Critical points and approximation

1 Sketch a function

Sketch the graph of a function (any function you like, no need to specify a functional form) that is:

a. Continuous on $[0, 3]$ and has the following properties: an absolute minimum at 0, an absolute maximum at 3, a local maximum at 1 and a local minimum at 2.
b. Do the same for another function with the following properties: 2 is a **critical number** (i.e. $f'(x) = 0$ or $f'(x)$ is undefined), but there is no local minimum and no local maximum.

2 Find critical values

Find the critical values of these functions:

a. $f(x) = 5x^{3/2} - 4x$
b. $s(t) = 3t^4 + 4t^3 - 6t^2$
c. $f(r) = \dfrac{r}{r^2 + 1}$
d. $h(x) = x \log(x)$

3 Find absolute minimum/maximum values

Find the absolute minimum and absolute maximum values of the functions on the given interval:

a. $f(x) = 3x^2 - 12x + 5, [0, 3]$
b. $f(t) = t\sqrt{4 - t^2}, [-1, 4]$
c. $s(x) = x - \log(x), [1/2, 2]$
d. $h(p) = 1 - e^{-p}, [0, 1000]$

4 A function with no local minima/maxima

Demonstrate that the function $f(x) = x^5 + x^3 + x + 1$ has no local maximum and no local minimum.

5 Approximate root-finding

Show that the equation

$$x^7 - 6x + 4 = 0$$

has a root between 0 and 1.

a. Find an initial approximation by ignoring the term x^7.

b. Use Newton's method to find the root correct to 3 decimal places.

6 Apply the mean value theorem

Does a continuous, differentiable function exist on $[0, 2]$ such that $f(0) = -1$, $f(2) = 4$, and $f'(x) \leq 2 \; \forall x$? Use the mean value theorem to explain your answer.