Random Numbers

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Contents

Abstract—**This manual provides a simple introduction to digital signal processing.**

1 Software installation

1.1 Run the following commands.

```
sudo apt−get update
sudo apt−get install libffi−dev libsndfile1
    python3−scipy python3−numpy python3−
    matplotlib
sudo pip install cffi pysoundfile
```

2 Digital Filter

2.1 Download the sound file from

```
wget https://github.com/jarpula−Bhanu/
    EE3900/blob/main/Ass/soundfiles/
    Sound_Noise.wav
```

2.2 You will find a spectogram at https://academo.org/demos/spectrum-analyzer. Upload the sound file that you downloaded in problem 2.1 in the spectrogram and play. Observe the spectogram. What do you find?
Solution: There are a lot of yellow lines betweeen 440Hz to 5.1KHz. These represent the synthesizer key tones. Also, the key strokes are audible along with background noise.

2.3 Write the python code for removal of out of band nosie and execute the code.
Solution: Download and run the following code.

```
wget https://github.com/jarpula−Bhanu/
    EE3900/blob/main/Ass/Codes/2.3_noise.
    py
```

run the above code using the command

```
python3 2.3_noise.py
```

2.4 The output of the python scripy in problem 2.3 is the audio file Sound_With_ReducedNoise.wav. Play the file in the spectogram in problem 2.2. What do you observe?
Solution: The key strokes as well as background noise is subdued in the audio. Also the signal is blank for frequencies above 5.1KHz.

3 Difference Equation

3.1 Let

$$x(n) = \left\{ \underset{\uparrow}{1}, 2, 3, 4, 2, 1 \right\} \tag{3.1}$$

Sketch x(n).
Solution: Download and run the following code.Below code plots fig(3.1)

```
wget https://github.com/jarpula−Bhanu/
    EE3900/blob/main/Ass/Codes/3.1.py
```

run the above code using the command

```
python3 3.1.py
```

3.2 Let

$$y(n) + \frac{1}{2}y(n-1) = x(n) + x(n-2), y(n) = 0, n < 0 \tag{3.2}$$

Sketech y(n).
Solution: Download and run the following code.Below code plots fig(3.2)

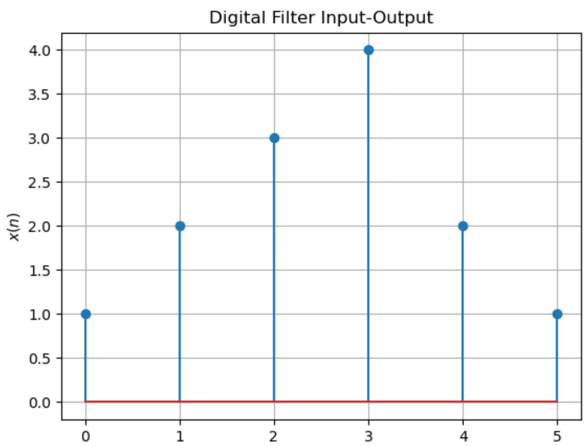

Fig. 3.1: Sketch of $x(n)$

```
wget https://github.com/jarpula-Bhanu/
    EE3900/blob/main/Ass/Codes/3.2.py
```

run the above code using the command

```
python3 3.2.py
```

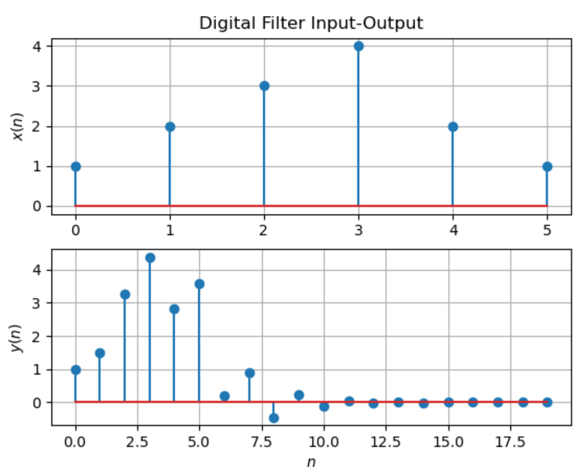

Fig. 3.2: Sketch of $x(n)$ and $y(n)$

3.3 Repeat the above exercise using C code.
Solution: Download and run the following code.Below code plots fig(3.3)

```
wget https://github.com/jarpula-Bhanu/
    EE3900/blob/main/Ass/Codes/3.3.c
wget https://github.com/jarpula-Bhanu/
    EE3900/blob/main/Ass/Codes/3.3_plot.py
```

run the above code using the command

```
gcc 3.3.c -o 3.3.out
./3.3.out
python3 3.3_plot.py
```

