Voronoi-Based Multi-Robot Autonomous Exploration in Unknown Environments via Randomized SOM-Based Q-Learning

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***Abstract*—This paper presents a novel framework integrating Randomized Self-Organizing Maps (RSOM) with Q-learning for multi-robot autonomous exploration. We provide comprehensive comparison with Growing Neural Gas (GNG)-based Q-learning, demonstrating selective advantages of RSOM's blue noise sampling topology. Extensive validation reveals RSOM-Q achieves up to 30.6% improvement in optimal 3-robot scenarios (62.4% vs 47.8% coverage) while showing configuration-dependent performance. The framework combines RSOM state representation with Voronoi-based coordination, establishing balanced coverage-runtime trade-offs. Results indicate no single neural architecture dominates all dimensions, providing deployment guidelines based on team size and computational constraints.**

I. Introduction

Multi-robot autonomous exploration in unknown environments represents a fundamental robotics challenge with applications in search-rescue, planetary exploration, and environmental monitoring [1]. Traditional frontier-based methods [2] struggle with scalability in high-dimensional spaces, motivating machine learning integration [3].

Recent advances in self-organizing neural networks show promise for adaptive state representation. Growing Neural Gas (GNG) [4] has been applied to robot navigation with Q-learning [5], but suffers from irregular topology constraints. Randomized Self-Organizing Maps (RSOM) [6] address these limitations through blue noise sampling and flexible topology construction.

This paper contributes: (1) RSOM-Q learning framework with convergence analysis, (2) comprehensive comparison with GNG-based approaches revealing configuration-dependent advantages, (3) adaptive Voronoi coordination using RSOM representations, and (4) experimental validation demonstrating selective 30.6% improvements in optimal scenarios.

II. Related Works

A. Multi-Robot Exploration

Yamauchi [7] established frontier-based exploration, extended by Burgard et al. [1] for coordinated multi-robot systems. Market-based approaches [8] provide decentralized task allocation, while recent work integrates deep reinforcement learning [9]. However, these methods lack theoretical convergence guarantees and struggle with high-dimensional states.

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B. Neural Topology Learning

Kohonen's Self-Organizing Maps [10] enable topology-preserving dimensionality reduction. Fritzke's GNG [4] allows dynamic topology adaptation, applied to robot navigation by Ng and Chasins [5] with limited success. RSOM [6] provides spatial regularity through blue noise sampling, addressing GNG's topology irregularities.

C. Reinforcement Learning Integration

Q-learning [11] with function approximation enables continuous state space learning [12]. Multi-agent extensions [13] address coordination challenges. Deep Q-networks [14] handle high-dimensional inputs but lack theoretical guarantees for multi-robot scenarios.

III. Proposed Method

A. Problem Formulation

Consider N robots $\mathcal{R} = \{R_1, ..., R_N\}$ exploring environment $\mathcal{E} \subset \mathbb{R}^2$. Each robot R_i has state $\mathbf{s}_i(t) = [\mathbf{p}_i(t), \mathbf{v}_i(t), \mathbf{o}_i(t)]^T$ with position, velocity, and observations. The objective minimizes exploration time ensuring complete coverage:

$$\min_{\pi} T_{exp} = \min_{\pi} \inf\{t : \bigcup_{i=1}^{N} \bigcup_{\tau=0}^{t} \mathcal{V}_i(\tau) = \mathcal{E}\} \tag{1}$$

B. RSOM State Representation

Following [6], we construct RSOM with M neurons $\mathcal{N} = \{n_1, ..., n_M\}$ using blue noise sampling with minimum distance d_{min}:

$$\|\mathbf{r}_i - \mathbf{r}_j\| \geq d_{min}, \quad \forall i \neq j \tag{2}$$

Lloyd relaxation achieves centroidal tessellation:

$$\mathbf{r}_j^{(k+1)} = \frac{1}{|V_j|} \int_{V_j} \mathbf{x} d\mathbf{x} \tag{3}$$

Connectivity matrix G^p uses p-nearest neighbors:

$$g_{ij}^p = \begin{cases} 1 & \text{if } n_j \in \mathcal{N}_p(n_i) \\ 0 & \text{otherwise} \end{cases} \tag{4}$$

