the first dynode by an electric field upon impact it causes the emission of several more electrons through secondary emission, These secondary electrons are then accelerated towards the next dynode to repeat the process starting an electron cascade process. Now that there is a large number of electrons the signal is strong enough for the electronics box to pick up and process as presented in the next section in more detail. The muons that decay in the detector have a typical energy of 160 MeV [4] and emit one to two photons; they slow down in the detector and decay. The two neutrinos go undetected by the detector but because the electron is charged and has large energy compared to its rest mass, the electron will also emit photons. The time difference between the photon bursts is what the electronics box processes to find the muon lifetime.

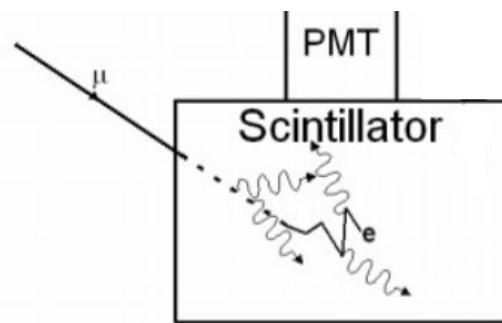


Figure 4.2: Image from (Coan & Ye, 2016) of a muon decaying in the scintillator

#### 4.1.2 Detector Electronics

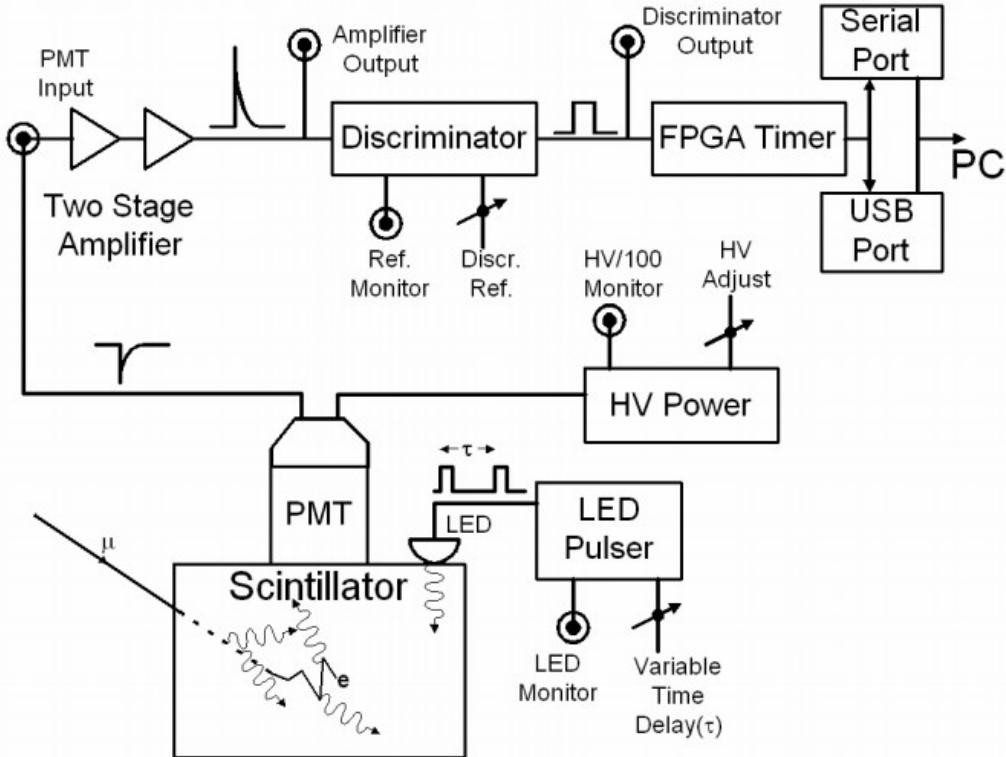


Figure 4.3: Diagram of the electronics from [4]

There are several electronic components in the muon detector that can be seen in the figure above. The signal starts in the scintillator, where a photon is produced by a muon entering the scintillator, as previously discussed. However, there can be another source of light besides a particle: an LED Pulser. The Pulser is there to make sure that the electronics are working properly. The user can turn it on and off, and it is left off during an actual run. The LED produces pairs of light pulses at 100Hz, and the time between the pulses is adjustable. The photons from either the Pulser or from a muon event are detected by the PMT in the process discussed in the previous section. The signal from the PMT is sent to a two-stage amplifier that turns the signal from the PMT into a larger signal which is then sent to a voltage comparator labelled “discriminator” in the diagram. The voltage comparator produces a transistor-transistor logic pulse or TTL pulse. A TTL pulse is a signal defined as a 0 or 1, signifying yes or no; if the incoming signal is above an adjustable threshold the TTL pulse is 1 and triggers the FPGA (field programmable gate array) timer to start counting. If another signal is above the threshold and within  $20\mu s$  of the first it sends the time between the two pulses to the computer. This means there was a muon decay. If there is no other signal within  $20\mu s$  then the timer resets, meaning there was no decay.

#### 4.1.3 Detector Output

It is important to understand how the raw data looks to move on to the data analysis. The raw data is saved on my computer as a data file with two columns; (the left column represents an event or events and the right column is the time that the events happened.) If the number in the left column is greater or equal to 40000, it means that there was no decay in that second; if the number is less than 40000 it means there was a decay and the value indicates how many nanoseconds the decay took. The right column is simply the Unix time that the event happened, the number of seconds since the Unix Epoch on January 1st, 1970 at UTC . Below is an example of what the raw data looks like

```
40009 1621360439
40003 1621360440
40008 1621360441
40002 1621360442
40004 1621360443
40003 1621360444
40007 1621360445
40005 1621360446
40005 1621360447
40004 1621360448
40003 1621360449
1720 1621360450
40006 1621360450
40009 1621360451
```

Figure 4.4: Sample raw data represented in an output text file.

The first line represents nine muons passing there the detector on May 18<sup>th</sup> 2021 17:53:59 GMT+0000 and the 12<sup>th</sup> line represents a muon decay that took 1720 ns occurring on May 18<sup>th</sup> 2021 17:54:10 GMT+0000. a sample of 1000 seconds of detector data is represented in Appendix B.

## 4.2 Other Equipment

While the detector is the main piece of equipment, there are also other pieces to the experiment. In order to take the muon detector outside, a pure sine wave AC power source was used, as well as a computer to receive the data from the detector as shown in figure 4.5.

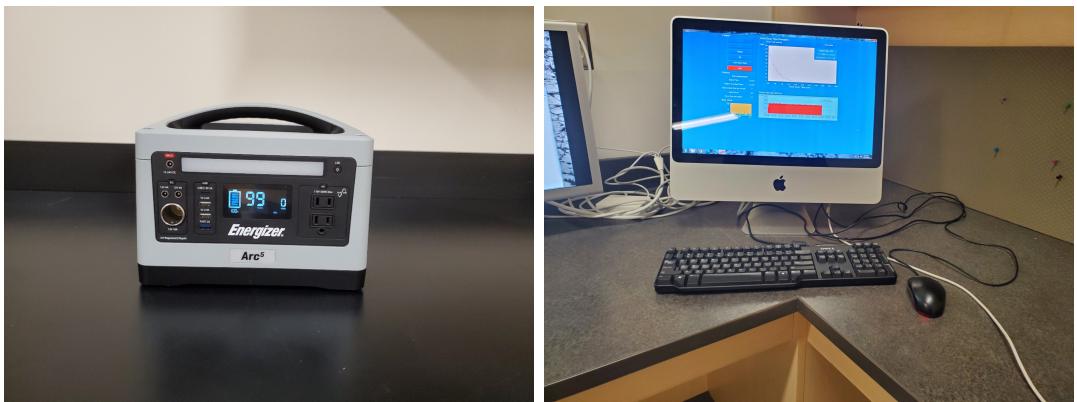


Figure 4.5: The left image is the Pure sine AC power source. The right image to right is the computer used for the experiment.

## CHAPTER 5

# Muon flux in the Antigonish area

### 5.1 Theories and Math

The theory behind the muon flux calculations is very straightforward. The detector is simply set up and let run; then the total number of muons counted is divided by the total time, to get the muon rate. Finding the mean muon lifetime is not as straightforward. Using the decay equation again

$$N = N_0 e^{-t\lambda} \quad (5.1)$$

$\lambda$  can be defined as the reciprocal of the average life. Taking the derivative, we find that

$$-\frac{dN}{dt} = N_0 \lambda e^{-t\lambda} \quad (5.2)$$

this is what we can plot with a histogram. On the X-axis plot how long it took a muon to decay and on the Y-axis put how many decayed in that time interval ( $dt$ ) where  $dt$  is just the bin size of the histogram. A fit of this histogram would then provide us with the decay constant and from there the average life. Below in figure 5.1 you can see the histogram produced from my code.

