# Report: comparison of 9 global optimization methods on several test problems classes

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## 1 List of the algorithms

- Algorithm of global search (AGS) (`https://github.com/sovrasov/ags_nlp_solver`)
- Multi Level Single Linkage (MLSL) (`https://nlopt.readthedocs.io/en/latest/NLopt_Algorithms/#mlsl-multi-level-single-linkage`)
- DIRECT (`https://nlopt.readthedocs.io/en/latest/NLopt_Algorithms/#direct-and-direct-l`)
- Locally-based DIRECT (DIRECT$l$) (`https://nlopt.readthedocs.io/en/latest/NLopt_Algorithms/#direct-and-direct-l`)
- Dual Simulated Annealing (`https://github.com/sgubianpm/sdaopt`)
- Differential Evolution (`https://docs.scipy.org/doc/scipy/reference/generated/scipy.optimize.differential_evolution.html#scipy.optimize.differential_evolution`)
- Controlled Random Search (`https://nlopt.readthedocs.io/en/latest/NLopt_Algorithms/#controlled-random-search-crs-with-local-mutation`)
- Simple (`https://github.com/chrisstroemel/Simple`)
- StoGO (`https://nlopt.readthedocs.io/en/latest/NLopt_Algorithms/#stogo`)

All parameters of the methods can be found in *experiments/solve_different_methods.py* script. Since NLOpt hasn't an API to control parameters of the algorithms from Python, it was built with $\varepsilon = 10^{-4}$ for DIRECT and DIRECT$l$ methods.

## 2 List of the test problems

- Functions from $F_{GR}$ class. It consists of 100 multi-extremal problems of the same structure. The description can be found in `https://core.ac.uk/download/pdf/82313177.pdf`.
- Functions from classes generated by the GKLS generator (`http://wwwinfo.deis.unical.it/yaro/GKLS.html`).

Each class consists of 100 multi-extremal problems with 10 and more local minima. Problem is considered solved when optimization method placed a new trial point in the $\Delta$-vicinity of the known global optima $x^*$: $\|x^* - \widetilde{x}\|_{\text{inf}} \leq \Delta$.

## 3 Results on the $F_{GR}$ class

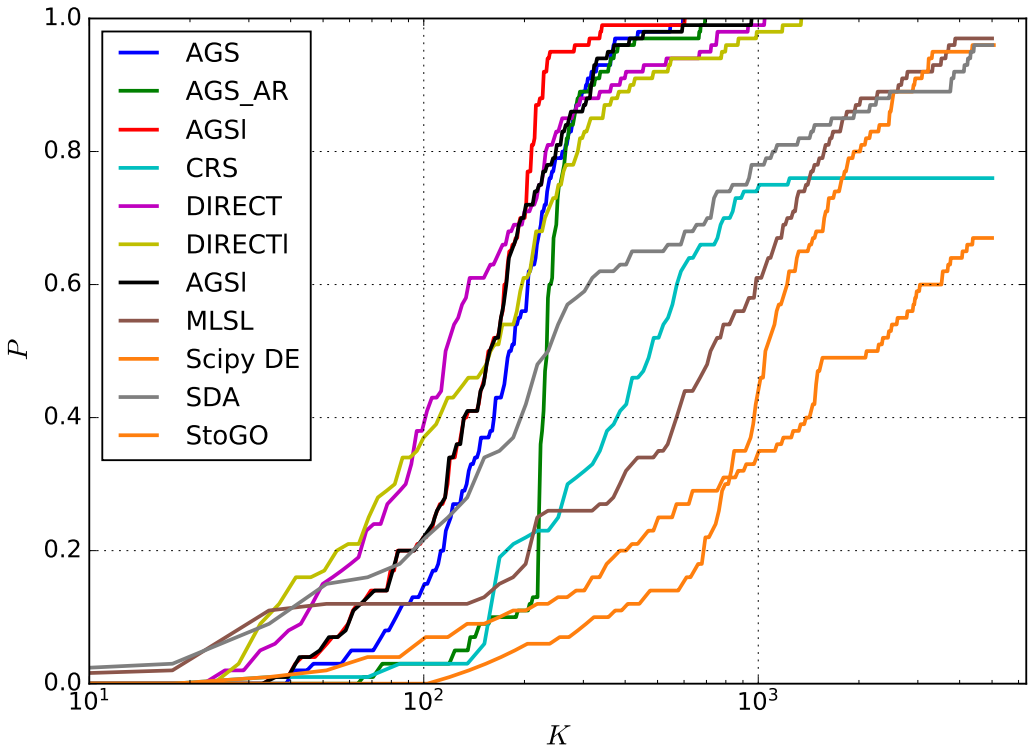


Figure 1: $\Delta = 10^{-2}$

| Method | Average number of trials | Problems solved |
|---|---|---|
| AGS | 193.11 | 100 |
| AGS_AR | 248.29 | 100 |
| AGSl | 158.30 | 100 |
| AGSl | 180.14 | 100 |
| CRS | 400.30 | 76 |
| DIRECT | 182.25 | 100 |
| DIRECTl | 214.92 | 100 |
| MLSL | 947.18 | 97 |
| SDA | 691.24 | 96 |
| Scipy DE | 1257.34 | 96 |
| StoGO | 1336.78 | 67 |