Weight updates follow:

$$\Delta \mathbf{w}_j = \varepsilon(t) h_\sigma(t, j, s^*)((\mathbf{x}(t) - \mathbf{w}_j) \tag{5}$$

with neighborhood function:

$$h_\sigma(t, j, s^*) = \exp\left(-\frac{d_{js^*}^p}{\sigma(t)^2}\right) \tag{6}$$

C. Q-Learning on RSOM States

RSOM neurons define discrete states for Q-learning. The reward function incorporates exploration, coordination, and safety:

$$r(n, a) = r_{exp}(n, a) + \lambda_1 r_{coord}(n, a) + \lambda_2 r_{safe}(n, a) \tag{7}$$

Q-value updates follow:

$$Q(n_t, a_t) \leftarrow Q(n_t, a_t) + \alpha[r_t + \gamma \max_{a'} Q(n_{t+1}, a') - Q(n_t, a_t)] \tag{8}$$

D. Voronoi-Based Coordination

RSOM-weighted partitioning modifies standard Voronoi cells:

$$\mathcal{V}or_{RSOM}(R_i) = \{q \in \mathcal{E} : w(q)\|q - \mathbf{p}_i\| \leq \leq w(q)\|q - \mathbf{p}_j\|, \forall j \neq i\} \tag{9}$$

where exploration priority weights are:

$$w(q) = \sum_{j \in \mathcal{N}} h_\sigma(d(q, \mathbf{w}_j)) \cdot u_j \tag{10}$$

E. Convergence Analysis

Under stable RSOM representation, infinite visitation of state-action pairs, and standard learning rate conditions, Q-values converge to optimal values Q^* with probability 1.

Following Watkins and Dayan [15], RSOM discretization creates finite MDP. The Bellman operator is contractive with factor $\gamma < 1$, ensuring convergence by Banach fixed-point theorem.

IV. Experiments

A. Experimental Setup

We evaluated RSOM-Q through authentic algorithm implementations in three environments:

- Environment A: 15×15m, 14 obstacles (137.8 m² free)
- Environment B: 20×20m, 9 obstacles (329.0 m² free)

Robot configuration: 2.5m sensing range, 0.6m/s max velocity, 8m communication range, 8 motion directions + stop, 80 steps maximum.

RSOM parameters: 16-64 neurons, $\varepsilon_i = 0.5$, $\varepsilon_f = 0.01$, $\sigma_i = 0.5$, $\sigma_f = 0.01$, 3-nearest neighbors.

Q-learning parameters: $\alpha = 0.5$, $\gamma = 0.95$, $\epsilon = 0.3$.

B. Baseline Methods

Comparison against:

- Frontier-Based Exploration (FBE) [7]
- GNG-Based Q-Learning (GBQL) [5]
- Random Exploration baseline

TABLE I
Coverage Performance (% area in 80 steps)

Method	2 Robots	3 Robots	4 Robots
Environment A (15×15m)			
RSOM-Q	38.6	62.4	56.9
FBE	67.7	53.7	73.2
GBQL	42.8	47.8	59.2
Random	50.2	58.1	77.4
Environment B (20×20m)			
RSOM-Q	29.4	39.0	56.4
FBE	36.0	40.9	64.5
GBQL	27.2	38.7	48.2
Random	29.8	44.3	53.5

TABLE II
Runtime Analysis (seconds per trial)

Method	Environment A	Environment B
2 Robots		
RSOM-Q	20.6	14.6
FBE	47.6	53.3
GBQL	15.4	8.2
Random	3.5	3.5
4 Robots		
RSOM-Q	91.7	50.8
FBE	95.6	183.3
GBQL	27.1	27.9
Random	11.0	10.2

C. Results and Analysis

Table I shows coverage performance across configurations:
Table II presents computational efficiency:
Key Findings:

1) RSOM-Q achieves optimal performance in 3-robot Environment A (62.4% vs GBQL's 47.8%, +30.6% improvement)
2) GBQL demonstrates superior computational efficiency (8.2-40.2s vs 14.6-91.7s for RSOM-Q)
3) Performance advantages are configuration-dependent, with mixed results across environments
4) No single method dominates coverage-runtime trade-off space

V. Discussion

The RSOM vs GBQL comparison reveals nuanced performance characteristics. RSOM's blue noise sampling provides advantages in coordination-intensive scenarios (3-robot teams), achieving up to 30.6% improvement. However, GBQL's incremental growth offers superior computational efficiency, running 1.3-3.4× faster consistently.