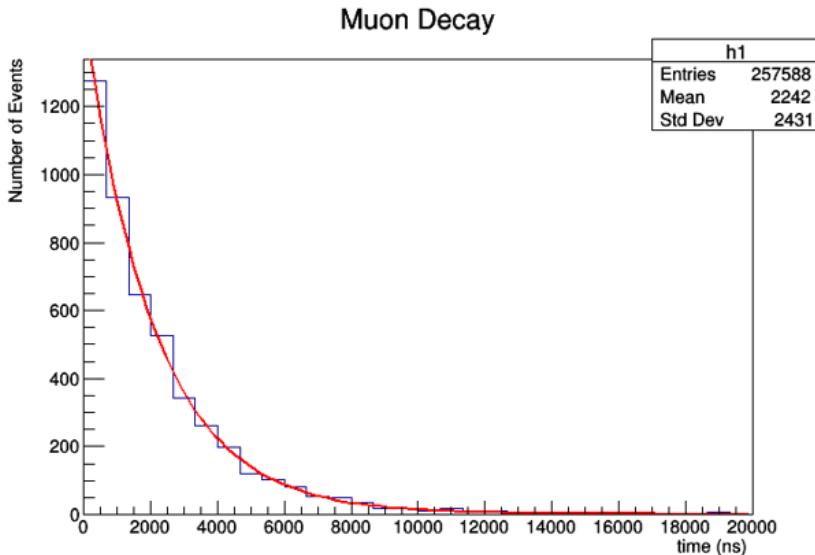


Figure 5.1: Histogram with fit produced in methods discussed later.

## 5.2 Data Collection

To begin our experiment, we needed to ensure that the detector was accurately calibrated to ensure reliable data collection. Initially, we attempted to calibrate the detector using the techniques described in the manual, however, these methods proved ineffective and will be further elaborated on in the error analysis section.

Instead, we took a different approach and calibrated the detector to achieve well-established quantities such as 1 muon per second per  $cm^2$ . Once we had successfully calibrated the detector, we connected it to the computer and downloaded the necessary muon software. I selected the correct port and initiated data collection by pressing the start button.

The software provided several options for analysis. However, for our experiment, we only used the software for data collection purposes, while the analysis was done using a separate platform.

To ensure the validity of our results, we ran the experiment multiple times, letting the software run for a week each time. This ensured that our results could be replicated consistently and that any anomalies or discrepancies could be identified and addressed. The table of data can be seen below.

### 5.2.1 Data Table

Table 5.1: Total data taken from muon detector.

Unix Timestamp	Date taken	File size	Run time (seconds)	Run time
1678844435	March 15, 2023	1724KB	103440	1 days, 04:44:00
1621360450	May 18, 2021	12712 KB	762720	8 days, 19:52:00
1651586462	May 03, 2022	10197KB	611820	7 days, 01:57:00
1667671586	November 05, 2022	18 KB	1080	0 days, 00:18:00
1667670743	November 05, 2022	25 KB	1500	0 days, 00:25:00
1667670303	November 05, 2022	32 KB	1920	0 days, 00:32:00
1667670455	November 05, 2022	39 KB	2340	0 days, 00:39:00
1667670303	November 05, 2022	13 KB	780	0 days, 00:13:00
1667664407	November 05, 2022	50 KB	3000	0 days, 00:50:00
1667660685	November 05, 2022	61 KB	3660	0 days, 01:01:00
1667659403	November 05, 2022	17 KB	1020	0 days, 00:17:00
1667657683	November 05, 2022	44 KB	2640	0 days, 00:44:00
1667656541	November 05, 2022	19 KB	1140	0 days, 00:19:00
1667582720	November 04, 2022	197 KB	11820	0 days, 03:17:00
1663436783	September 17, 2022	7,382 KB	442920	5 days, 03:02:00
1659973347	August 08, 2022	15773KB	946380	10 days, 22:53:00
1658852730	July 26, 2022	10,130 KB	607800	7 days, 00:50:00
1657902464	July 15, 2022	5,837 KB	350220	4 days, 01:17:00
1651586462	May 03, 2022	10,197 KB	611820	7 days, 01:57:00
1655831365	June 21, 2022	47 KB	2820	0 days, 00:47:00
1655815234	June 21, 2022	264 KB	15840	0 days, 04:24:00
1655585418	June 18, 2022	2362 KB	141720	1 days, 15:22:00
1655476212	June 17, 2022	1,829 KB	109740	1 days, 06:29:00
1655395017	June 16, 2022	134 KB	8040	0 days, 02:14:00
1655231710	June 14, 2022	1,202 KB	72120	0 days, 20:02:00
1655142553	June 13, 2022	1,313 KB	78780	0 days, 21:53:00
1635521232	October 29, 2021	4,431 KB	265860	3 days, 01:51:00
1635429315	October 28, 2021	1,496 KB	89760	1 days, 00:56:00
1634261498	October 15, 2021	10,866 KB	651960	7 days, 13:06:00
1635427207	October 28, 2021	319 KB	19140	0 days, 05:19:00
1633995050	October 11, 2021	3479 KB	208740	2 days, 09:59:00
1632260374	September 21, 2021	8052 KB	483120	5 days, 14:12:00
1632140553	September 20, 2021	1,963 KB	117780	1 days, 08:43:00
1629310647	August 18, 2021	2710 KB	162600	1 days, 21:10:00

Unix Timestamp	Date taken	File size	Run time (seconds)	Run time
1629141987	August 16, 2021	2803 KB	168180	1 days, 22:43:00
1628873786	August 13, 2021	4444 KB	266640	3 days, 02:04:00
1628083101	August 04, 2021	192 KB	11520	0 days, 03:12:00
1628002723	August 03, 2021	187 KB	11220	0 days, 03:07:00
1627566268	July 29, 2021	7,326 KB	439560	5 days, 02:06:00
1627480852	July 28, 2021	1,358 KB	81480	0 days, 22:38:00
1627403418	July 27, 2021	1,169 KB	70140	0 days, 19:29:00
1626885871	July 21, 2021	8642 KB	518520	6 days, 00:02:00
1626185382	July 13, 2021	1,464 KB	87840	1 days, 00:24:00
1625668858	July 07, 2021	8555 KB	513300	5 days, 22:35:00
1625588743	July 06, 2021	1,046 KB	62760	0 days, 17:26:00
1625497068	July 05, 2021	1,512 KB	90720	1 days, 01:12:00
1624998035	June 29, 2021	4275 KB	256500	2 days, 23:15:00
1624033121	June 18, 2021	10404 KB	624240	7 days, 05:24:00
1622486774	May 31, 2021	11,250 KB	675000	7 days, 19:30:00
1622214521	May 28, 2021	3,770 KB	226200	2 days, 14:50:00
1622141911	May 27, 2021	1,202 KB	72120	0 days, 20:02:00
1621360437	May 18, 2021	4,271 KB	256260	2 days, 23:11:00
1627403418	July 27, 2021	1,169KB	70140	0 days, 19:29:00
1651586462	May 03, 2022	10197Kb	611820	7 days, 01:57:00
1652209377	May 10, 2022	2751 KB	165060	1 days, 21:51:00
1626185382	July 13, 2021	1,464 KB	87840	1 days, 00:24:00
1622044462	May 26, 2021	10251 KB	615060	7 days, 02:51:00
1654011664	May 31, 2022	1,404 KB	84240	0 days, 23:24:00
1653922894	May 30, 2022	1,472 KB	88320	1 days, 00:32:00
1653062992	May 20, 2022	14,359 KB	861540	9 days, 23:19:00
1652797826	May 17, 2022	4,434 KB	266040	3 days, 01:54:00
1651586462	May 03, 2022	1019KB	61140	0 days, 16:59:00
1654786086	June 09, 2022	5,837 KB	350220	4 days, 01:17:00
1654540258	June 06, 2022	4,077 KB	244620	2 days, 19:57:00
1652209377	May 10, 2022	2751 KB	165060	1 days, 21:51:00
1652374364	May 12, 2022	944 KB	56640	0 days, 15:44:00
1651843970	May 06, 2022	67 KB	4020	0 days, 01:07:00
1651842637	May 06, 2022	70 KB	4200	0 days, 01:10:00