## 4 Results on the $GKLS$ problems

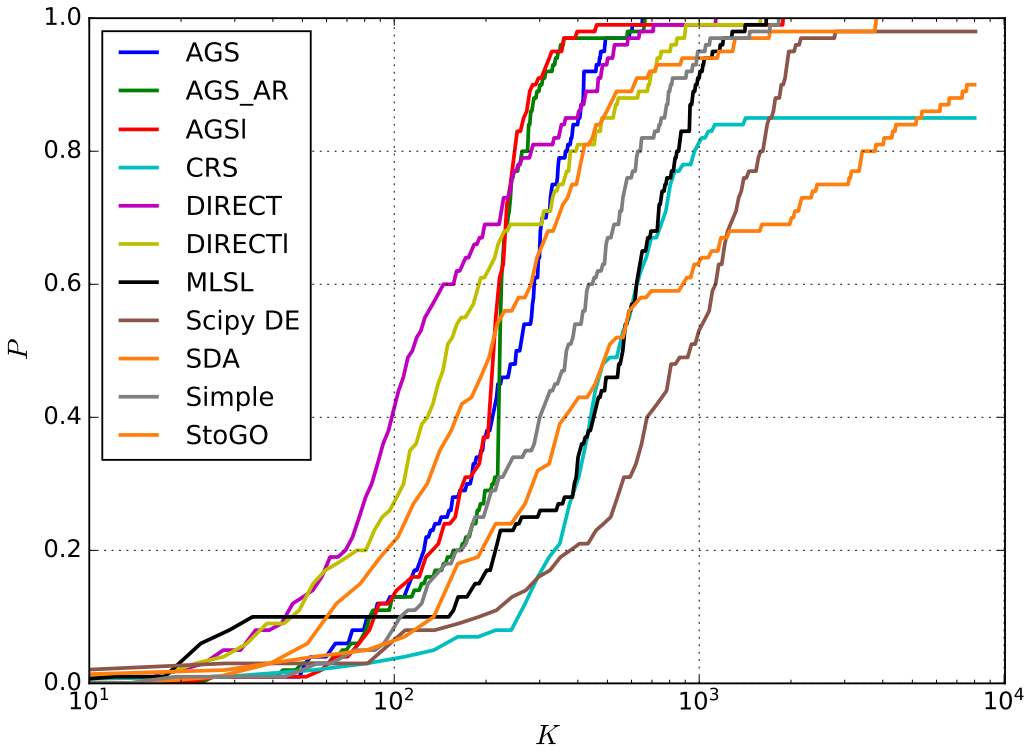


Figure 2: Class GKLS Simple 2d. $\Delta = 2 \cdot 10^{-2}$

| Method | Average number of trials | Problems solved |
|---|---|---|
| AGS | 254.89 | 100 |
| AGS_AR | 221.63 | 100 |
| AGSl | 217.60 | 100 |
| CRS | 510.61 | 85 |
| DIRECT | 189.03 | 100 |
| DIRECTl | 255.21 | 100 |
| MLSL | 556.83 | 100 |
| SDA | 356.30 | 100 |
| Scipy DE | 952.16 | 98 |
| Simple | 440.63 | 100 |
| StoGO | 1251.52 | 90 |

