# Dark energy, gravitation and supernovae

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**Abstract.** The discovery of the acceleration of the rate of expansion of the Universe fosters new explorations of the behavior of gravitation theories in the cosmological context. Either the GR framework is valid but a cosmic component with a negative equation of state is dominating the energy–matter contents or the Universe is better described at large by a theory that departs from GR. In this review we address theoretical alternatives that have been explored through supernovae.

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#### 1. Introduction

The discovery of the acceleration of the expansion of the universe [1, 2] has revived questions concerning the domain of validity of Einstein's General Relativity. At present, the data gathered on the expansion rate do not disclose whether the acceleration is due to a component formally equivalent to the cosmological constant introduced by Einstein [3], or whether we are finding the effective behavior of a theory of a wider scope whose low energy limit slightly departs from GR.

The bare Einstein's equations (without cosmological constant) for a Universe with a FRW metric and dust–like matter imply a continuous deceleration of the expansion rate. However, a Universe containing a fluid with an equation of state  $p = w\rho$  with index w < -1/3 overcomes the deceleration when the density of this fluid dominates over that of the dust–like matter. In this context, the cosmological constant, if it is positive and added to the equations, balances the deceleration by acting as a fluid with an equation of state  $p = -\rho$ .

The cosmological constant was originally in the metric part of Einstein's equations, and alternative metric theories of gravity provide a wealth of terms in that side. When describing a homogeneous and isotropic universe within a FRW model, it is frequent to come as close as possible to the standard Friedmann–Einstein equation for the expansion rate by bringing terms to the right–hand side within the so–called "effective index"  $w_{\text{eff}}$  of the cosmic fluid whose equation of state is  $p = w_{\text{eff}} \rho$ .

In the field of gravitation, ideas that appeared in the classical domain several decades ago, are now being brought back within the different context of quantum gravity theories. Scalar—tensor theories of gravitation are being reconsidered. Scalar fields coupled to gravity were proposed as a way to incorporate Mach inertial induction. They gave rise to the Brans—Dicke theory of gravitation [4], which is well restricted by solar system experiments[5]. Superstring theory and M—theory introduce a number of quantum scalar fields, the dilaton, the radion, the moduli [6]. They can play a role in the late acceleration of the Universe.

On the other hand, brane worlds inspired by M-theory provide modifications to gravity and new equations of cosmic expansion [7]. In the brane world context, gravity lives in the bulk of 3+1+d dimensions while the rest of the interactions are confined to the 3+1 brane. Some braneworld proposals do not explain the observed cosmic expansion. However, it was realized [8, 9] that it is possible to obtain late-time acceleration with one extra dimension. This possibility is interesting as it accounts for the observed cosmic expansion without the need of a cosmological constant term. Acceleration results from the induced gravity term in the brane. The number and size of additional dimensions has been explored in a wide range of proposals. The existence of additional dimensions and their size are constrained by tests of the gravity law [10, 11, 12].

At the time when gravity theories of the scalar–tensor type and others were examined, the solar system tests were the most restrictive ones. Nowadays, the theories have to confront the cosmological observations. The theories have to allow Big Bang nucleosynthesis [13]. They have to fullfil the constraints from the growth of density perturbations and give correct predictions for the evolution of the growth factor. Such growth function of density perturbations requires careful evaluation for several modified gravity scenarios [14, 15]. Ultimately, gravity theories have to be consistent with the CMB data and with the expansion rate of the Universe provided by supernovae.

Supernovae enable to measure the rate of the expansion of the Universe along redshift z. They do so by tracing the luminosity distance along z for a standard candle. If the Universe is flat, as inferred from WMAP [16], the  $d_L$  relation is simply:

$$d_L(z) = cH_0^{-1}(1+z) \int_0^z \frac{dz'}{H(z')}$$
(1)

while in the general case,

$$d_L(z) = cH_0^{-1}(1+z)|\Omega_K|^{-1/2}\sin\left(|\Omega_K|^{1/2}\int_0^z \frac{H_0 dz'}{H(z')}\right)$$
(2)

where sin(x) = sin(x), x, or sinh(x) for closed, flat, and open models respectively, where the curvature parameter  $\Omega_K$ , is defined as  $\Omega_K = \Omega_T - 1$ .

H(z) is the Hubble parameter. For a FRW universe containing matter and a general form of matter-energy X with index of the equation of state  $w_X$ , H(z) is given by:

$$H^{2}(z) = H_{0}^{2} \left\{ \Omega_{K}(1+z)^{2} + \Omega_{M}(1+z)^{3} + \Omega_{X}(1+z)^{3(1+w_{X})} \right\}$$
(3)

where  $H_0$  is the present value of the Hubble parameter and

$$\Omega_K \equiv \frac{-k}{H_0^2 a_0^2} \tag{4}$$

where k accounts for the geometry and  $a_0$  is the present value of the scale factor.

$$\Omega_M = \frac{8\pi G}{3H_0^2} \rho_M \tag{5}$$

 $\Omega_X$  is the density parameter for dark energy.

When w(z) varies along z, H(z) can be expressed as:

$$H(z)^{2} = H_{0}^{2} \left\{ \Omega_{K}(1+z)^{2} + \Omega_{M}(1+z)^{3} + \Omega_{X} \exp \left[ 3 \int_{0}^{z} \frac{1+w(z')}{1+z'} dz' \right] \right\} 6)$$

One sees from (1–2), that  $d_L$  contains w(z) through a double integral. This leads to limitations in recovering w(z) if  $d_L$  data along z are scarce and with large errorbars. Data are often given as m(z) which relates to  $d_L(z)$  as:

$$m(z) = 5 \log d_L(z) + [M + 25 - 5 \log(H_0)] \tag{7}$$

The quantity within brackets is  $\mathcal{M} \equiv M - 5 \log H_0 + 25$ , the "Hubble-constant-free" peak absolute magnitude of a supernova. The data are given sometimes in the form of distance moduli:

$$\mu = 5 \log d_L + 25 \tag{8}$$

The measurement of w(z) through  $d_L(z)$  is the standard approach, though possibilities of measuring directly H(z) have been investigated (for a review see [17]).

This Hubble law in (3) is derived from the Einstein-Hilbert action. We will review other possibilities related to expansion histories arising from modified gravity theories and effective theories which contain scalar fields accelerating the cosmic expansion. In the present overview we concentrate on the empirical evaluation of those theories. Some of the models to be explored here might contain a ghost problem or have a subclass amongst them that present quantum instabilities. We refer for this topic to recent updates in [18, 19, 20, 21, 22].

## 2. Gravity and extra dimensions

The idea of allowing gravity to live in extra dimensions arose to explain the hierarchy problem and constitutes one of the major challenges to the classical picture of a 4–dimensional description of gravity [7]. The Planck scale of  $10^{19}$  GeV, relevant for gravitation, is much larger than the electroweak scale, of about 1 TeV of the standard model of particle physics. According to the proposal by Arkani-Hamed, Dimopoulos and Dvali [7], the Planck scale is not a fundamental scale, but its enormity is simply a consequence of the large size of new extra dimensions. While gravitons can propagate in the new extra dimensions, the standard model fields are confined to our submanifold of 3 + 1 dimensions, the brane, which is embedded in a higher dimensional bulk.