Unix Timestamp	Date taken	File size	Run time (seconds)	Run time
1635521232	October 29, 2021	3616 KB	216960	2 days, 12:16:00
1635429315	October 28, 2021	1,496 KB	89760	1 days, 00:56:00
1635521232	October 29, 2021	4,431 KB	265860	3 days, 01:51:00
1629904936	August 25, 2021	363 KB	21780	0 days, 06:03:00
1628688567	August 11, 2021	308Kb	18480	0 days, 05:08:00
1628608746	August 10, 2021	366 KB	21960	0 days, 06:06:00
1627566268	July 29, 2021	7,513 KB	450780	5 days, 05:13:00
1627403418	July 27, 2021	512 KB	30720	0 days, 08:32:00
1627403418	July 27, 2021	741 KB	44460	0 days, 12:21:00
1627403418	July 27, 2021	1,481 KB	88860	1 days, 00:41:00
1626194185	July 13, 2021	6912 KB	414720	4 days, 19:12:00
1622044462	May 26, 2021	16251 KB	975060	11 days, 06:51:00
1621360437	May 18, 2021	4,271 KB	256260	2 days, 23:11:00
1624033210	June 18, 2021	170 KB	10200	0 days, 02:50:00

## 5.3 Data Analysis

### 5.3.1 Choosing an analysis software

The Muon detector from Teach Spin came with CD analysis tools, but the built-in rate calculator provided values that did not match those on the monitor. Upon examination of the raw data file, it was discovered that over 71 hours had elapsed between the first and last data points, while the monitor indicated that the detector had only run for less than 70 hours. Muon decay was calculated based on the time between the first and last event, yielding the same result as the rate calculator. This discrepancy suggested that the timing was off. Even after accounting for the timer discrepancy, the results are close to the right answer but not exact. This can be seen by doing a small sample by hand. All of this shows that a custom analysis software was necessary.

To ensure that more data, including the average lifetime, could be accessed, a decision was made to write custom code for the data analysis. After considering various analysis tools, CERN's ROOT [25] was chosen due to its high-power analysis capabilities and built-in C++ interpreter. One of the main reasons for selecting ROOT was its above-average fitting software, which was critical not only for fitting the lifetime but also for reducing background noise. In the following section, a custom fit will be described, which effectively reduces background noise. While the TeachSpin software provided a rough estimate for background noise, having it built into the fit in ROOT proved to be an advantage. I report the ROOT code in appendix C.

### 5.3.2 Analysis Process

The code opens the data file and assigns numbers in the right column to Y and those in the left column to X, then if X is greater than 40000, it checks to see how much greater and increases the muon count. If X is less than 40000, it adds the X value to a histogram and increments the decay count. It also finds the time passed by adding the current Y value and subtracting the previous Y value. This is better than incrementing by one for two reasons: (1) if there is both a decay and muons passing by in the same second, it will add 0, and (2) the time jump may be greater than 1. The program also will not increase the time if the time jump is greater than 100, so that the results of two runs can be merged without interpreting the data if the muon detector was running the whole time between the two runs with no events. The program then calculates the muon rate and decay rate and fits the histogram as an exponential plus a constant. The constant

is to filter out background noise. For example, if two muons both entered the detector within  $20 \mu s$  of each other, that would (incorrectly) count as a decay; these incorrectly counted events are assumed to be uniform, so a constant is added. Then it calculates the Fermi coupling constant using [4]

$$\tau = \frac{192\pi^3 \hbar^7}{G_f^2 m^5 c^4} \quad (5.3)$$

which can be rearranged to find  $G_f$

$$G_f = \sqrt{\frac{192\pi^3 \hbar^7}{\tau m^5 c^4}} \quad (5.4)$$

The program then prints out the calculated values shown in figure 5.2.

Pseudo-code is shown below and full code in appendix D.

```

1      open file
2      for (all lines of the document)
3      {
4
5
6      if (x < 40000)
7      {
8          add to histogram
9          |           add 1 to decay count
10     }
11    else
12    {
13        |   add x - 40000 to count
14    }
15    get time passed
16}
17    fit histogram as exponential + constant
18    find lifetime from fit
19    modify the decay rate to get rid of background noise
20    find fermi coupling constant
21    print out findings

```

Figure 5.2: Pseudo-code for Data analysis

## 5.4 Error Analysis

In order to quantify the uncertainty in our measurements and calculations, we performed an error analysis on our data. There are several sources of error in our experiment, including statistical uncertainties, systematic errors, and uncertainties in our equipment and measurements.

### 5.4.1 Statistical Uncertainties

The statistical uncertainty in our measurements can be quantified using standard error propagation techniques. The statistical uncertainty can be calculated using the formula:

$$\delta z = \sqrt{\left(\frac{\partial f(x, y)}{\partial x} \delta x\right)^2 + \left(\frac{\partial f(x, y)}{\partial y} \delta y\right)^2}, \quad (5.5)$$

Where  $z$  is some value that is calculated by a function of  $x$  and  $y$  ( $f(x, y)$ ) and  $\delta z, \delta x$ , and  $\delta y$  are the error in  $z, x$  and  $y$  respectively for example say you have an equation that is  $z = 2x + y$  and  $\delta x = 0.1$  and  $\delta y = 0.15$  to get the  $\delta z$  we can use equation 5.5 and find

$$\delta z = \sqrt{\left(\frac{\partial}{\partial x}(2x + y)0.1\right)^2 + \left(\frac{\partial}{\partial y}(2x + y)0.15\right)^2}, \quad (5.6)$$

$$\delta z = \sqrt{(2 \times 0.1)^2 + (1 \times 0.15)^2}, \quad (5.7)$$

$$\delta z = \sqrt{0.08}, \quad (5.8)$$

form using equation 5.5 we can find the error for all of our values.

$$\delta \tau = \delta \lambda \cdot \tau^2 \quad (5.9)$$

Where  $\tau$  and  $\lambda$  is the decay constant and the error from that is given from the fit from the histogram.

$$\delta G_f = 0.2727498648 \times 10^{-3} \cdot \frac{\delta \tau}{\tau^{1.5}} \quad (5.10)$$

Where  $G_f$  is the fermi coupling content

$$\text{Muon rate Error} = \sqrt{\frac{\sigma}{t}} \quad (5.11)$$

$$\text{Preliminary Decay rate Error} = \sqrt{\frac{\sigma}{t}} \cdot 60 \quad (5.12)$$

Where  $\sigma$  is the standard deviation. Preliminary Decay rate Error is the decay rate before you account for the background Noise.

$$\text{Decay rate Error} = 60 \cdot \sqrt{\left(\frac{\text{Background error}}{\text{time}}\right)^2 + \left(\frac{\text{Preliminary Decay rate Error}}{\text{time}}\right)^2} \quad (5.13)$$

$$N/D = \sqrt{\left(\frac{\delta N}{D}\right)^2 + \left(\frac{N \cdot \delta D}{D^2}\right)^2} \quad (5.14)$$

where N is the number of muons and D is the number of Decays.

#### 5.4.2 Calibration and Systematic Uncertainties

Keep in mind that all of that is just statistical error, to account for all errors we also need to account for Systematic errors. Systematic errors can arise from factors such as calibration errors, environmental factors, or measurement biases. To quantify the systematic uncertainties in our measurements, we performed a series of sensitivity tests in which we varied the experimental conditions and recalculated our results.

In our results, we found that slight calibration differences can result in major differences in results. This shows us how for two separate runs you need to be very careful with your measurements. When implementing the calibration techniques described in the manual we produced un-physical results seen below the life time is  $0.474247 \pm 0.0114609 \mu s$  the fermi coupling constant in natural units is  $2.50491e-05 \pm 3.02676e-07 GeV^{-2}$  the muon rate is  $117.031 \pm 0.0391556$  muons a second  
the decay rate is  $26.5867 \pm 2.34934$  muon decays per minute  
the decay rate subtract background is  $5.06837 \pm 0.200186$  muon decays per minute  
the muon rate is  $39.7319 cm^2$  pre minute  $\pm 0.0132933$   
the ratio of no decays to decays is  $264.112 \pm 1.43878$