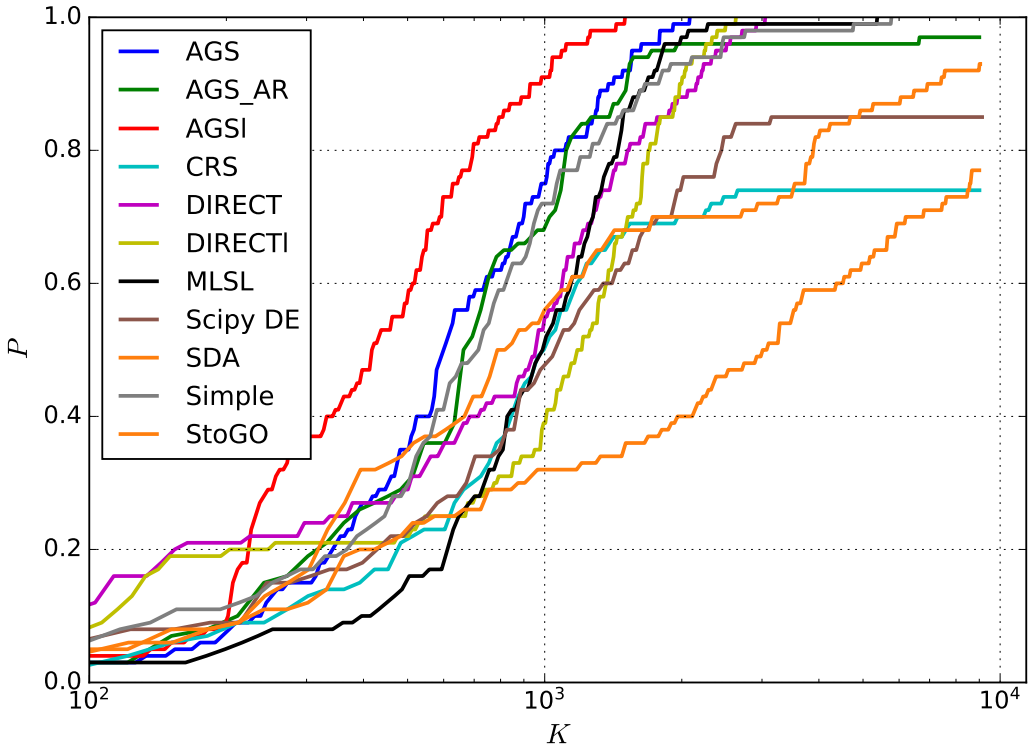


Figure 3: Class GKLS Hard 2d. $\Delta = 2 \cdot 10^{-2}$

| Method | Average number of trials | Problems solved |
|---|---|---|
| AGS | 728.71 | 100 |
| AGS_AR | 784.97 | 97 |
| AGSl | 487.96 | 100 |
| CRS | 844.74 | 74 |
| DIRECT | 985.44 | 100 |
| DIRECTl | 1126.65 | 100 |
| MLSL | 1042.54 | 100 |
| SDA | 1637.92 | 93 |
| Scipy DE | 1041.12 | 85 |
| Simple | 898.19 | 100 |
| StoGO | 2532.23 | 77 |

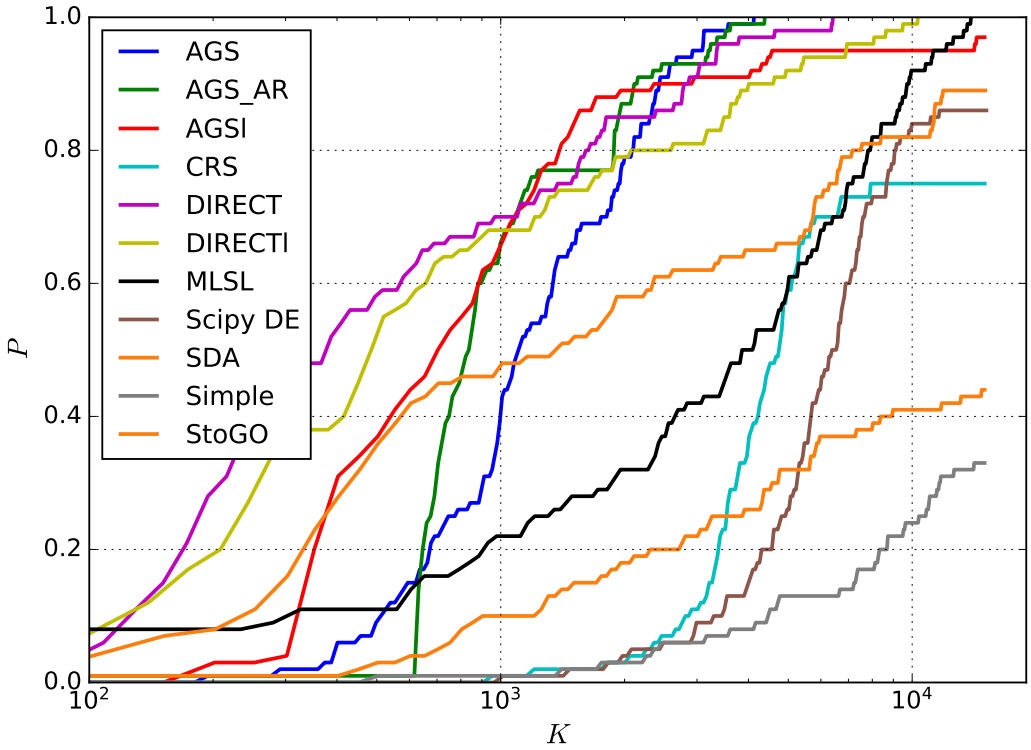


Figure 4: Class GKLS Simple 3d. $\Delta = 2 \cdot 10^{-2}$

| Method | Average number of trials | Problems solved |
|---|---|---|
| AGS | 1372.13 | 100 |
| AGS_AR | 1169.54 | 100 |
| AGSl | 1195.32 | 97 |
| CRS | 4145.81 | 75 |
| DIRECT | 973.64 | 100 |
| DIRECTl | 1477.79 | 100 |
| MLSL | 4609.17 | 100 |
| SDA | 2706.52 | 89 |
| Scipy DE | 5956.94 | 86 |
| Simple | 7098.45 | 33 |
| StoGO | 3856.11 | 44 |