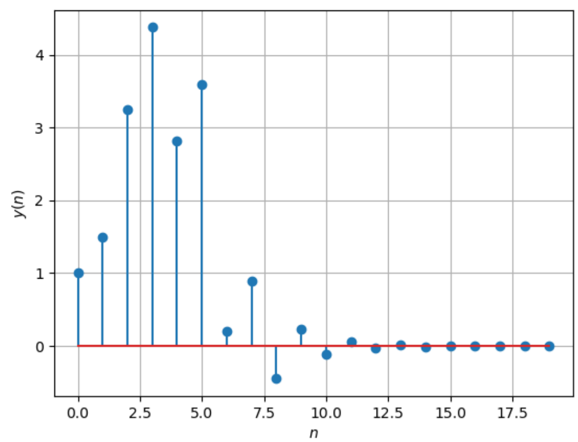

Fig. 3.3: Sketch of $y(n)$

4 $\mathcal{Z}$-transform

4.1 The Z-transform of $x(n)$ is defined as

$$X(z) = \mathcal{Z}\{x(n)\} = \sum_{n=-\infty}^{\infty} x(n)z^{-n} \tag{4.1}$$

Show that

$$\mathcal{Z}\{x(n-1)\} = z^{-1}X(z) \tag{4.2}$$

and find

$$\mathcal{Z}\{x(n-k)\} \tag{4.3}$$

Solution: From (4.1)

$$\mathcal{Z}\{x(n-k)\} = \sum_{n=-\infty}^{\infty} x(n-1)z^{-n} \tag{4.4}$$

$$= \sum_{n=-\infty}^{\infty} x(n)z^{-n-1} = z^{-1}\sum_{n=-\infty}^{\infty} x(n)z^{-n} \tag{4.5}$$

resulting in (4.2). Similarly, it can be shown that

$$\mathcal{Z}\{x(n-k)\} = z^{-k}X(z) \tag{4.6}$$

$\mathcal{Z}$- transform of $x(n)$ is

$$\mathcal{Z}\{x(n)\} = 1 + 2z^{-1} + 3z^{-2} + 4z^{-3} + 2z^{-4} + z^{-5} \tag{4.7}$$

$$\mathcal{Z}\{x(n-k)\} = z^{-k}(1 + 2z^{-1} + 3z^{-2} + \tag{4.8}$$
$$4z^{-3} + 2z^{-4} + z^{-5}) \tag{4.9}$$
$$\mathcal{Z}\{x(n-k)\} = z^{-k} + 2z^{-(k+1)} + 3z^{-(k+2)} \tag{4.10}$$
$$+ 4z^{-(k+3)} + 2z^{-(k+4)} + z^{-(k+5)} \tag{4.11}$$

4.2 Obtain $X(z)$ for $x(n)$ in problem (3.1)
Solution:

$$\mathcal{Z}\{x(n)\} = 1 + 2z^{-1} + 3z^{-2} + 4z^{-3} + 2z^{-4} + z^{-5} \tag{4.12}$$

$$\mathcal{Z}\{x(n-k)\} = z^{-k}(1 + 2z^{-1} + 3z^{-2} + \tag{4.13}$$
$$4z^{-3} + 2z^{-4} + z^{-5}) \tag{4.14}$$
$$\mathcal{Z}\{x(n-k)\} = z^{-k} + 2z^{-(k+1)} + 3z^{-(k+2)} \tag{4.15}$$
$$+ 4z^{-(k+3)} + 2z^{-(k+4)} + z^{-(k+5)} \tag{4.16}$$

4.3 Find

$$H(z) = \frac{Y(z)}{X(z)} \tag{4.17}$$

from (3.2) assuming that the Z-transform is a linear operation.
Solution: Applying (4.6) in (3.2),

$$Y(z) + \frac{1}{2}z^{-1}Y(z) = X(z) + z^{-2}X(z) \tag{4.18}$$
$$\implies \frac{Y(z)}{X(z)} = \frac{1 + z^{-2}}{1 + \frac{1}{2}z^{-1}} \tag{4.19}$$

4.4 Find the Z transform of

$$\delta(n) = \begin{cases} 1 & n = 0 \\ 0 & \text{otherwise} \end{cases} \tag{4.20}$$

and show that the Z-transform of

$$u(n) = \begin{cases} 1 & n \geq 0 \\ 0 & \text{otherwise} \end{cases} \tag{4.21}$$

is

$$U(z) = \frac{1}{1 - z^{-1}}, \quad |z| > 1 \tag{4.22}$$

Solution: It is easy to show that

$$\delta(n) \overset{\mathcal{Z}}{\rightleftharpoons} 1 \tag{4.23}$$

and from (4.21),

$$U(z) = \sum_{n=0}^{\infty} z^{-n} \tag{4.24}$$
$$= \frac{1}{1 - z^{-1}}, \quad |z| > 1 \tag{4.25}$$

using the fomula for the sum of an infinite geometric progression.