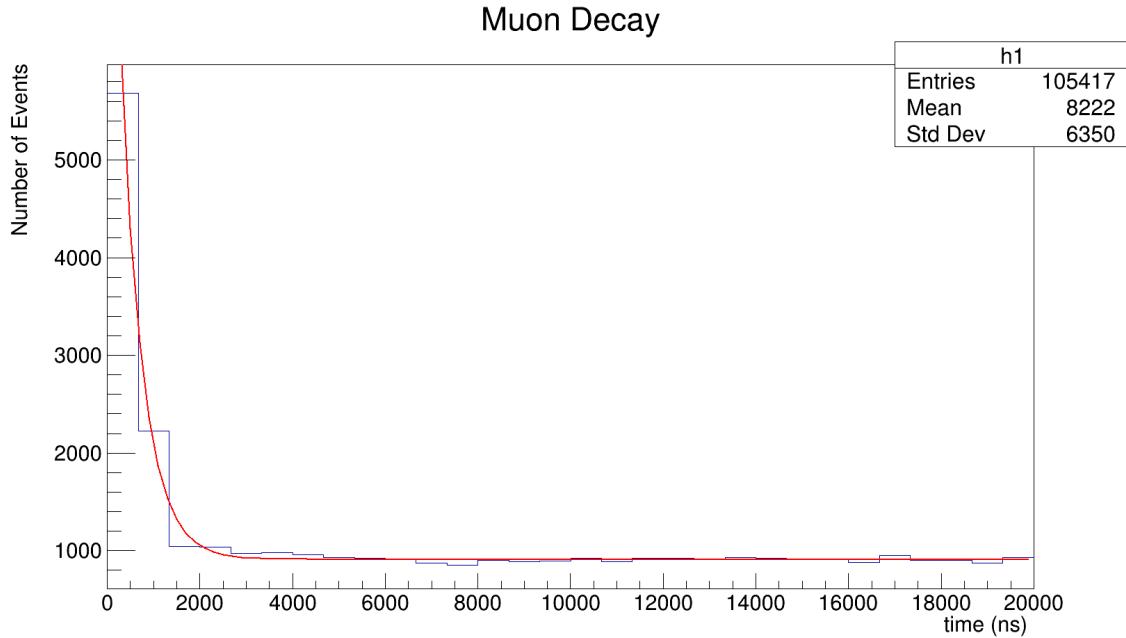


Figure 5.3: Histogram from teachspins calibration method.

as you can see the theoretical values are nowhere near the error margin given from the statistical error with the muon rate having 3973 % error

## 5.5 Findings

	Experimental results	Theoretical
$\lambda$	$0.471 \pm 0.0060 \mu\text{s}^{-1}$	$(0.4551700513 \pm 0.00000046) \mu\text{s}^{-1}$
$\tau$	$(2.12 \pm 0.0270) \mu\text{s}$	$(2.1969811 \pm 0.0000022) \mu\text{s}$
$G_F/(\hbar c)^3$	$(1.1834 \pm 0.0075) \times 10^{-5} \text{ GeV}^{-2}$	$(1.1663788 \times 10^{-5} \pm 510) \text{ GeV}^{-2}$
Muon rate	$(1.082 \pm 0.00078) \text{ cm}^{-2} \text{ min}^{-1}$	$1 \text{ cm}^{-2} \text{ min}^{-1}$
$\mu_+/\mu_-$	$1.21838 \pm 0.858389$	1

Table 5.2: Comparison of Experimental Results and Theoretical values.(If there is no error it means it is an approximate global average). The Theoretical values for  $\tau$ ,  $\lambda$  and  $G_F$  are from [5]. The expected value for  $\mu_+/\mu_-$  is from [4]. The muon rate expected value is from [6]

Here is the output from my code: the muon rate is close the expected value, as the theoretical value is around 1 per square cm per minute, while the lifetime and Fermi coupling constant appear to be off, for good reason: the  $2.2 \mu\text{s}$  lifetime is for muons in a vacuum, not in matter. When muons pass through matter, they have a decay mode where a muon and proton can decay into a neutron and a muon neutrino. This means there is a second interaction mechanism for negative muons. According to [4], the lifetime of a negative muon in the detector is  $2.043 \pm 0.003 \mu\text{s}$ , and the ratio of negative to positive

muons is very close to 1 according to [5]. This implies that the value from the detector should be the average between the two lifetimes.

$$\tau_d = \frac{\tau_- + \tau_+}{2} \quad (5.15)$$

$$\tau_d = \frac{2.043 + 2.19703}{2} = 2.120\mu s \quad (5.16)$$

This value matches the experimental value much better. However, this does mean that the Fermi coupling constant is not accurate because the weak force is not the only interaction mechanism as they have the secondary mechanism previously discussed.

	Experimental results	Expected values
$\lambda$	$0.471 \pm 0.0060 \mu s^{-1}$	$(0.4717 \pm 0.00067) \mu s^{-1}$
$\tau$	$(2.12 \pm 0.0270) \mu s$	$(2.120 \pm 0.003) \mu s$
$G_F/(\hbar c)^3$	$(1.1834 \pm 0.0075) \times 10^{-5} \text{ GeV}^{-2}$	$(1.1663788 \times 10^{-5} \pm 510) \text{ GeV}^{-2}$
Muon rate	$(1.082 \pm 0.00078) cm^{-2} min^{-1}$	$1 cm^{-2} min^{-1}$
$\mu_+/\mu_-$	$1.21 \pm 0.86$	1

Table 5.3: Comparison of experimental results with expected values, taking into account the effect of muons travelling through matter rather than free space. (If there is no error it means it is an approximate global average). The expected value for  $\tau$  is from [4] and also derived earlier.  $\lambda$  is found from  $\tau$ . The expected value for  $G_F$  is from [5]. The expected value for  $\mu_+/\mu_-$  is from [4]. The muon rate expected value is from [6]

## CHAPTER 6

# Finding Muon Energy

### 6.1 Theories and Math

To find the average energy of muons, one can run an experiment to measure the muon rate at two different elevations. Because muons decay, we can find the ratio of muon flux at the two elevations and use the now-found lifetime and the decay equation to calculate the time passed in the muon's reference frame. The  $\gamma$  can be found by comparing the time passed in the muon's reference frame with the time passed in the earth's reference frame.

#### 6.1.1 Math

Starting with the decay formula,

$$N = N_0 e^{\frac{-t'}{\tau}} \quad (6.1)$$

this equation can be rearranged to find the following.

$$\frac{N}{N_0} = e^{\frac{-t'}{\tau}} \quad (6.2)$$

$$\ln(N/N_0) = \frac{-t'}{\tau} \quad (6.3)$$

$$-\tau \ln(N/N_0) = t' \quad (6.4)$$

According to Einstein,

$$t = \gamma t' \quad (6.5)$$

$t$  is the time from our reference frame.

The speed of the muon is the speed of light times  $\beta$ . Time in the earth's reference frame can now be determined.

$$t = \frac{H}{\beta c} \quad (6.6)$$

where  $H$  is the height difference.

The equations can be combined to find  $\gamma$ .

$$\gamma = \frac{t}{t'} = -\frac{H}{\tau \ln(N/N_0) \beta c} \quad (6.7)$$

From  $\gamma$ , the speed and the energy can be found,

$$\gamma = \frac{1}{\sqrt{1 - \beta^2}} \quad (6.8)$$

$$\frac{1}{\sqrt{1 - \beta^2}} = -\frac{H}{\tau \ln(N/N_0) \beta c} \quad (6.9)$$

where  $\beta$  has its usual meaning as  $\frac{v}{c}$ . Rearranging for  $\beta$ :

$$\beta = \frac{H}{\sqrt{c^2 \tau^2 \ln^2(\frac{N}{N_0}) + H^2}} \quad (6.10)$$

So trivially,

$$v = \frac{Hc}{\sqrt{c^2 \tau^2 \ln^2(\frac{N}{N_0}) + H^2}} \quad (6.11)$$

Energy can be found using

$$E = \left( \frac{1}{\sqrt{1 - \beta^2}} \right) E_0 \quad (6.12)$$

## 6.2 Experimental set up

In order to run at different elevations, the detector needed to be operated outside the lab; this meant that a pure sine wave AC power source was needed to power the muon detector outside. The higher elevation used was the tallest mountain in the Antigonish area with a drivable road to the top: Keppoch Mountain, with an elevation of 260m. At the top, the detector was powered by the battery and operated as normal to get the muon flux.

