Theorem 3.1

WTS.

Proof. Let ν be a signed measure, and fix any increasing sequence $E_j \nearrow E = \bigcup E_{j\geq 1}$ of sets. This induces a disjoint sequence in $\{F_n\}$. Define $F_1 = E_1$, and if $n \geq 2$,

$$F_n = E_n \setminus \bigcup E_{j\leq n-1}$$

and from this, the finite It is clear that $\bigcup F_{n\geq 1} = E$, and let us assume $\nu(E)$ is of finite measure.
By countable additivity, and the absolute convergence of the series $\sum_{j\leq n} \nu(F_j)$

$$\begin{aligned}\nu\left(\bigcup E_{j\geq 1}\right) &= \sum_{j\geq 1} \nu(F_j)\\ &= \lim_n \sum_{j\leq n} \nu(F_j)\\ &= \lim \nu(E_n)\end{aligned}$$

□