In this scenario, the Planck scale in the bulk can be much smaller than in the 4–dimensional brane as

$$M_{\rm P,4}^2 = R^n M_{\rm P,n+4}^{\rm n+2} \tag{9}$$

The existence of extra dimensions in which gravity propagates produces a transition in the gravitation law at the compactification radius, proposed to be of the submillimetre scale.

In the cosmological context, this idea has some interesting consequences since it predicts a departure from GR. As soon as the cosmological model was implemented for n=1 extra dimensions [23, 24, 9, 8], it was noted that the brane cosmology leads to Friedmann–like equations very different from the standard ones [24]. In the 5–dimensional model by Dvali, Gabadadze and Porrati (DGP) whose cosmology is explored by Deffayet and collaborators in [9], the competition between the five–dimensional term of the Einstein–Hilbert action and the four–dimensional term induced in the brane leads to a modified Friedmann equation which simulates an acceleration term without the need to include a cosmological constant. The transition from the gravity law  $r^{-2}$  to

the modified one is characterized by a scale  $r_c$ , the so-called cross-over scale defined as [9]:

$$r_c \equiv \frac{M_{\rm P}^2}{2M_{(5)}^3}$$
 (10)

At large distances, the five dimensional term takes over and gravity gets weaker [9]. If the parameter  $r_c$  is chosen to be of the order of  $H_0^{-1} \sim 10^{29}$  mm, this is a choice for  $M_{(5)} \sim 10$ –100 MeV. At short distances  $r \ll r_c$ , Newton's law is modified by a logarithmic repulsion. At large distances  $r \gg r_c$ , the modification involves a factor  $\frac{r}{r_c}$ .

The Hubble expansion law shows significant departures from the standard Friedmann expansion law:

$$H^{2}(z) = H_{0}^{2} \left\{ \Omega_{K}(1+z)^{2} + \left( \sqrt{\Omega_{r_{c}}} + \sqrt{\Omega_{r_{c}} + \sum_{\alpha} \Omega_{\alpha}(1+z)^{3(1+w_{\alpha})}} \right)^{2} \right\} (11)$$

where the sum over  $\alpha$  stands for a sum over each component with equation of state  $w_{\alpha}$ ,  $\Omega_{\alpha}$  being the density parameter:

$$\Omega_{\alpha} \equiv \frac{\rho_{\alpha}^{0}}{3M_{\rm Pl}^{2} H_{0}^{2} a_{0}^{3(1+w_{\alpha})}} \tag{12}$$

and

$$\Omega_{r_c} \equiv \frac{1}{4r_c^2 H_0^2} \tag{13}$$

In the matter-dominated epoch, the expansion equation is:

$$H^{2}(z) = H_{0}^{2} \left\{ \Omega_{K}(1+z)^{2} + \left(\sqrt{\Omega_{r_{c}}} + \sqrt{\Omega_{r_{c}} + \Omega_{M}(1+z)^{3}}\right)^{2} \right\}$$
(14)

In the original 5D model in [8],  $M_5$  was estimated to be  $\sim 1$  TeV, which corresponds to a cross-over scale  $r_c$  of  $10^{15}$  cm. Such  $r_c$  led to incompatibilities with the solar system data. When  $M_5$  is taken to be  $M_5 \lesssim 1$  GeV, i.e.  $r_c \gtrsim 10^{25}$  cm, the conflict disappears.

High-z supernovae give useful constraints to the parameter  $\Omega_{rc}$  in (10) [25, 26]. In [26], two samples of high-z supernovae are used in conjunction with measurements from baryon oscillations [27] and the CMB shift parameter [28] to estimate the favored values for  $\Omega_{rc}$  and  $\Omega_{M}$ .  $\Omega_{rc} = 0.125$  for the supernova data in the Riess sample from 2004 [29] and  $\Omega_{rc} = 0.13$  for the SNLS data [30]. It is found in [26] that, for a flat Universe, this 5D modified gravity model gives a worse fit than GR plus cosmological constant. Further supernova samples would reexamine the empirical compatibility of the DGP model and other modified gravity models involving more dimensions.

Those models can be explored through a phenomenological description. The Hubble law arising from gravity with extra dimensions can be expressed by adding  $H^{\alpha}$  terms to the standard Friedmann equation [31]:

$$H^2 - \frac{H^\alpha}{r_c^{2-\alpha}} = \frac{8\pi G\rho_M}{3} \tag{15}$$

where  $r_c$  is the cross-over radius, i.e. the transition radius from a regime to another in the gravity law, and  $\alpha$  is the parameter to be empirically determined. From redshift z=0 to z=2, the above Friedmann equation gives an effective index of the equation of state [31]:

$$w_{\text{eff}} \approx -1 + 0.3\alpha \tag{16}$$

Such effective equation of state evolves too fast to be compatible with present SNe Ia data, unless  $\alpha$  is small. Present data favor  $\alpha$  close to 0 [25].

Though the most simple implementations lack empirical confirmation, room is left for several models. Cosmological SNe Ia are useful tests in restricting the cross—over radius that relates to departures from GR.

A different kind of model in which the brane is embedded in a 5D bulk has been proposed by Randall and Sundrum [32, 33]. Contrary to the previous approach, here the bulk has a  $AdS_5$  geometry as opposed to the flat geometry of the DGP model.

In the original Randall–Sundrum (R–S) model, the brane has tension  $\sigma$ , i.e. cosmological constant, as well as the bulk  $\Lambda_5$ . The bulk cosmological constant is negative. The resulting Hubble expansion in the brane is the result of the compensating effect of both terms. To make zero the effective cosmological constant in the brane,  $\Lambda_4$ , a fine tuning between the brane tension and the bulk cosmological constant is required. The original proposal sets the effective cosmological constant,  $\Lambda_4$ , to zero. Generally, a tiny deviation from exact compensation [34] would lead to an effective cosmological constant at late times. Modifications of the original R–S proposal include in the brane, in addition to tension (cosmological constant), a matter–energy density  $\rho$ . The total brane density  $\rho_B$  is:

$$\rho_B = \sigma + \rho \tag{17}$$

This gives the modified Friedmann equation [35]:

$$H^{2} = \left(\frac{\kappa^{4}}{36}\sigma^{2} - \frac{1}{l^{2}}\right) + \frac{\kappa^{4}}{18}\sigma\rho + \frac{\kappa^{4}}{36}\rho^{2} + \frac{\mathcal{C}}{a^{4}}$$
(18)

where  $\kappa$  is the inverse Planck mass  $M_{\rm P}^{-1}$  and l is related to  $\Lambda_5$ .

Generalizations of the Randall–Sundrum scenario in cosmology consider the  $AdS_5$  bulk geometry containing the fifth–dimensional cosmological constant  $\Lambda_5$ , but explore arbitrary densities in the brane and in the bulk.

Explorations of braneworld ideas that allow a late acceleration of the Universe can be found in [36, 37, 38].

## 3. The dark energy scalar field and scalar–tensor gravity

## 3.1. Early developments

Before the development of braneworld scenarios, dark energy as associated to a scalar field was considered in analogous way to research done within the context of inflation [40]. This theoretical possibility was entertained a decade before there was evidence of a non-zero cosmological constant from supernovae and that the CMB results indicated a flat Universe. The favored  $\Omega_T = 1$  value by inflation and the evidence that  $\Omega_M < 1$  led to think about a possible  $\Lambda$  term. In the recent expansion history a scalar field could play a role in analogy with inflation, where a scalar field dubbed the inflaton gives rise to the exponential phase of early acceleration. The scalar field responsible for the late acceleration, later named quintessence, is proposed as a new massless scalar field which induces a cosmological constant behavior at late times.

