Cooperative Multi-Finger Manipulation of Force-Sensitive Objects via Graph Rigidity

Xinan Rong[1], Changhuang Wan[2], Aochen He[1], Xiaolong Li[1] and Gangshan Jing[1]

***Abstract*— In-hand trajectory tracking with multi-finger dexterous hands is essential for achieving human-like precise manipulation. However, due to the high complexity of multi-finger coordination and limited sensing capabilities, existing methods still face significant challenges in precise and safe manipulation of force-sensitive objects. To address these challenges, we propose a novel manipulation framework based on graph rigidity theory, for coordinated contact force planning of multiple fingers. In the absence of tactile sensing and direct torque control, a dual-layer in-hand manipulation algorithm is proposed, which not only ensures a stable grasp and avoids deformation of the object during the manipulation, but also drives the object to track a desired trajectory precisely. Finally, we validate the effectiveness of our method by manipulating a yarn shape, a plastic cup, and a raw egg, respectively, through a customized dexterous hand platform.**

Fig. 1. Precise in-hand movement of force-sensitive objects: (A) Manipulating a soft yarn while maintaining its shape during the movement process. (B) Manipulating a disposable plastic cup to move the AprilTag along a predefined trajectory; (C) Manipulating a raw egg to move the AprilTag along a predefined trajectory;

I. INTRODUCTION

In-hand manipulation with multi-finger robotic hands is a pivotal topic in robotics, aiming to replicate human dexterity in complex tasks [1], with applications spanning medical procedures, industrial assembly, and assistive technologies for the disabled [2]–[4]. Despite progress, robotic hands still exhibit low manipulation precision due to the complexity of multi-finger coordination and limited perceptual capabilities.

To deal with the multi-finger coordination problem, optimization modeling can be employed to integrate kinematic constraints, trajectory planning, and collision avoidance [5], [6]. Other approaches, such as integrating mechanism analysis with particle filtering, enhance self-recognition and control capabilities [7], [8]. Data-driven methods like reinforcement learning [9], [10] and imitation learning [11] further enable complex dexterous manipulation through policy optimization. However, these works focus only on hand joints control, leaving a significant gap in addressing the precise manipulation of force-sensitive objects, which require meticulous force planning.

Force planning in manipulation traditionally relies on modeling contact constraints [12] and applying force optimization to evaluate feasibility [13]. Learning-based approaches, by contrast, embed force optimization into policy learning [14]. In addition, tactile sensors provide crucial feedback for force control and have gained extensive attention [15], [16]. However, their high cost, complex integration, signal noise, and processing delays limit wide practical deployment. Furthermore, most of the commercial dexterous hands lack torque control [17], [18], which further complicates precise force management.

Assuming certain physical properties of the object are known, we propose a framework for precise in-hand trajectory tracking of force-sensitive objects without relying on tactile sensors or torque control. As illustrated in Fig. 1, our approach integrates visual feedback with dynamics models to estimate and regulate contact forces, enabling real-time adjustment through model-based control. Experiments on a custom-designed dexterous hand show improved tracking accuracy and reduced risk of object damage compared with conventional methods.

The main contributions are: (i) A graph rigidity-based force planning method that ensures stable grasping of force-sensitive objects during dynamic manipulation; (ii) A dual-layer framework that achieves precise in-hand trajectory tracking via penetration-based force control, without requiring torque or tactile sensing.

II. MAIN WORK

A. Dynamics Modeling and Problem Statement

As illustrated in Fig. 2, the goal is to control the manipulation frame $\{\mathcal{M}\}$ so that the object follows a desired

This work was supported in part by the National Key Research and Development Program of China under Grant 2022YFB4701400, the National Natural Science Foundation of China under grant (62203073), and the Natural Science Foundation of Chongqing under Grant (CSTB2022NSCQ-MSX0577). (Corresponding Author: Gangshan Jing)

[1]School of Automation, Chongqing University, rongxinan2023 @163.com, riemhac@163.com, 20222032@stu.cqu.edu.cn, jinggangshan @cqu.edu.cn

[2]Department of Mechanical Engineering, City University of Hong Kong, changwan@cityu.edu.hk

trajectory through multi-finger coordination. The task-space dynamics of the hand are

$$M_c\dot{v}_c + C_c v_c + g_c = -f_c + \left(J_h^{+}\right)^{\top}\tau, \tag{1}$$

where v_c is the fingertip velocity, f_c the contact force, τ the joint torque, and M_c, C_c, g_c denote the task-space inertia, Coriolis, and gravity terms.