4.5 Show that

$$a^n u(n) \overset{\mathcal{Z}}{\rightleftharpoons} \frac{1}{1 - az^{-1}} \quad |z| > |a| \tag{4.26}$$

Solution:

$$a^n u(n) \overset{\mathcal{Z}}{\rightleftharpoons} \sum_{n=0}^{\infty} (az^{-1})^n \tag{4.27}$$
$$= \frac{1}{1 - az^{-1}} \quad |z| > |a| \tag{4.28}$$

4.6 Let

$$H(e^{\jmath\omega}) = H(z = e^{\jmath\omega}). \tag{4.29}$$

Plot $|H(e^{\jmath\omega})|$. Comment. $H(e^{\jmath\omega})$ is known as the *Discret Time Fourier Transform* (DTFT) of $x(n)$.
Solution: Download and run the following code. The following code plots Fig. 4.6.

```
wget https://github.com/jarpula-Bhanu/
    EE3900/blob/main/Ass/Codes/4.5.py
```

run the above code using the command

```
python3 4.5.py
```

We observe that $|H(e^{\jmath\omega})|$ is periodic with fundamental period 2π.

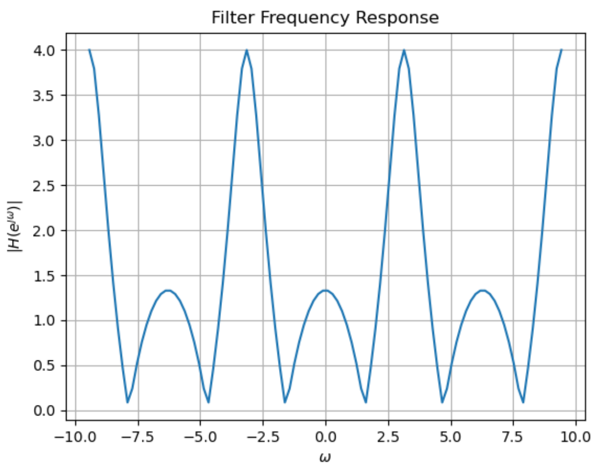

Fig. 4.6: $|H(e^{\jmath\omega})|$

$$H(e^{\jmath\omega}) = \frac{1+e^{-2\jmath\omega}}{1+\frac{1}{2}e^{-\jmath\omega}} \tag{4.30}$$

$$\implies |H(e^{\jmath\omega})| = \frac{\left|1+\cos 2\omega - \jmath \sin 2\omega\right|}{\left|1+\frac{1}{2}\cos\omega - \frac{1}{2}\sin\omega\right|} \tag{4.31}$$

$$= \sqrt{\frac{(1+\cos 2\omega)^2 + (\sin 2\omega)^2}{(1+\frac{1}{2}\cos\omega)^2 + (\frac{1}{2}\sin\omega)^2}} \tag{4.32}$$

$$= \sqrt{\frac{2+2\cos 2\omega}{\frac{5}{4}+\cos\omega}} \tag{4.33}$$

$$= \sqrt{\frac{2(2\cos^2\omega)4}{5+4\cos\omega}} \tag{4.34}$$

$$= \frac{4\,|\cos\omega|}{\sqrt{5+4\cos\omega}} \tag{4.35}$$

period of $|cos\omega|$ is π and period of $\sqrt{5+4\cos\omega}$ is 2π.
Now period of $|H(e^{\jmath\omega})|$ is $\frac{LCM(\pi,2\pi)}{HCF(\pi,2\pi)} = \frac{2\pi}{1} = 2\pi$

4.7 Express $h(n)$ in terms of $H(e^{j\omega})$
Solution: $h(n)$ is given by the inverse DTFT of $H(e^{j\omega})$

$$h(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H(e^{j\omega})e^{j\omega n}d\omega \tag{4.36}$$

$$= \frac{1}{2\pi}\sum_{n=-\infty}^{\infty}\int_{-\pi}^{\pi} h(k)e^{-j\omega k}e^{j\omega n}d\omega \tag{4.37}$$

$$= \frac{1}{2\pi}\sum_{n=-\infty}^{\infty} h(k)\int_{-\pi}^{\pi} e^{j\omega(n-k)}d\omega \tag{4.38}$$

$$= \frac{1}{2\pi}\sum_{n=-\infty}^{\infty} h(k)2\pi\delta[n-k] \tag{4.39}$$

$$= h(n) \tag{4.40}$$

Since

$$\int_{-\pi}^{\pi} e^{j\omega n}d\omega = 2\pi\delta[n] \tag{4.41}$$

$$and \tag{4.42}$$

$$\delta[n-k] = \begin{cases} 1 & n = k \\ 0 & \text{otherwise} \end{cases} \tag{4.43}$$