In this original setting, the field is minimally coupled to gravity, i.e., thus the inclusion of the field remains within the framework of GR. The interaction of the field with the matter fields is set to be negligible.

The scalar field responsible for dark energy can be identified by recovering its potential, or equivalently its effective equation of state. The presence of such scalar field  $\phi$ , would induce an action:

$$S = \int d^4x \sqrt{-g} \left[ \frac{1}{2} g^{\mu\nu} \partial_{\mu} \phi \partial_{\nu} \phi - V(\phi) \right]$$
 (19)

The field contributes to the stress-energy momentum tensor with an effective mass density and pressure:

$$\rho_{\phi} = \frac{1}{2}\dot{\phi}^2 + V(\phi)$$

$$p_{\phi} = \frac{1}{2}\dot{\phi}^2 - V(\phi)$$
(20)

In this case, the effective value of the equation of state  $w_{\text{eff}}$  depends on the form of the potential  $V(\phi)$  and it can evolve with time [39].

$$w_{\text{eff}} = \frac{p_{\phi}}{\rho_{\phi}} \tag{21}$$

A search for general properties of this potential, in analogy to the search for the inflation potential, leads to suggestions for potentials with tracking behavior [40, 41, 42, 43]. Several potentials have been proposed, from the original  $V = \kappa/\phi^{\alpha}$  [40] to other recent proposals [44].

The dark energy scalar field that does not couple with matter has the feature of avoiding effects on variations of constants and other effects subject to experimental evidence [45, 46]. There is already a reconstruction of what could be the potential of the minimally coupled field  $V(\phi)$  obtained from the SNLS SNe Ia [47] following reconstruction ideas given in [48, 49]. Another reconstruction of  $V(\phi)$  using SNe Ia from various collaborations and considering Padé approximant expansions of the potential is found in [50]. By now, many proposals have been discarded, since the equation of state between z=0 and z=1.5 evolves very slowly as shown by high–z SNe Ia data [2, 51, 29, 52].

## 3.2. Scalar-tensor gravity

One can still consider the case of a scalar field that is coupled to other matter fields or to gravity. Some of these proposals come from the ballpark of superstring theory.

The most well–known scalar–tensor gravity proposal, i.e. Brans–Dicke theory, incorporated the coupling of a field to gravity to account for the Mach induction on Newton's constant. In Brans–Dicke gravity [53, 4], Newton's constant is replaced by a dynamical field G leading to the generalized Friedmann equation:

$$H^{2} = \frac{8\pi}{3} \frac{\omega}{2\pi\phi^{2}} \left( \rho + \frac{1}{2}\dot{\phi}^{2} + V(\phi) - \frac{3}{2\omega} H\phi\dot{\phi} \right)$$
 (22)

The Brans–Dicke action contains a  $\phi^2 R$  term:

$$S = \int \sqrt{-g} \ d^4x \left[ \pm \frac{1}{8\omega} \phi^2 R \mp \frac{1}{2} g^{\mu\nu} \phi_{,\mu} \phi_{,\nu} - V(\phi) + \mathcal{L}_{\text{matter}} \right]$$
 (23)

The theory has been well tested [5]. Observationally, for other scalar–tensor theories, we aim at retrieving the self–interaction potential of the scalar  $V(\phi)$ . and the coupling function to matter and gravity  $A(\phi)$  in the matter term of the action [54]:

$$S_{\text{matter}}(\psi, A^2(\phi)g_{\mu\nu}) \tag{24}$$

In the case of quintessence (minimally coupled field)  $A(\phi) = 1$ . The present supernova data have explored the potential for a minimally coupled field [47]. In the non-minimally coupled case, also referred to as coupled quintessence or coupled dark energy [55, 56], the CMB data can put constraints on the strength of scalar gravity as compared to ordinary tensorial gravity [55]. However, there are limitations to the recovery of the form of the coupling and the coupled dark energy potential from CMB observations [55]. Supernovae together with observational constraints on the growth of density perturbations would allow to retrieve  $A(\phi)$  [57, 58].

We expand  $A(\phi)$  in its derivatives  $\alpha_0$ ,  $\beta_0$  and higher orders:

$$\ln A(\phi) \equiv \alpha_0(\phi - \phi_0) + \frac{1}{2}\beta_0(\phi - \phi_0)^2 + \mathcal{O}(\phi - \phi_0)^3$$
 (25)

By exploring the empirical evidence it is found [54] that solar system tests and binary pulsar tests allow to impose precise bounds on the first and second derivatives of the matter–scalar coupling function, while SNe Ia could "a priori" allow to reconstruct the full shape of the function  $A(\phi)$  or the higher order derivatives. It is argued in [57, 58, 59] that the knowledge of the luminosity distance and the density fluctuations  $\delta \rho/\rho$  as functions of redshift z is sufficient to reconstruct the potential  $V(\phi)$  and the coupling function  $A(\phi)$ .

These results can be generalized to proposed actions that can be re–expressed as a theory of interacting matter fields in general relativity [60]. Actions containing  $e^{-\Phi}R$  are motivated from the lowest order effective action including the dilaton  $\Phi$  [61]. They can be brought by redefinition of the field to an Einstein frame in which the field is minimally coupled to the metric.

A number of interesting cases have been examined and have been expressed in terms of a scalar field which is minimally coupled to the metric, but nonminimally coupled to the other fields [62, 63, 64]. The self–interaction potential shows an explicit energy transfer between the scalar field and the matter fields.

Related to the previous exploration is that of f(R) gravity or extended curvature gravity [65], also known as "c-essence" when including an inverse power of R [67]. Those modified gravity theories with a Lagrangian containing an arbitrary function of R are equivalent to a particular class of scalar-tensor theories of gravity. For instance, the action

$$S = \frac{1}{2\kappa^2} \int d^4x \sqrt{-g} \left( R - \frac{\mu^4}{R} \right) + S_{\text{matter}}(g_{\mu\nu})$$
 (26)

is found to be equivalent to [67]:

$$S = \frac{1}{2\kappa^2} \int d^4x \sqrt{-g} \left( \left( 1 + \frac{\mu^4}{\phi^2} \right) R - \frac{2\mu^4}{\phi} \right) + S_{\text{matter}}(g_{\mu\nu})$$
 (27)

The equivalence is generalized to an arbitrary function of R [60, 65]:

$$S = \frac{1}{2\kappa^2} \int d^4x \sqrt{-g} F(R) + S_{\text{matter}}(g_{\mu\nu})$$
 (28)

with an equivalent action [60, 65]:

$$S = \frac{1}{2\kappa^2} \int d^4x \, \sqrt{-g} \, (F(\phi) + F'(\phi)(R - \phi)) + S_{\text{matter}}(g_{\mu\nu})$$
 (29)

where  $F'(\phi) = dF/d\phi$ .