The object dynamics are [19]:

$$M_o(x)\dot{\mathcal{V}}_o + C_o(x,\dot{x})\mathcal{V}_o + g_o = Gf_c, \tag{2}$$

where x is the object pose, M_o, C_o, and g_o are the object inertia, Coriolis, and gravity terms, and G is the grasp matrix.

The objective is to design a joint trajectory $q(t)$ such that $\{\mathcal{M}\}$ tracks the desired trajectory $x_d(t)$ within bounded error, while maintaining a stable grasp and avoiding object deformation. We assume point contact without rolling, negligible elastic deformation, and quasi-static dynamics for light or slow motions.

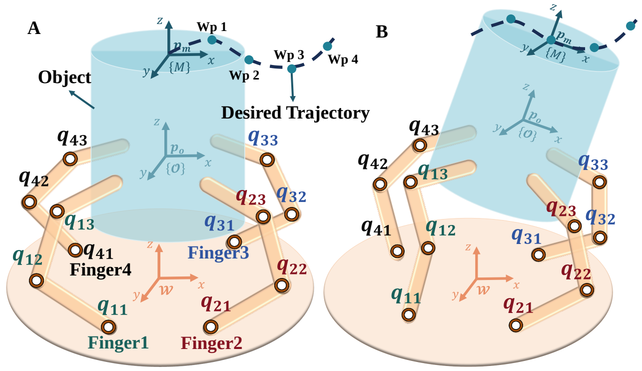

Fig. 2. Precision in-hand object movement: (A) The object moves along a predefined trajectory, with manipulation frame $\{\mathcal{M}\}$ following each waypoint (denoted as $Wp1, Wp2, \ldots$). (B) The motion of $\{\mathcal{M}\}$ at the second waypoint.

B. Rigid Contact Framework

To ensure stable grasp of force-sensitive objects, we introduce a rigid contact framework that preserves the relative distances between fingertips in $\mathbb{R}^3$. The contact framework is represented as a graph $\mathcal{G} = (V, E)$, where vertices $V = \{1, \ldots, m\}$ denote contact points and edges $(i,j) \in E$ encode distance constraints. The rigidity condition can be expressed as

$$R(x(t))v_c(t) = 0, \tag{3}$$

where $v_c(t)$ is the fingertip velocity and $R(x(t))$ is the rigidity matrix derived from $\mathcal{G}$ and the fingertip positions. This enforces that the pairwise distances $\|p_i(t) - p_j(t)\|$ remain constant for all $(i,j) \in E$. Fig. 3 illustrates a four-point example in 3D space.

C. Contact Force Planning and Execution

We achieve stable in-hand manipulation by decomposing the contact force f_c into manipulation force f_{ope}, rigidity internal force $f_{\text{int},R}$, and friction internal force $f_{\text{int},\mu}$, i.e., $f_c = f_{\text{ope}} + f_{\text{int},R} + f_{\text{int},\mu}$, with $f_{\text{int},R}$ derived from our

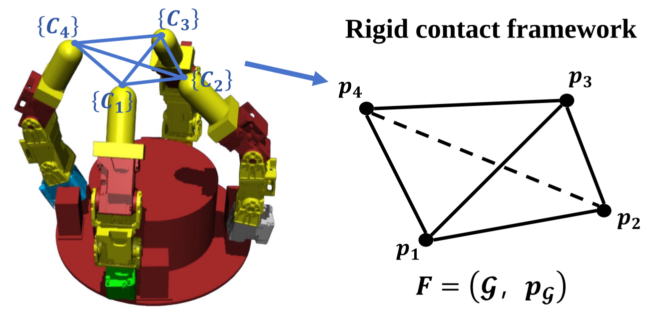

Fig. 3. A graph-based abstraction of geometric constraints among contact points, modeling a rigid configuration that remains invariant during grasp execution.

proposed rigid contact framework and computed via a constrained least-squares formulation, and $f_{\text{int},\mu}$ optimized to satisfy friction cone and admissible force limits.