5 Impulse Response

5.1 Using long division, find

$$h(n), \quad n < 5 \tag{5.1}$$

for $H(z)$ in 4.18
Solution:

$$H(z) = \frac{1+z^{-2}}{1+\frac{1}{2}z^{-1}} \tag{5.2}$$

Substitute $z^{-1} = x$

$$\begin{array}{r|l} & \quad 2x-4 \\ \frac{1}{2}x+1) & x^2 \qquad +1 \\ & -x^2-2x \\ \hline & \qquad -2x+1 \\ & \qquad\ \ 2x+4 \\ \hline & \qquad\qquad 5 \end{array}$$

$$\implies 1+z^{-2} = \left(1+\frac{1}{2}z^{-1}\right)\left(-4+2z^{-1}\right)+5 \tag{5.3}$$

$$\implies H(z) = -4+2z^{-1}+\frac{5}{1+\frac{1}{2}z^{-1}} \tag{5.4}$$

On applying the inverse Z-transform on both sides of the equation

$$H(z) \overset{\mathcal{Z}}{\rightleftharpoons} h(n) \quad (5.5)$$

$$-4 \overset{\mathcal{Z}}{\rightleftharpoons} -4\delta(n) \quad (5.6)$$

$$2z^{-1} \overset{\mathcal{Z}}{\rightleftharpoons} 2\delta(n-1) \quad (5.7)$$

$$\frac{5}{1+\frac{1}{2}z^{-1}} \overset{\mathcal{Z}}{\rightleftharpoons} 5\left(-\frac{1}{2}\right)^n u(n) \quad (5.8)$$

$$(5.9)$$

Therefore,

$$h(n) = -4\delta(n)+2\delta(n-1)+5\left(-\frac{1}{2}\right)^n u(n) \quad (5.10)$$

Download the following Python code that plots Fig. 5.1.

```
wget https://github.com/jarpula−Bhanu/
    EE3900/blob/main/Ass/Codes/5.1.py
```

Run the code by executing

```
python 5.1.py
```

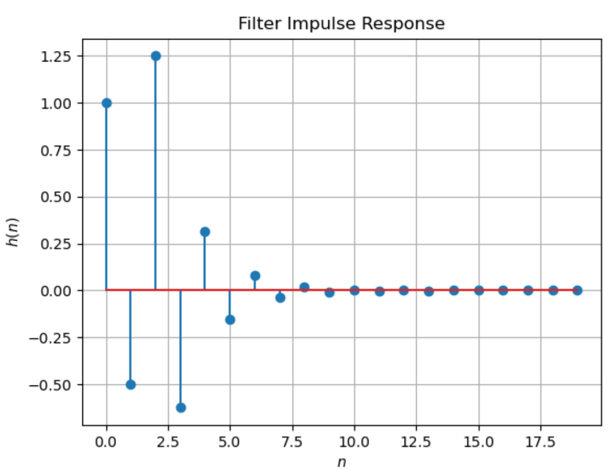

Fig. 5.1: Plot of $h(n)$

5.2 Find an expression for $h(n)$ using $H(z)$, given that

$$h(n) \overset{\mathcal{Z}}{\rightleftharpoons} H(z) \quad (5.11)$$

and there is a one to one relationship between $h(n)$ and $H(z)$. $h(n)$ is known as the *impulse response* of the system defined by (3.2).

Solution: From (4.19),

$$H(z) = \frac{1}{1+\frac{1}{2}z^{-1}} + \frac{z^{-2}}{1+\frac{1}{2}z^{-1}} \quad (5.12)$$

$$\implies h(n) = \left(-\frac{1}{2}\right)^n u(n) + \left(-\frac{1}{2}\right)^{n-2} u(n-2) \quad (5.13)$$

using (4.26) and (4.6).

5.3 Sketch $h(n)$. Is it bounded? Justify theoretically.

Solution: Download and run the following code.The following code plots Fig. 5.3.

```
wget https://github.com/jarpula−Bhanu/
    EE3900/blob/main/Ass/Codes/5.2.py
```