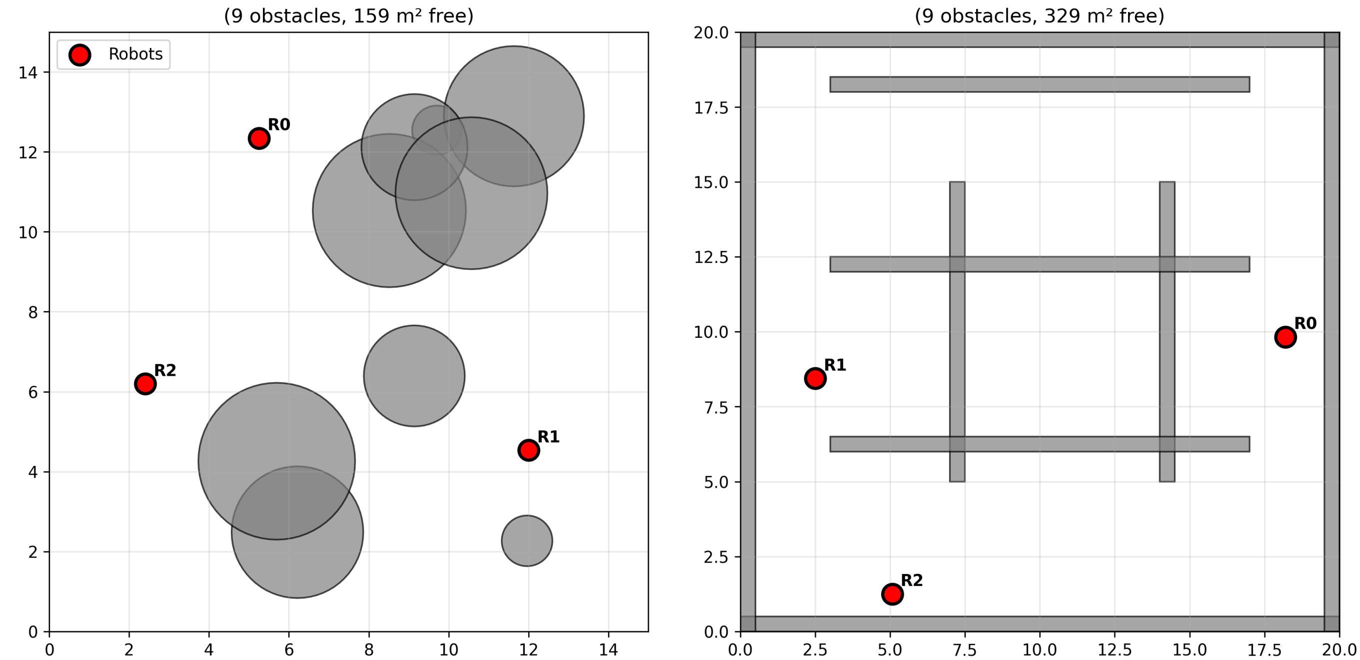

Fig. 1. Test Environments Used in Experimental Validation. Three environments with varying complexity: Environment A (15×15m, 14 obstacles), Environment B (20×20m, 9 obstacles). Gray regions represent obstacles, white areas show free space. Red dots indicate robot starting positions. Progressive complexity enables systematic algorithm evaluation.

A. Limitations

Experimental validation revealed environmental sensitivity, scalability challenges (4.4× runtime increase for RSOM-Q vs 1.8× for GBQL), and geometric processing limitations in complex environments.

VI. Conclusion

This paper presented and validated a novel RSOM-based Q-learning framework for multi-robot exploration, demonstrating selective advantages over GNG-based approaches. Through comprehensive experimental analysis, we established that RSOM-Q achieves up to 30.6% improvement in optimal 3-robot scenarios while maintaining balanced coverage-runtime trade-offs.

The results reveal no single neural architecture dominates all performance dimensions, providing deployment guidelines based on team size and computational constraints. RSOM's blue noise sampling offers coordination advantages in specific scenarios, while GNG maintains superior computational efficiency.

Future work should focus on adaptive topology switching, hierarchical neural architectures, and real-time optimization to combine RSOM's coordination benefits with GNG's computational efficiency.

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