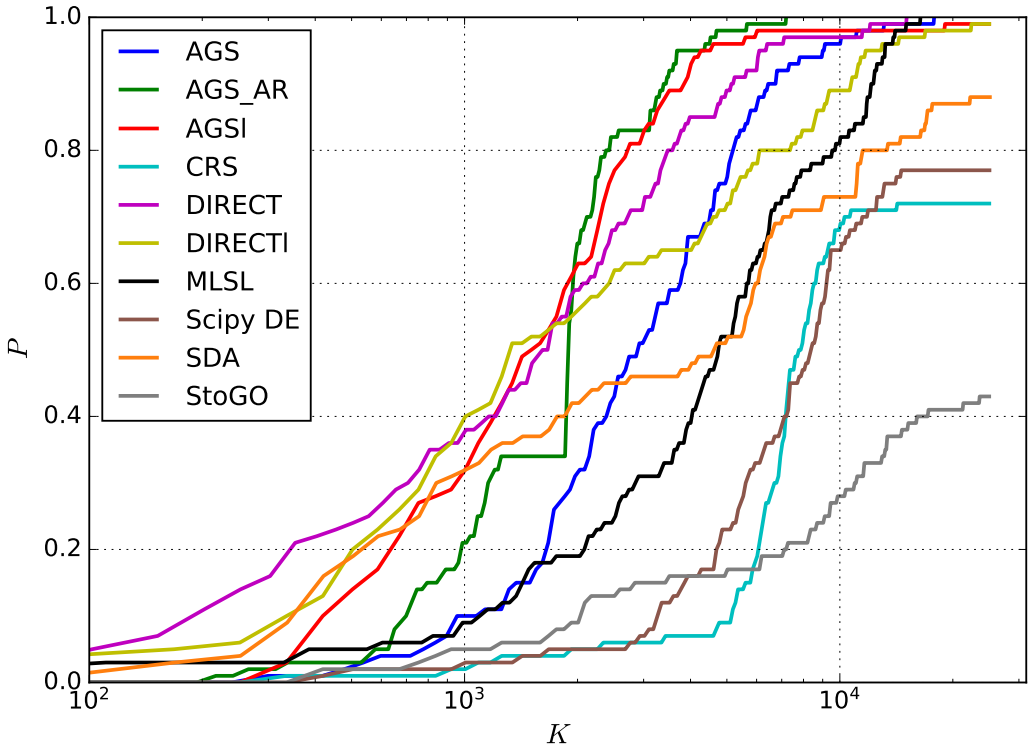


Figure 5: Class GKLS Hard 3d. $\Delta = 2 \cdot 10^{-2}$

| Method | Average number of trials | Problems solved |
|---|---|---|
| AGS | 3636.12 | 100 |
| AGS_AR | 1952.10 | 100 |
| AGSl | 1930.49 | 99 |
| CRS | 6786.96 | 72 |
| DIRECT | 2298.74 | 100 |
| DIRECTl | 3553.33 | 99 |
| MLSL | 5640.10 | 100 |
| SDA | 4708.43 | 88 |
| Scipy DE | 6914.34 | 77 |
| StoGO | 7843.23 | 43 |

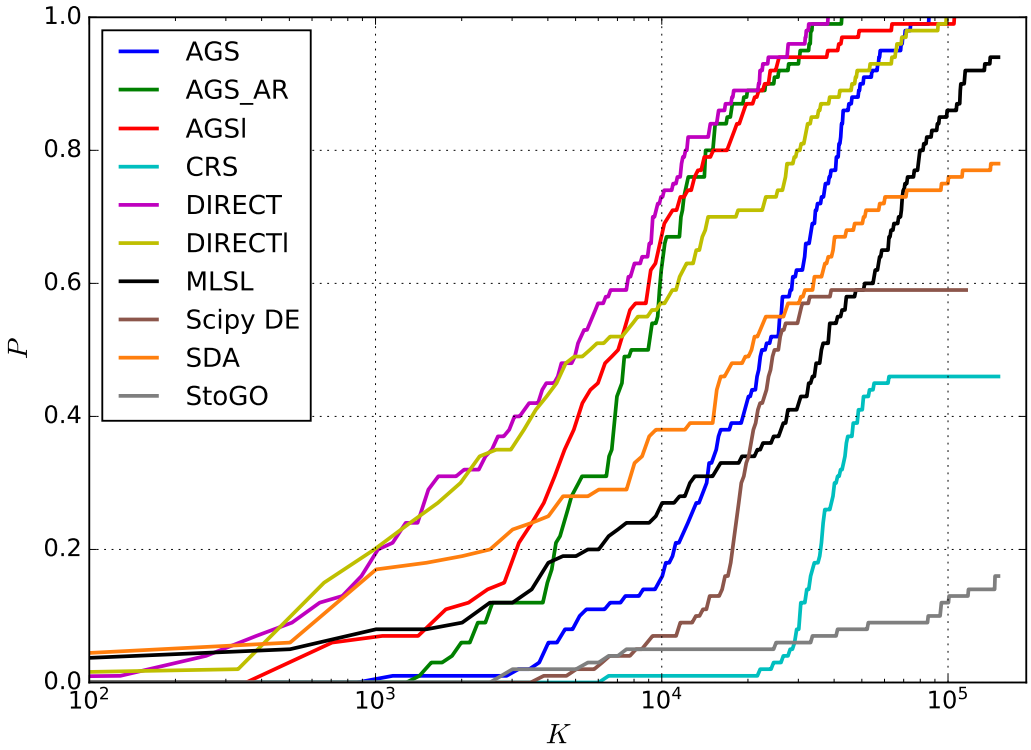


Figure 6: Class GKLS Simple 4d. $\Delta = 2 \cdot 10^{-2}$

| Method | Average number of trials | Problems solved |
|---|---|---|
| AGS | 26654.07 | 100 |
| AGS_AR | 10571.13 | 100 |
| AGSl | 11095.65 | 100 |
| CRS | 37436.76 | 46 |
| DIRECT | 7824.32 | 100 |
| DIRECTl | 15994.11 | 100 |
| MLSL | 41514.32 | 94 |
| SDA | 21417.90 | 78 |
| Scipy DE | 19157.73 | 59 |
| StoGO | 59895.44 | 16 |