## 6.3 Data Analysis

The data analysis to find the muon flux is essentially the same as the previous section, except it is done once for each file and the data is stored in a list. The equation to compute the lifetime is independent of elevation, so the histogram includes values from both runs. The software then calculates the same values as it did in the lab and performs the calculations described in section 6.1.1. Pseudo-code for my data analysis is shown below

```
open file
for both files
{
    for (all lines of the document)
    {
        if (x < 40000)
        {
            add to histogram
            add 1 to decay count
        }
        else
        {
            add x - 40000 to count
        }
        get time passed
    }
    store muon rate and decay rate
}
fit histogram from both files as exponential + constant
find lifetime from fit
get N / N0
Do the math from previous section
print out findings
```

Figure 6.1: Pseudo-code for Data analysis

## 6.4 Additional Error Analysis

Using the same procedure as before we can find the error for the values in this section. Again keep in mind that this is just statistical error and not systematic error calculations.

$$\delta R = \frac{\sqrt{N^2(\delta N_0)^2 + N_0^2(\delta N)^2}}{N_0^2} \quad (6.13)$$

Here R is the ratio of  $\frac{N}{N_0}$ . Recall that N and  $N_0$  are the muon rates at the two elevations so their errors can be found using the equations in the first error analysis section.

$$\delta\beta = c^2 \sqrt{\frac{(\ln(R))^2 (R^2 (H^2 \delta\tau^2 + \tau^2 \delta H^2) (\ln(R))^2 + H^2 \tau^2 \delta R^2)}{(c^2 \tau^2 (\ln(R))^2 + H^2)^3}} \left| \frac{\tau}{R} \right| \quad (6.14)$$

Where c is the speed of light, d is the distance,  $\tau$  is the life time in seconds, and H is the elevation difference.

$$\delta t = \frac{\sqrt{H^2(\delta\beta)^2 + \beta^2(\delta H)^2}}{\beta^2 c} \quad (6.15)$$

Where t is the time in the earth's reference frame.

$$\delta t' = \sqrt{(\log R)^2 (\delta\tau)^2 + \frac{\tau^2 (\delta R)^2}{R^2}} \quad (6.16)$$

$$\delta\gamma = \frac{\sqrt{t^2(\delta t'^2) + \tau^2(\delta t)^2}}{t'^2} \quad (6.17)$$

Now that we found  $\delta\gamma$  we can easily find  $\delta KE$  and  $\delta E$  where we assume that  $\delta E_0 \approx 0$  because the  $\delta\gamma \gg \delta E_0$ .

$$\delta KE = \delta E = E_0 \times \delta\gamma \quad (6.18)$$

## 6.5 Findings

Parameter	Antigonish	Approximate Average on Earth
$\gamma$	$4.8 \pm 0.37$	38
$\beta$	$0.978 \pm 0.0032$	0.9996
KE	$(403.5 \pm 38.79)\text{Mev}$	3894.3 Mev
E	$(509.2 \pm 38.79)\text{Mev}$	4000 Mev

Table 6.1: The approximate average for Energy is from [6] .  $\beta$ ,  $\gamma$  and KE are found from E.

The  $\beta$  values here deviate from the expected value of 0.99965, because of three main sources of error. (1) Time frame: At the higher elevation, the detector only ran for 6 hours, whereas in the lab it ran for several days. (2) Calibration: I realized that one of the adjustable threshold dials was moved while transporting the detector, and this could have skewed the data. (3) Insufficient elevation difference: Keppoch Mountain might not be a high enough elevation to notice a statistically significant difference. Once error analysis is completed, the statistical significance can be evaluated.

## CHAPTER 7

### Conclusion

I repeat here the results found in this experiment.

	Experimental results	Expected values
$\lambda$	$0.471 \pm 0.0060 \mu\text{s}^{-1}$	$(0.4717 \pm 0.00067) \mu\text{s}^{-1}$
$\tau$	$(2.12 \pm 0.0270) \mu\text{s}$	$(2.120 \pm 0.003) \mu\text{s}$
$G_F/(\hbar c)^3$	$(1.1834 \pm 0.0075) \times 10^{-5} \text{ GeV}^{-2}$	$(1.1663788 \times 10^{-5} \pm 510) \text{ GeV}^{-2}$
Muon rate	$(1.082 \pm 0.00078) \text{ cm}^{-2} \text{ min}^{-1}$	$1 \text{ cm}^{-2} \text{ min}^{-1}$
$\mu_+/\mu_-$	$1.21 \pm 0.86$	1
$\gamma$	$4.8 \pm 0.37$	38
$\beta$	$0.978 \pm 0.0032$	0.9996
KE	$(403.5 \pm 38.79) \text{ Mev}$	3894.3 Mev
E	$(509.2 \pm 38.79) \text{ Mev}$	4000 Mev

Table 7.1: Comparison of experimental results with expected values from both parts of this paper. (If there is no error it means it is an approximate global average). The expected value for  $\tau$  is from [4] and also derived earlier.  $\lambda$  is found from  $\tau$ . The expected value for  $G_F$  is from [5]. The expected value for  $\mu_+/\mu_-$  is from [4]. The expected values for Energy and muon rate are from [6],  $\beta$ ,  $\gamma$  and KE are found from E.

Although the data obtained in the previous sections did not exactly meet my initial expectations, I remain optimistic about the outcomes of this study. Despite the presence of various sources of error, the beta factor calculated in our analysis still confirms the necessity of special relativity for the behaviour of muons. This research has also led to several unintended yet valuable findings that can contribute to improvements in experimental physics.

Firstly, our study indirectly resulted in the enhancement and refinement of certain design elements related to the software used by Teach spin, a leading company supplying physics instruments worldwide. As we investigated our experimental setup, we identified areas for improvement and subsequently proposed potential solutions.

Moreover, we discovered a more efficient and accurate method for estimating background noise, which can be beneficial in future experiments where reducing noise is crucial for obtaining reliable data. We also identified an issue with the built-in timer of Teach spin's instruments and provided feedback to the company, thereby enabling them to address the problem and improve the accuracy of their products.

In addition, our research revealed that the calibration techniques described by Teach spin

produced results that were not consistent with physical principles.

Overall, while the data we collected may not have been ideal. Not only did we confirm the necessity of special relativity for muons, but we also uncovered opportunities to enhance the technology and methodology used in related experiments. These improvements will likely have a positive impact on future research, leading to more accurate and reliable results in the study of Cosmic Muons using Teach Spins detector.

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## Appendix A

---

```
#include "Riostream.h"
#include "TH2.h"
#include "TF1.h"
#include "TStyle.h"
#include "TBranch.h"
#include "TCanvas.h"
#include "TPaveStats.h"
#include "TLine.h"
#include "TMath.h"
#include <iostream>
#include <time.h>
#include <stdio.h>
#include <stdlib.h>
using namespace std;
// defining math operations

Double_t Log(Double_t x)
{
    return log(x);
}
Double_t Log10(Double_t x)
{
    return log10(x);
}
Double_t SinH(Double_t x)
{
    return sinh(x);
}
Double_t CosH(Double_t x)
{
    return cosh(x);
}
Double_t Cos(Double_t x)
{
    return cos(x);
}
Double_t TanH(Double_t x)
{
    return tanh(x);
```

```

}

Double_t Exp(Double_t x)
{
    return exp(x);
}

Double_t Power(Double_t x, Double_t y)
{
    return pow(x, y);
}

Double_t Sech(Double_t x)
{
    return (1/Cosh(x));
}

///////////////////////////////
/////////////////////////////
double Eng(double h ,double A) //engery as a function of hight
{
    double C = 200000;
    double alpha = 0.0065;
    double t0 = 288.16;
    double rho0 = 1.225;
    double n = 5.2561;
    double E0 = 105700000;
    // this is the intergal of C0 times air dencity as a
    // function of hight
    return (-C * rho0 * t0 * (Power((-alpha * h* Cos(A) + t0)
        / t0, n) - 1) / (alpha * n))/Cos(A)+(4000000000);
}

double eq(double h , double A)//time as a function of hight
{
    double c = 299792458;
    double E0 = 105700000;
    double gamma = Eng(h,A) / E0;
    double rgamma = 1 / gamma;
    double beta = Power((1-(rgamma* rgamma)),0.5);
    //cout << "beta = " << beta <<" gamma = " << gamma << "
    engery = "<< Eng(h) << endl;
    return 1/(c*beta*gamma) ;// equation from page 12 of muon
                           manual
}

```

```

void intergal2()
{

    auto Deacy = new TGraph();
    auto Eh = new TGraph();
    auto Et = new TGraph();
    auto ht = new TGraph();
    gStyle->SetPadTickX(1);
    gStyle->SetPadTickY(1);

    cout << "\n" << endl;
    // making variables
    double pi = 3.14159265359;
    double angle = 0;
    double left = 0;
    double tau = 0.0000022;
    double right = 15000/(Cos(angle));
    double Ng = 1;
    double y;
    double e = 2.718281828459045;
    double N = 0;
    double inc = 0.1;//smaller the better but also longer run
                    time.
    double Larea = 0; double Rarea = 0; double Aarea = 0;
    for (double x = left; x < right; x += inc)//Left hand
    {

        y = eq(x , angle);//time as a function of hight
        Larea += (y * inc);
        Eh->AddPoint(x, Eng(x,angle)/1000000);
        ht->AddPoint(Larea *1000000, 15000 - x);

    }
    double NO = Ng * Power(e, Larea / tau);
    for (double x = right; x > left; x -= inc)//Right hand
    {
        y = eq(x , angle);
        Rarea += (y * inc);
        //cout << Rarea << endl;
        N = NO * Power(e, -Rarea / tau);
    }
}