The introduction of a canonical scalar field  $\varphi$  such that  $F'(\phi) = \exp(\sqrt{2/3} \kappa \varphi)$ , allows to rewrite the action with F(R) as:

$$S = \int d^4x \sqrt{-g_E} \left( \frac{1}{2\kappa^2} R_E - \frac{1}{2} (\nabla \varphi)^2 - V(\varphi) \right) + S_{\text{matter}} (g_{\mu\nu}^E / F'(\phi(\varphi)),$$

$$V(\varphi) = (\phi(\varphi) F'(\phi(\varphi)) - F(\phi(\varphi))) / 2\kappa^2 F'(\phi(\varphi))^2$$
(30)

The gravity described by  $g_{\mu\nu}^E$  is the Einstein–scalar system and  $g_{\mu\nu}$  (=  $g_{\mu\nu}^E/F'(\phi)$ ).  $g_{\mu\nu}$  is a metric of scalar–tensor gravity which is, in this case, subject to constraints coming from solar system experiments. Again, a classification of when one recovers an action equivalent to the Einstein–Hilbert case and when one recovers a Brans–Dicke–type action in f(R) gravity is given in [60]. When considering a general function of R, it is not always possible to obtain a description equivalent to Einstein GR plus additional fields minimally coupled to the metric but coupled to other fields. When this is possible, we have in the matter term the coupling between the matter fields  $\psi$  and the scalar field  $\phi$ . Examples of f(R) casted in terms of fields can be found in [62, 64, 66, 67].

For the Starobinsky model [66] with  $F(R) = R + R^2/M^2$  with  $M \sim 10^{12}$  GeV, for instance, the effective potential can be rewritten, in terms of the scalar field  $\varphi$ , as

$$V(\varphi) = \frac{M^2 e^{-2\sqrt{2/3} \kappa \varphi}}{8\kappa^2} (e^{\sqrt{2/3} \kappa \varphi} - 1)^2$$
 (31)

Actions with generalized inverse powers of R have been investigate as well [68, 69, 70, 71]. The supernovae gathered up to now are compatible with some f(R) gravity models and incompatible with others [68, 70, 72].

More general scalar–tensor theories than the ones here defined, such as those where the potential depends on several scalar fields  $V(\phi^n)$  and matter–scalar coupling  $A(\phi^n)$ , give rise to a wider phenomenology.

In the superstring scenario, nonminimal coupling of the scalar fields to the spacetime curvature leads to effective actions with similarities to scalar–tensor proposals. Some scalars present in the effective action arise from compactification to four dimensions. Those are structure moduli which do not couple directly with the spacetime curvature tensor. One modulus field  $\sigma$  associated with the overall size of the internal compactification couples to Einstein gravity via Riemann curvature invariants, such as the Gauss–Bonnet invariant [74]. If one considers the Gauss–Bonnet term, at least an extra function, the coupling to the Gauss–Bonnet term,  $W(\sigma)$  needs to be determined. In the following, we review attempts to include such enlarged action and their test with cosmological  $\mathrm{d}_L$  data.

3.2.1. Effective low energy action with  $R_{GB}$  and other terms. The Gauss-Bonnet term can be found in the effective low energy string Lagrangian and in brane theories [73, 74, 75, 35, 76]. The case of the simplest action containing the Gauss-Bonnet term can be written as:

$$S = \int \sqrt{-g} \left\{ \frac{R}{2\kappa^2} - \frac{1}{2} (\partial_\mu \phi)^2 \right\} - \int \sqrt{-g} W(\sigma) R_{GB}^2 + S_{\text{matter}}[\text{matter}; g_{\mu\nu}]$$
(32)

This Lagrangian includes the Gauss–Bonnet term coupled to a field  $\phi \equiv \sigma$ , for instance the modulus field in [74] where  $W(\sigma)$  is the coupling of the modulus field to the Gauss–Bonnet term,  $R_{GB}$ .

$$R_{GB}^2 \equiv R^{\mu\nu\rho\sigma} R_{\mu\nu\rho\sigma} - 4R^{\mu\nu} R_{\mu\nu} + R^2 \tag{33}$$

The possible coupling of a scalar field to the Gauss-Bonnet topological invariant can be constrained if one takes into account cosmological supernovae and solar system tests. From supernova data gathered in [29] and with a parameterized analysis of the evolution, it is found that the present value of the equation of state  $w_0$  is below -1. Though for the measurement of the index of the equation of state at z=0 one awaits a large enough nearby supernova sample, the results presented in [29, 52] move to examine the possibility of  $w_{\text{eff}} < -1$ . In the original exploration of the idea [77], a Universe which evolves towards  $w_{\text{eff}} < -1$  would suffer a super-aceleration phase, that has come to be called the Big Rip. From an action which includes the Gauss-Bonnet term, one can obtain  $w_{\text{eff}} < -1$  for a canonical scalar field (positive kinetic energy) [78, 35]. It avoids the Big Rip for  $w_{\text{eff}} < -1$ . To add a word on this, this is not the main motivation for

Gauss–Bonnet gravity, since variations of the braneworld scenario can also get  $w_{\text{eff}} < -1$  and avoid the Big Rip [79, 80, 81]. The interest on the GB term is justified by its presence in a variety of gravity proposals from string and M–Theory. Recently some tests using supernovae have already been done [82]. The extra GB term opens naturally many possibilities for the evolution of the scale factor. The supernova test succeeds for a range of effective low energy actions including a Gauss-Bonnet term [82].

3.2.2. Fifth force and equivalence principle constraints The fields that interact with the dark energy scalar field are constrained by a large number of tests [45]. The possibility that the dark energy scalar couples to nucleons and photons has been investigated in relation to implications for a fifth force [83, 84]. Within current scenarios of particle physics such as MSSM and MSUGRA the role of this dark energy field shows some observational signatures of the fifth force type as well [85]. For instance, the interaction of the dark energy scalar field  $\phi$  with the Higgs doublet  $H_{\rm u}$  and  $H_{\rm d}$  leads to an action where the field couples differently to each of the Higgs  $\psi_{\rm u}$  and  $\psi_{\rm d}$  and one expects a violation of the weak equivalence principle. In the string scenario, where radion, dilaton and moduli can be responsible for the early and late acceleration, one has as well some possible strong couplings of the fields with matter [86, 75, 87]. Thus, the cosmological context has to be considered together with classical gravitational tests on Earth. Thus far, the torsion balance experiments set strict limits on the variation of Newton's law using different substances and locations. There is a wide range of tests being done to examine the predictions from scalar—tensor proposals and modified gravity ideas [12, 10, 11, 88, 89]. The empirical bounds on some types of couplings are tight and worth considering together with the cosmological probes.

## 4. Cosmological constant?

Finding that the negative pressure component gives w = -1, would likely be the biggest challenge to gravitation theories. At present, SNe Ia have not provided such evidence, though averaged values of w are close to -1 at redshifts up to  $z \sim 1$ . To probe that w stays -1 up to the surface of last scattering, involves refining several other methods (growth of density perturbations, integrated Sachs-Wolfe effect) [90].

From the view point of string theorists [91], there is no natural explanation for the vanishing or extreme smallness of the vacuum energy. A way to address the small value of the cosmological constant in the context of a stringy landscape is to consider the anthropic principle [92]. However, invoking the anthropic principle as a way out of the problem entails to abandon the quest for a conventional scientific explanation [91].