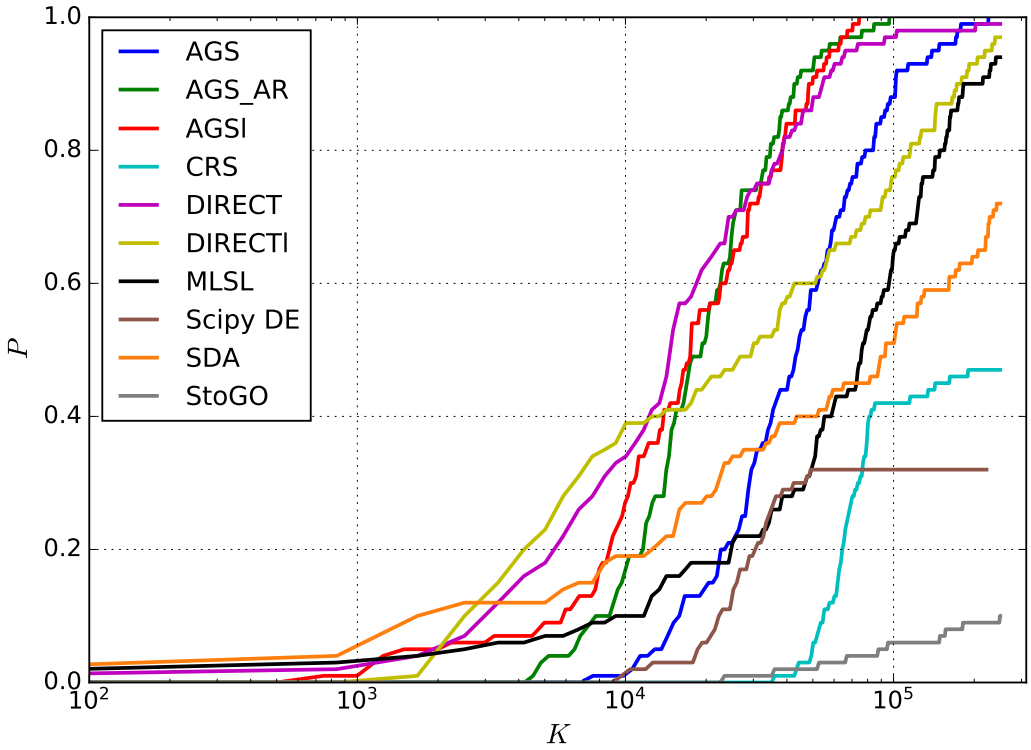


Figure 7: Class GKLS Hard 4d. $\Delta = 2 \cdot 10^{-2}$

| Method | Average number of trials | Problems solved |
|---|---|---|
| AGS | 54536.84 | 100 |
| AGS_AR | 23614.70 | 100 |
| AGSl | 23167.84 | 100 |
| CRS | 73779.32 | 47 |
| DIRECT | 23204.38 | 99 |
| DIRECTl | 54489.92 | 97 |
| MLSL | 80247.19 | 94 |
| SDA | 68815.53 | 72 |
| Scipy DE | 27466.06 | 32 |
| StoGO | 109328.10 | 10 |

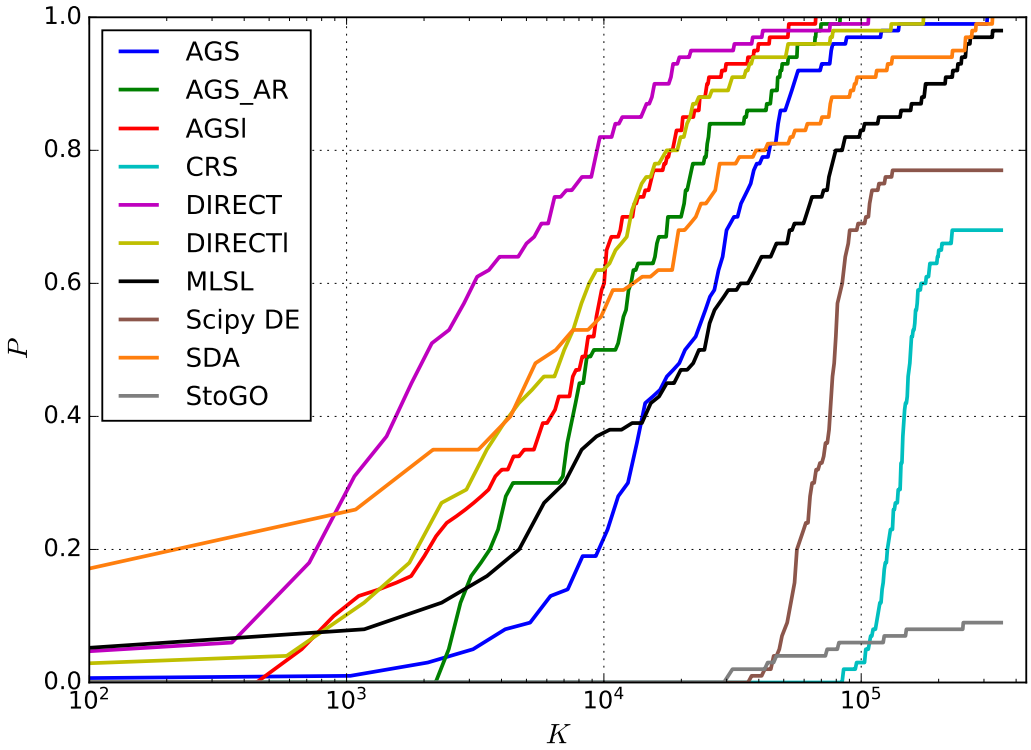


Figure 8: Class GKLS Simple 5d. $\Delta = 2 \cdot 10^{-2}$

| Method | Average number of trials | Problems solved |
|---|---|---|
| AGS | 29809.99 | 100 |
| AGS_AR | 16884.40 | 100 |
| AGSl | 11529.03 | 100 |
| CRS | 143574.99 | 68 |
| DIRECT | 7166.49 | 100 |
| DIRECTl | 13970.53 | 100 |
| MLSL | 52647.63 | 98 |
| SDA | 34255.31 | 100 |
| Scipy DE | 73074.52 | 77 |
| StoGO | 91580.44 | 9 |