Each fingertip is assigned a virtual target point, mapping the planned contact forces f_c to fingertip displacements through $D_i = \text{diag}(k_x, k_y, k_z)f_{c_i}$. Finger joint angles are then optimized to track these targets under forward kinematics and joint limits. This approach converts force control into a low-dimensional trajectory tracking problem, enabling precise, stable, and efficient multi-finger manipulation without direct torque control.

III. EXPERIMENT

We conduct three experiments to evaluate the proposed rigid contact model, force control strategy, and object trajectory tracking.

A. Hardware Setup

Experiments are performed on a fully actuated four-finger robotic hand, modified from the LEAP Hand [18] . Each finger shares a uniform kinematic structure with independent actuation. No tactile sensors are used; the system relies on visual feedback from an overhead Intel RealSense D405 camera and joint encoders. AprilTag markers on objects enable accurate pose estimation.

Optimization is solved online using the SLSQP algorithm with analytically derived gradients [6]. Object mass M is measured directly, and friction coefficient μ is set according to material properties.

B. Experiment Design and Results

1) Yarn Shape Manipulation: To validate the rigid contact model, yarn is attached to the fingertips as a visual marker. Fingertip motions tracing a square trajectory are tracked, and configuration changes are quantified by the *Total Relative Deviation* (TRD):

$$\text{TRD} = \sum_{(i,j)\in E} \left| \frac{d_{ij} - d_{0,ij}}{d_{0,ij}} \right| \times 100\%,$$

where $d_{0,ij}$ and d_{ij} denote initial and real-time distances between fingertip pairs. Minimal TRD and negligible yarn deformation (Fig. 4) confirm that the proposed method effectively maintains a rigid grasp.

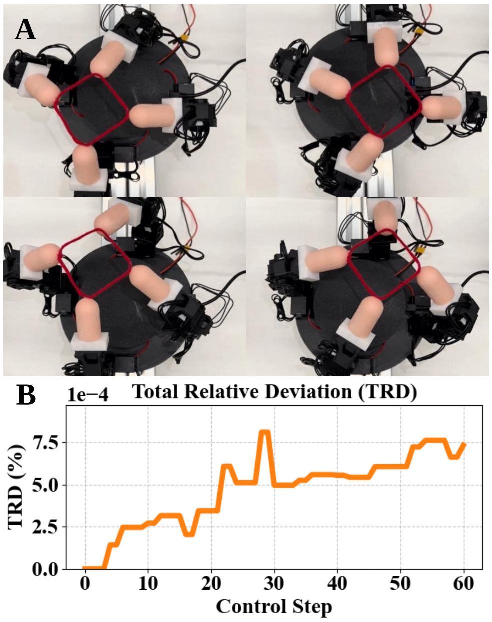

Fig. 4. Rigid contact model verification experiment: (A) Actual hardware execution showing robotic finger motion in real environment; (B) Fingertip relative distance curves evaluating the effectiveness of rigid contact constraints.

2) Plastic Cup Manipulation: A disposable plastic cup with an AprilTag on the rim is manipulated to validate the force control strategy. Quasi-static motion is assumed due to low object velocity, and visual plus joint encoder feedback guides the AprilTag along target trajectories.

For an empty cup ($M = 0.008\,\text{kg}$, $[k_x, k_y, k_z] = [2, 2, 5]$, $\mu = 0.65$), the system tracks two waypoints stably, maintaining cup integrity, while the baseline method [6] causes deformation and unstable grasping. For a water-filled cup ($M = 0.058\,\text{kg}$, $[k_x, k_y, k_z] = [3.5, 3.5, 9]$) and three waypoints, our method still achieves stable grasping and accurate trajectory tracking without slippage (Fig. 5).

3) Egg Manipulation: To validate high-precision trajectory tracking, a raw egg with an AprilTag on top is manipulated to write the letters "RAL". The system controls finger joints in real time to guide the AprilTag through predefined waypoints. The egg mass is $0.0525\,\text{kg}$, with stiffness coefficients $[k_x, k_y, k_z] = [3, 3, 9]$ and friction $\mu = 0.6$.

