Notes on Chapter 3

WTS. *Prove two things,*

1. $\limsup_{r\to R}\phi(r) = \lim_{\varepsilon\to 0}\sup_{0<|r-R|<\varepsilon}\phi(r) = \inf_{\varepsilon>0}\sup_{0<|r-R|<\varepsilon}\phi(r)$,
2. $\lim_{r\to R}\phi(r) = c \iff \limsup_{r\to R}|\phi(r)-c| = 0$

Proof. □

WTS. *If* $U \subseteq B(1,0) = \{|x| < 1\}$*, and* $U \in \mathbb{B}$*, and if* $m(U) > 0$*, then the family of sets*

$$E_r = \left\{ x + ry,\ y \in U \right\}$$

shrinks nicely to $x \in \mathbb{R}^n$*.*

Proof. Let $r > 0$ be fixed then $\forall z \in E_r \looparrowright z = x + ry$. Hence,

$$\begin{aligned} d(x,z) &= d(x, x + ry) \\ &= |r| d(0, y) < |r| \end{aligned}$$

by translation invariance. $\square$