run the above code using the command.

```
python3 5.2.py
```

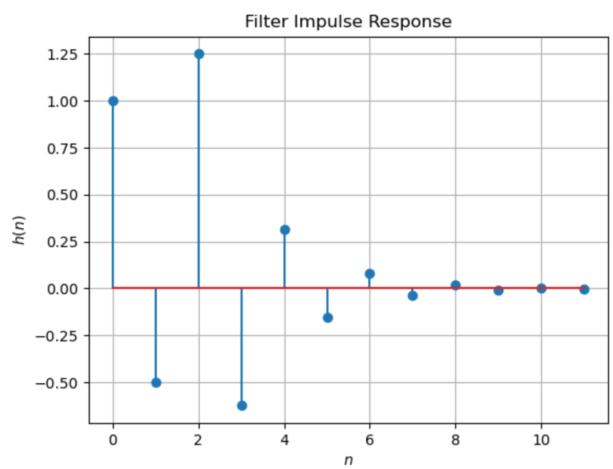

Fig. 5.3: $h(n)$ as the inverse of $H(z)$

From the plot, it is clear that $h(n)$ is bounded. Theoretically,

$$|u(n)| \le 1 \quad (5.14)$$

$$\left|\left(-\frac{1}{2}\right)^n\right| \le 1 \quad (5.15)$$

$$\implies \left|\left(-\frac{1}{2}\right)^n u(n)\right| \le 1 \quad (5.16)$$

Similarly,

$$\left|\left(-\frac{1}{2}\right)^{n-2} u(n-2)\right| \le 1 \quad (5.17)$$

$$\implies h(n) \le 2 \quad (5.18)$$

Therefore $h(n)$ is bounded.

5.4 Convergent? Justify using the ratio test.
Solution: The ratio test for convergence

$$\lim_{n\to\infty}\left|\frac{h(n+1)}{h(n)}\right| = \lim_{n\to\infty}\left|\frac{\left(-\frac{1}{2}\right)^{n-1}\left(\frac{1}{4}+1\right)}{\left(-\frac{1}{2}\right)^{n-2}\left(\frac{1}{4}+1\right)}\right| \tag{5.19}$$

$$= \lim_{n\to\infty}\left|-\frac{1}{2}\right| \tag{5.20}$$

$$= \frac{1}{2} < 1 \tag{5.21}$$

Therefore, $h(n)$ is convergent which implies that it is bounded.

5.5 The system with $h(n)$ is defined to be stable if

$$\sum_{n=-\infty}^{\infty} h(n) < \infty \tag{5.22}$$

Is the system defined by (3.2) stable for the impulse response in (5.11)?
Solution: Note that

$$\sum_{n=-\infty}^{\infty} h(n) = \sum_{n=-\infty}^{\infty}\left(-\frac{1}{2}\right)^{n} u(n) + \left(-\frac{1}{2}\right)^{n-2} u(n-2) \tag{5.23}$$

$$= 2\left(\frac{1}{1+\frac{1}{2}}\right) = \frac{4}{3} \tag{5.24}$$

Thus, the given system is stable.

5.6 Verify the above result using a python code.
Solution: Download the following code.

```
wget https://github.com/jarpula−Bhanu/
    EE3900/blob/main/Ass/Codes/5.6.py
```

Run the code by executing

```
python 5.6.py
```

5.7 Compute and sketch $h(n)$ using

$$h(n) + \frac{1}{2}h(n-1) = \delta(n) + \delta(n-2), \tag{5.25}$$

This is the definition of $h(n)$.
Solution: The following code plots Fig. 5.7. Note that this is the same as Fig. 5.3.

```
wget https://github.com/jarpula−Bhanu/
    EE3900/blob/main/Ass/Codes/5.4.py
```

run the above code using the command.

```
python3 5.4.py
```

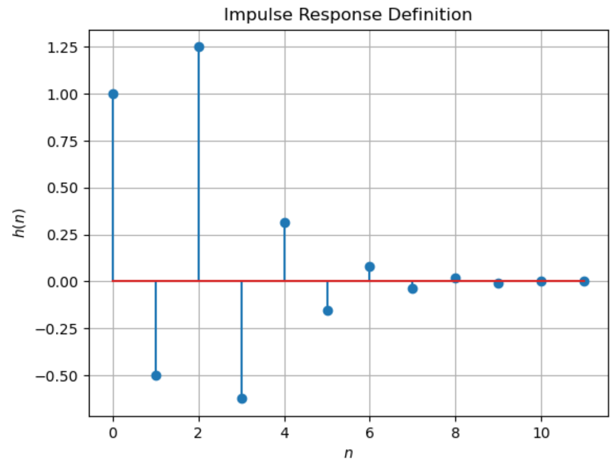

Fig. 5.7: $h(n)$ from the definition

5.8 Compute

$$y(n) = x(n) * h(n) = \sum_{n=-\infty}^{\infty} x(k)h(n-k) \tag{5.26}$$

Comment. The operation in (5.26) is known as *convolution*.
Solution: The following code plots Fig. 5.8. Note that this is the same as $y(n)$ in Fig. 3.2.

```
wget https://github.com/jarpula−Bhanu/
    EE3900/blob/main/Ass/Codes/5.5.py
```