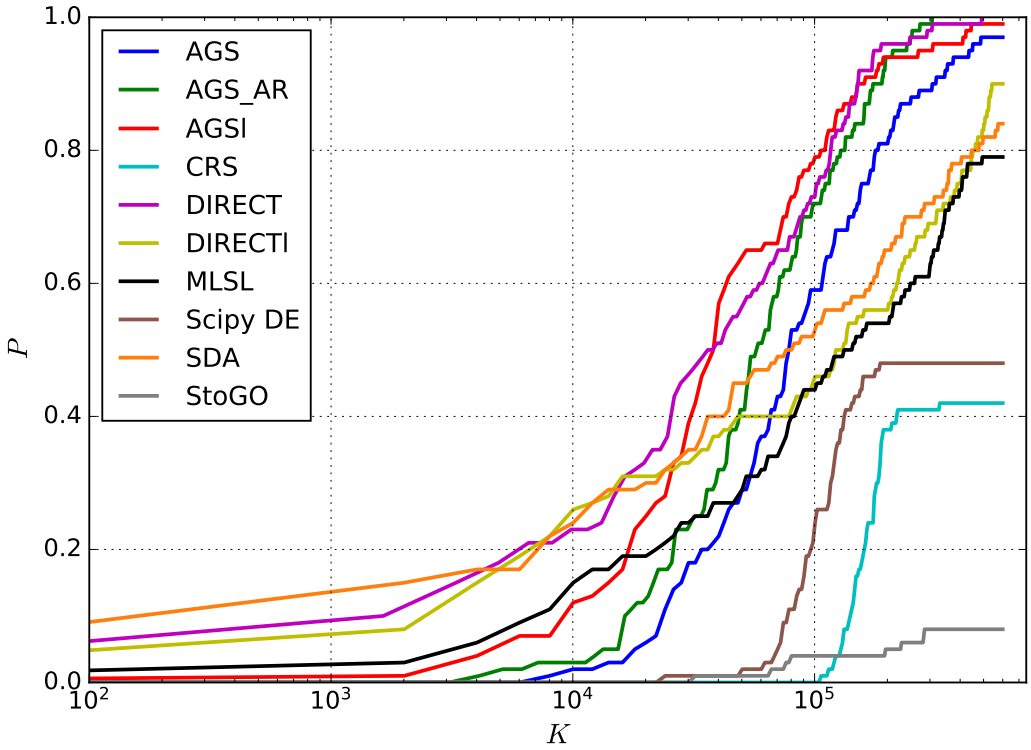


Figure 9: Class GKLS Hard 5d. $\Delta = 2 \cdot 10^{-2}$

| Method | Average number of trials | Problems solved |
|---|---|---|
| AGS | 113129.08 | 97 |
| AGS_AR | 80761.69 | 100 |
| AGSl | 67652.72 | 99 |
| CRS | 165192.76 | 42 |
| DIRECT | 66327.42 | 100 |
| DIRECTl | 164390.63 | 90 |
| MLSL | 138766.23 | 79 |
| SDA | 116973.10 | 84 |
| Scipy DE | 105496.88 | 48 |
| StoGO | 155123.75 | 8 |

