PHY499: Introduction to Python for Scientists

Homework Assignment 7 (18 Nov, 2016)

(NOTE: This assignment will be graded!)

Send your code to `michael.mommert@nau.edu`.

1 Bacterial Growth

A biology undergrad has to finish an experiment for their research project late at night. In the experiment, a bacterial culture is implanted into a petri dish. The student's task is to measure the area covered by bacteria as a function of time and to determine that point in time when 90% of the petri dish is covered with bacteria.

The students starts the experiment and measures the area occupied with bacteria to be 0.74% after one minute, 1.1% after 5 minutes, 1.8% after 10 minutes ... and then the student falls asleep. Waking up about two hours later, the whole petri dish is covered with bacteria and there is no time left to redo the experiment...

Help the student to derive the time at which 90% of the area of the petri dish is covered with bacteria by fitting a population model to the three data points available. Bacterial growth can be approximated using a logistic growth model:

$$A(t) = \frac{A_0}{1 + \exp(-k(t - t_{1/2}))},$$

where $A(t)$ is the area covered at time t, A_0 the total area available in the petri dish, k the growth coefficient, and $t_{1/2}$ the time at which 50% of the area is occupied.

Task: Fit this logistic growth model to the three available data points, plot the growth model for the first 120 minutes, and derive the time at which 90% of the petri dish is occupied by bacteria.

Hint: Use the fact that at some point the petri dish is entirely covered with bacteria, i.e., $A_0 = 1$ and $A(t)$ is simply the fractional area that is covered with bacteria.

2 Flying on a Plane (a Monte-Carlo simulation)

Monte-Carlo simulations, which use repeated random sampling to obtain numerical results, are useful to simulate complex systems. Consider the following case:

Imagine, you are on a 10 hr flight across the Atlantic Ocean. The plane has 200 seats, but not all of them are equally desirable. 10 of the seats are close to the toilets, 10 are seats where the entertainment system does not work, and 10 seats are broken in some way (assume that only one of the above applies to each seat). And then, of course, there are other people on the plane: 20 people smell funny, 30 people talk all the time, and 10 people are flaming supporters of that political party that you dislike and they love to talk about it (again, assume that no single person has more than one of these features). All passengers, including yourself are distributed randomly across the plane.

Task: Use a Monte-Carlo simulation to calculate the probability that you will have a pleasant flight, i.e., none of the above applies to your seat.

Hint: Simplify the situation: assume that the plane seats are numbered sequentially (seat 199 is next to seat 0). Let's say seats 0–10 are next to the toilets, etc. (we can use this simplification, since people are assigned to seats randomly). In an array with 200 elements, the element index can be used as the seat number. Next, generate an array of numbers from zero to 200 representing the passengers and use the `numpy.random.permutation()` function to scramble them across the seats. Define yourself as passenger '0'. In order to find if you have a seat near the toilet for one modeling run, check if '0' is in the first ten elements of the permutated seat allocation (since we defined seats 0–10 to be near the toilets). Do this over many runs, and you will get the probability that you have a seat near the toilet (the number of runs must be large enough so that if you repeat the simulation, you get pretty much the same result). For the final result, calculate the probability that you get assigned a pleasent seat - and then multiply that with the probability that you have a pleasant seat neighbor.

Bonus Task: Determine the probability for the worst case scenario: you have a bad seat, one bad neighbor, and your other neighbor is that one German guy that won't stop complaining about something for the entire flight.

If you want this assignment to count into your final grade, please submit it to `michael.mommert@nau.edu` before 2 Dec, 23:59!