run the above code using the command.

```
python3 5.5.py
```

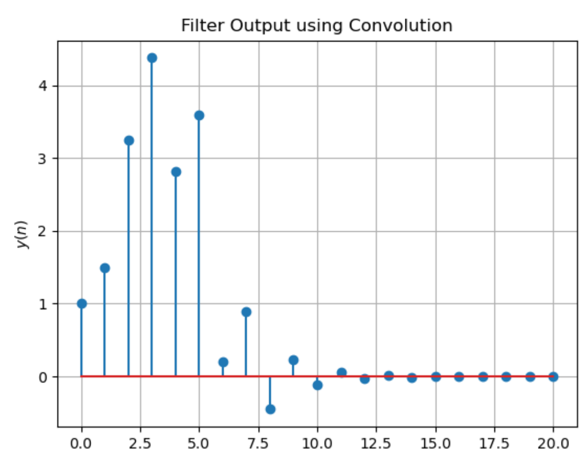

Fig. 5.8: $y(n)$ from the definition of convolution

5.9 Express the above convolution using a Teoplitz matrix.
Solution: Let

$$\mathbf{x} = \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 2 \\ 1 \end{pmatrix} \qquad \mathbf{h} = \begin{pmatrix} 1 \\ -0.5 \\ 1.25 \\ -0.62 \\ 0.31 \\ -0.16 \end{pmatrix} \tag{5.27}$$

Their convolution is given by the product of the following Toeplitz matrix **T**

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ -0.5 & 1 & 0 & 0 & 0 & 0 \\ 1.25 & -0.5 & 1 & 0 & 0 & 0 \\ -0.62 & 1.25 & -0.5 & 1 & 0 & 0 \\ 0.31 & -0.62 & 1.25 & -0.5 & 1 & 0 \\ -0.16 & 0.31 & -0.62 & 1.25 & -0.5 & 1 \\ 0 & -0.16 & 0.31 & -0.62 & 1.25 & -0.5 \\ 0 & 0 & -0.16 & 0.31 & -0.62 & 1.25 \\ 0 & 0 & 0 & -0.16 & 0.31 & -0.62 \\ 0 & 0 & 0 & 0 & -0.16 & 0.31 \\ 0 & 0 & 0 & 0 & 0 & -0.16 \end{pmatrix} \tag{5.28}$$

and **x**

$$\mathbf{y} = \mathbf{x} \circledast \mathbf{h} = \mathbf{T}\mathbf{x} = \begin{pmatrix} 1 \\ 1.5 \\ 3.25 \\ 4.38 \\ 2.81 \\ 3.59 \\ 0.12 \\ 0.78 \\ -0.62 \\ 0 \\ -0.16 \end{pmatrix} \tag{5.29}$$

Download the following code .

```
wget https://github.com/jarpula−Bhanu/
    EE3900/blob/main/Ass/Codes/5.9.py
```

Run the code by executing

```
python 5.9.py
```

5.10 Show that

$$y(n) = \sum_{n=-\infty}^{\infty} x(n-k)h(k) \tag{5.30}$$

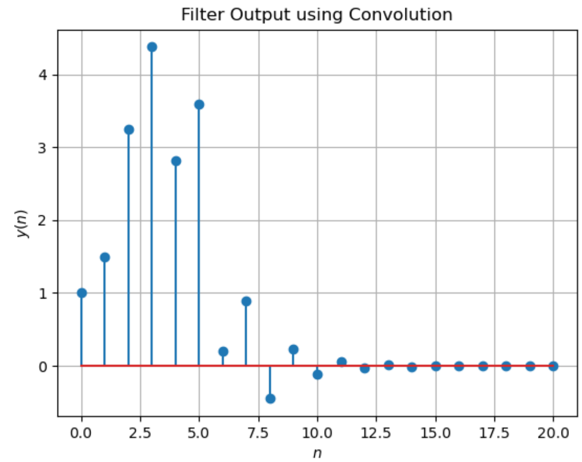

Fig. 5.9: Plot of the convolution of $x(n)$ and $h(n)$

Solution: from 5.26, we substitute $k := n - k$ to get

$$y(n) = \sum_{k=-\infty}^{\infty} x(k)h(n-k) \tag{5.31}$$

$$= \sum_{n-k=-\infty}^{\infty} x(n-k)h(k) \tag{5.32}$$

$$= \sum_{k=-\infty}^{\infty} x(n-k)h(k) \tag{5.33}$$

6 DFT and FFT

6.1 Compute

$$X(k) \triangleq \sum_{n=0}^{N-1} x(n)e^{-\jmath 2\pi kn/N}, \quad k = 0, 1, \dots, N-1 \tag{6.1}$$

and $H(k)$ using $h(n)$.
Solution: The following code plots Fig. 6.1.

```
wget https://github.com/jarpula−Bhanu/
    EE3900/blob/main/Ass/Codes/6.1.py
```

run the above code using the command.

```
python3 6.1.py
```

6.2 Compute

$$Y(k) = X(k)H(k) \tag{6.2}$$

Solution: Download and run the following code.

```
wget https://github.com/jarpula−Bhanu/
    EE3900/blob/main/Ass/Codes/6.2.py
```