From the observational point of view, the answer is near to w = -1, but possible departures from w = -1 are presently being examined through expansions of the equation of state along the scale factor and other various approaches. A expansion

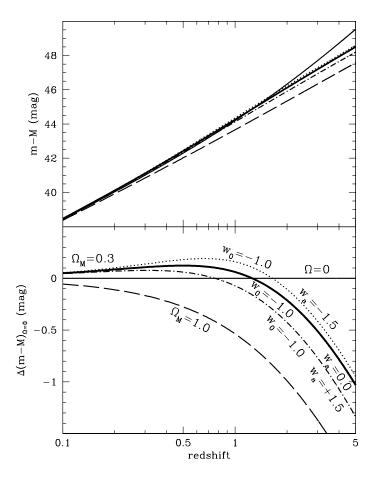


Figure 1. The cosmological constant case (bold line) is compared with evolving models close to w = -1, i.e. a model with  $w_0 = -1.0$  and  $w_a = -1.5$  (short dash line) and a model with  $w_0 = -1.0$  and  $w_a = +1.5$  (dash-dotted line)

often used is [93, 94]:

$$w(a) = w_0 + w_a(1 - a) (34)$$

where the scale factor  $a = (1+z)^{-1}$ , so  $w' = w_a/2$  when evaluated at z = 1.

Differences between the cosmological constant and departures from it in the equation of state are measurable nowadays (see Fig 1). To reach a higher level of discrimination between dark energy models depends on the progress made in the accuracy of the measurements. The aim of future projects is to determine distances with 1 % accuracy, i.e. with errors in magnitudes of 0.02 mag [95]. If the equation of state of dark energy is very close to -1, one can anticipate decades of improving the supernova measurements. The development that we might witness could be akin to that of the postnewtonian tests of GR where higher levels of precision are being targeted since the middle of the last century into the present.

## 5. The $d_L$ test from SNe Ia

#### 5.1. Overview

A long path has brought us to the present situation where we can already aim at accuracies of 1% in distance. Back in 1968, a first attempt to draw the Hubble diagram from SNe resulted in a spread of  $\sigma \sim 0.6$  mag [96]. Such large scatter was due to the inclusion of other types of supernovae, which were later recognized as being of a different type (SNI b/c) and characterized by a large scatter in luminosities [97].

SNe Ia are not standard candles either, but they follow a well known correlation between maximum brightness and rate of decline of the light curve. In 1977, Pskovskii [98] first described the correlation between the brightness at maximum and the rate of decline of the light curve: the brighter SNe Ia have a slower decline of their light curves whereas fainter ones are the faster decliners. A systematic follow—up of SNe Ia confirmed the brightness—decline relation [99, 100]. The intrinsic variation of SNe Ia is written as a linear relationship:

$$M_B = M_{B0} + \alpha \left( \Delta m_{15}(B) - \beta \right) + 5 \log(H_0/65) \tag{35}$$

 $\Delta m_{15}$ , the parameter of the SNe Ia light curve family, is the number of magnitudes of decline in 15 days after maximum. The value of  $\alpha$ , as well as the dispersion of that relation, have been evaluated in samples obtained from 1993 to the present [100].  $M_{B0}$  is the absolute magnitude in B for a template SNIa of  $\beta$  rate of decline. An intrinsic dispersion of 0.11 mag in that law is found when information in more than one color is available.

In fact, the 0.11 mag scatter in the brightness—rate of decline is found when one includes supernovae affected by extinction. The scatter is smaller in environments where dust in the host galaxies of the SNe Ia is a lesser effect [101]. Additionally, the family of SNe Ia form a sequence of highly resembling spectra with subtle changes in some spectral features correlated with the light curves shapes [102, 103, 104, 105, 106, 107, 108]. The multiple correlations allow to control errors within the family of SNe Ia [108].

The parameter  $\Delta m_{15}$  requires to have observed the maximum in the light curve of the supernova. This is not always possible, and therefore the various SN collaborations have formulated the brightness-decline correlation in different ways. The High-Z SN Team uses the full shape of the light curve with respect to a template, method refered as MLCS2k2 [109, 110]. The Supernova Cosmology Project collaboration, on the other hand, has introduced the *stretch-factor*, s, as a parameter to account for the brightness-decline relationship [111, 2, 112]. The stretch parameter fits the SNIa light curve from premaximum to up to 60 days after maximum. Thus, even if the maximum brightness phase is not observed, the whole light curve provides the decline value. More recently, other ways to measure the decline have been proposed [113, 114, 115].

Measuring the evolution of the rate of expansion of the Universe does not require knowledge of its present value,  $H_0$ . Large projects have been devoted to the determination of  $H_0$  by attempting the absolute calibration of  $M_B$  from nearby SNe

Ia that occur in galaxies with Cepheid–measured distances [116, 117]. Controversy persists, though the most recent values quoted by the two collaborations using this method are closer (in the range 65–75 km s<sup>-1</sup> Mpc<sup>-1</sup>) [116, 117]. Though largely ignored as an argument, the mere fact that a SNIa is a explosion of white dwarf bounds very efficiently the lower and upper possible values of  $H_0$  [118]. On the empirical side, the emission at late phases of SNe Ia gives a precise value of  $H_0$ , since such emission is very sensitive to the density profile, the mass of the supernova and the amount of <sup>56</sup>Ni synthesized in the explosion. Those quantities are easily measurable from the spectra [119]. The value favored by this method is  $H_0 = 68 \pm 6$  (syst) km s<sup>-1</sup> Mpc<sup>-1</sup>.

This disgresion done, we emphasize the *nuisance role* of the absolute magnitude of SNe Ia when it comes to dark energy cosmology. The apparent magnitude of supernovae along z give us the cosmology,  $m_B$  as [2, 51]:

$$m_B = \mathcal{M} + 5\log \mathcal{D}_{\mathcal{L}}(z; \Omega_M, \Omega_\Lambda) - \alpha(s-1)$$
(36)

where s is the stretch value for the supernova,  $\mathcal{D}_{\mathcal{L}} \equiv H_0 d_L$  is the "Hubble-constant-free" luminosity distance,  $d_L$  is given by (1) and  $\mathcal{M} \equiv M_B - 5 \log H_0 + 25$  is the "Hubble-constant-free" B-band peak absolute magnitude of a s = 1 SN Ia with true absolute peak magnitude  $M_B$ . Similar use of the nuisance parameter in their light curve fitting is made in [29, 52].

In the first determinations of dark energy cosmology, there were four parameters in the fit: the matter density parameter  $\Omega_M$ , the cosmological constant density parameter  $\Omega_\Lambda$ , as well as the two nuisance parameters,  $\mathcal{M}$  and  $\alpha$  (either  $\alpha$  itself as used by the SCP or a equivalent parameter by other collaborations). The four-dimensional  $(\Omega_M, \Omega_\Lambda, \mathcal{M}, \alpha)$  space, explored as a grid, yields  $\chi^2$  and  $P \propto e^{-\chi^2/2}$  of the fit of the luminosity distance equation to the peak B-band magnitudes and redshifts of the supernovae. After normalization by integrating over the "nuisance" parameters, the confidence regions in the  $\Omega_M$ - $\Omega_\Lambda$  plane can be obtained.