The results show that the maximum tracking error remains within 0.3 mm, demonstrating precise trajectory control. The execution time between waypoints ranges from 2.79 s to 6.89 s (Fig. 6). The framework is theoretically applicable to arbitrary orientations, as varying gravitational loads do not affect the rigid contact solution.

IV. Conclusion

In this work, we proposed a graph rigidity-based framework for in-hand trajectory tracking with multi-finger dexterous hands, enabling precise and safe manipulation of force-sensitive objects without tactile sensing. Experiments demonstrate stable multi-finger force control. In future work, we will use learning-based models to automatically map contact forces and penetration, and apply online system identification to estimate object parameters, reducing manual tuning and prior dependence on physical properties.

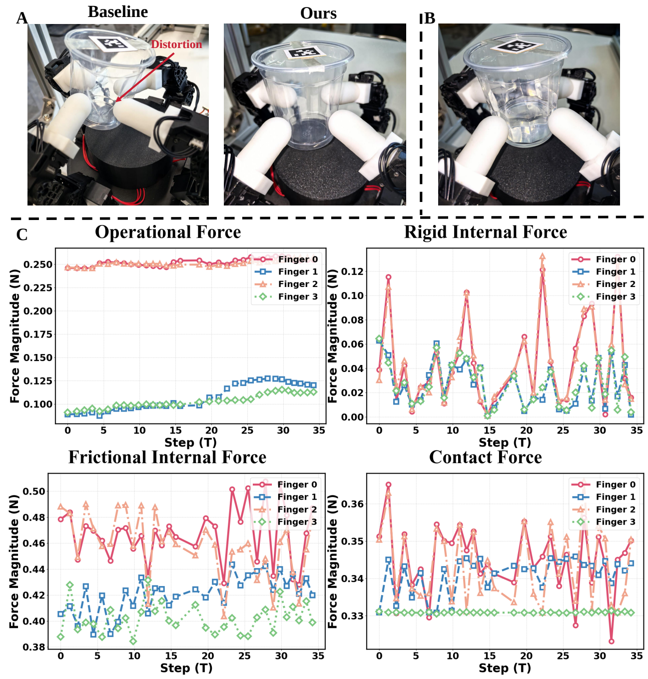

Fig. 5. Force control algorithm validation experiment: (A) Manipulation of an empty disposable plastic cup. The baseline grasps unstably and deforms the cup, while ours remains stable and intact. (B) Safe manipulation of a water-filled cup. (C) Contact forces and its decompositions during the manipulation. The observed discreteness primarily arises from the step-wise nature of MPC and the sparse waypoint setting adopted in the experiments.

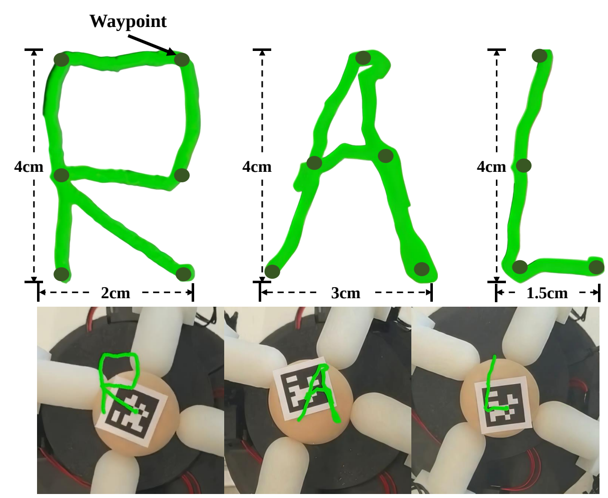

Fig. 6. Trajectory tracking algorithm validation experiment

References

[1] A. Billard and D. Kragic, "Trends and challenges in robot manipulation," *Science*, vol. 364, no. 6446, p. eaat8414, 2019.

[2] R. H. Taylor, A. Menciassi, G. Fichtinger, P. Fiorini, and P. Dario, "Medical robotics and computer-integrated surgery," in *Springer Handb. Robot.*, B. Siciliano and O. Khatib, Eds., 2016, pp. 1657–1684.

[3] M. Kyrarini, M. A. Haseeb, D. Ristić-Durrant, and A. Gräser, "Robot learning of industrial assembly task via human demonstrations," *Autonomous Robots*, vol. 43, pp. 239–257, 2019.