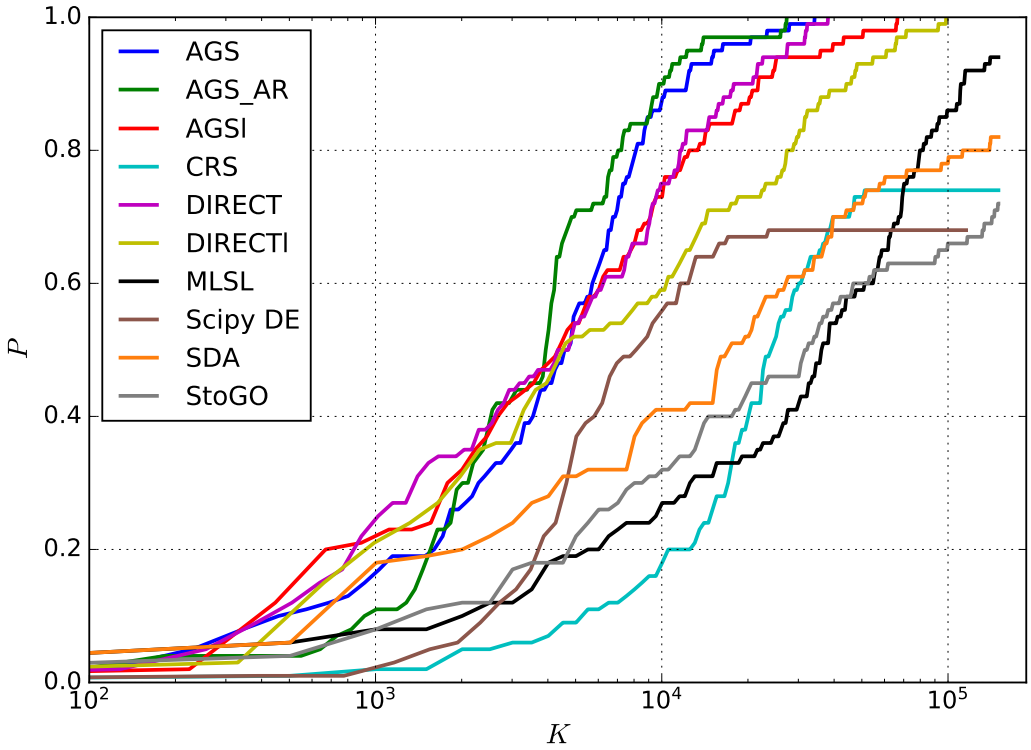


Figure 10: Class GKLS Simple 4d. $\Delta = 0.0632$

| Method | Average number of trials | Problems solved |
|---|---|---|
| AGS | 5729.82 | 100 |
| AGS_AR | 4919.08 | 100 |
| AGSl | 8847.40 | 100 |
| CRS | 19883.59 | 74 |
| DIRECT | 7328.78 | 100 |
| DIRECTl | 15010.01 | 100 |
| MLSL | 41484.80 | 94 |
| SDA | 22065.96 | 82 |
| Scipy DE | 6271.24 | 68 |
| StoGO | 29359.22 | 72 |

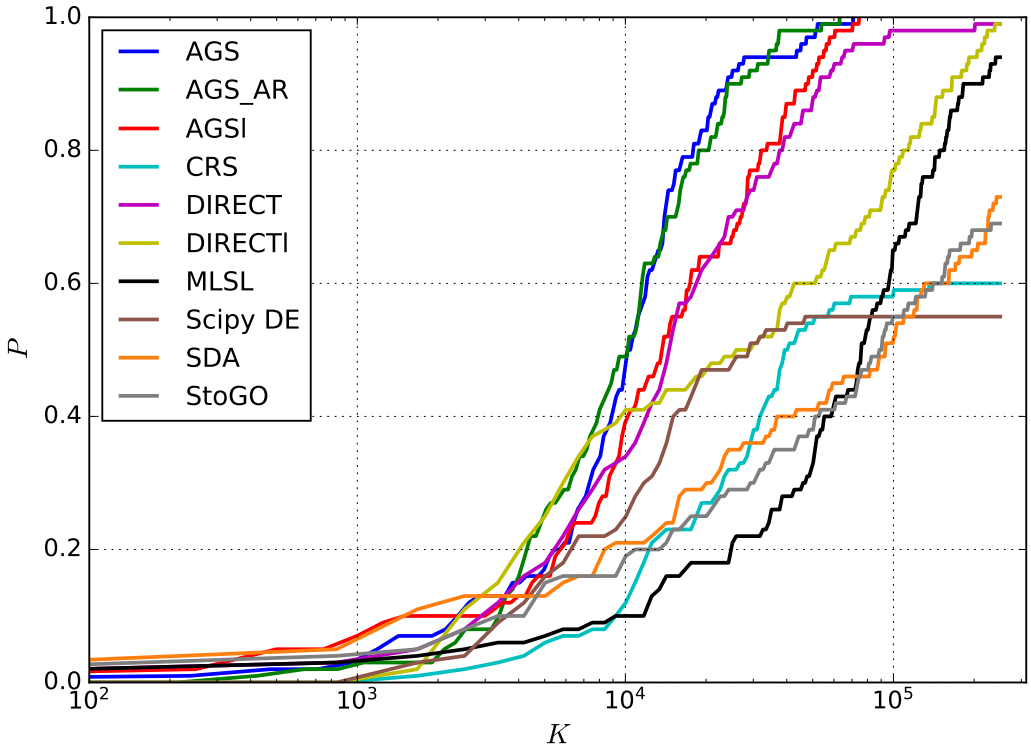


Figure 11: Class GKLS Hard 4d. $\Delta = 0.0632$

| Method | Average number of trials | Problems solved |
|---|---|---|
| AGS | 13113.40 | 100 |
| AGS_AR | 12860.10 | 100 |
| AGSl | 19826.36 | 100 |
| CRS | 27137.40 | 60 |
| DIRECT | 22884.35 | 99 |
| DIRECTl | 55596.07 | 99 |
| MLSL | 80220.11 | 94 |
| SDA | 68048.01 | 73 |
| Scipy DE | 12487.64 | 55 |
| StoGO | 58925.54 | 69 |