This use of the magnitude–redshift relation m(z) as a function of  $\Omega_M$  and  $\Omega_{\Lambda}$  with a sample of high–z SNe Ia of different z, is a very powerful pointer to the allowed values in the  $\Omega_M$ – $\Omega_{\Lambda}$  plane [120, 121]. By 1998, such use yielded important results, impling that  $\Omega_{\Lambda} > 0$  at the  $3\sigma$  confidence level. For a flat universe  $(\Omega_T = 1)$ , the results from the Supernova Cosmology Project meant  $\Omega_M$ =0.28 $^{+0.09}_{-0.08}$ (statistical) $^{+0.05}_{-0.04}$  (systematics), and the High–Z Supernova Team obtained for a flat universe  $\Omega_M$ =0.24±0.1. The outcoming picture of our universe is that about 20–30% of its density content is in matter and 70–80% in cosmological constant. According to the allowed  $\Omega_M$  and  $\Omega_{\Lambda}$  values, the Universe will expand forever, accelerating its rate of expansion.

## 5.2. Results on w

If the size of the SNe Ia sample and its redshift range do allow it, one can aim at determining the average value of the index of the equation  $\langle w \rangle$  or, going a step further, aim at determining the confidence regions on the  $w_0$ –  $w_a$  plane. For this later purpose,

using the information on the global geometry as given by WMAP provides a helpful prior.

In Knop et al. (2003) [51], the supernova results were combined with measurements of  $\Omega_M$  from galaxy redshift distorsion data and from the measurement of the distance to the surface of last scattering from WMAP. The confidence regions provided by the measurement of the distance to the surface of last scattering and the SNe Ia confidence regions cross in the  $\Omega_M$ -w plane, thus providing very good complementarity. This work considers an averaged  $\langle w \rangle$  (given the range in z of the SNe Ia observed). From the fitting assuming w constant, the value is  $\langle w \rangle = -1.06^{+0.14}_{-0.21} \text{(statistical)}^{+0.08}_{-0.08} \text{ (identified systematics)}$ .

The analysis of the SNLS [30] also aims at an average value of w, allowed by the z range of this survey. Their 71 high–z SNeIa give  $\langle w \rangle = -1.023 \pm 0.090$  (statistical)  $\pm$  0.054 (identified systematics). Recently, the results from ESSENCE, using 60 SNe Ia centered at z= 0.5 point to a similar result with  $\langle w \rangle = -1.05^{+0.13}_{-0-12}$  (statistical)  $\pm$  0.13 (systematics) [122]. In this last work, the supernovae from various collaborations are joined with two separate light curve fitting approaches giving a consistent result for the average value of w.

High–z SNe Ia beyond z > 1 enable to have a first look at possible evolution of w(z). The first campaign by Riess et al. (2004) gave the results in the  $w_0$ –w' plane, using the Taylor expansion for w(z) as  $w(z) = w_0 + w'z$ . The best estimates for  $w_0$  and w' are  $w_0 = -1.36^{+0.12}_{-0.28}$  and  $w' = 1.48^{+0.81}_{-0.98}$ . Those results [29, 52] exclude a fast evolution in the equation of state, and therefore they rule out a number of modified gravity proposals. Moreover, the results are confirmed in [122] where the available SNe Ia at large z, which can constrain  $w_0$  and  $w_a$ , are incorporated to test with a joined sample encompassing a wide range of z.

Significant limits have been placed on the evolution of w(z). Now the potential of the method relies on an improved control of systematic effects. In this area, there has been substantial progress along the last years.

#### 5.3. Results on systematic uncertainties

Huge advances have taken place in controlling the uncertainties due to extinction by dust, the universality of the supernova properties, the control of lensing by dark matter and the observational process leading to the the construction of the Hubble diagram of SNe.

5.3.1. Dust The study with the HST of the host galaxies of the SNe Ia sample of the Supernova Cosmology Project [101] allowed to subclassify the galaxy hosts of the P99 sample [2]. It was found that, at high z, early—type galaxies show a narrower dispersion in SNe Ia properties than late—type galaxies, as they do at low z ([123]).

This result is very encouraging as it supersedes previous dispersion values obtained in samples of supernovae in all galaxy types. Supernovae in dust–rich environments,

such as spiral galaxies, are more affected by extinction. The supernova magnitudes are corrected regularly from Galactic extinction [124], but the correction by dust residing in the host galaxy or along the line of sight is often not included in the standard fit, as it would require to have extensive color information that might not be available for high–z SNe Ia. When it is included, it is still subject to the uncertainty in the extinction law [110].

Let us, for instance, consider the correction of dimming by dust as entering in the term  $A_B$ , the magnitudes of extinction in the B band. We take, for instance, a supernova of a given stretch s, and we place its  $m_{\text{eff}}$  (the magnitude of the equivalent supernova of s = 1) in the Hubble diagram. We might use, as in [2, 51]:

$$m_{\text{eff}} = m_B + \alpha(s-1) - A_B \tag{37}$$

The extinction in the canonical band B of the spectrum  $(A_B)$  is determined by the extinction coefficient  $R_B$  in that band. Any uncertainty in this coefficient as due to variation of dust properties, leads to uncertain estimates of  $A_B$ .  $R_B$  is of the order of  $4.1 \pm 0.5$  as properties of dust might change going to high z. If we have an error of 0.1 in the observable E(B-V) due to dust extinction, it gets amplified as:

$$A_B \equiv R_B \times E(B - V) \tag{38}$$

The present way of dealing with SNe Ia, including information in several photometric bands of the spectrum where extinction coefficients  $(R_V, R_I)$  are lower and one gets smaller values in the corresponding  $A_V$  and  $A_I$ , allows to control the dust problem. In fact, the coefficients  $R_B$ ,  $R_V$  and  $R_I$  are also determined by methods determining distances to SNe Ia. In a sample of 133 nearby SNe Ia, it is found in [110], that the dust in their host galaxies is well described by a mean extinction law with  $R_V \simeq 2.7$ . In all this, the level at which SNe Ia are treated, including how many bands are used, becomes critical. A use of SNe Ia without taking into account color information leads to a dispersion of 0.17mag. When taking into account color information, the limit goes down to 0.11mag. Limiting the effect of extinction, as in SNeIa in ellipticals should bring down the dispersion. This finding is motivating the ongoing SN searches in clusters of elliptical galaxies [125]. In environments where dust is controlled, such as SNe in clusters rich in elliptical galaxies, the systematic effect introduced by dust extinction can be better controlled. There, one can also control the evolution of dust-properties. In Table I, the level of accuracy in the first uses of SNe Ia back in 1998 is shown, with the contributions of each systematic and statistical error. The improvement is given for a future mission able to control the systematics in the best possible way. It is foreseen that several sources of systematic errors will become negligible within the next decade.

5.3.2. Evolution or Population drift This question resides on whether we know that SNe Ia properties evolve with z. Much has been done along this path. Nowadays, while we know that dust is the unavoidable systematics, several studies confirm the universality of the light curve properties of SNe Ia. The study of a significant sample

of SNe Ia at various z by the Supernova Cosmology Project reveals that the rise times to maximum of high–z SNe Ia are similar to the low–z SNe Ia ([126, 112]). The same is found in the SNLS sample ([127]). Statistical evaluation is so far consistent with no difference between the low–z and high–z samples.

Moreover, the possibility of existence of an extra parameter in the maximum brightness–rate of decline relation has been carefully examined. Among possible influences, metallicity was one of the obvious ones to consider. Examining supernovae in galaxies with a gradient of metallicity, it is found [128] that there is no evidence for metallicity dependence as an extra parameter in the light curve correlation of SNe Ia. This result comes from an analysis of the SNe Ia light curves of a sample of 62 supernovae in the local Universe. The SNe Ia belong to different populations along a metallicity gradient. This check has been done as well at high–z [129] using 74 SNe Ia (0.17 < z < 0.86) from the SCP sample. No significant correlation between peak SNIa luminosity and metallicity is found.