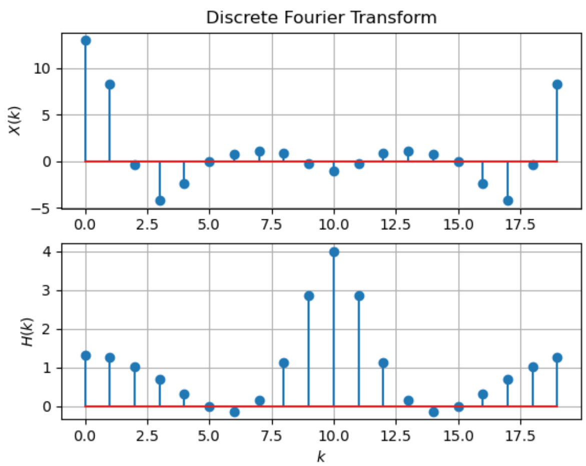

Fig. 6.1: Plots of the real parts of the DFT of $x(n)$ and $h(n)$

run the above code using the command.

```
python3 6.2.py
```

6.3 Compute

$$y(n) = \frac{1}{N} \sum_{k=0}^{N-1} Y(k) \cdot e^{\mathrm{j}2\pi kn/N}, \quad n = 0, 1, \ldots, N-1 \tag{6.3}$$

Solution: The following code plots Fig. 5.8. Note that this is the same as $y(n)$ in Fig. 3.2.

```
wget https://github.com/jarpula−Bhanu/
    EE3900/blob/main/Ass/Codes/6.3.py
```

run the above code using the command.

```
python3 6.3.py
```

6.4 Repeat the previous exercise by computing $X(k), H(k)$ and $y(n)$ through FFT and IFFT.
Solution: Download the code from

```
wget https://github.com/jarpula−Bhanu/
    EE3900/blob/main/Ass/Codes/6.4.py
%
```

and execute it using

```
$ python3 6.4.py
```

Observe that Fig. (6.4) is the same as $y(n)$ in Fig. (3.2).

6.5 Wherever possible, express all the above equations as matrix equations.

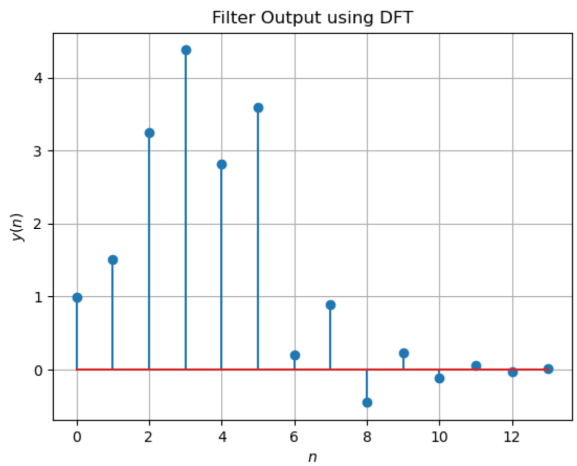

Fig. 6.3: $y(n)$ from the DFT

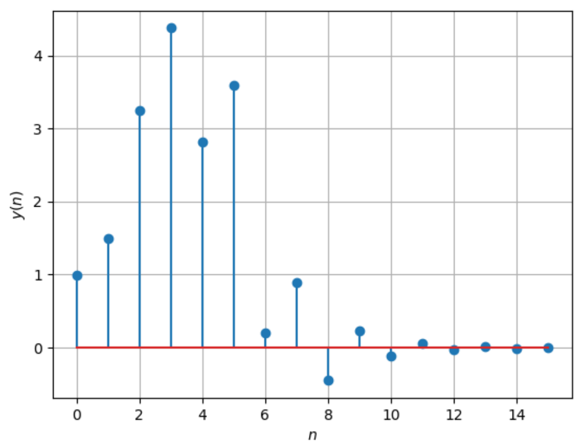

Fig. 6.4: $y(n)$ using FFT and IFFT

Solution: We use the DFT Matrix, where $\omega = e^{-\frac{j2k\pi}{N}}$, which is given by

$$\mathbf{W} = \begin{pmatrix} \omega^0 & \omega^0 & \ldots & \omega^0 \\ \omega^0 & \omega^1 & \ldots & \omega^{N-1} \\ \vdots & \vdots & \ddots & \vdots \\ \omega^0 & \omega^{N-1} & \ldots & \omega^{(N-1)(N-1)} \end{pmatrix} \tag{6.4}$$

i.e. $W_{jk} = \omega^{jk}$, $0 \leq j, k < N$. Hence, we can write any DFT equation as

$$\mathbf{X} = \mathbf{W}\mathbf{x} = \mathbf{x}\mathbf{W} \tag{6.5}$$

where

$$\mathbf{x} = \begin{pmatrix} x(0) \\ x(1) \\ \vdots \\ x(n-1) \end{pmatrix} \tag{6.6}$$