```

```

        Deacy->AddPoint(x, N);
        Et->AddPoint(Rarea * 1000000, Eng(x, angle) /
                      1000000);

    }

Aarea = (Rarea + Larea) / 2;// only have the negitive
                           sign for the spsifc thing im doing now get rid of it
                           for normal intergals

//cout << "area is " << Aarea << endl;
cout << "avrage area under is " << Aarea << " left hand
      side is " << Larea << " right hand side is " << Rarea
      << endl;

Deacy->SetLineColor(2);
Deacy->SetTitle("Muon rate; hight (m) ;events per minute
                  per cm^2");
Deacy->Draw();

Eh->SetLineColor(2);
Eh->SetTitle("Energy(h); hight (m) ;Energy (Mev)");
//Eh->Draw();

Et->SetLineColor(2);
Et->SetTitle("Energy(t); time (micro seconds) ;Energy (
                  Mev)");
//Et->Draw();

ht->SetLineColor(2);
ht->SetTitle("hight(t); time (micro seconds) ;hight(m)");

//ht->Draw();

// test
double testh = 0;
double c = 299792458;
double E0 = 105700000;
double gamma = Eng(testh, angle) / E0;
double rgamma = 1 / gamma;

```

```
double beta = Power((1 - (rgamma * rgamma)), 0.5);
cout << "\nAt sea level engergy is " << Eng(testh, angle)
    / 1000000 << " Mev, Beta is " << beta
    << " and gamma is, " << gamma << endl;
testh = right;
c = 299792458;
E0 = 105700000;
gamma = Eng(testh, angle) / E0;
rgamma = 1 / gamma;
beta = Power((1 - (rgamma * rgamma)), 0.5);
cout << "\nAt production engergy is " << Eng(testh,
angle) / 1000000 << " Mev, Beta is " << beta
    << " and gamma is, " << gamma << " the change in
    engergy is " << (Eng(testh, angle) - Eng(0,
angle)) / 1000000 << " Mev the time it takes
    is " << Aarea *
    1000000 << " us\n " << endl;
}
```

---

## Appendix B

first 1000 seconds of the Detector output. Note that it is in a slightly different format for the sake of conciseness. where there is normally a space there is a comma and instead of a new line, there is a colon.

```
40000 ,1621360437 : 40000 ,1621360438 : 40009 ,1621360439 : 40003 ,1621360440 :  
40008 ,1621360441 : 40002 ,1621360442 : 40004 ,1621360443 : 40003 ,1621360444 :  
40007 ,1621360445 : 40005 ,1621360446 : 40005 ,1621360447 : 40004 ,1621360448 :  
40003 ,1621360449 : 1720 ,1621360450 : 40006 ,1621360450 : 40009 ,1621360451 :  
40010 ,1621360452 : 40003 ,1621360453 : 40003 ,1621360454 : 40006 ,1621360455 :  
40006 ,1621360456 : 40007 ,1621360457 : 40005 ,1621360458 : 40007 ,1621360459 :  
40005 ,1621360460 : 40008 ,1621360461 : 40004 ,1621360462 : 40003 ,1621360463 :  
40006 ,1621360464 : 40002 ,1621360465 : 40004 ,1621360466 : 40003 ,1621360467 :  
40003 ,1621360468 : 40005 ,1621360469 : 40006 ,1621360470 : 40007 ,1621360471 :  
40003 ,1621360472 : 40003 ,1621360473 : 40006 ,1621360474 : 40003 ,1621360475 :  
40005 ,1621360476 : 40000 ,1621360477 : 40005 ,1621360478 : 40006 ,1621360479 :  
40003 ,1621360480 : 40004 ,1621360481 : 40004 ,1621360482 : 40006 ,1621360483 :  
40004 ,1621360484 : 40005 ,1621360485 : 40004 ,1621360486 : 40002 ,1621360487 :  
40005 ,1621360488 : 40006 ,1621360489 : 40006 ,1621360490 : 40005 ,1621360491 :  
40001 ,1621360492 : 7860 ,1621360493 : 40004 ,1621360493 : 40006 ,1621360494 :  
40002 ,1621360495 : 40006 ,1621360496 : 40010 ,1621360497 : 40004 ,1621360498 :  
40003 ,1621360499 : 40006 ,1621360500 : 40003 ,1621360501 : 40004 ,1621360502 :  
40007 ,1621360503 : 40005 ,1621360504 : 40002 ,1621360505 : 40003 ,1621360506 :  
40009 ,1621360507 : 40005 ,1621360508 : 40002 ,1621360509 : 40003 ,1621360510 :  
40005 ,1621360511 : 40002 ,1621360512 : 40004 ,1621360513 : 40003 ,1621360514 :  
40006 ,1621360515 : 40004 ,1621360516 : 40005 ,1621360517 : 40009 ,1621360518 :  
40005 ,1621360519 : 40006 ,1621360520 : 40005 ,1621360521 : 40000 ,1621360522 :  
40002 ,1621360523 : 40010 ,1621360524 : 40008 ,1621360525 : 40004 ,1621360526 :  
40004 ,1621360527 : 40012 ,1621360528 : 40003 ,1621360529 : 40005 ,1621360530 :  
40005 ,1621360531 : 40005 ,1621360532 : 40009 ,1621360533 : 40008 ,1621360534 :  
40004 ,1621360535 : 40004 ,1621360536 : 40003 ,1621360537 : 40003 ,1621360538 :  
40002 ,1621360539 : 40006 ,1621360540 : 40006 ,1621360541 : 40007 ,1621360542 :  
40005 ,1621360543 : 6200 ,1621360544 : 40002 ,1621360544 : 40003 ,1621360545 :  
40004 ,1621360546 : 40007 ,1621360547 : 40006 ,1621360548 : 40005 ,1621360549 :  
40004 ,1621360550 : 40004 ,1621360551 : 40003 ,1621360552 : 40006 ,1621360553 :  
40005 ,1621360554 : 40002 ,1621360555 : 40004 ,1621360556 : 40002 ,1621360557 :  
40005 ,1621360558 : 40002 ,1621360559 : 40005 ,1621360560 : 40004 ,1621360561 :  
40004 ,1621360562 : 40004 ,1621360563 : 40010 ,1621360564 : 40006 ,1621360565 :
```

40007 ,1621360566 : 40008 ,1621360567 : 40003 ,1621360568 : 40001 ,1621360569 :  
40002 ,1621360570 : 40006 ,1621360571 : 40003 ,1621360572 : 40007 ,1621360573 :  
40004 ,1621360574 : 40004 ,1621360575 : 40005 ,1621360576 : 40004 ,1621360577 :  
40002 ,1621360578 : 40005 ,1621360579 : 40004 ,1621360580 : 40002 ,1621360581 :  
40003 ,1621360582 : 40001 ,1621360583 : 40001 ,1621360584 : 40008 ,1621360585 :  
40003 ,1621360586 : 40003 ,1621360587 : 40003 ,1621360588 : 40007 ,1621360589 :  
40006 ,1621360590 : 40003 ,1621360591 : 40005 ,1621360592 : 40003 ,1621360593 :  
40002 ,1621360594 : 40010 ,1621360595 : 40002 ,1621360596 : 40006 ,1621360597 :  
40003 ,1621360598 : 40002 ,1621360599 : 40005 ,1621360600 : 40004 ,1621360601 :  
40006 ,1621360602 : 40005 ,1621360603 : 40005 ,1621360604 : 40003 ,1621360605 :  
40007 ,1621360606 : 40007 ,1621360607 : 40007 ,1621360608 : 40006 ,1621360609 :  
40006 ,1621360610 : 40012 ,1621360611 : 40006 ,1621360612 : 40003 ,1621360613 :  
40006 ,1621360614 : 40003 ,1621360615 : 40006 ,1621360616 : 40002 ,1621360617 :  
40004 ,1621360618 : 40007 ,1621360619 : 40006 ,1621360620 : 40004 ,1621360621 :  
40008 ,1621360622 : 40006 ,1621360623 : 40002 ,1621360624 : 40005 ,1621360625 :  
40003 ,1621360626 : 40008 ,1621360627 : 40001 ,1621360628 : 40005 ,1621360629 :  
40007 ,1621360630 : 40003 ,1621360631 : 40006 ,1621360632 : 40005 ,1621360633 :  
40004 ,1621360634 : 40004 ,1621360635 : 40003 ,1621360636 : 40006 ,1621360637 :  
40002 ,1621360638 : 40004 ,1621360639 : 40005 ,1621360640 : 40004 ,1621360641 :  
40004 ,1621360642 : 40005 ,1621360643 : 40005 ,1621360644 : 40004 ,1621360645 :  
40004 ,1621360646 : 40002 ,1621360647 : 40003 ,1621360648 : 40004 ,1621360649 :  
40005 ,1621360650 : 40004 ,1621360651 : 40004 ,1621360652 : 40009 ,1621360653 :  
40009 ,1621360654 : 360 ,1621360655 : 40009 ,1621360655 : 40007 ,1621360656 : 40008  
,1621360657 : 40004 ,1621360658 : 40006 ,1621360659 : 40002 ,1621360660 : 40004  
,1621360661 : 40002 ,1621360662 : 40000 ,1621360663 : 40005 ,1621360664 : 40003  
,1621360665 : 40006 ,1621360666 : 40006 ,1621360667 : 40005 ,1621360668 : 40004  
,1621360669 : 40005 ,1621360670 : 40004 ,1621360671 : 40001 ,1621360672 : 40002  
,1621360673 : 40002 ,1621360674 : 40007 ,1621360675 : 40005 ,1621360676 : 40004  
,1621360677 : 40007 ,1621360678 : 40002 ,1621360679 : 40003 ,1621360680 : 40002  
,1621360681 : 3960 ,1621360682 : 40006 ,1621360682 : 40007 ,1621360683 : 40007  
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## Appendix C