[4] A. M. Cook, B. Bentz, N. Harbottle, C. Lynch, and B. Miller, "School-based use of a robotic arm system by children with disabilities," *IEEE Trans. Neural Syst. Rehabil. Eng.*, vol. 13, no. 4, pp. 452–460, 2005.

[5] B. Sundaralingam and T. Hermans, "Relaxed-rigidity constraints: kinematic trajectory optimization and collision avoidance for in-grasp manipulation," *Auton. Robots*, vol. 43, pp. 469–483, 2019.

[6] M. Yu, Y. Jiang, C. Chen, Y. Jia, and X. Li, "Robotic in-hand manipulation for large-range precise object movement: The rgmc champion solution," *arXiv:2502.07472*, 2025.

[7] K. Hang, W. G. Bircher, A. S. Morgan, and A. M. Dollar, "Manipulation for self-identification, and self-identification for better manipulation," *Science Robotics*, vol. 6, no. 54, p. eabe1321, 2021.

[8] J. T. Grace, P. Chanrungmaneekul, K. Hang, and A. M. Dollar, "Direct self-identification of inverse jacobians for dexterous manipulation through particle filtering," in *Proc. IEEE Int. Conf. Robot. Autom. (ICRA)*, 2024, pp. 13 862–13 868.

[9] M. Andrychowicz, B. Baker, M. Chociej, R. Jozefowicz, B. McGrew, J. Pachocki, A. Petron, M. Plappert, G. Powell, A. Ray *et al.*, "Learning dexterous in-hand manipulation," *Int. J. Robot. Res.*, vol. 39, no. 1, pp. 3–20, 2020.

[10] S. Suresh, H. Qi, T. Wu, T. Fan, L. Pineda, M. Lambeta, J. Malik, M. Kalakrishnan, R. Calandra, M. Kaess *et al.*, "Neuralfeels with neural fields: Visuotactile perception for in-hand manipulation," *Science Robotics*, vol. 9, no. 96, p. eadl0628, 2024.

[11] W. Wei, P. Wang, S. Wang, Y. Luo, W. Li, D. Li, Y. Huang, and H. Duan, "Learning human-like functional grasping for multi-finger hands from few demonstrations," *IEEE Trans. Robot.*, 2024.

[12] M. Buss, H. Hashimoto, and J. B. Moore, "Dextrous hand grasping force optimization," *IEEE Trans. Robot. Autom.*, vol. 12, no. 3, pp. 406–418, 1996.

[13] D. Prattichizzo, M. Malvezzi, M. Gabiccini, and A. Bicchi, "On motion and force controllability of precision grasps with hands actuated by soft synergies," *IEEE Trans. Robot.*, vol. 29, no. 6, pp. 1440–1456, 2013.

[14] W. Hu, B. Huang, W. W. Lee, S. Yang, Y. Zheng, and Z. Li, "Dexterous in-hand manipulation of slender cylindrical objects through deep reinforcement learning with tactile sensing," *Robot. Auton. Syst.*, vol. 186, p. 104904, 2025.

[15] Y. Yan, Z. Hu, Z. Yang, W. Yuan, C. Song, J. Pan, and Y. Shen, "Soft magnetic skin for super-resolution tactile sensing with force self-decoupling," *Science Robotics*, vol. 6, no. 51, p. eabc8801, 2021.

[16] N. F. Lepora, Y. Lin, B. Money-Coomes, and J. Lloyd, "Digitac: A digit-tactip hybrid tactile sensor for comparing low-cost high-resolution robot touch," *IEEE Robot. Autom. Lett.*, vol. 7, no. 4, pp. 9382–9388, 2022.

[17] D. Sharma, K. Tokas, A. Puri, and K. Sharda, "Shadow hand," *J. Adv. Res. Appl. Sci.*, vol. 2208, p. 2352, 2014.

[18] K. Shaw, A. Agarwal, and D. Pathak, "Leap hand: Low-cost, efficient, and anthropomorphic hand for robot learning," *arXiv:2309.06440*, 2023.

[19] M. Pfanne, *In-Hand Object Localization and Control: Enabling Dexterous Manipulation with Robotic Hands*. Springer Nature, 2022, vol. 149.