Often we address the question of the spread of the SNe Ia samples in galaxies of different types: on how many fast SNe Ia versus intermediate or slow decliners are found in the various morphological types. Spirals at low or moderate redshifts should encompass all ranges of variation in the SNe Ia properties since they contain populations with a wide spread in age. The cosmological SN collaborations find that in those samples the one–parameter correlation does give a good description of the variation of the SNe Ia light curves: there is no residual correlation after the light–curve shape correction [112]. One can go a step further and test the SNIa objects themselves. This has been attempted in a intermediate z sample and it is a feasible project at all z [108]. The physical diagrams of SNe Ia at intermediate z and at low z seem to be similar [108]. Those diagrams reveal the composition, velocity gradient and radiation properties of SNeIa [108]. Looking at the inside of the SNe Ia is the best way to test whether they are the same at all z. Such step forward is giving definitive results which reaffirm the validity of the SNe Ia method. The progress done in this domain is summarized in Table I.

5.3.3. K-corrections There is another element yet to be introduced here: it deals with the K correction. The K-correction is necessary to calculate the apparent magnitude in a y filter band of a source observed in a x filter. As we are comparing the observed flux of supernovae in a given spectral band along the expansion history, we need to take into account the effect that the photons of the supernova have shifted to redder wavelengths and spread over a different spectral wavelength range and the observed flux is different from that at emission.

$$m_Y(z) = m_X(z=0) + K_{xy}(z)$$
 (39)

where

$$K_{xy} = 2.5 \log \left\{ (1+z) \frac{\int F(\lambda) S_x(\lambda) d\lambda}{\int F[\lambda/(1+z)] S_y(\lambda) d\lambda} \right\} + \mathcal{Z}_y - \mathcal{Z}_x$$
 (40)

where  $F(\lambda)$  is the source spectral energy distribution (a SN in this case), and  $S_x(\lambda)$  is the transmission of the filter x. The  $\mathcal{Z}_y - \mathcal{Z}_x$  term accounts for the different zero points of the filters [130]. At present, this term amounts to an uncertainty of 0.02mag in the overall budget of SNeIa as distance indicators, but it can be decreased to 0.01 mag. The comparisons of supernovae with the fine time sequences of spectra along all phases available, are turning the uncertainty in the K-correction into a decreasingly small figure. The large amount of spectral data on SNe Ia, and the good correlation with photometric properties, can indeed make this uncertainty very small.

- 5.3.4. Gravitational lensing of SNe Ia Gravitational lensing causes dispersion in the Hubble diagram for high redshift sources. There was an early concern on the systematics and bias that this effect could cause in the determination of the cosmological parameters using SNeIa [131]. It has been shown through the study of lensing in the highest z SNe Ia sample [132] that the magnification distribution of SNeIa matches very well the expectations for an unbiased sample, i.e., their mean magnification factor is consistent with unity. The effect can be very well controlled by studying the galaxy fields [132]. Moreover, SNe Ia should show a negligible systematics caused by gravitational lensing when having a few SNeIa per redshift bin [122]. This means that in future missions that effect should not have impact in limiting the accuracy of w(z).
- 5.3.5. The limit of SNeIa as calibrated candles The better we deal with the light curve and spectral information of SNe Ia, the lower becomes the dispersion in the Hubble diagram. If SNe Ia have a systematic error in their use (independently from evolution), this is likely well below the accuracy to which we would like to determine distances (1%). We have not yet hit that brick wall. Using multi-epoch spectral information and accurate light curves in various filters should enable to test the power of these candles and their limiting irreduceable error [108].
- 5.3.6. A joined sample Supernovae collected along years show the effect of observations treated with various degrees of accuracy. The dispersion around the best fit reflects that fact. For instance, some samples would have information on only two colors and at very few epochs, making it difficult to control reddening, or supernovae had no spectrum confirming that they were SNe Ia. The new samples will naturally have a higher degree of accuracy in magnitude and extinction for each supernova than previous samples. The ones with poorer information have been placed in [29, 52] into the "silver category (as opposed to "gold" category). However, even within the Gold sample the requirements have changed: the latest definition of Gold sample demands higher quality standards than in [29]. In some of these SNe Ia lists, distance moduli of supernovae are assembled but they have been obtained through procedures which deal differently with dust extinction. The Hubble diagram resulting from those compilations would naturally show a large dispersion. In [133], it is argued that a few supernovae of an early High Z SN Search sample are pulling w(z) towards evolution. The exploration

done with a method independent from that in [29] showed that the central value in the  $1\sigma$  confidence varied according to the light curve and reddening fitter of those SNe Ia [134]. Fortunately, supernovae enable to compare results obtained with different light curve fitting approaches. The published photometry by the various collaborations can be used to bring together the SNe Ia by using a consistent procedure into the Hubble diagram. This was done for the SNe Ia considering available up to 2005 in [134]. This is done in [122, 135, 136] for samples including the latest data from the Higher Z collaboration.

## 6. Testing the adequacy of a FRW metric

In most of the above discussion on dark energy, the FRW metric is taken as the right metric for our Universe. Modern cosmology supports that the Universe is highly isotropic on average on high scales. The main evidence is coming from the statistical isotropy of the Cosmic Microwave Background radiation. There are also cosmological observations supporting a matter distribution homogeneous on large scales, of the order of 100 h<sup>-1</sup> Mpc. Given the observed structures of matter in voids and filaments, it is worth considering whether departures from the homogeneous and isotropic Universe could alter the present discussion, in particular, the possibility that there were no need for dark energy, but instead, that the observations are the result of an effect of inhomogeneity [137].

As pointed out in [138], large samples of supernovae will soon allow to test the luminosity distance relation in various directions and redshift bins. One can consider the luminosity distance  $d_L$  as a function of direction  $\mathbf{n}$  and redshift z [138].

The direction–averaged luminosity distance is:

$$d_L^{(0)}(z) = \frac{1}{4\pi} \int d\Omega_{\mathbf{n}} dL(z, \mathbf{n}) = (1+z) \int_0^z \frac{dz'}{H(z')}$$
(41)

This is the usual value retrieved by the SN collaborations. But its directional dependence  $d_L(z, \mathbf{n})$  can be a test of the validity of the isotropy assumption. The authors in [138] propose to expand the luminosity distance in terms of spherical harmonics, to obtain the observable multipoles  $C_l(z)$ . This idea was also investigated in [109], where the motion of our Local Group with respect to the CMB was measured. Results in [138] are in agreement with the results in [109].

Moreover, we can think of using the dispersion in the magnitude of SNe Ia in each direction  $\mathbf{n}$  to put constrains on anisotropic models.

As discussed in [139], the dispersion of the magnitude in the Hubble diagram is at odds with some simple inhomogeneous models that predict a higher spread in m(z) and also a non–linear behavior.

The anisotropic and inhomogeneous models have to confront present and future values on the SNeIa dispersion at each redshift interval  $z_{bin}$  and in every direction **n**:

$$\frac{\sigma(d_L(z_{bin}, \mathbf{n}))}{d_L(z_{bin}, \mathbf{n})} = \frac{\ln(10)}{5} \sigma(m(z_{bin}, \mathbf{n}))$$
(42)

The scaling solutions of the regionally averaged cosmologies can simulate the presence of a quintessence field or other negative pressure component [137, 142, 141]. But, any backreaction from inhomogeneity and anisotropy would show in the dispersion in the Hubble diagram. Those challenges have to be met by the non–FRW proposals.