---

```
#include "Riostream.h"
#include "TH2.h"
#include "TF1.h"
#include "TStyle.h"
#include " TBranch.h"
#include "TCanvas.h"
#include "TPaveStats.h"
#include "TLine.h"
#include "TMath.h"
#include <iostream>

using namespace std;
double Sqrt(double x) {
    return pow(x, 0.5);
}

void ratecalc2e() {
    cout << "\n\n" << endl;

    //opening file
    ifstream in;
    string file;
    cout << "file name:" ;
    cin >> file;
    in.open("/mnt/c/Users/matt/OneDrive/Desktop/muon/" + file + ".txt");// change this to where your files are
    //making tuple

    double_t x, y;
    Int_t nlines = 0;
    TNtuple ntuple("ntuple", "data from ascii file", "x:y");
    TCanvas* c = new TCanvas("c", "Graph2D example", 600, 400);
    //making variables
    double cont = 0;
    float temp;
    double BackEdcount;
    double BackEdrate = 1;
```

```

double rate;
double time = 0;
double first;
float dcont = 0;
double last;
double prev;
double e = 2.71828;
double sddList[21755];
double sdr=0;
double sumdc;
double sdd=0;
int sddc=0;
float drate;
float ratio;
float lifetime;
float con;
double FgS;
double Fg;
double row;
double us_lifetime;
double tempcount;
int Dprev;
int Dprev2;
//making histogram
int bin = 30;
TH1* h1 = new TH1F("h1", "x distribution", bin, 0.0, 20000.0)
;

//goes over documant
while (1) {
    in >> x >> y;
    if (nlines == 0) { first = y; }//gets first Y

    // fills in histogram
    h1->Fill(x);
    ntuple.Fill(x, y);

    if (x < 40000) //counts decays
    {
        dcont++;
        cont++;
    }
}

```

```

// if ((y - Dprev2 == 0)&&(x>100)) { cout << std::
    setprecision(10) << y << endl; }
//Dprev2 = Dprev;           // use this to see how many
    outlier evets
//Dprev = y;

}

if (x >= 40000) //gets number of muons
{
    //if (x > 40013) {cout << "over 13 " << endl; } //
        use this to see how many outlier evets
    temp = (x - 40000); //muons in 1 second
    cont = (cont + temp); //add to total count
    sddList[sddc] = temp;
    sddc++;
    tempcount += temp;
}

if ((y - prev) < 100 && nlines != 0 && (y - prev)) //you
    need to have this for merging documents
{
    time += (y - prev);

}
prev = y;

nlines++;
if (time > 3600)
{
    // break;
}
if (!in.good())
{
    //cout << "here" << endl;
    last = y; //gets last value
    break; //ends loop
}

```

```

rate = (cont / time); //muon rate
drate = (dcont / time)*60; //decay rate (*60 so it's in
    minutes)
ratio = (cont / dcont);

h1->SetTitle("Muon Decay; time (ns) ;Number of Events");
TF1* f1 = new TF1("f1", "[0]+exp([1]+[2]*x)", 0, 20000); //
    this fit is a constant plus an exponential
h1->Fit("f1","Q"); // fits histogram
Double_t p0 = f1->GetParameter(0);
Double_t p2 = f1->GetParameter(2);
Double_t e2 = f1->GetParError(2);
Double_t e0 = f1->GetParError(0);
Double_t p1 = f1->GetParameter(1);
lifetime = (-1 / p2); // gets life time
us_lifetime = lifetime / 1000;
BackEdcount = dcont - (p0 * bin); // decays - the backgroundS
BackEdrate = ((BackEdcount / time) * 60);

// fermi coupling constant GFS is it squared
//equation is FgS = (Gf^2)/(Hbar^6 x c^6) = (192 *pi^3
    Hbar)/(lifetime * mass^5 * c^10)
FgS = (((192) * (pow(3.14159, 3)) * (6.582 * pow(10, -16))) /
    (((lifetime / 1000000000))*(1.316799478 * pow(10, 40))));
Fg = pow(FgS, 0.5) * pow(10, 18); // pow(10, 18) is so its in
    GeV^-2
row = -(2.19703 / 2.043)*(2.043 - us_lifetime) / (2.19703 -
    us_lifetime); //ration of posive to negitve muon using
    values in T.E. Coan and J. Ye

//Error caluclations
for (int i = 0; i < sddc; i++)
{
    sumdc += (sddList[i] - rate) * (sddList[i] - rate);
}
double std = pow((sumdc/sddc), 0.5);
cout << sddc << " HERE " << std << endl;

double dcontError = Sqrt(dcont);
double contError = Sqrt(cont);

```

```

double LambdaError = e2;
double lifetimeError = LambdaError*lifetime*lifetime; // 
    error in decay contant / (decay contant)^2
double us_lifetimeError= lifetimeError/1000;
double FgError= 0.2727498648e-3 * lifetimeError / pow(
    lifetime, 1.5);
double rateError= std/pow(sddc, 0.5);
double drateError= pow(rate / time, 0.5)*60;
double BackEdrateError= 60 * Sqrt(pow(bin * e0 / time, 2) +
    pow(dcontError / time, 2));
double RatePerCmError= rateError* 0.3395;
double ratioError= Sqrt(pow(contError / dcont, 2) + pow(cont
    * dcontError / pow(dcont, 2), 2));
double rowError= Sqrt(pow((-1.075394028 / (2.19703 -
    us_lifetime) + (1.075394028 * (2.043 - us_lifetime)) / pow
    (2.19703 - us_lifetime, 2)) * us_lifetimeError, 2));

//prints out values
cout << "deacy contant " << p2 << " +/- " << LambdaError << "
ns^-1" << endl;
cout << "the life time is " << us_lifetime << " +/- " <<
us_lifetimeError << " us" << endl;
cout << "the fermi coupling constant in natural units is " <<
    Fg << " +/- " << FgError << " GeV^-2" << endl;
cout << "the muon rate is " << rate << " +/- " << rateError
    << " muons a second " << endl;
cout << "the decay rate is " << drate << " +/- " <<
    drateError << " muon decays per minute" << endl;
cout << "the decay rate subtract background is around " <<
    BackEdrate << " +/- " << BackEdrateError << " muon decays
    per minute" << endl;
cout << "the muon rate is " << rate * 0.3395 << " cm^2 pre
    minute" << " +/- " << RatePerCmError << endl;
cout << "the ratio of no decays to decays is " << ratio << "
    +/- " << ratioError << endl;
cout << "time is " << time << " seconds " << endl;
cout << "the ratio of positive to negative muons is " << row
    << " +/- " << rowError << endl;
cout << "here" << endl;
}