Using (6.3), the inverse Fourier Transform is given by

$$\mathbf{x} = \mathcal{F}^{-1}(\mathbf{X}) = \mathbf{W}^{-1}\mathbf{X} = \frac{1}{N}\mathbf{W}^{\mathbf{H}}\mathbf{X} = \frac{1}{N}\mathbf{X}\mathbf{W}^{\mathbf{H}} \tag{6.7}$$

$$\implies \mathbf{W}^{-1} = \frac{1}{N}\mathbf{W}^{\mathbf{H}} \tag{6.8}$$

where H denotes hermitian operator. We can rewrite (6.2) using the element-wise multiplication operator as

$$\mathbf{Y} = \mathbf{H} \cdot \mathbf{X} = (\mathbf{Wh}) \cdot (\mathbf{Wx}) \tag{6.9}$$

6.6 Verify the above equation by generating the DFT matrix in python.
Solution:

7 Exercises

Answer the following questions by looking at the python code in Problem 2.3.

7.1 The command

```
output_signal = signal.lfilter(b, a,
    input_signal)
```

in Problem 2.3 is executed through the following difference equation

$$\sum_{m=0}^{M} a(m)\, y(n-m) = \sum_{k=0}^{N} b(k)\, x(n-k) \tag{7.1}$$

where the input signal is $x(n)$ and the output signal is $y(n)$ with initial values all 0. Replace **signal.filtfilt** with your own routine and verify.
Solution: Solution: Download the code from

```
wget https://github.com/jarpula−Bhanu/
    EE3900/blob/main/Ass/Codes/7.1.py
```

and execute it using

```
$ python3 7.1.py
```

7.2 Repeat all the exercises in the previous sections for the above a and b.
Solution: The filter frequency response is plotted at

```
wget https://github.com/jarpula−Bhanu/
    EE3900/blob/main/Ass/Codes/7.2.1.py
```

The impulse response function is plotted at

```
wget https://github.com/jarpula−Bhanu/
    EE3900/blob/main/Ass/Codes/7.2.2.py
```

We see that $h(n)$ is bounded and convergent.Also,since 1 is not a pole of the transfer function, the system is stable.
run the above codes using

```
python3 7.2.1.py
python3 7.2.2.py
```

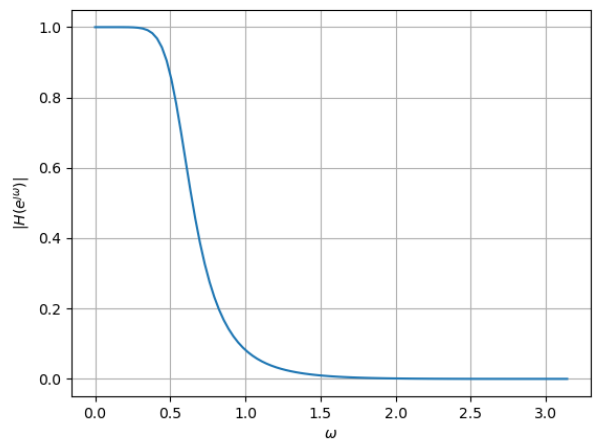

Fig. 7.2: Filter frequency response

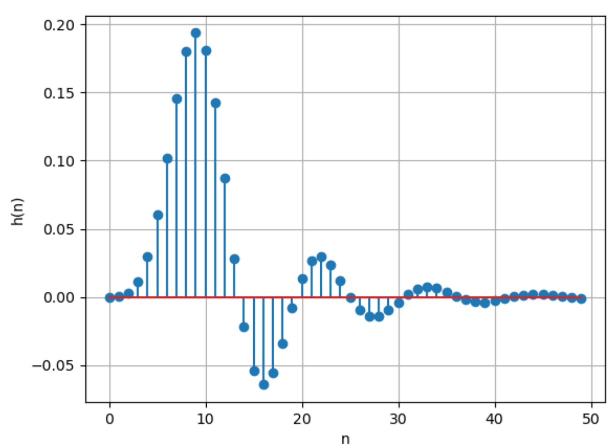

Fig. 7.2: Plot of $h(n)$

7.3 What is the sampling frequency of the input signal?
Solution: Sampling frequency(fs)=44.1kHZ.

7.4 What is type, order and cutoff-frequency of the above butterworth filter
Solution: The given butterworth filter is low pass with order=4 and cutoff-frequency=4kHz.
7.5 Modifying the code with different input parameters and to get the best possible output.
Solution: A better filtering was found on setting te order of the filter to be 7