Up to now, the exploration of toy models calculating the effect of inhomogenities in the scale factor, find irregularities in the expansion rate beyond what is observed in the SN Ia Hubble diagram. The dispersion in the Hubble diagram of a standard candle is increased [139].

The situation concerning the predictions of inhomogeneous models [137, 140, 141] is summarized in [142]. It is concluded that the inhomogeneities that would be able to amount to a  $d_L(z)$  effect as observed from supernovae must be subhorizon and of strong type and they require a treatment beyond first—order perturbation theory. Research in this field will continue towards quantitative predictions to improve the connection with observational cosmology [143].

## 7. Next decade experiments

The aim of discriminating between various possibilities for dark energy places stringent requirements on the use of standard candles [144]. Within the two decades from the discovery (1998) and the gathering of space mission data, it is feasible to reduce largely the errors, as mentioned before, and this would reflect into an error of only 1%– 2% in  $d_L(z)$ .

Errors much bigger than this 1% of error in  $d_L$  result in the impossibility to distinguish between models. Even at the level of 1% in  $d_L(z)$  one finds some degeneracy [144, 145]. Space projects aimed at determining the nature of dark energy are counting on a systematic error in the method around 0.02 mag (i.e. 1% in the distance) [95].

A different question is whether from ground, within the planned experiments at work in the present decade, one can converge towards that limit.

These questions have been investigated in detail by using Monte Carlo simulations including the systematic errors in m(z) (or  $d_L(z)$ ) to be found in the evaluation of the present value of the equation of state of dark energy  $w_0$  and its first derivative in the scale factor  $w_a$  [95] (see expression 33).

SNAP is a mission that has been specifically designed to gather about 2000 SNe Ia up to  $z \sim 1.7$ . As a single mission gathering supernova data with the same instrumentation, it can eliminate possible photometric offsets and include high–precision photometry out to 1.7 microns[146].

With such large number of SNe Ia, statistical errors in every redshift bin of  $\Delta z = 0.1$  become negligible. The error is dominated by the systematics. The total error in each redshift bin is given by:

$$\sigma_{bin} = \sqrt{\frac{\sum \sigma_{st}^2}{N_{bin}} + \sigma_{sys}^2} \tag{43}$$

with expected systematic error;

$$\sigma_{sys} = 0.02 \ z/z_{max} \tag{44}$$

Table I gives a progress evaluation in the control of high–redshift supernova distance uncertainties.

Systematic Uncertainties $(1\sigma)$	Magnitude	Statistical Uncertainties $(1\sigma)$	Magnitude
Photometric System Zero Point	0.05	Zero Points, S/N	0.17
Evolution	< 0.17	K-corrections	0.03
Evolution of Extinction Law	0.02	Extinction	0.10
Gravitational Lensing	0.02	Light curve sampling	0.15

PROSPECTS TOWARDS 1% ERROR IN  $d_L(z)$  in a space mission

Systematic Uncertainties $(1\sigma)$	Magnitude	Statistical Uncertainties $(1\sigma)$	Magnitude
Photometric System Zero Point <sup>a</sup>	0.01	Zero Points, S/N	0.00*
Evolution	0.00*	K-corrections	0.00*
Evolution of Extinction Law	< 0.02	Extinction	0.00*
Gravitational Lensing	< 0.01	Light curve sampling	0.00*

<sup>&</sup>lt;sup>a</sup> 0.00\* means values of the order of 10<sup>-3</sup>. The statistical errors are for the redshift bin.

Ground-based projects will not be able to achieve such accuracy. They aim at a reduction of systematic errors to 0.04–0.05 mag. Much of what will be feasible from now until the next space era of determination of the equation of state, depends on the use of all the information already gathered in the best way. Spectra gathered so far can be exploited further than what it has been done up to now, in order to reduce systematic errors.

In the coming years, a lot can be done to improve the present dispersion of the Hubble diagram of SNe Ia and aim at a scatter of 1% in  $d_L$ . The above limit is only reachable if all the systematic effects are dealt with to the best.

#### 8. Complementarity

The SNe Ia test of w(z) benefits from using information on  $\Omega_M$  obtained by some other method. This can come from weak lensing or from large scale structure [147, 148, 149]. Baryon acoustic oscillations can also provide a measurement of distance along z, in this case, angular distances  $d_A(z)$  and of  $E(z) = H(z)/H_0$  [27, 150, 151]. Generally, for any geometry, one has:

$$A = \frac{\sqrt{\Omega_M}}{E(z)^{\frac{1}{3}}} \left[ \frac{1}{z\sqrt{|\Omega_K|}} S\left(\sqrt{|\Omega_K|} \int_0^z \frac{dz'}{E(z')}\right) \right]^{\frac{2}{3}}$$

$$\tag{45}$$

 $<sup>^</sup>b$  In the Table for prospects in a space mission, the statistical error from light curve sampling of SNe Ia is reduced due to the large number of SNe Ia data per redshift bin.

A measurement of  $d_A$  has been obtained [27].

$$A = \frac{\sqrt{\Omega_M}}{E(z_1)^{1/3}} \left[ \frac{1}{z_1} \int_0^{z_1} \frac{dz}{E(z)} \right]^{2/3} = 0.469 \pm 0.017$$
 (46)

where  $z_1 = 0.35$  is the redshift at which the acoustic scale has been measured in the redshift sample.

Combining BAO and SNeIa distances one can aim at tracing dark energy all through a high z range. The value of  $w(z=z_{rec})$  is provided by WMAP and Planck. In between, other methods such as studying the growth of large scale structure (LSS) obtained from Ly $\alpha$  clouds and clusters would depict the growth factor of structure along z. Considering the information at high z derived from LSS, SNe Ia and CMB should lead to learn about early dark energy [152, 153, 90, 148, 154, 155].

#### 9. Conclusions

Type Ia supernovae can test GR against other alternative theories of gravitation. Theories of higher dimensional gravity have different Friedmann equations from those originating from GR. Some of those modified gravity proposals have already been tested and ruled out through the Hubble diagram of SNeIa up to very high z.

If dark energy is due to a scalar field either minimally coupled to gravity (i.e. quintessence) or non–minimally coupled to gravity (arising from scalar–tensor gravity or the effective low–energy superstring proposals), the Hubble diagram up to z>1 obtained by the supernova collaborations gives the opportunity for exhaustive tests. In what concerns scalar–tensor gravity, supernovae are a precision experiment in a way analogous to the observations of the PSR B1913+16 or to solar system tests.

Moreover, supernovae have already provided hints on the form of the potential of the hypothetical scalar field (for the case of minimally coupled field) that could be associated to the acceleration of the Universe.

Nonetheless, GR plus a form of cosmological constant is not ruled out observationally. Finding out how is w close to z=0 is one of the most intriguing questions. Hopefully, this will be clarified by the samples to come. The goal of obtaining a consistent picture on w(z) up to  $z \sim 1.5$  from the present samples is approached through the various SN procedures used by the existing collaborations.

Ultimately, supernovae together with complementary methods just recently discovered shall select the correct ideas on dark energy and gravity. If any modified gravity theory has a cosmological effect, those methods should start to see it.

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