```

---

## Appendix D

---

```
#include <iostream>

#include "Riostream.h"
#include "TH2.h"
#include "TF1.h"
#include "TStyle.h"
#include "TBranch.h"
#include "TCanvas.h"
#include "TPaveStats.h"
#include "TLine.h"
#include "TMath.h"
#include <iostream>
#include<Math/Integrator.h>
#include <gsl/gsl_integration.h>
#include <stdio.h>
#include <math.h>
#include <gsl/gsl_sf_gamma.h>

using namespace std;

void SRcalc2()
{
    //making variables

    double cont = 0;
    float temp;
    float rate;
    double time = 0;
    double first;
    float dcont = 0;
    double last;
    double prev;
    float drate;
    float ratio;
    float lifetime;
    float con;
    double FgS;
    double Fg;
    double testtime;
```

```

double row;
double us_lifetime;
double lim;
double background;
int bin = 30;
double MuRatio;
double MuDRatio;
double ratelist[2];
double dratelist[2];
double rateErrorlist[2];
double drateErrorlist[2];
double t;
double tprime;
double lifetimeSec;
double distance = 250;
double lifetimeusec;
double lifeList[2];
double avLifeNs;
double c = 299792458; // speed of light
double trest; // earth refrancne fram
double gamma;
double beta;
double KE;
double restE = 105.6583745; // rest energy from PDG in MeV
double restM = 1.883531627 * pow(10, -28); // rest mass in kg
double dcontError;
double contError;
double LambdaError;
double rateError;
double drateError;
double RatePerCmError;

Double_t p0;
Double_t p2;
Double_t p1;
Double_t e0;
Double_t e2;

printf("\n\n\n");
TH1* h1 = new TH1F("h1", "x distribution", bin, 0.0, 20000.0)
; // makes histogram

```

```

TF1* f1 = new TF1("f1", "[0]+exp([1]+[2]*x)", 0, 20000); //
    this fit is a constant plus an exponential
for (int doc = 0; doc < 2; doc++)
{

    cont = 0;
    time = 0;
    dcont = 0;
    //opening file
    ifstream in;
    string file;
    cout << "file name:" ;
    cin >> file;
    in.open("/mnt/c/Users/matt/OneDrive/Desktop/muon/" + file
        + ".txt"); // change this to where your files are
    //making tuple

    double_t x, y;
    Int_t nlines = 0;
    TNtuple ntuple("ntuple", "data from ascii file", "x:y");

    //goes over document
    while (1) {
        in >> x >> y;
        if (nlines == 0) first = y;//gets first Y

        // fills in histogram
        h1->Fill(x);
        ntuple.Fill(x, y);

        ntuple.Fill(x, y);

        if (x < 40000) //counts decays
        {
            dcont = (dcont + 1);
        }

        if (x >= 40000) //gets number of muons
        {
            temp = (x - 40000); //muons in 1 second
        }
    }
}

```

```

        cont = (cont + temp); //add to total count
    }

    if ((y - prev) < 100 && nlines != 0) //you need to
        have this for merging documents
    {
        time += (y - prev);

    }
    prev = y;

    nlines++;

    if (!in.good())
    {
        last = y; //gets last value
        break; //ends loop
    }
}

rate = (cont / time); //muon rate
drate = (dcont / time) * 60; //decay rate (*60 so it's in
    minutes)
ratio = (cont / dcont);
ratelist[doc] = rate; // adds to list
dratelist[doc] = drate;

/* IMPORTANT NOTE : on the second run this histogram is
NOT the histogram
of the second file if you want that you need to zero the
histogram
(use for loop to go over all bins and set each to 0)
this is the combined histogram of both files as lifetime
is the same at any eleavation*/

h1->Fit("f1", "0");// fits histogram
p0 = f1->GetParameter(0);
p2 = f1->GetParameter(2);

```

```

p1 = f1->GetParameter(1);
e2 = f1->GetParError(2);
e0 = f1->GetParError(0);

lifetime = (-1 / p2); // gets life time in ns
us_lifetime = lifetime / 1000;
lifeList[doc] = lifetime; // note I dont use this but you
    might if you zero hist

//Error caluclations
dcontError = sqrt(dcont);
contError = sqrt(cont);
LambdaError = e2;
rateError = pow(rate / time, 0.5);
drateError = pow(rate / time, 0.5) * 60;
RatePerCmError = rateError * 0.3395;
rateErrorlist[doc] = rateError;
drateErrorlist[doc] = drateError;

//prints out values.
cout << endl;
cout << "for file " << doc + 1 << endl;
cout << "the muon rate is " << rate << " +/- " <<
    rateError << " muons a second " << endl;
cout << "the decay rate is " << drate << " +/- " <<
    drateError << " muon decays per minute" << endl;
cout << "the muon rate is " << rate * 0.3395 << " cm^2
    pre minute" << " +/- " << RatePerCmError << endl;
cout << "time elpaced " << time << " seconds" << endl;

}

us_lifetime = lifetime / 1000;
// fermi coupling constant GFS is it squared
//equation is FgS = (Gf^2)/(Hbar^6 x c^6) = (192 *pi^3
    Hbar)/(lifetime * mass^5 * c^10)
FgS = (((192) * (pow(3.14159, 3)) * (6.582 * pow(10, -16))) /
    (((lifetime / 1000000000) * (1.316799478 * pow(10, 40)))) ;

```

```

Fg = pow(FgS, 0.5) * pow(10, 18); // pow(10, 18) is so its in
GeV^-2
row = -(2.19703 / 2.043) * (2.043 - us_lifetime) / (2.19703 -
us_lifetime); //ration of posive to negitve muon using
values in T.E. Coan and J. Ye

MuRatio = ratelist[0] / ratelist[1];
MuDRatio = dratelist[0] / dratelist[1];
lifetimeSec = lifetime / pow(10, 9);
t = -lifetimeSec * Log(MuRatio); // muons refrance frame
beta = (distance / pow(c * c * lifetimeSec * lifetimeSec *
Log(MuRatio)* Log(MuRatio) + distance * distance, 0.5));
trest = distance / (beta * c);
gamma = trest / t; // relitivistic meaning
KE = (gamma - 1) * restE; // kenitic engergy

//Error caluclations
double lifetimeError = LambdaError * lifetime * lifetime; ///
error in decay contant / (decay contant)^2
double us_lifetimeError = lifetimeError / 1000;
double lifetimeSecError = lifetimeError / pow(10, 9);
double FgError = 0.2727498648e-3 * lifetimeError / pow(
lifetime, 1.5);
double MuRatioError = sqrt(ratelist[0] * ratelist[0] * pow(
rateErrorlist[1], 2) + ratelist[1] * ratelist[1] * pow(
rateErrorlist[0], 2)) / (ratelist[1] * ratelist[1]);
double MuDRatioError = sqrt(dratelist[0] * dratelist[0] * pow(
drateErrorlist[1], 2) + dratelist[1] * dratelist[1] * pow(
drateErrorlist[0], 2)) / (dratelist[1] * dratelist[1]);
double TError = sqrt(pow(log(MuRatio), 2) * pow(
lifetimeSecError, 2) + pow(lifetimeSec, 2) * pow(
MuRatioError, 2) / pow(MuRatio, 2));
double distanceError = 15;
double betaError = c * c * t * sqrt(distance * distance * pow(
(TError, 2) + t * t * pow(distanceError, 2)) / pow(c * c *
t * t + distance * distance, 1.5));
double trestError = sqrt(distance * distance * pow(betaError,
2) + beta * beta * pow(distanceError, 2)) / (beta * beta
* c);

```

```

double gammaError = sqrt(trest * trest * pow(TError, 2) + t *
    t * pow(trestError, 2)) / (t * t);
double KEError= restE* gammaError;
double EError = KEError;

//print out stuff
cout << "\nthe life time is " << us_lifetime << " micro
seconds" << endl;
cout << "Muon Ratio=" << MuRatio << " +/- " << MuRatioError
<< endl;
cout << "MuDRatio=" << MuDRatio << " +/- " << MuDRatioError
<< endl;
cout << "the fermi coupling constant is " << Fg << " +/- " <<
FgError << " GeV^-2" << endl;
cout << "gamma is " << gamma << " +/- " << gammaError << endl
;
cout << "beta is " << beta << " +/- " << betaError << endl;
cout << "KE is " << KE << " MeV" << " +/- " << KEError <<
endl;
cout << "engergy is " << gamma * 105.7 << " +/- " << EError
<< " MeV" << endl;
cout << "Time in the muons refrance fram is" << t << " +/- "
<< TError << " seconds" << endl;
cout << "Time in the earths refrance fram is" << trest << "
+/- " << trestError << " seconds" << endl; ;
}

```

---