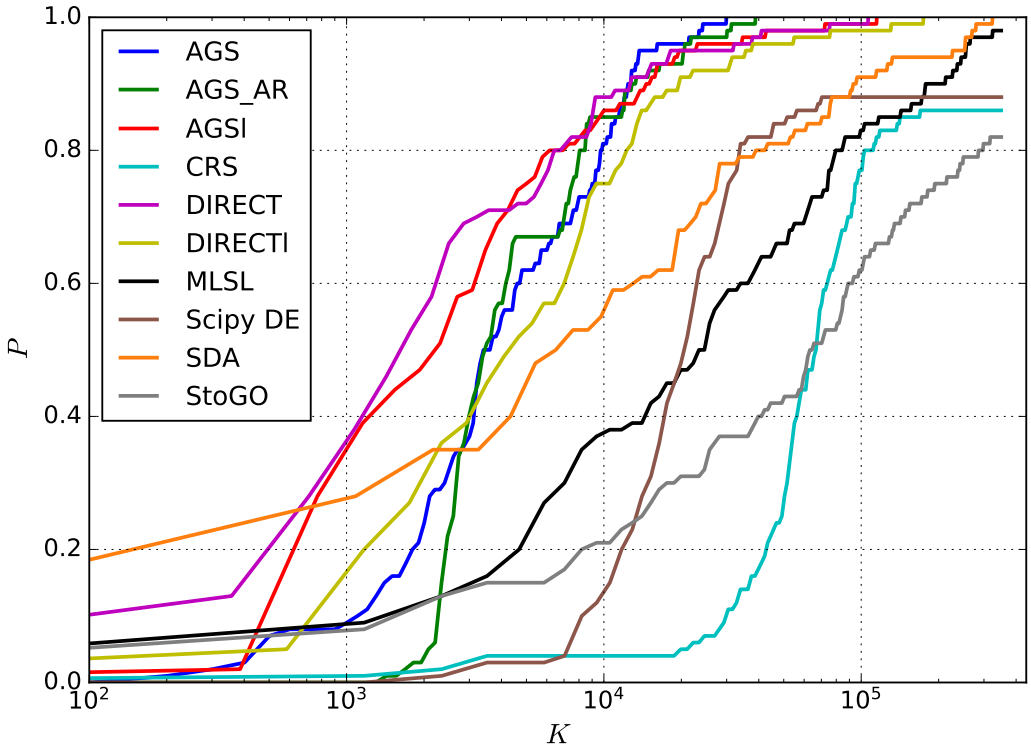


Figure 12: Class GKLS Simple 5d. $\Delta = 0.0796$

| Method | Average number of trials | Problems solved |
|---|---|---|
| AGS | 5821.47 | 100 |
| AGS_AR | 6241.28 | 100 |
| AGSl | 6314.25 | 100 |
| CRS | 62921.69 | 86 |
| DIRECT | 5966.13 | 100 |
| DIRECTl | 10795.46 | 100 |
| MLSL | 52609.18 | 98 |
| SDA | 34208.83 | 100 |
| Scipy DE | 20859.38 | 88 |
| StoGO | 69206.76 | 82 |

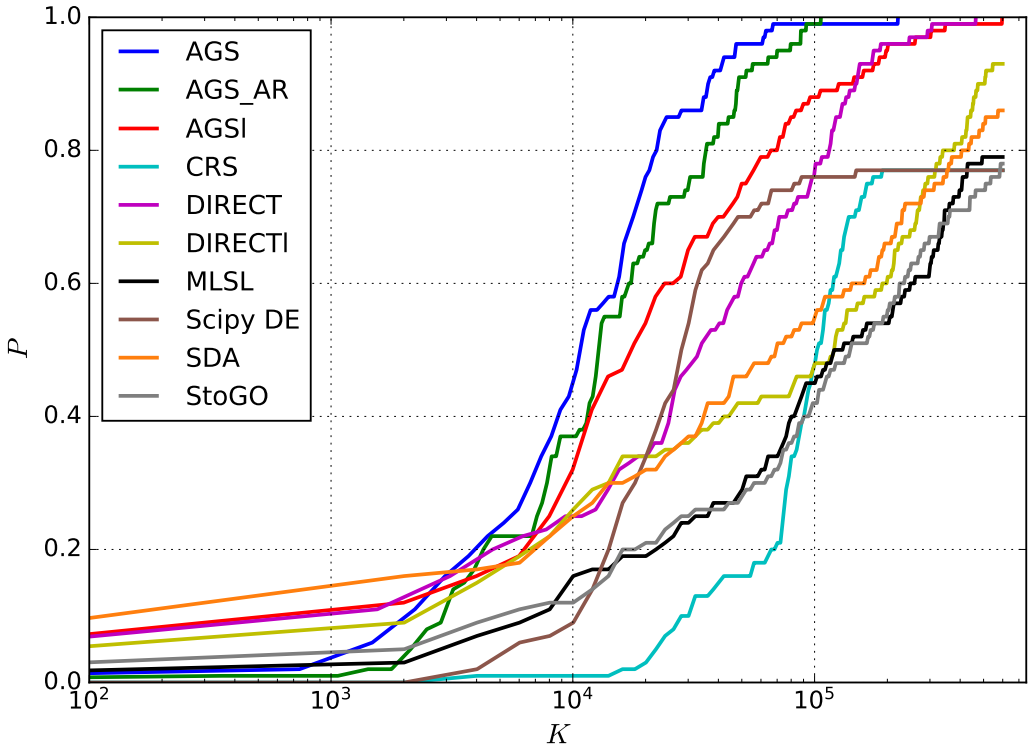


Figure 13: Class GKLS Hard 5d. $\Delta = 0.0796$

| Method | Average number of trials | Problems solved |
|---|---|---|
| AGS | 17008.61 | 100 |
| AGS_AR | 21555.09 | 100 |
| AGSl | 48514.29 | 100 |
| CRS | 87563.88 | 77 |
| DIRECT | 61657.32 | 100 |
| DIRECTl | 148637.82 | 93 |
| MLSL | 138011.78 | 79 |
| SDA | 115634.59 | 86 |
| Scipy DE | 26850.04 | 77 |
| StoGO | 141886.49 | 78 |